WHY FIBONACCI SEQUENCE FOR PALM LEAF SPIRALS?

T. ANTONY DAVIS
Indian Statistical Institute, Calcutta, India

On account of their very large, prominently-stalked leaves, palms are ideal material to study phyllotaxis, which means the arrangement of leaves on the trunk. The leaves of palms are produced one after another, and their arrangement is termed alternate, which is the case with the majority of plants that display spiral phyllotaxis. That is, two consecutive leaves are placed on the stem at different heights with an angular deflection of less than 180°. Some other plants like most grasses subtend an angle of 180° between two consecutive leaves, and this arrangement is known as distichous of 1/2 phyllotaxis, as two leaves are produced in one complete rotation. Even a rare palm (Wallichia disticha) shows this peculiarity. In alternate phyllotaxis, some plants will have five leaves produced before completing two complete revolutions. This system is referred to as 2/5 phyllotaxis. Also, other plants may show 3/8, 5/13, 8/21, 13/34, 21/55, ···, or 2/3, 3/5, 5/8, 8/13, 13/21, 21/34, 34/55, and so on phyllotaxis. The numerators or the denominators of this series, when considered alone, form the successive stages of the Fibonacci sequence. It is known [1-5] that the Fibonacci phyllotaxis gives optimum illumination to the photosynthetic surface of plants since the leaves overlap least.

2. VARYING NUMBERS OF LEAF SPIRALS IN PADS

Different species of palms display different numbers of leaf spirals, and the numbers always match with Fibonacci numbers. For example, in the arecanut palm (Areca catechu) (Fig. 1), or the ornamental Ptychosperma macarthurii palm, only a single foliar spiral is discernible, while in the sugar palm (Arenga saccharifera) (Fig. 2), or Arenga pinnata, two spirals each are visible. In the palmyra palm (Borassus flabellifer) (Fig. 3), or Corypha elata, as well as a number of other species of palms, three clear spirals are visible. The coconut palm (Cacos nucifera) as well as Copernicia spirals (Fig. 4) have five spirals, while the African oil palm (Elaeis guineensis) (Fig. 5) bears eight spirals. The wild date palm (Phoenix sylvestris) and a

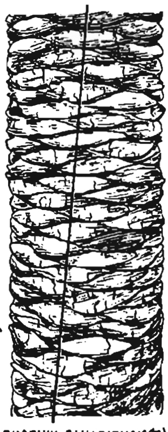
ARECA CATECHU(1)
Figure 1

ARENGA PINNATA(2)
Figure 2

BORASSUS FLABELLIFER(3)
Figure 3

COPERNICIA SP.(5)
Figure 4

ELAEIS GUINEENSIS(8)
Figure 5



PHOENIX CANARIENSIS(13)
Figure 6

few other species of palms also show eight spirals. On the stout trunks of the Canary Island palm (Phoenix canariensis) (Fig. 6), thirteen spirals can be observed. Also in some of these plams, twenty-one spirals can be made out. It is surprising that all the above-mentioned numbers (1, 2, 3, 5, 8, 13 and 21) happen to be Fibonacci numbers. Palms bearing 4, 6, 7, 9, 10, 11 or 12 obvious leaf spirals are not known.

3. FIBONACCI NUMBERS IN VERTICAL PALMS

In some exceptional palms, the arrangement of leaves does not show any spiral mechanism. Instead, the leaves are arranged one vertically above another along two or more rows. As mentioned, in Wallichia disticha, there are two vertical rows of leaves and the angle between any two consecutive leaves is 180°. In Madagascar's three-sided palm, Neodypsis decaryi, the leaves fall along three vertical rows, and the successive leaves are formed at a constant angular deflection of 120° (narrow angle). In the case of most individuals of Syagrus treubiana, five impressive vertical rows of persisting leaf bases can be seen. The angular deflection between any two successive leaves is 135°. Also, it is possible to observe on some trunks of the Canary Island palm, leaf scars that match with thirteen vertical rows, the angular deflection between their successive leaves will be 138.5°. If a palm showing twenty-one vertical rows of leaves can be detected, the angular deflection between two consecutive leaves will be 137.14°. Incidentally, the number of vertical rows of leaves in the above-mentioned palms also turn out to be all Fibonacci numbers. If the figures of the above angular deflections (180, 120, 144, 135, 138.5, 137.14, ···) are examined, one finds that the alternate numbers turn out to be more than 137.5° and the others less, and the difference between two numbers progressively gets narrower. This narrow angle makes with the remaining angle (222.5°) to complete one full circle, a proportion of 0.618, which is the golden proportion. This phenomenon exactly demonstrates one of the specific properties of the Fibonacci Sequence. That is, the proportion between any two consecutive Fibonacci numbers is alternately more (or less) than the golden proportion. If the values 137.14, 138.5, 135, 144, 120 and 180 are subtracted by the golden proportion angle of 137.5, we get -0.36, 1.0, -2.5, 6.5, -17.5, 42.5, which when multiplied by 2 gives values approximating to alternate Fibonacci numbers.

