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A SYSTEM OF DEMAND EQUATIONS BASED ON PRICE INDEPENDENT GENERALIZED LINEARITY*

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1. INTRODUCTION

This paper proposes a theoretically plausible static demand system based on the concept of Price Independent Generalized Linearity (PIGL) of commodity budget share functions proposed by MuellBauer [1975]. PIGL, as is well-known, arose in the context of an exact non-linear aggregation, viz. consistent aggregation, of commodity budget share functions across a population of consumers with identical preferences, and is equivalent to commodity engel curves being of the Generalized Working-Leser (GWL) form.[2] Here, starting with the system of two-parameter GWL engel curves, we have modelled price responses through making the parameters of the engel curves dependent on prices in a manner such that the resulting system of budget share functions is integrable. The full-specification of the resulting static demand system contains $4n-1$ independent parameters (n being the number of commodities considered) — there being three other variants obtained as special cases of the full-specification.

The motivation for proposing the new system of demand equations emerges primarily from empirical considerations. The pattern of income responses displayed by PIGL appears to be empirically quite plausible, and therefore, the corresponding Generalised Gorman polar form of cost function should be attractive for conveniently generating complete systems of demand functions that can incorporate non-linear income responses. Indeed, Deaton and Muellbauer [1980] proposed the Almost Ideal Demand System (AIDS) on this line. The AIDS, in addition to allowing for nonlinear income effects, is reasonably general as it can give an arbitrary first order approximation to any demand system, satisfies the axiom of choice and aggregates perfectly over consumers. However, even under conditions of symmetry and homogeneity this system involves a large number of parameters — a feature shared by many other static demand systems having flexible functional forms. The empirical implication of this is clear — the estimates of the price responses based on such systems may be affected by the large sampling errors of the estimated parameters if the sample size is not large

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[2] That is, given the commodity prices, budget share for each commodity is a linear function of a fixed power of income or total expenditure. A special case of PIGL corresponding to Working-Leser engel curves has been called Price Independent Generalized Log-Linearity (PIGLOG).

enough and fairly disaggregated groups of commodities are considered. Thus, it seems worthwhile to search for a system of demand functions, within the PIGL class, that would describe consumer behaviour reasonably well through a relatively parsimonious parametrization of PIGL and without making unduly restrictive assumptions about the preference structure. Such an attempt has been made in this paper. In a sense, our proposed system has some formal resemblance to a non-additive generalisation of Stone's Linear Expenditure system (LES) — called the Simple Non-Additive Model (SNAM), suggested in Deaton [1975, 1976]. The empirical work reported here is basically directed towards a comparison of the AIDS, a modified version of SNAM and our proposed system on the basis of Indian consumer expenditure data. The format of presentation is as follows: Section 2 describes the proposed system; Section 3 briefly describes the nature of data used in the empirical exercises, summarises the estimation procedure and presents the results; finally, Section 4 gives some concluding observations.

2. THE PROPOSED SYSTEM

Muellbauer's [1975] definition of exact nonlinear aggregation, viz., consistent aggregation, of demand relationships across consumers with identical preferences leads to the following class of Generalized Linear (GL) form of budget share equations

$$(1) \qquad w_i = \alpha_i(p) + \beta_i(p)V(y, p); \qquad i = 1, 2, \ldots, n,$$

with $\sum_i \alpha_i(p) = 1$ and $\sum_i \beta_i(p) = 0$, where w_i: budget share for the i-th commodity, there being n commodities; y: the total expenditure; p: the n-vector of prices; and $\alpha_i(p)$, $\beta_i(p)$ and $V(y, p)$ are functions of total expenditure and/or prices. Precisely, for (1) there exists a unique representative level of y, viz., $y^*[=f(\underset{\sim}{y}, p)$, where $\underset{\sim}{y}$ is the vector of total expenditure distribution across consumers] such that y^* and the average budget share $\bar{w}_i \left[= \sum_h \frac{w_{ih} y_h}{\sum_h y_h}\right.$, w_{ih} and y_h being the budget share for the i-th commodity and total expenditure for the h-th consumer respectively$\left.\right]$ satisfy (1) for all i. PIGL and PIGLOG are special cases of GL corresponding to the case where y^* is independent of p, and are equivalent to (1) being

$$(2) \qquad w_i = \alpha_i(p) + \beta_i(p)y^{\varepsilon}, \quad \varepsilon \neq 0, \quad \text{and}$$

$$(3) \qquad w_i = \alpha_i(p) + \beta_i(p)\log y \qquad \text{for} \quad \varepsilon = 0$$

respectively. The cost functions underlying (2) and (3) are of the following Generalized Gorman polar form

$$(4) \qquad C(u, p) = [a^*(p) + b^*(p)u]^{\frac{1}{\varepsilon}}$$

where u represents a specified utility level, and $a^*(p)$, $b^*(p)$ are homogeneous functions in p of degree ε.

