Optimal Technique in Multi-Sector Economy

Ajit K Biswas

THE criterion of surplus[1] has been widely advocated in recent discussions concerning the optimal choice of investment techniques for economic development. In most cases[2], arguments relate to macro-economic aggregates like national income, investment, consumption, etc, or else to individual economic entities in isolation from the rest of the economy. Both of these fail to reveal the inter-sectoral relationships in the economic system. It seems there is a gap here which badly needs being filled.

Optimal Technique in a Simple Economy[3]

Let us consider the case of a closed economy producing one consumption good with the aid of primary factors of production and an investment good. The investment good in its turn is supposed to be producible by means of only primary factors of production. We shall also assume that a unit of investment good produced during the current period is available for use in the production of consumption good only in the next period. Evidently investment good which is being produced today cannot simultaneously be used for the production of consumption good; hence this unit period time-lag. The crucial factor for economic development in such an economy is the maximum amount of surplus available from the production of consumption good (i.e., the excess of production of consumption good over consumption of those engaged in its production) which in a planned economy is ideally mobilisable to feed newly employed labour in the production of investment good. The new investment goods produced (neglecting replacement problems) may be used in the next period to augment the production of consumption good, leading to a process of growth. Obviously what is absolutely essential to set the economy growing is that there must be a positive surplus of consumption good over the consumption of those who are responsible for its production. It can be shown rigorously that in our postulated economy, the growth of output of consumption good will be given by

$$O_c(t) = O_c(O) \quad [1+p_i \ (p_c-w)/w]^t \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

where

$O_c(t)$ = output of consumption good in period t

$O_c(o)$ = output of consumption good in initial period 'O'

p_c = productivity per unit of labour of the consumption good

p_i = productivity per unit of labour of the investment good (where the investment good unit is so defined as to make investment good/labour ratio in the production of consumption good 1 : 1)

w = real wage rate per unit of labour measured in consumption good units (assumed to be the same for both the sectors), all of which is supposed here to be consumed in the current period

Evidently p_c and p_i are parameters which are fixed for any given technique relating to the production of consumption and investment goods. It is seen that if $p_c = w$ (no surplus condition) then $O_c(t) = O_c(o)$, i.e., there is no change in the output of consumption goods over time and that agrees with what we should intuitively expect. Further, it may be verified that in our simple system, the production of investment good, labour employed in the production of consumption and investment goods —all grow at the rate given by $r = 1+p_i(p_c-w)/w$.

Corresponding to each of any finite number of techniques, 1, 2,........, n we get a pair, p_c and p_i. Denoting by p_{cj} and p_{ij} the productivities per unit of labour of consumption good and investment good respectively associated with j^{th} technique and regarding for the moment that real wage rate is the same for all techniques, we can write $r_j = p_{ij} \ (p_{cj}-w)/w$ to denote the rate of growth resulting from the j^{th}, technique. Now the r's can be arranged on a scale given by $r_1 \leqslant r_2 \leqslant \ldots\ldots\ldots\ldots \leqslant r_n$. If the equality relationship holds for any pair of r's, then it is evidently a matter of indifference from the point of view of maximising the rate of growth which technique we choose. On the other hand, if on looking across the scale of r's, there is found a last r_j which is higher than its predecessors, we choose the technique j. The above may be summed up in one sentence,—among the array of r's corresponding to a finite number of techniques, choose the maximum for that maximises the rate of growth of output.

There is no necessity that the real wage rate w must be regarded as the same for all techniques, nor is there any need to assume that real wage rate is the same in both consumption and investment good sectors. It is necessary, however, to regard real wage rate as functionally independent of output. Since fixed coefficients of production are assumed, any interesting observation regarding real wage rate should be based on the conditions of labour supply in the economy. I do not intend to go into those details here.

We introduce two additional notations w_{cj} and w_{ij} to indicate real wage rate in consumption and investment goods corresponding to the j^{th} technique. The rate of growth of output corresponding to the j^{th} technique is now given by $r_j = p_{ij} \ (p_{cj} - w_{cj})/w_{ij}$. Again, maximisation of the rate of output implies the choice of maximum r and hence the corresponding technique.

