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ON RECENT DEVELOPMENTS OF MARRIAGE MODELS AND THEIR APPLICATIONS TO INDIAN NUPTIALITY

INTRODUCTION

In recent years increasing attention has been given on nuptiality primarily because it is one of the important components of population growth. Mathematical demography has been enriched substantially by encompassing nuptiality in its domain of study. An understanding of the fertility process of a population has become comparatively easy as a result of the development of marriage models. Though substantial progress has been made towards the development of nuptiality models, not much work has been done in the field of Indian nuptiality. An attempt has, therefore, been made here to review the different models and to apply them over Indian nuptiality data.

SOME MARRIAGE MODELS

Henry (1972) was one of the pioneers to contribute some fundamental ideas towards the study of nuptiality. He set out a novel and illuminating model of marriage formation. A more crystallised shape was given by Coale (1971) who brought revolution in this field by his discovery of the common age patterns of first marriage. He was the first to note the similarity in the schedules of first marriage frequency in population belonging to varied cultures. By trial and error Coale showed that the risk of first marriage (of those who marry) for the standard population is very closely fitted by a double exponential function:

$$r_s(x) = .174\ Exp\ [-4.411\ Exp\ (-.309\ x)]$$

The standard schedule was based on Swedish nuptiality for the period 1865-69. For a cohort where marriages begin at age a_0 and where the time scale of marriage is compressed by a factor k, the risk of marriage $r(a)$ at age a, among

those who marry, is

$$r(a) = (.174/k)\, Exp\,[-4.411\, Exp\,\{-\frac{.309}{k}\,(a - a_0)\}]$$

Later Coale and McNeil (1972) found an analytical expression for the study of first marriage that fits the Swedish standard. The mathematical model is given by

$$g_s(x) = .1946\, Exp\,[-.174\,(x - 6.06) - Exp\,\{-.2881\,(x - 6.06)\}]$$

where $g_s(x)$ is the probability density function of the age at first marriage in the standard population. It has been shown that the standard distribution can be approximated by the convolution of three exponential distributions with the smallest exponents and a normal distribution. The standard frequencies as well as the standard proportion ever married by age have been tabulated by Coale at intervals of one tenth of a year. He has also given a method by which an observed schedule of nuptiality can be related to the standard schedule by choosing the origin, the horizontal scale and the vertical scale. Coale has described these transformation in terms of three parameters a_0, K and C: a_0 is the location parameter or the age at which a consequential number of first marriages occur, k is the scale parameter and may be interpreted as the rate at which marriage occurs relative to the standard and C is the proportion of the cohort ultimately marrying. Besides giving a satisfactory representation of different marriage schedules, Coale's model offers a unique opportunity to explore the underlying behavioural pattern of nuptiality. In this connection it may be in order to quote Coale (1977):

First marriage consists of arriving at an age of marriageability (an age with a distribution that is approximately normal), followed by passage through 2 or 3 stages, the probability of passage through the next state being approximately constant within each stage. In a Western society becoming marriageable might be equated with starting to "date" regularly; the end of the first stage might be marked by meeting the ultimate spouse; the second stage might end with engagement, and the third with marriage.

Ewbank (1974) advanced the idea of convolution by using Swedish nuptiality data.

An alternative formulation of Coale's model was made by Rodriguez and Trussell (1980). In their view, the three parameters a_0, K, C are not easily interpretable (an argument on which the present author differs) and does not provide a convenient basis for comparing across cohorts or populations. A new standard with mean 0 and variance 1 has been obtained with the probability density function of age at first marriage:

$$g_0(z) = 1.2813\, Exp\,[-1.145\,(z + .805) - Exp\,\{-1.896\,(z + .805)\}]$$

The cumulative distribution function has been expressed in terms of incomplete gamma function. Rodriguez-Trussell formulation shows that the age at first marriage is distributed as a linear function of the logarithm of a standard gamma random variable. They have also described a method of estimation from household data as well as from individual data on ever married women.