The number of green leaves a palm bears at a time generally indicates the number of foliar spirals exhibited by the species. Palm species having fewer green leaves manifest smaller numbers of foliar spirals, and those bearing larger numbers of leaves show greater numbers of spirals. This situation can be easily explained by the help of the schematic representation of a palm crown shown in Fig. 7. There is also an indication that the mean number of gren leaves of a palm is more or less a Fibonacci number.

4, MAKING A PALM CROWN

The centralmost point in the schematic crown (Fig. 7) represents the serial view of a palm trunk, and the radial lines, its leaves. The outermost leaf which is the oldest, is numbered 1. Leaf No. 2 is drawn at an arbitrary angular deflection of 137.5° to the left of leaf No. 1. Since leaf No. 2 is nearer to leaf No. 1 by the left-hand side of an observer looking from the tip of leaf No. 1, this crown maybe regarded as representing a left-spiralled palm. Similarly, leaf No. 3 is nearer to leaf No. 2 by the left, and the subsequent leaves are also similarly located. In another palm, leaf No. 2 can as well be nearer to leaf No. 1 by the right, in which case the diagram will represent a right-spiralled crown. The two types of individuals for any species of palms are distributed more or less equally in any locality, although for the coconut, there is an excess of one kind of individual in the Northern hemisphere, while the Southern hemisphere has more of the other kind, the hemispherical difference being significant statistically [6].

The younger leaves are represented progressively by shorter radial lines, and leaf No. 90 is the youngest visible leaf in this crown. The tips of all these leaves are connected by a line which forms a clockwise (left-handed) coil, and this will represent the only visible spiral in some palms such as the areca. In a palm showing two foliar spirals, one spiral will comprise leaves 1, 3, 5, 7, 9, and so on, while the second spiral will comprise all the even-numbered leaves. It is to be noticed that both the spirals move counter-clockwise. In a palm bearing three clearly visible spirals, the following leaves constitute the three spirals. Spiral one will connect

