# PRODUCTIVITY OF LABOUR AND CAPITAL IN INDIAN MANUFACTURING INDUSTRIES DURING 1951 TO 1961

In this paper, the aggregate industrial production, labour supply, and capital stock of Indian manufacturing for the period 1951 to 1961 are analysed by fitting Cobb-Douglas production function after empirical verification of its suitability, and the marginal productivities of labour and capital along with certain theoretical properties of the function are discussed.

In considering the exponents of labour and capital in the fitted production function, it is found that an increase in labour by 1 per cent with capital constant would increase output by 0·46 per cent and an increase in capital stock by 1 per cent with labour constant would increase output by 0·54 per cent, and increase of both labour and capital by 1 per cent would increase output by 1 per cent. That the two components add to unity indicates that the production function operates with constant return to scale.

While calling attention to the marginal productivity of labour it is shown that there is a broad agreement between marginal net productivity of labour and average wage rate and this is in keeping with the marginal productivity theory or wages.

The population of India has increased from 359 million to 439 million or by nearly 22 per cent between 1951 and 1961. During the same period employment index in manufacturing has also increased by 22 points; that is, the ratio of labour force employed in manufacturing to total population has remained constant after first two five year plans and that is a striking fact. For it means that the programme of industrialization did not affect the ratio of labour force in manufacturing to population.

The index of fixed capital has increased by 140 points and the intensity of capital per labour year has increased by 97 points during the period 1951 to 1961. It is also observed that the indices of gross product and net product at constant prices have increased by 91 and 73 points respectively. Also, net productivity per labour year have increased by 53 and 52 points respectively at current and constant prices and during the same period, it is observed that the real wages and salaries per labour year has increased by 56 points.

The subsequent analysis shows that wages per unit of labour is determined by the marginal net productivity of labour, total wage bill is more or less a fixed share of gross or net product and productivity per labour is determined by capital invested per labour. Also the rate of growth of wages and net output have practically remained the same so that share of net output going to labour has remained the same.

As an adequate description of reality we start with a model of Cobb-Douglas type

$$(1) \quad P = AL^{\alpha} C^{\beta} U$$

where P is the index of output, A a constant, U a disturbance term, and L and C the indices of inputs of labour and capital. $\alpha$, $\beta$, the exponents of labour and capital, indicate the proportionate changes in output (P) for given proportionate changes in factor (L or C).

We have, (2) $\alpha = \frac{L}{P} \frac{dP}{dL}$ (elasticity with respect to labour input)

(3) $\beta = \frac{C}{P} \frac{dP}{dC}$ (elasticity with respect to capital input)

(4) $\frac{dP}{dL} = \alpha \frac{P}{L}$ (marginal productivity of labour input)

(5) $\frac{dP}{dC} = \beta \frac{P}{C}$ (marginal productivity of capital input)

and P/L and P/, the average productivity of labour and capital input respectively and

$$(6) \quad L \frac{dP}{dL} = \alpha P, \; C \frac{dP}{dC} = \beta P$$

the absolute shares of the two factors of production.

The parameters $\alpha$ and $\beta$, in addition to being elasticities, possess other important attributes for economic analysis and it is an important economic question whether $\alpha + \beta$ is less than, equal to or greater than unity. Actually, the sum $\alpha + \beta$ shows the degree of hogogeneity of the function and the return to scale in production; that is,

(7) $\alpha + \beta < 1$ (decreasing returns to scale)

(8) $\alpha + \beta = 1$ (constant returns to scale)

(9) $\alpha + \beta > 1$ (increasing returns to scale)

When $\alpha + \beta$ equal to unity holds, the production function, homogeneous of the first degree, takes the form

$$(10) \quad P = AL^{\alpha} C^{1-\alpha} U \qquad 0 < \alpha < 1$$

and we may write the production function as

$$(11)\quad P = L\frac{dP}{dL} + C\frac{dP}{dC},$$

that is the marginal productivity of labour multiplied by the number of units of labour plus the marginal productivity of capital multiplied by the number of units of capital would equal output. This means that the remuneration of productive agents, labour and capital, according to marginal productivities exhaust the output.

If we plot P/C against C, we get different values of productivity for different values of capital investment and thus we get average productivity curve of capital invested and from this curve we can get the elasticity of the average productivity curve. If A.P. and M.P. denote average and marginal productivity at a given output then, we have

$$(12)\quad M.P. = A.P.\left(1-\frac{1}{\eta}\right) \text{ i.e., } \eta = \frac{A.P.}{A.P. - M.P.}$$

and in our earlier notation.

$$(13)\quad \eta = \frac{P/C}{P/C - \frac{dC}{dP}}$$

From production function in (10) we get

$$(14)\quad \frac{dP}{dC} = \frac{P}{C}\,(1-\alpha)$$

and in that case (13) becomes

$$(15)\quad \eta_c = \frac{P/C}{P/C - P/C\,(1-\alpha)} = \frac{1}{\alpha}$$

In the same way, we get the elasticity of average productivity curve of labour as

$$(16)\quad \eta_l = \frac{P/L}{P/L - P/L\,(\alpha)} = \frac{1}{1-\alpha}$$

*Verification of the Model and Estimation of Parameters*

If we suppose that the industrial, production of Indian manufacturing can be represented by a linear homogeneous function in terms of labour and capital and it takes the form (10) then, with $1-\alpha=\gamma$

$$(17)\quad (P/L) = A\ (C/L)^{\gamma}$$

That is, output per unit of labour input is linearly related to capital per unit of labour input in double logarithmic scale.