Another marriage model has been used by Gudmund Hernes (1972) to describe the process of entry into first marriage of a cohort's members as they age. The model incorporates simple sociological assumptions which generate a curve of the general shape which is universally observed. The model rests on the differential equation:

$$\frac{d}{dt} P_t = Ab^t (1 - P_t) P_t$$

where P_t = proportion of the cohort already married at time 't'. Under this model each person starts with a certain marriage potential A, but this potential declines with a constant proportion b for each time unit. The solution of the above differential equation leads to $\frac{P_t}{1 - P_t} = Ka^{b^t}$, a Gompertz function. The familiar logit function of the proportion already married at time t is represented by a modified exponential. The model has been fitted to data on cumulative first marriages taken from US census population 1960 and the fit has been found satisfactory. The parameters for entry into first marriage of US citizens 35 and 75 years of age in 1960 (marriageability, constant of deterioration and asymptotic value for the percentage ever married) have also been presented.

There are certain other models less frequently used. The curve with some theoretical justification considered by Nydel and Wicksell (1924) is the lognormal curve to describe the age at marriage in a population. Hyrenius et al. (1967) tried various functions to graduate the number of single women at different ages and the best fit has been obtained by the logistic curve. To get a skew distribution of marriage ages only a part of the curve was used and the fit was found satisfactory. Again in the estimation of adult mortality levels from information on widowhood, Hill (1977) has used simple polynomials to represent the first marriage distribution. He has suggested two functions:

$$f(t) = t^{1.8} (30 - t)^4 \quad \text{for females and}$$

$$f(t) = t^{1.2} (30 - t)^3 \quad \text{for males}$$

where $f(t)$ is proportional to the number of marriages at age t. Both the functions have the same range of 30 years. The mean of the female function is 6.3 years with 90 percent married after 12.8 years while that of the male function is 8.3 years with 90 percent married after 15.5 years.

As will be revealed from the above account, most of the nuptiality models

are one sex models. Among researchers who have incorporated both sexes in the marriage analysis, mention may be made of Hoem (1969), Pollard (1971) and Keyfitz (1971). As an illustration, according to Pollard's model, the number of marriages X_{ij} between males aged i and females aged j is obtained by maximising a linear function of these marriages:

$$\sum_i \sum_j C_{ij} X_{ij}$$

subject to certain constraints. This is a linear programming problem and, apart from data limitations, such models seem to be too sophisticated and are not readily amenable to numerical procedures.

APPLICATIONS TO INDIAN NUPTIALITY

Most of the marriage models deal with cohort nuptiality. But such data are not readily available particularly for developing countries. As we are aware, in periods of stable nuptiality, a cross sectional schedule may be taken as representative of a real cohort, we have considered the cross sectional data for males and females taken from the censuses of India (1971, 1981). Coale's model curve and Hernes' model with logit function have been applied to proportions ever-married in five year age groups. The present author (Malaker, 1978) has already made some preliminary applications of different models like the logistic, the lognormal and Coale's standard curve to Indian marriage data, 1971. Again in the course of a recent investigation (Malaker, 1985) on modelling of Indian nuptiality with state level data for India, 1971 the author has noted that the method suggested by Coale does not work with many of the states in India. He has developed a program using minimum absolute deviations for filtering the model schedules of Coale. As the findings seem to have a great value in understanding the age at first marriage in India, summary tables of the above investigation have been reproduced in table 1.

Regarding the application of Coale's model to cross sectional schedules – India 1971 and 1981 we have used yet another simple device based on method of moments for estimation of the parameters a_0 and K. C has been estimated from the inspection of the run of values of proportion ever married. The mean and standard deviation of the age schedule at first marriage have been calculated (for mean age at marriage see Hajnal, 1953; for standard deviation of age at marriage see Malaker, 1981). They have been equated with the corresponding model values: i.e.,

$$\text{mean} = a_0 + 11.36\, k$$
$$\text{s.d.} \quad = 6.583\, k$$

The expected proportions single in different quinquennial age groups as obtained by Coale's and Hernes' model along with the observed proportions are shown in table 2. We have also presented the parameters and the goodness of fit statistic *('I')* in the same table.