For our proposed system, we specify the following algebraic forms of (2) and (3) respectively

(5) $$w_i = \frac{p_i\alpha_i(p)}{\sum_j p_j\alpha_j(p)} + \beta_i(p)\left[\frac{\left(\frac{y}{\sum_j p_j\alpha_j(p)}\right)^{\varepsilon} - 1}{\varepsilon}\right], \quad \varepsilon \neq 0, \text{ and}$$

(6) $$w_i = \frac{p_i\alpha_i(p)}{\sum_j p_j\alpha_j(p)} + \beta_i(p)\log\left[\frac{y}{\sum_j p_j\alpha_j(p)}\right] \quad \text{for} \quad \varepsilon = 0$$

for $i=1, 2,\dots, n$; where p_j: price of the j-th commodity and $\alpha_j(p)$ and $\beta_j(p)$ are such that $\sum_j p_j\alpha_j(p)>0$ and $\sum_j \beta_j(p)=0$ for all p[3].

The forms of $\alpha_j(p)$ and $\beta_j(p)$ in (5) and (6) are taken as follows

(7) $$\alpha_j(p) = a_j + c_j \log\frac{p_j}{\pi_1}; \quad \pi_1 = \frac{\sum_k c_k p_k}{\sum_k c_k}, \text{ and}$$

(8) $$\beta_j(p) = b_j + d_j \log\frac{p_j}{\pi_2}; \quad \log \pi_2 = \frac{\sum_k d_k \log p_k}{\sum_k d_k},$$

and $\sum_j b_j=0$ ensures the adding-up property of the system.

Equations (7) has been used in Deaton [1975] in developing the SNAM and also in Deaton [1976] to generate a class of non-additive models of demand equations belonging to the Gorman family. Equation (8) is a similar functional form for the β-parameters of the proposed system in which the deflator price index π_2 has been taken as a geometric average of prices in order to guarantee the adding-up criterion.

The system described in (5)–(8) above thus has the parameters a_j, b_j, c_j and d_j, $j=1, 2,\dots, n$ and ε. It may be noted that in the more general form (5), the parameter ε controls the non-linearity of the budget shares with respect to y. In some preliminary experiments based on the restrictive versions of (5)–(6) with $\alpha_j(p)=a_j$ and $\beta_j(p)=b_j$, the estimate of ε turned out to be close to zero. This in effect suggests that (6) is a reasonable specification.[4] In what follows, therefore, we shall consider only (6) i.e., the PIGLOG form of the system.

The explicit form of the cost function underlying (6) is

[3] If we consider special cases of (5) and (6) in which $\alpha_j(p)=a_j$ and $\beta_j(p)=b_j$ for all j, the resulting systems come very close to the LES in appearance. However, to allow for additional flexibility in respect of price variations into the system, we suggest that $\alpha_j(p)$'s and $\beta_j(p)$'s be treated as functions of prices.

[4] This observation appears reasonable. Indeed, in the data budget shares for different commodities may show up different degrees of nonlinearity with respect to y, requiring ε to vary across equations. In such a situation, if ε is forced to be the same across equations, the resulting estimate of the common ε may be close to the modal value of ε, possibly $\varepsilon=0$. However, there is always a scope for experimentation with the value of ε in empirical exercises.

(9) $$C(u, p) = \alpha(p) \exp [u \prod_j p_j^{\beta_j^*(p)}]$$

where $\beta_j^*(p) = \frac{1}{2}[b_j + \beta_j(p)]$, $\alpha(p) = \sum_j p_j \alpha_j(p)$ and $C(u, p)$ is the cost of obtaining u at prices p. Equation (9) is evidently a specific form of the Generalized Gorman polar form cost function. The indirect utility function corresponding to (9) is

(10) $$u = \log Z / \prod_j p_j^{\beta_j^*(p)},$$

where $Z = y/\alpha(p)$ and $\alpha(p) > 0$. Thus, given the prices, u is monotonically non-decreasing in y if and only if $y > \alpha(p)$. $\alpha(p)$, therefore, may be interpreted as the cost of attaining the level of satisfaction for which the utility index is zero, given the prices.

Let us now turn to the expressions for the elasticities implied by (6). The engel elasticity is

(11) $$\eta_i = 1 + [\beta_i(p)/w_i].$$

Thus, given $w_i > 0$, a commodity is a luxury if $\beta_i(p) > 0$ and a necessary/inferior good if $\beta_i(p) < 0$. Also, given the prices, η_i falls as y increases.

The expression for non-compensated price elasticities for (6) is as follows

(12) $$\mu_{ij} = -\frac{1}{w_i}\Big[\theta_i(p)\{\theta_j(p) - \delta_{ij}\}\phi(p) + \alpha_j^*(p)\alpha_i^*(p) + \beta_i(p)\alpha_j^*(p) + \Big\{\delta_{ij}\beta_i(p) - d_i\Big(\delta_{ij} - \frac{d_j}{\sum_k d_k}\Big)\Big\}\log Z\Big]$$

where

$$\theta_k(p) = \frac{p_k c_k}{\sum_j p_j c_j}$$

$$\alpha_k^*(p) = \frac{p_k \alpha_k(p)}{\alpha(p)}$$

$$\phi(p) = \frac{\sum_j p_j c_j}{\sum_j p_j \alpha_j(p)}$$

and δ_{ij} is the Kronecker delta.