With the help of production data of Indian manufacturing industries for the period 1951 to 1961 (a note on data of Indian manufacturing industries and definition of variables are given in the appendix), the indices of deflated gross product[2] per labour year against the indices of deflated capital[3] per labour year were plotted in double logarithmic scale and the graph was found to be linear. We then conclude that the data of industrial production of Indian manufacturing industries can reasonably be represented by a linear homogeneous function of first degree in terms of the two factors of production, labour and capital.

The parameters $\gamma$ and A in equation (17) were evaluated from indices of deflated gross product per labour year against the indices of deflated capital per labour year by the method of least squares[3] and we obtain $\gamma = 0{\cdot}54$ and $\log A = 0{\cdot}96$.

The correlation coefficient between observed values of indices of deflated gross product per labour year and their corresponding expected values is obtained as $R = 0{\cdot}87$.

To test the statistical significance of the estimate of $\gamma$, analysis of variance was performed and mean square due to regression (d.f. = 1) was compared with mean square due to deviation from regression (d.f. = 9) and we got an F value of 22·09 and F (1, 9) is significant at 1% level. With the estimate of mean square error in the analysis of variance table, the variance of regression coefficient $\gamma$ was estimated and variance ($\gamma$) = 0·013 is a reasonable figure.

*Interpretation of Results*

If the production function be homogeneous of first degree then (3) and (10) we have

$$(18) \qquad (1-\alpha) = \frac{dP}{dC}\,\frac{C}{P} = \frac{\frac{dP}{dC}}{P/C}$$

If the marginal productivity of capital $\frac{dP}{dC}$ is assumed to be constant over a period and since $\alpha$ is constant for a given structure of industries over a period, then the capital-output ratio can also be assumed

[2] The gross industrial product was deflated by the index of 'manufactured goods' (finished products) and the increment in the value of capital stock $C_{t+1} - C = \Delta C_{t+1}$ was deflated by a 'general index' and in both the indices 1951 was used as base year.

[3] Multicollinearity as a tendency of different economic time series to move together in the same trend is likedly to be present between capital and and labour in the production function. While fitting the production function by the method of least squares we have made no attempt to eliminate the effect of multicollinearity.

to be constant for the same structure of industries over the same period. In industrially and technologically advanced countries the marginal productivity of capital is likely to be low and therefore the ratio P/C will also have a lower value than in the relatively less advanced countries. In an ideally industrialised country where maximum level of production has been attained any fresh addition to the existing stock of capital will not result in any corresponding increase in total net output. In that case as C tends to its maximum value, $P/C \rightarrow$ to a value near about zero and $\frac{dP}{dC}$ $\rightarrow$ zero and therefore $1-\alpha \rightarrow 0$ and $\alpha \rightarrow 1$.

From the above arguments it transpires that $\alpha$ varies between 0 and 1 in accordance to the industrial and technological advancement of the country; $1-\alpha$ takes lower value in an industrially and technologically advanced country and higher value in a semi industrialised country or a country where process of industrialisation has just started.

In a semi-industrialised and developing country like India we have obtained the equation of industrial production as

$$(19) \quad P = AL^{0.46} \quad C^{0.54}$$

in which the exponents of capital (C) = 0·54 is an index of the state of industrialisation of Indian economy.

Murti and Sastri (1957), based on cross-section Indian data for 1952, obtained the exponents of labour and capital as 0·53 and 0·50 respectively.[4]

$$P = AL^{0.75} \quad C^{0.25}$$

Before we discuss further the economic significance of the exponents of labour and capital we will interpret constant 'A' mathematically and give its economic significance. The value of logA indicates the value of logP when logL and logC are zero and thus if L and C are one unit each the theoretical value will be indicated by 'A' and therefore 'A' indicates the product attributed to one unit of labour with one unit of capital. The value 'A' thus symbolises at a given technology, the state of industrial development reached with a given technique of production for a given industrial structure in the economy.