Coming back again to the case where real wage rate is considered the same for all techniques and for both the sectors of consumption and investment goods, we find that in case of two techniques 1 and 2 such that $p_{i_2} > p_{i_1}$ and $p_{c_2} > p_{c_1}$ it necessarily follows that $r_2 > r_1$. This is rather a singularly uninteresting case. Evidently a technique which implies increased productivity in both the sectors is decidedly a better one. Question of real choice arises when $p_{i_2} < p_{i_1}$ and $p_{c_2} > p_{c_1}$. Thus technique 2 ensures increased labour productivity in the consumption good

---

[1]. The precise meaning of the word surplus as used here will be clarified in the following discussion.

[2]. See, for instance, M. Dobb, "A Note on the So-called Degree of Capital Intensity of Investment in Under-developed Countries", in On Economic Theory and Socialism, London, by the author.

Also, Galenson and Leibenstein, "Investment Criteria, Productivity and Economic Development," the Quarterly Journal of Economics, Vol. 69, 1955.

[3]. The assumptions and line of arguments leading to the results in this section constitute part of a more comprehensive paper by the author,

sector at the cost of lower productivity in the investment good sector. The magnitude of real wage rate here is crucial; which rate of growth is higher will depend decisively on the real wage rate. It may be verified, however, that if real wage rate exceeds a certain critical magnitude, technique 2 (which ensures increased productivity of consumption good) will attain superiority over technique 1. An economy which is continually facing pressures in favour of a levelling up of the real wage rate will find at some point advisable to switch from technique 1 (if it was initially superior) to technique 2.

Relation with Harrod-Domar System

There is a striking formal similarity between the system presented above and the Harrod-Domar model which has become the stock-in-trade of growth theorists. In the Harrod-Domar model, the optimal or equilibrium rate of growth is given by

$$y(t) = y(o)\ [1+s/g]^{t} \qquad \ldots\ldots (ii)$$

where

$y(t)$ = income in period t

$y(o)$ = income in initial period 'O'

s = marginal propensity to save

g = acceleration coefficient (to be interpreted here as the marginal capital-output ratio or the inverse of Domar's σ, namely the productivity of investment.

It follows from (ii) that any increase in the marginal propensity to save s or a lowering of the capital-output ratio g (i.e., increase in the productivity of investment) will increase the optimal rate of growth.

We shall now interpret our system in terms of the familiar categories of marginal capital-output ratio and marginal propensity to save. Productivity of consumption good per unit of labour is p_c and the real wage rate is held fixed at w (all of which is consumed) so that marginal propensity to save is given by $(p_c-w)/p_c$. One unit of labour (whose remuneration is w) produces p_i units of investment good which in its turn produces $p_i\ p_c$ units of consumption good. Thus an investment of w results in $p_i\ p_c$ units of consumption goods, such that the marginal capital-output ratio is given by $w/p_i\ p_c$. The rate of growth of output r can now be written as

$$r = \frac{p_c - w}{p_c} \Big/ \frac{w}{p_i\ p_c} = \frac{s}{g} \qquad \ldots\ldots\ldots\ldots\ldots (iii)$$

While this formal similarity between the Harrod-Domar model and our system is revealing, there is an important difference. In the former model, s and g are capable of independent variation and they are not functionally related. In our system, however, s and g are functionally related for both of them involve p_c and w. For two techniques 1 and 2 $g_1 < g_2$ (i.e., marginal capital-output ratio of technique 1 is lower than that of technique 2), the Harrod-Domar model implies an increase in the equilibrium rate of growth for technique 1. In our system, $g_1 < g_2$ implies $p_{c_1} p_{i_1} > p_{c_2} p_{i_2}$ so that if $p_{c_2} > p_{c_1}$, we have $s_1 < s_2$. Thus a decrease in the marginal capital-output ratio under technique 1 is to some extent offset by a lower marginal propensity to save and as we have seen before, in case of real wage rate exceeding a certain critical level, technique 2 will ensure a higher rate of growth in spite of its higher capital-output ratio.

Optimal Technique in a More Complex Economy

Now I shall consider a somewhat more complex economy. Let us suppose that there are three sectors engaged in productive activity consisting of

(1) consumption goods, supplying households with consumption goods, (2) means of production, supplying raw materials (or intermediate goods or current inputs) and finished investment goods to other sectors, and (3) households, which supply other sectors with labour services. Each of these three sectors requires for its current production materials or services drawn from one or more (at least one) of the three sectors, together with non-produced means of production (which are exogenous to the system). These non-produced means of production are available in any required quantity for continued expansion of the economy.

In view of the nature of our sectors, it will be postulated that all of consumption goods production in any period is allocated to households. Means of production are allocated to consumption goods and to itself both as inputs to be currently used up in course of production and for capital formation, yielding return in the next period and thereafter. Household services are used for the production of consumption goods and means of production, providing required labour inputs.