By construction, the system described above satisfies the homogeneity, symmetry and adding-up restrictions. In regard to the negative-definiteness of the matrix of compensated price effects, nothing can be said definitely. This and equivalently the concavity of the cost function may, therefore, be examined empirically for given price and income data.

3. SOME EMPIRICAL RESULTS

In this section, we shall report on the comparison of the empirical performances of three systems, viz., the AIDS, a modified version of the SNAM, and the PIGLOG version of our proposed system.

For the AIDS, the budget share functions are

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{y}{\bar{P}}\right), \qquad i = 1, 2, \ldots, n \tag{13}$$

with

$$\log \bar{P} = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \log p_j \log p_k \tag{14}$$

where α_i, β_i and γ_{ij}'s are the parameters. The adding-up property requires $\sum_j \alpha_j = 1$, $\sum_j \beta_j = 0$ and $\sum_i \gamma_{ij} = 0$ for all j. Homogeneity and symmetry of the slutsky substitution matrix additionally require $\sum_j \gamma_{ij} = 0$ for all i and $\gamma_{ij} = \gamma_{ji}$ for every pair of i, j, respectively. Thus, the homogeneity and symmetry constrained version of the system contains $\frac{(n-1)(n+4)}{2}$ independent parameters, while the unrestricted version involves $(n-1)(n+2)$ independent parameters. For the present exercise, we have estimated the unrestricted version of the system for which the sample values of $\log \bar{P}$, i.e., $\log \bar{P}_t$, $t = 1, 2, \ldots, T$, haven been approximated by the Divisia index $\log \bar{P}_t = \sum_j w_{jt} \log p_{jt}$ (w_{jt} and p_{jt} being respectively the observed budget share and price for item j at period t) following the suggestion in Deaton and Muellbauer [1980].

The modified version of the SNAM considered here is of the form

$$p_i q_i = p_i \alpha_i(p) + \beta_i(p)\{y - \alpha(p)\}, \qquad i = 1, 2, \ldots, n \tag{15}$$

where q_i is the quantity of the i-th commodity, $\alpha_i(p)$, $\beta_i(p)$ and $\alpha(p)$ are the same as those already defined above. However, here the adding-up property requires $\sum_i b_i = 1$. Equation (15) becomes the SNAM of Deaton [1976] when $\beta_i(p) = b_i$ and $\alpha_i(p) = a_i + c_i \frac{\log p_i}{\pi_i} - \log \sum_k c_k$. For (15), the following there versions have been estimated: the LES, i.e., $\alpha_i(p) = a_i$ and $\beta_i(p) = b_i$; a $3n-1$ parameter variant in which $\beta_i(p) = b_i$; and the $4n-1$ parameter full specification of (15). We shall call these variants LES, SNAM 1 and SNAM 2 respectively.

For the PIGLOG version of our proposed system also, the three corresponding variants have been estimated, viz., the $2n-1$ parameter specification in which $\alpha_i(p) = a_i$ and $\beta_i(p) = b_i$; the $3n-1$ parameter specification with $\beta_i(p) = b_i$; and finally, the $4n-1$ parameter full specification given by (6). In what follows, we shall refer to these specifications as Variant 1, Variant 2 and Variant 3 respectively. Formally, there may be another $3n-1$ parameter specification of both the modified SNAM and our proposed system, i.e., the ones with $\alpha_i(p) = a_i$.

For our proposed system, this may be referred to as Variant 4. From the cost function (9) it is clear that this variant would not allow any substitution possibility for the poorest consumers — a phenomenon that does not sound very sensible. Thus, Variant 4 may not be empirically very appealing compared to Variant 2[5].

The alternative variants of the modified SNAM and our proposed system have been estimated in their budget share function form by Non Linear Full Information Maximum Likelihood (NLFIML) method assuming a multivariate normal distribution for the additive equation disturbances with a constant non-diagonal covariance matrix. The unrestricted AIDS, on the other hand, has been estimated by single equation Ordinary Least Squares (OLS) method. As done in other similar studies, we have treated the per capita total consumer expenditure (PCE) as an exogenous variable in the estimation. For the rural data, this assumption may be questioned as most of the rural households in India perhaps link up their consumption decisions to their production and/or labour supply decisions, so that PCE may not necessarily be exogenous.[6]

The consumption data used here consist of the estimates of all-India average monthly per capita consumer expenditure on nine broad groups of items and the corresponding per capita total consumer expenditure (PCE) separately for rural and urban households. These have been taken from the published reports of the National Sample Survey (NSS) Organization for its 7th to 28th round of surveys, excluding the 26th and the 27th rounds for which estimates are not available. The period covered is from October 1953–March 1954 (7th round) to October 1973–June 1974 (28th round). For each round of the NSS survey, we have computed estimates of average per capita expenditures for three population groups based on ranking by the level of PCE, viz., the poorest 30 per cent, the middle 40 per cent and the richest 30 per cent, separately for the rural and the urban sector. In effect, thus, we have a time series of cross-section data on consumer expenditure.