TABLE 1

Parameters of Coale's model curve and a "Goodness of fit statistic" (T) by States India, 1971

State	Male			Female		
	a_0	K	T'(*)	a_0	K	T'(*)
Andhra Pradesh	16.5	.58	.0542	10.5	.51	.0258
Arunachal Pradesh	14.0	1.00	.0423	12.2	.63	.0325
Assam	16.7	.80	.0225	12.4	.54	.0075
Bihar	11.1	.79	.0356	9.6	.50	.0130
Gujarat	14.1	.71	.0452	14.0	.40	.0269
Haryana	12.6	.71	.0307	9.9	.61	.0623
Himachal Pradesh	16.8	.58	.0584	12.8	.44	.0279
Jammu and Kashmir	14.7	.80	.0359	12.8	.44	.0253
Kerala	18.4	.74	.0089	13.9	.63	.0114
Madhya Pradesh	10.2	.84	.0527	9.2	.51	.0162
Maharashtra	16.6	.64	.0490	11.6	.53	.0232
Manipur	16.8	.83	.0515	13.4	.80	.0545
Meghalaya	16.1	.83	.0233	12.3	.68	.0079
Mysore	17.7	.67	.0346	12.1	.51	.0298
Nagaland	16.8	.92	.0394	14.6	.80	.0300
Orissa	16.5	.56	.0435	12.3	.44	.0131
Punjab	15.9	.67	.0276	14.5	.50	.0139
Rajasthan	11.3	.76	.0588	8.8	.57	.0406
Sikkim	13.3	1.00	.0746	12.2	.80	.0707
Tamil Nadu	18.6	.65	.0101	14.5	.45	.0087
Tripura	17.0	.74	.0353	11.9	.57	.0102
UP	9.8	.87	.0430	9.0	.58	.0338
West Bengal	15.5	.82	.0389	11.2	.58	.0143

(*) T' = proportionate error.

Source: C.R. Malaker, 1985.

It should be noted that it is not strictly possible to test the validity or the behavioural aspect of a model with nuptiality data in such broad age groups particularly when marriages in India are concentrated in a very narrow age range for both sexes. To probe further, we have used another set of data giving age distribution of first marriage by single year age for males and females on nuptiality experience of the decade 1961-71 (Malaker, 1973). In the absence of availability of nuptiality data by single year age we will assume these data sets to represent the experience of a real cohort exposed to the marriage rates of the decade 1961-71 and henceforth refer them as observed data.

TABLE 2

Proportions single in the censuses of India 1971 and 1981

Age group	1971			1981		
	Observed	Coale	Hernes	Observed	Coale	Hernes
			Males			
10 - 15	.953	.986	.979	.974	.997	.989
15 - 20	.814	.802	.768	.875	.880	.833
20 - 25	.499	.432	.363	.561	.495	.416
25 - 30	.186	.197	.151	.215	.216	.163
30 - 35	.070	.095	.076	.071	.096	.076
35 - 40	.040	.052	.047	.034	.049	.044
40 - 45	.031	.036	.034	.027	.032	.030
45 - 50	.027	.026	.027	.023	.023	.024
Parameters						
a_0		11.96	–		13.53	–
K		.92	–		.86	–
A		–	.65		–	.71
b		–	.921		–	.922
T'		.0633	.0988		.0503	.0982
			Females			
10 - 15	.882	.869	.908	.934	.930	.950
15 - 20	.437	.406	.372	.559	.496	.485
20 - 25	.095	.134	.084	.100	.157	.109
25 - 30	.023	.044	.026	.033	.046	.030
30 - 35	.010	.014	.018	.012	.015	.012
35 - 40	.008	.005	.007	.006	.004	.006
Parameters						
a_0		9.27	–		10.67	–
K		.70	–		.66	–
A		–	.68		–	.72
b		–	.921		–	.925
T'		.0763	.0783		.0863	.0620

Hernes' model and the simple polynomials $f(t) = t^{1/3}\ (30 - t)^4$ (females) and $f(t) = t^{1/2}\ (30 - t)^3$ (males) have been applied to the age distribution of first marriage. Before applying the simple polynomials some preliminary tests were made and it was revealed that the observed data sets might be amenable to such models. The results of the above analysis are presented in table 3.

