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A method for testing stability in heterogeneous environments

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The presence of variety-environment interaction poses problems in identifying the significant superiority of any variety specially with heterogeneous environments. A method for selection of stable varieties and an exact method for testing the homogeneity of variety means, even with heterogeneous environments, is described in this paper.

METHOD

The method is a direct adaptation of the work of James (1951) and Welch (1951) to the problem under consideration. The following proposition emerges from their work.

Let t_{hj} denote the estimate of the j-th variety effect at h-th environment ($h = 1, 2, \ldots, p$; $j = 1, 2, \ldots, v$) and be distributed independently and normally with mean $T_{1j}, T_{2j}, \ldots, T_{pj}$ and variances $D_{1j}, D_{2j}, \ldots, D_{pj}$ respectively. Let $D_{1j}, D_{2j}, \ldots, d_{pj}$ be the estimates of $D_{1j}, D_{2j}, \ldots, D_{pj}$ based on $f_1, f_2, \ldots, f_p$ degrees of freedom respectively, the d_{hj} being distributed independently of the t_{hj} and of each other as χ^2 variates. Then for testing the hypothesis.

$$Ho : T_{1j} = T_{2j} = \ldots = T_{pj} \quad (1)$$

the test statistic

$$F = \frac{\sum_h W_{hj} (t_{hj} - t_j)^2/(p-1)}{[1 + \frac{2(p-2)}{p^2-1} \Sigma f_h (1 - \frac{W_{hj}}{W_j})^2]} \quad (2)$$

is distributed as variance ratio to order $1/f_h$ with (p-1) and

$$[\frac{3}{p^2-1} \Sigma \frac{1}{f_h} (1 - \frac{W_{hj}}{W_j})^2]^{-1} \text{ df}$$

where

$$W_{hj} = \frac{1}{d_{hj}}, \; W_j = \sum_h W_{hj} \text{ and}$$

$$t_j = \frac{\sum_h W_{hj} t_{hj}}{W_j} \quad (3)$$

The proof is discussed by Welch (1951). James (1951) proved that under the null hypothesis (1) the statistic

$$\sum_h W_{hj} (t_{hj} - t_j)^2 \quad (4)$$

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is distributed as χ^2_{p-1} provided f_h are large. If f_h is not large enough, James suggested comparison of the statistic (4) with

$$\chi^2[1+\frac{3\chi^2+(p+1)}{2(p^2-1)}\Sigma\ \frac{1}{f_h}(1-\frac{W_{hj}}{W_j})^2]$$

where χ^2 is the table value of χ^2 at $\alpha\%$ level of significance with (p–1) df. The hypothesis (1) indicates the stability of the j-th variety over all the environments. To identify the stable varieties, the F statistic is to be computed for all the v varieties, where insignificant F-ratio will indicate that the j-th variety is stable.

The analysis is now performed with the stable varieties. This will obviate heterogeneous interaction variances. The selection of stable varieties by the above tests may sometimes transfer the heteroscedastic data into homogeneous one, as was observed in our analysis of data on wheat. The combined estimate of variety offset with heteroscedastic data is given by t_j in (3). The estimate of variance of t_j is given by

$$v(t_j) = \frac{1}{W_j}[1+\frac{4}{W^2_j}\Sigma\frac{1}{f'_h}W_{hj}(W_j-W_{hj})]$$

where $f'_h = f_h - \frac{4(p-2)}{p-1}$

This formula is due to Meier (1953) with an adjustment due to Cochran and Carroll (1953).

To test the hypothesis

$$Ho: T_1 = T_2 = \ldots = T_q,\ q \leq v, \quad (5)$$

there are p test statistics for p environments and all these statistics are independent with p different probabilities. Then for a single test for the hypothesis (5) the p probabilities of these p test statistics will be combined. Fisher (1941) was apparently the first author to discuss such a combined test. According to him,

$$Z = -2\sum_h \log p_h \quad (6)$$

is distributed as χ^2 with 2P df. where p_h is the probability of the h-th test statistic for the hypothesis (5). Such a combination of tests will provide an exact test even under heteroscedastic environments. The combination of tests is also available for testing the hypothesis

$$Ho: T_j = T_j'$$
$$j \neq j' = 1, 2, \ldots, q \leq v$$

As an illustrative example of the methods discussed above, we analyse the data on 2 experiments on wheat conducted at 2 experimental stations in West Bengal during 1976. The object of the experiment was to determine the critical stages of irrigation in a variety of dwarf wheat. In the experiments N: P: K = 100: 50: 50 kg/ha was applied as a basal dose. The experiment was laid out in a randomized-block design with 3 replications where each replicate contained 15 plots, each of which received irrigation at different stages.

RESULTS

The error variances of the 2 experiments were $S^2_1 = 0.89346$ and $S^2_2 = 0.04757$ and these were heterogeneous. The values of t_{hj} (h = 1, 2) are given below.

Levels of irrigation	t_{1j}	t_{2j}	Levels of irrigation	t_{1j}	t_{2j}
1	1.711	3.034	8	0.546	0.934
2	0.826	0.934	9	−0.364	−0.300
3	0.699	−0.133	10	0.126	−0.366
4	1.381	2.200	11	−0.436	−0.766
5	0.983	0.434	12	−1.066	−1.533
6	1.094	1.134	13	−1.462	−0.666
7	−0.152	0.334	14	−1.976	−2.800
			15	−1.912	−2.433

The value of $d_{1j} = 0.27796$ and $d_{2j} = 0.01480$ ($j = 1, 2, \ldots, 15$). By (2) the observed F values for the 15 levels of irrigation are given below

Levels of irrigation	F value	Levels of irrigation	F value
1	5.970	9	0.014
2	0.040	10	0.830
3	2.340	11	0.370
4	2.290	12	0.750
5	1.030	13	2.160
6	0.005	14	2.320
7	0.805	15	0.930
8	0.512		

The F value for first level of irrigation only was greater than the value of $F_{0.05}$ at 1 and 31 df. Thus only the first level of irrigation was unstable over the 2 experimental stations. The F value for testing the hypothesis.

$$Ho : T_2 = T_3 = \ldots = T_{15}$$

from the first and second experimental stations were 4.27 and 381.53 respectively. By computing the statistic (6), the above hypothesis was found to be valid showing that among 15 stages of irrigation, 14 were stable over both the experimental stations and the combined effect of these stages of irrigation was significant in increasing the performance of the variety of dwarf wheat.

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