The nine item groups considered here are 'cereals and cereal substitutes'; 'milk and milk products'; 'edible oils'; 'meat, fish and egg'; 'sugar etc.'; 'other food items'; 'clothing'; 'fuel and light'; and 'other non-food items'. The 'other food items' and the 'other non-food items' are the residual categories. The former includes pulses and products, salt, spices, vegetables, beverages, processed food etc., while the latter consists of expenditure on such non-durables as pan (i.e., betel leaves), tobacco and intoxicants, alcoholic beverages, transport, medical

[5] For completeness, we have estimated Variant 4 and reported its log-likelihood values along with those for the other variants in Table 1, for rural and urban India. As the log-likelihood values for Variant 4 are much smaller than those for Variant 2, this specification has not been pursued further.

[6] It should be noted here that as in the present study, when data on allocation of consumer's time are not available, the assumption of separability of the utility function in 'leisure' and 'other commodities' is invoked to rationalize the econometric estimation of demand functions using consumer expenditure and price data only. This, however, does not ensure exogeneity of PCE [Blackorby, *et al*, 1978, pp. 288.]

care, education, recreation, rents and taxes etc., and also expenditure on durables such as footwear, utensils, ornaments, household equipments, furniture etc. The durable expenditures in the 'other non-food items' category, however, account for only about 1.5 per cent of the total expenditure for either sector. It should be mentioned that the groups other than the two residual groups are fairly standard item groups for which the NSS organization regularly publishes expenditure estimates. However, for some of these groups, the observed budget shares are frequently very small – e.g., for items like 'meat, fish and egg', 'edible oils', and 'sugar etc.', the average budget shares are of the order of 3 per cent of PCE.[7] Yet, this grouping of the items has been retained essentially because of the fact that such a grouping is considered in most of the disaggregated studies based on Indian consumer expenditure data (e.g., Radhakrishna, Murthy and Shah [1979], Bhattacharya and Maitra [1970]).

The price data used here have been taken from Radhakrishna, Murthy and Shah [1979] who compiled the time series of price indices for these nine broad groups of items separately for rural and urban India on the basis of wholesale price relatives for a more detailed list of item groups. Briefly, their procedure of compilation of these price indices was as follows: for individual items in a broad item group, monthly average all-India wholesale price relatives (published on a regular basis by the Office of the Economic Adviser, Government of India) were first averaged over months covered by each NSS round. The price index for a broad group of items was then constructed as weighted average of the item price relatives, the weights being the item-specific expenditure share within the broad group. These expenditure weights were worked out on the basis of the detailed item estimates of average monthly consumer expenditure for the NSS 13th round (September 1957–May 1958). Separate weighting diagrams were used for the rural and the urban sector to construct separate price indices for rural and urban consumers.[8]

Let us now examine the empirical results. Table 1 presents the 2 log-likelihood values for the alternative specifications of the modified SNAM and our proposed system.[9]

[7] For observed budget shares of such small order, the assumption of normality made in the estimation may be questioned. However, whether such an assumption is reasonable would depend on the magnitude of the variance of the disturbance term in the functions corresponding to these items, and the inconsistency of the estimates arising from not assuming truncated distribution for the disturbances may actually be negligible.

[8] These price data compiled on the basis of wholesale price relatives may not reflect fully the intertemporal movements in corresponding retail consumer prices, and thus can give distorted estimates of price responses. However, these have been used in the absence of a more appropriate set of prices.

[9] For the SNAM specifications, in three out of the six cases, one of the c_j parameters failed to converge, while all others did. In such cases, we fixed the relevant c_j parameter to a reasonable value and obtained the restricted maximum likelihood estimates. Deaton [1976] also reported this type of identification problem for similar systems and ascribed this phenomenon to the broad grouping of items considered. No such problem, however, occured in our proposed system.

TABLE 1

TWICE LOG-LIKELIHOOD VALUES FOR THE ALTERNATIVE VARIANTS OF THE SNAM AND THE PROPOSED MODEL: ALL-INDIA RURAL AND URBAN, NSS 7TH-28TH ROUND

Model	Number of Parameters	All-India	
		Rural	Urban
LES	18	3523.6	3534.7
SNAM 1	27	3568.9	3607.9
SNAM 2	36	3690.2	3686.3
Variant 1	18	3511.6	3465.9
Variant 2	27	3688.0	3726.1
Variant 3	36	3758.2	3778.2
Variant 4	27	3614.4	3678.7

Table 1 shows that the introduction of prices through $\alpha_i(p)$ and $\beta_i(p)$ improves performance of both the modified SNAM and our proposed system. Formal likelihood ratio tests indicate significant improvement as one moves from the $2n-1$ parameter specification through the $4n-1$ parameter full-specification of either system for both rural and urban India. The performance of the two alternative $3n-1$ parameter specifications of the proposed system, viz., Variant 2 and Variant 4, is clearly much better than that of Variant 1. As between Variant 2 and Variant 4, the former appears to be a superior specification.[10] The log-likelihood values for the similar specifications of the modified SNAM and our proposed system are, strictly speaking, noncomparable. However, using such comparisons only as rough indications, one would observe that the $2n-1$ parameter specification of the modified SNAM (i.e., the LES) performs better than the corresponding specification of our proposed system, while the order of performance is reversed in the extended specifications.