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[4] Professor Cobb and Douglas from the indices of production, labour and capital in the manufacturing industries of United States for the years 1900 to 1922 evaluated the production function as

In interpreting the exponents of labour and capital we find the laws of factor return to capital and labour. These relate to the question: if labour increases by 1 per cent but capital remain constant or capital increases by 1 per cent while labour remains constant, will the increase in output be equal to, or more than or less than 1 per cent? That is, factor return is concerned with changes in the ratio of output to input of a factor as its quantity is increased in relation to other factor or factors. Secondly, if both labour and capital increase by 1 per cent will the increase in output be equal to, greater than or less than 1 per cent?

We now discuss the exponents of labour (L) and capital (C) and thereby the marginal efficiencies of factor inputs. In the light of the properties of Cobb-Douglas function we may interprete the exponents of equation (19) of Indian manufacturing as follows: (i) an increase in labour (L) by 1 per cent with capital (C) constant would increase output (P) by 0·46 per cent, (ii) an increase of capital stock (C) by 1 per cent with labour (L) constant would increase the output (P) by 0·54 per cent and (iii) an increase in both labour (L) and capital (C) by 1 per cent would increase the output (P) by 1 per cent.

In table (1), are shown marginal gross and net productivity of labour (vide footnote 6) at current and constant prices together with gross marginal productivity of capital at current prices. Wages and salaries per labour year at current and constant prices are also shown in the table. The results show that there is broad agreement between net productivity per labour year and the average wages and salaries per labour year and this may be considered to show a broad confirmation of the marginal productivity theory of wages. Further, since average productivity per labour year is much higher than average wages and salaries per labour year, producers are always on the safe side of making reasonable profits. We also estimate the elasticity of average productivity curve of capital and labour as 2·2 and 1·8 from equations (15) and (16).

Table (2) shows that during the period of 1951 to 1961 the indices of employment in manufacturing have gone up by 22 points. During the same period the index of population has increased by 22 points[5]. That is, the ratio of labour force employed in manu-

[5] The population in India increased from 359 million to 439 million or by nearly 80 million between 1951 and 1961. That is, the population increased during the period at a geometric growth rate of 2 per cent per annum.

facturing to total population has remained fixed at the end of first two five qear plans. In the same table we find that the indices of fixed capital and invested capital (i.e. fixed capital plus working capital) have increased by 140 and 106 points respectively and the indices of gross product and net product have gone up by 106 and 86 points respectively at current prices and by 91 and 73 points respectively at constant prices (deflated by indices of manufactured goods). The above results show that the industrial production in India during the period is affected mainly by factor return of capital.

In table (3), we find the capital intensity has practically doubled during the period, that is, the index of fixed capital per labour year has increased by 97 points. The ratios gross and net output per unit of capital have practically remained constant during the period. In the same table we also find stability of the ratios of net output to gross output[9] and also the approximate stability of ratio of fixed capital to invested capital. The ratios of fixed capital to gross output and invested capital to gross output are also somewhat stable.

Table (4) shows that during 1951 to 1961 the indices of gross and net productivity per labour year have increased by 69 and 53 points respectively at current prices and by 57 and 52 points respectively at constant prices. We also see that wages and salaries per labour year have increased by 65 points at current prices and by 56 points at constant prices (deflated by general index) in table (5). That is, the increase in wages and salaries per unit of labour is given, so to say, from increase in productivity per unit of labour. It is also seen from table (5) that the ratios of wages and salaries to gross output and wages and salaries to net output have practically remained constant.

From these we find that the real wages and salaries per worker is determined practically by the productivity per unit of labour and the total wages and salaries are a fixed share of the gross product or net product. Also, the productivity per labour is determined broadly by the capital investment per unit of labour. This, in fact, is implicit in equation (19) fitted above. This also means that the share of the output going to labour has remained practically constant.

[9] Due to stability of net output to gross output ratio, or its complementary part input to output ratio, Cobb-Douglas production function may be fitted either to gross output or net output and the exponents of labour and capital would remain same in both the cases.

## CONCLUDING REMARKS

From empirical consideration we have represented the production of Indian manufacturing industries by the Cob-Douglas production function. As a logical extension we may search a function to represent the production of whole economy but this is a difficult task. We may possibly argue that if Cobb-Douglas function holds for industrial sector, it may alsco hold for agricultural sector and also for the whole economy. Of course, the values of parameters of the aggregate production function for the whole economy depend upon the way in which the factors are distributed over separate sectors and their technological relationship.

If we take the whole economy and if the real income per capita is to increase with the increase in population the capital stock and labour force in employment would have to increase at a faster rate than population increase. Also, if we include in the levels of consumption, the health service and education service, then the investment in such services would have to increase at a faster rate than population increase so as to raise the levels of consumption. If capital includes natural resources (*e.g.* land) then the man-made capital must increase at a faster rate than the population increase to make up for the failure to increase natural resources equally with population.

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[1] The increase in population may be affected by (i) increase in birth rates and ii) improvement in health measures. In the first measure there is a lag in increase of effective labour force for a few years whereas in the second measure the ratio of labour force to population may be increased or decreased according to the age group affected by health measures.