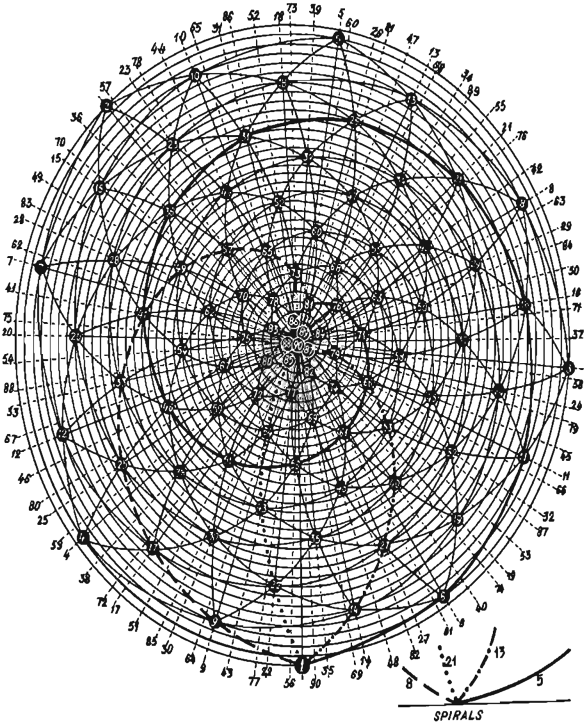


Figure 7

leaves 1, 4, 7, 10, 13, and so on. The second spiral will have leaves 2, 5, 8, 11, 14, and so on, while the third spiral will comprise leaves 3, 6, 9, 12, 15, and so on. All the three spirals run clockwise as opposed to the direction of the two spirals. No palm shows four clear spirals. This is in part due to the fact that leaves 1 and 5 which should form the two consecutive leaves of one of the four spirals, are located almost opposite each other. In a five-spiralled crown, leaves 1, 6, 11, 21, and so on, constitute one of the spirals, the other four starting with leaves 2, 3, 4, and 5, respectively. All the five spirals clearly move counter-clockwise which is opposite to the direction of the three spirals. In a palm with eight spirals, leaves 1, 9, 17, 25, 33, and so on, will form one of the spirals (shown in bold broken line) and the remaining spirals commence from leaves 2, 3, 4, 5, 6, 7 and 8. The eight spirals move opposite to the five spirals. In a like manner, if the diagram is to represent a thirteen-spiralled palm (one spiral comprising leaves 1, 14, 27, 40, 53, 66, etc.), the spirals will move opposite to the eight spirals, and similarly, the twenty-one spirals (shown in dots) move slantingly opposite to the thirteen spirals. Thus, in this diagram, the more obvious numbers of foliar spirals synchronize with Fibonacci numbers. Foliar spirals representing numbers 1, 3, 8, 21 move clockwise and the others counter-clockwise. This situation is in conformity with some properties of the Fibonacci Sequence.

In a palm crown having four or five leaves, only the single spiral is discernible, and two spirals may be clear if the leaf number goes up to seven or eight. Three spirals may be made out in a crown having ten to twelve leaves, and five spirals in a crown having about twenty leaves. Therefore, as the number of green leaves in a crown increases (a situation which normally necessitates a proportional increase in the girth of the trunk), higher orders of foliar spirals are displayed. Moreover, from a crown showing, say, eight spirals, those representing the two neighboring Fibonacci numbers (5 and 13) could also be made out. The leaf bases on the *Elaeis guineensis* trunk (Fig. 5) and the leaf scars on the stem of *Phoenix canariensis* (Fig. 6) bear testimony to this.

The angular deflection of 137.5° has been chosen arbitrarily because this would provide no two leaves in the diagram exactly superimposing each other till the 145th leaf. No palm is likely to have one hundred functional leaves at a time, and so, this angular deflection gives the leaves scope for maximum exposure to sunlight.

From observations made on a few species of palms having large numbers of leaves, the smaller angle subtended by any two consecutive leaves has been found to be 137.5° which makes a 0.618 proportion with the larger angle of 222.5°. The proportion between any two consecutive Fibonacci numbers (excepting the few smaller ones) also turns out to be 0.618. However, it is difficult to explain why most palms conform to this rule of golden proportion in the provision of the angular deflection between consecutive leaves. Obviously, it is genetically controlled. In Nature, small variations are noticed in the deflection between leaves in some palms, and sometimes within a palm at different heights. The foliar spirals of a tree at different heights tend to move along varied curves which is caused by the varying lengths of the internodes between leaves as well as the changes in the thicknesses of the stem. However, the numbers of these spirals are constant for a species.

5. FURTHER PECULIARITIES

If the schematic crown (Fig. 7) is examined critically, one will find that one of the five spirals connecting leaves 1, 6, 11, 16, ···, and one of the eight spirals connecting leaves 1, 9, 18, 25, ··· meet first at leaf No. 41, i.e., after forty leaves, and again at regular intervals of forty leaves. Similarly, if spirals of stages No. 1 (connecting leaves 1, 2, 3, 4, ···) and 2 (connecting leaves 1, 3, 5, 7, ···) are considered, they will meet first at leaf No. 3; likewise, spirals stages Nos. 2 and 3 meet at leaf 7; spirals 3 and 5 meet at leaf 16; spirals 8 and 13 at leaf 105, and so on. Each of the numbers of the above-mentioned leaves minus the first one gives the product of the two spirals and they are always Fibonacci numbers. This situation is applicable not only between leaf spirals representing consecutive stages, but also to any two different spirals. For example, the spirals Nos. 3 and 8 meet first at leaf 25 (i.e., after 24 leaves), while spirals Nos. 3 and 13 which meet at leaf 40 (i.e., after 39 leaves). Therefore, these numbers are always the products of any set of two Fibonacci numbers.

The 1/3, 2/5, 3/8, 5/13, 8/21, ··· phyllotaxis mentioned earlier can be clearly made out from the drawing. After leaf No. 1, three leaves are formed by the time the spiral has completed one full revolution (and a little more); five leaves are formed in two revolutions (a little less); eight leaves are formed after three revolutions; twenty-one leaves are formed after eight revolutions, and so on. It is also clear from the figure that the figures 1/3, 2/5, 8/21, ··· are not absolute figures since the specified numbers of leaves cover a little more or a little less distance than what they actually stand for. This is again in accordance with the mathematical properties of Fibonacci numbers.

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