A comparison of the goodness of fit of the different estimated systems based on the items squared correlation coefficient between observed and predicted budget shares (R_w^2) gives the following picture: (i) The performance of the LES and Variant 1 are broadly similar — LES performing marginally better for Urban India. Both the systems have, however, smaller R_w^2 values compared to the AIDS for most items; (ii) Introduction of $\alpha_i(p)$ improves R_w^2, the improvement being slightly more for Variant 2; (iii) Introduction of both $\alpha_i(p)$ and $\beta_i(p)$ brings the modified SNAM and our proposed system closer to the AIDS, Variant 3 being somewhat closer than SNAM 2.

Next, the presence of serial correlation in the residuals of the three best fitting systems (viz., SNAM 2, Variant 3 and AIDS) were examined. The Durbin-Watson test, however, could not be used because of the repeated cross-section

[10] Since these two are non-nested specifications, their log-likelihood values are not directly comparable. As a rough indicator, however, the improvements in log-likelihood values for these specifications over that of Variant 1 are compared here.

nature of the data. In the absence of a suitable procedure for testing serial correlation, we have estimated the first order autocorrelation coefficient of the estimated residuals for each item-population group combination and applied the standard test for correlation coefficient to examine their statistical significance. Strictly speaking, this is not a valid procedure for the present case, and this has been used only for rough guidance. The results of these tests for SNAM 2, Variant 3 and the AIDS are summarised in Table 2 below.

TALBE 2

RESULTS OF TEST FOR PRESENCE OF FIRST ORDER AUTOCORRELATION OF RESIDUALS NUMBER OF REJECTIONS OF THE NULL HYPOTHESIS OF NO AUTOCORRELATION FOR DIFFERENT SYSTEMS AND POPULATION GROUPS

System	Number of rejections out of a total of 9 cases for					
	Rural India			Urban India		
	Population Group 1 (Lowest 30%)	Population Group 2 (Middle 40%)	Population Group 3 (Upper 30%)	Population Group 1 (Lowest 30%)	Population Group 2 (Middle 40%)	Population Group 3 (Upper 30%)
SNAM 2	6	1	6	6	6	3
Variant 3	5	3	4	4	4	3
AIDS	2	1	1	4	2	2

Briefly, the problem of serial correlation is far more serious for SNAM 2 and Variant 3 than for the AIDS, results for which indicate that for rural India the problem is not at all serious. Some systematic patterns in the serial correlation for the other two systems may be noted,viz., it is significant for most of the items for population groups 1 and 3 in rural India and for population groups 1 and 2 in urban Inida. Such a systematic pattern is perhaps indicative of the fact that the observed serialco rrelation are largely due to misspecification of the functional form rather than omission of relevant dynamic elements.[11] Ideally, given this type of data, one should consider a suitable error structure to take into account the intergroup correlation at a point of time and/or the intragroup serial correlation over time. No such refinements, however, have been attempted here.

[11] Apparently the observed serial correlations of the residuals for the systems noted above would call for reestimation of the systems taking into account the possible autocorrelation of the stochastic disturbances, or else, suitably dynamised versions of the static systems, that can take care of the observed serial correlation, may be estimated. However, if the observed serial correlations arise from misspecification of the functional form, the other standard assumptions of the model, viz., that the expectation of the error is zero, and that the error is independent of the regressors are likely to be violated.

In such a situation these usual methods of reestimation may not be very helpful [Maddala, 1977, Judge *et al*, 1980].

As the cost functions underlying the three systems, viz., SNAM 2, Variant 3 and the AIDS, are not necessarily concave, the regularity conditions were also checked empirically. Precisely, for each sample point, the negative semi-definiteness of the matrix of compensated price effects have been examined. Thus, for each population group in either sector, twenty such checks have been done for every system. The regularity conditions were satisfied only for SNAM 2 in 7 and 5 cases respectively for the two richer urban population groups.

To give some idea about the magnitudes of the parameters estimated for Variant 3, the full set of estimated parameters for this system are presented in Table 3. To save space, the estimates of parameters of the other systems have not been reported here. Some general comments on the estimates of the parameters of Variant 3 may be made. First, judged by the asymptotic standard errors, all

TABLE 3

ESTIMATED PARAMETERS FOR VARIANT 3: ALL-INDIA RURAL AND URBAN, NSS 7TH–28TH ROUND

(FIGURES IN PARENTHESES ARE ASYMPTOTIC STANDARD ERRORS)