The following notations will be used:

$C(t)$ = output of consumption goods in period t

$M(t)$ = output of means of production in period t

$H(t)$ = output of households (measured in man-hours or some other convenient unit) in period t

$Ch(t)$ = consumption goods allocated to households in period t

$Mc(t)$ = means of production allocated to consumption goods in period t

$Mm(t)$ = means of production allocated to itself in period t

$Hc(t)$ = household services allocated to the consumption goods sector in period t

$Hm(t)$ = household services used for the production of means of production in period t.

Now the relations (identities) stated below must hold:

$$\begin{aligned} C(t) &= Ch(t) \\ M(t) &= Mc(t) + Mm(t) \qquad \ldots (iv) \\ H(t) &= Hc(t) + Hm(t) \end{aligned}$$

To bring our system to manageable dimensions we introduce structural coefficients based on fixed coefficients of production (the procedure followed in Leontief's input-output technique). We assume reasonably that the input of intermediate goods or services used per unit of production in any sector is a fixed proportion of total output of that sector, and input for capital formation in any sector is a fixed proportion of the increment of output of that sector. Thus we have,

$$\begin{aligned} Ch(t) &= a\ H(t) \\ Mc(t) &= b\ C(t) + c[C(t+1) - C(t)] \\ Mm(t) &= d\ M(t) + e[M(t+1) - M(t)] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (v) \\ Hc(t) &= f\ C(t) \\ Mm(t) &= g\ M(t) \end{aligned}$$

where a, b, c, d, e, f, g are constants; c and d are evidently capital coefficients and therefore that part of the requirements of means of production which is used for capital formation is expressed as the relevant capital coefficient times the increment in output. The rest of the structural coefficients relate to the current input of intermediate goods which are proportional to output of the sector where they are used. Substituting the values (v) in (iv) we obtain

$$\begin{aligned} C(t) &= a\ H(t) \\ M(t) &= b\ C(t) + c\ [C(t+1) - C(t)] \\ &\quad + d\ M(t) + e\ [M(t+1) - M(t)] \qquad \ldots\ldots\ldots (vi) \\ H(t) &= f\ C(t) + g\ M(t) \end{aligned}$$

The solution of the difference equation system (vi) is given by

$$C(t) = C(o)\left[1+\frac{(1-af)\ (1-d)-abg}{e(1-af)+cag}\right]^t \quad \ldots\ldots\ldots\ldots\ldots\ldots(vii)$$

C(O) being the output of consumption goods in the initial period. M(t) and H(t) are obtained from the following identities:

$M(t) = c(t)[(1-af)/ag]$ and
$H(t) = C(t)/a$

For each of the three sectors of the economy, there is a unique growth rate given by the second term of the bracketed expression in (vii), namely

$$r=[(1\text{-}af)\ (1-d)-abg]/[e(1-af)+cag] \quad \ldots.(viii)$$

Corresponding to any set of techniques encompassing the three sectors of our economy, we get a particular configuration of the structural constants a, b, ... g, and again the optimal set of technique is one which maximises the rate of growth r.

It is not possible here to go into the detailed interpretation of the expression for r given by (viii). We have already incorporated in this still very simple system some intersectoral relationships which complicate the picture. It may be easily seen, however, that our system (I) is a special case of system (vii). In system (I) investment good which corresponds here to sector 2 is producible by means of only primary factors of production (including labour) which implies that d=e=0. Also the consumption good (sector 1 here) is produced with the aid of only investment good and primary factors of production, so that b=0. Setting, then, b, d and e equal to 0, we obtain

$$r=(1\text{-}af)/cag \quad \ldots\ldots\ldots\ldots(ix)$$

This, we shall see, is equivalent to (I). Thus in order to produce a unit increment of consumption good, f units of labour are required which consume af units of consumption goods. Per unit increment of output of consumption goods leaves, therefore, a surplus given by 1—af, the term in the numerator of (ix). Again in order to produce an increment of consumption good, is required c units of investment good which in its turn use cg units of labour. Since one unit of labour consumes a units of consumption good, cg units of labour consume cga units of consumption good, which is the term in the denominator of (ix).

A clue to the generalisation of the systems considered here to take care of any finite number of sectors is provided by Professor Leontief's dynamic input-output system. Whether simple interpretations as above of more complicated expressions for the rate of growth (difference equations providing more than one root) are possible require investigation. This is however beyond the scope of this paper.