Item	All-India Rural				All-India Urban			
	a	*b*	*c*	*d*	*a*	*b*	*c*	*d*
Cereals, etc.	0.7742 (0.2057)	−0.1818 (0.0054)	−0.7107 (0.2101)	0.0999 (0.0355)	3.3351 (0.3937)	−0.1953 (0.0035)	−2.5499 (0.3900)	−0.0001 (0.0018)
Milk, etc.	−0.0716 (0.0112)	0.0527 (0.0018)	0.0551 (0.0097)	0.0094 (0.0097)	0.2272 (0.0853)	0.0337 (0.0024)	−0.3192 (0.1314)	−0.0028 (0.0016)
Edible Oil	0.0206 (0.0065)	−0.0015 (0.0007)	−0.0148 (0.0039)	0.0064 (0.0013)	0.2523 (0.0414)	−0.0109 (0.0014)	−0.1190 (0.0215)	−0.0046 (0.0023)
Meat, Fish, Egg	0.0084 (0.0089)	0.0040 (0.0006)	−0.0057 (0.0029)	0.0008 (0.0010)	0.1693 (0.0360)	0.0018 (0.0010)	−0.1261 (0.0274)	−0.0016 (0.0009)
Sugar, etc.	−0.0018 (0.0022)	0.0095 (0.0006)	−0.0138 (0.0055)	0.0149 (0.0018)	0.2019 (0.0357)	−0.0065 (0.0014)	−0.0324 (0.0285)	−0.0025 (0.0012)
Other Food	0.1292 (0.0038)	−0.0172 (0.0038)	−0.1558 (0.0385)	0.0629 (0.0137)	1.0868 (0.2257)	−0.0074 (0.0053)	−1.5279 (0.4213)	−0.0132 (0.0062)
Clothing	−0.0808 (0.0140)	0.0593 (0.0025)	0.0332 (0.0132)	0.0496 (0.0108)	0.0209 (0.0487)	0.0437 (0.0026)	0.0564 (0.0799)	0.0041 (0.0021)
Fuel and Light	0.1064 (0.0283)	−0.0252 (0.0010)	0.0746 (0.0163)	−0.0137 (0.0040)	0.5552 (0.0801)	−0.0200 (0.0007)	−0.4425 (0.0636)	0.0002 (0.0004)
Other Non-Food	−0.1305 (0.0236)	0.1002 (0.0052)	0.1061 (0.0220)	0.0244 (0.0088)	0.2153 (0.1906)	0.1613 (0.0063)	−1.4654 (0.5057)	0.0170 (0.0071)

the estimated "*b*" coefficients are significant. The estimates of "*a*" coefficients are significant except for 'sugar' for the rural sector and 'clothing' for the urban sector. The estimates of "*c*" are, by and large, significant (except, again for 'sugar' and 'clothing' for urban India) and are mostly negative.[12] However, the nature of estimates of "d" for the two sectors are somewhat different. While most of the estimated d's turn out is be significant, the d's for the rural sector are mostly positive while those for the urban sector are mostly negative.[13] However, very small magnitudes of the d's for the urban sector reflect the relative insensitivity of the income elasticities for this sector to variations in relative prices.

The items income and non-compensated own-price elasticities for the three systems were estimated at the sample mean of prices and PCE for a comparison.[14] Tables 4 and 5 present the estimates of income and own-price elasticities respectively, for the two sectors.[15] As regards the estimates of income elasticities, different variants of our proposed system (viz., Variant 1, Variant 2 and Variant 3) give estimates of comparable magnitude, that are, in turn, close to the corresponding AIDS estimates. The estimates based on the alternative SNAM specifications (viz., LES, SNAM 1 and SNAM 2) are close to each other, but differ to some extent from the other estimates. This could be largely due to the difference in the form of engel curves underlying the SNAM and the other systems.

The estimates of item-wise own price elasticity based on different systems presented in Table 5 show far less agreement. The magnitudes of the elasticities estimated from the different systems fail to discern any clear pattern of difference. On the whole, the estimates based on the AIDS may be seen to be often far off from those based on other systems. The AIDS estimates, however, need not be considered to be superior estimates, as these are based on the estimated γ_{ij} parameters of the system, most of which turned out to be statistically non-significant because of obvious multicollinearity problem encountered in the estimation. In Table 6 we have presented itemwise estimates of own-price elasticity separately for each population group based on the three systems, viz., SNAM 2, Variant 3 and the AIDS. A systematic pattern of variation of the elasticities based on Variant 3 is noteworthy. For most itemgroups these estimates tend to decline in magnitude with rising levels of real income for either sector. Such a pattern is, however, less prominent in the estimates based on the other systems.

4. CONCLUSIONS

We have proposed here a static demand system, based on Muellbauer's

[12] The negative sign of the estimated c parameters is reasonable as it indicates substitution possibilities at very low levels of living.

[13] A positive value of d means that with rising relative price of a commodity, its income elasticity goes up, which confirms Cramer's conjecture (Cramer, [1970]).

[14] A valid comparison of the elasticities for different systems based on the estimates at the sample mean requires the estimates of elasticities for different systems to follow a stable pattern of variations over all the sample points. Our limited experimentation confirms this stability; however, it is rather difficult to conclude definitely about the pattern of such variation.

[15] In a sense, the estimates of price elasticities for rural India reported here are incomplete, as they do not capture the profit effect of a rise in the agricultural prices (in addition to the usual income and substitution effects) that may arise for landed rural households in India.

TABLE 4

ESTIMATES OF ITEMWISE INCOME ELASTICITY FROM DIFFERENT EXPENDITURE: ALL-INDIA RURAL

ITEM	All-India Rural						
	LES	SNAM-1	SNAM-2	Variant 1	Variant 2	Variant 3	AIDS
Cereals, etc.	0.640	0.640	0.635	0.586	0.568	0.578	0.562
Milk, etc.	1.576	1.582	1.584	1.765	1.769	1.768	1.758
Edible Oil	0.996	1.003	1.026	1.031	0.999	1.024	1.006
Meat, Fish, Egg	1.156	1.148	1.173	1.239	1.155	1.169	1.165
Sugar, etc.	1.293	1.317	1.316	1.378	1.368	1.378	1.375
Other Food	0.945	0.959	0.959	0.910	0.913	0.933	0.925
Clothing	1.595	1.596	1.592	1.823	1.849	1.802	1.884
Fuel and Light	0.690	0.686	0.671	0.624	0.621	0.616	0.621
Other Non-Food	1.516	1.482	1.512	1.694	1.746	1.733	1.736

TABLE 5

ESTIMATES OF ITEMWISE OWN-PRICE ELASTICITY FROM DIFFERENT EXPENDITURE: ALL-INDIA RURAL AND

ITEM	All-India Rural						
	LES	SNAM-1	SNAM-2	Variant 1	Variant 2	Variant 3	AIDS
Cereals, etc.	−0.589	−0.699	−0.681	−0.213	−0.529	−0.489	−0.098
Milk, etc.	−1.067	−0.852	−0.832	−0.672	−0.384	−0.939	−0.590
Edible Oil	−0.690	−0.650	−0.509	−0.057	−0.435	−0.353	−0.604
Meat, Fish, Egg	−0.795	−0.882	−0.917	−0.223	−0.727	−0.818	−0.809
Sugar, etc.	−0.887	−0.764	−0.361	−0.339	−0.300	−0.395	−0.462
Other Food	−0.689	−0.598	−0.383	−0.065	−0.305	−0.232	−1.299
Clothing	−1.077	−0.820	−0.022	−0.718	−0.745	−0.132	−0.162
Fuel and Light	−0.493	−0.641	−1.284	0.238	−1.071	−1.097	−0.529
Other Non-Food	−1.027	−1.151	−0.816	−0.647	−0.928	−0.791	−0.648

SYSTEMS AT SAMPLE AVERAGE VALUES OF PRICES AND TOTAL
AND URBAN, NSS 7TH–28TH ROUND

All-India Urban						
LES	SNAM-1	SNAM-2	Variant 1	Variant 2	Variant 3	AIDS
0.387	0.381	0.375	0.123	0.286	0.279	0.269
1.331	1.332	1.334	1.484	1.415	1.424	1.414
0.888	0.907	0.931	1.144	0.860	0.896	0.860
1.104	1.109	1.087	1.336	1.108	1.120	1.106
0.942	0.959	0.950	1.191	0.892	0.929	0.887
1.031	1.047	1.052	1.344	1.044	1.073	1.049
1.489	1.486	1.488	1.415	1.711	1.692	1.642
0.756	0.761	0.752	0.883	0.682	0.676	0.675
1.397	1.378	1.382	1.417	1.578	1.556	1.590

SYSTEMS AT SAMPLE AVERAGE VALUES OF PRICES AND TOTAL
URBAN, NSS 7TH–28TH ROUND

All-India Urban						
LES	SNAM-1	SNAM-2	Variant 1	Variant 2	Variant 3	AIDS
−0.392	−0.556	−0.396	0.027	−0.626	−0.612	0.041
−1.111	−1.008	−0.637	−1.330	−1.036	−1.164	−1.025
−0.760	−0.764	−0.292	−0.415	−0.419	−0.481	−0.783
−0.928	−0.969	−0.960	−0.927	−0.879	−0.973	−0.806
−0.803	−0.509	−0.587	−0.535	−0.242	−0.145	−0.736
−0.895	−1.003	−1.380	−0.951	−1.642	−1.158	−1.069
−1.237	−0.841	−0.561	−1.142	−0.343	−0.555	−1.305
−0.657	−1.009	−0.956	0.223	−0.524	−0.708	−0.729
−1.118	−1.274	−1.160	−1.139	−1.179	−0.962	−0.844

TABLE 6

ESTIMATES OF ITEMWISE OWN PRICE ELASTICITY CALCULATED AT PRICES AND TOTAL EXPENDITURES OF NSS 28TH ROUND BY POPULATION GROUPS FOR SNAM 2, VARIANT 3 AND AIDS

Item	All-India Rural								
	SNAM 2			Variant 3			AIDS		
	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent
Cereals, etc.	−0.545	−0.628	−0.744	−0.683	−0.577	−0.316	−0.318	−0.197	0.138
Milk, etc.	−1.948	−0.985	−0.725	−1.233	−1.057	−0.964	−0.162	−0.506	−0.706
Edible oil	−0.491	−0.540	−0.581	−0.706	−0.636	−0.536	−0.672	−0.673	−0.674
Meat, Fish, Egg	−0.883	−0.884	−0.886	−1.061	−1.040	−1.014	−0.792	−0.807	−0.829
Sugar	−0.467	−0.395	−0.358	−0.383	−0.260	−0.134	−0.352	−0.463	−0.582
Other Food	−0.303	−0.381	−0.453	−0.454	−0.295	−0.032	−1.259	−1.267	−1.281
Clothing	−0.062	0.079	0.115	0.322	0.093	−0.021	4.181	0.861	−0.098
Fuel and Light	−0.791	−1.095	−1.522	−0.890	−0.975	−1.192	−0.586	−0.513	−0.310
Other Non-Food	−1.169	−0.812	−0.682	−1.155	−0.998	−0.907	−0.045	−0.496	−0.718

Item	All-India Rural								
	SNAM 2			Variant 3			AIDS		
	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent	Lowest 30 per cent	Middle 40 per cent	Upper 30 per cent
Cereals, etc.	−0.463	−0.433	−0.383	−0.746	−0.665	−0.400	−0.360	−0.161	0.534
Milk, etc.	−1.326	−0.764	−0.564	−1.254	−1.189	−1.140	−1.838	−1.029	−1.022
Edible Oil	−0.646	−0.501	−0.395	−0.680	−0.670	−0.655	−0.844	−0.835	−0.821
Meat, Fish, Egg	−1.242	−1.161	−1.109	−1.619	−1.574	−1.524	−0.783	−0.796	−0.812
Sugar, etc.	−0.221	−0.498	−0.710	−0.225	−0.256	−0.296	−0.789	−0.779	−0.762
Other Food	−0.762	−1.238	−1.546	−1.277	−1.157	−1.022	−1.070	−1.068	−1.066
Clothing	−0.848	−0.565	−0.497	−0.219	−0.447	−0.548	−2.258	−1.519	−1.279
Fuel and Light	−1.223	−1.073	−0.920	−0.780	−0.742	−0.664	−0.774	−0.737	−0.653
Other Non-Food	−1.801	−1.220	−0.970	−1.212	−0.871	−0.631	−0.649	−0.792	−0.869

PIGLOG, that has a close resemblance to the SNAM proposed as a non-additive generalization of the LES, and is parametically parsimonious compared to the AIDS — another well-known parametrization of PIGLOG of a very general nature. The four alternative variants of the proposed system display a varying degree of price flexibility of commodity budget shares. This and the nonlinearity of income responses implied by the underlying engel curves of the Working-Leser form should make the system flexible enough to provide reasonably satisfactory description of consumer expenditure pattern in applied works — particularly, when sample size is not large enough to permit a satisfactory application of demand systems having a flexible functional form.

Our empirical analyses were motivated towards a comparison of the performances of three systems, (viz., a modified version of the SNAM, the AIDS and our proposed system). Broadly, it was observed that with increased price flexibility the performance of the modified SNAM and our proposed system improved noticably. On the whole, the full-specification of our purposed system (i.e., Variant 3) performed better than the comparable specification of the SNAM. Also, compared to the AIDS, Variant 3 performed reasonably well for most commodity groups. However, in some cases even the AIDS specification failed in explaining the observed variations in budget share satisfactorily.

As a part of the empirical exercise, the validity of regularity conditions was examined empirically for the modified SNAM, the AIDS and our proposed system. In almost all cases, regularity conditions were violated, except for a few cases for the modified SNAM. However, no systematic pattern of violation over population groups was observed. This result is not unexpected given the grouped nature of data.

The estimates of income and own price elasticities obtained from the different systems were also compared. While the estimates of income elasticities from different systems were found to be broadly comparable (elasticities for the SNAM specifications being marginally different from those for the AIDS and our proposed system), the price elasticities were, by and large, less comparable. This is not quite surprising, as the way the price effects are modelled in these systems are widely different.

Our empirical results suffered rather heavily from the problem of serial correlation of the error term. This problem was observed to be far more serious for the SNAM and our proposed specifications than for the AIDS. Moreover, some systematic pattern of serial correlation was observed (e.g., the number of significant cases was systematically higher for population groups 1 and 3 in the rural sample and for populations groups 1 and 2 in the urban sample). This may be suggestive of some form of misspecification of the functional form rather than omission of dynamic factors in the analysis.

Our results thus perhaps reinforce the conjecture that for repeated cross-section data on consumption of such disaggregated item groups additional quadratic income terms in the budget share functions would improve the results substantially (see e.g. Pollak and Wales [1980] and Deaton [1984]). This, in general, points to the inadequacy of the PIGL specification in such a situation, and builds up a case for a quadratic extension of our proposed system. Such an extension is empirically rather straightforward. Although the quadratic system does not confrom to Muellbauer's notion of consistent aggregation any more, a theoretical basis of such systems can be found in Gorman [1981].

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