TABLE 3

Cumulative first marriage for Indian males and females: 1961-71
(of those who ever-marry)

Age	Males			Females		
	Observed	Poly-nomial	Hernes	Observed	Poly-nomial	Hernes
10	–	–	–	4.44	7.44	2.42
11	–	–	–	6.68	9.93	3.88
12	–	–	–	9.37	10.24	5.38
13	–	–	–	9.04	9.89	7.08
14	–	–	–	8.96	9.23	8.77
15	3.81	3.76	2.00	8.77	8.41	10.06
16	4.80	6.24	2.95	8.46	7.53	10.61
17	5.67	7.27	4.09	8.05	6.64	10.23
18	6.38	7.71	5.36	7.54	5.77	9.07
19	6.91	7.79	6.67	7.04	4.95	7.79
20	7.29	7.64	7.77	6.70	4.19	5.46
21	7.54	7.34	8.51	5.57	3.50	4.29
22	7.72	6.92	8.70	3.77	2.88	3.06
23	7.73	6.43	8.39	1.92	2.34	2.15
24	7.64	5.89	7.68	.85	1.86	1.49
25	7.58	5.33	6.71	.63	1.46	1.00
26	6.86	4.76	5.67	.48	1.12	.69
27	5.01	4.20	4.67	.37	.84	.46
28	3.13	3.66	3.77	.29	.61	.31
29	2.47	3.15	3.01	.23	.43	.21
30	1.96	2.66	2.40	–	–	–
31	1.61	2.22	1.90	–	–	–
32	1.31	1.81	1.50	–	–	–
33	1.07	1.45	1.19	–	–	–
34	.87	1.14	.95	–	–	–

CONCLUDING REMARKS

From a study of the tables presented in this paper we can make some general observations:

(1) Compared to international standard, the age at entry into the marriage market (a_0) is very low for both males and females in India. The rate at which marriage occurs (K) as well as the age at entry varies substantially over the states. Marriage is more or less universal in India. The states exhibiting very early marriage also exhibit very low age at entry into the marriage market. For females, marriage occurs at a faster rate compared to males. Relative to the Swedish standard, mar-

riage occurs at an accelerating rate.

(2) The initial marriageability *(A)* is slightly greater for women than for men in both 1971 and 1981. It has increased overtime for both sexes. There is no variation in the rate of decline of marriage capacity *(b)* either between the censuses or between the sexes.

(3) Comparing different models, Coale's standard curve gives a much better fit to Indian nuptiality relative to the other models. The parameters of the model curve obtained by minimising the sum of absolute deviations seem to give a very satisfactory representation of Indian nuptiality.

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SUMMARY

Nuptiality is one of the important components of population growth. Though in recent years increasing attention has been given towards the development of nuptiality models, not much work has been done in this field in India. An attempt has been made here to review the different marriage models and to apply them over Indian data.

Coale's model curve, Hernes' model and polynomials have been tried. Coale's method of fitting does not work with Indian marriage data. A program using minimum absolute deviations with proportions single in 5-year age groups has been developed for filtering the model schedule of Coale. This method works well with the state level data for India, 1971. Coale's model curve has also been applied to proportions single in 5-year age groups in the censuses of India, 1971 and 1981. Besides, Hernes model has been tried with these data. Again the age distribution of first marriage in single year age with the marriage experience of the decade 1961-71 has also been considered and Hernes' model and simple polynomials have been applied to this schedule.

It has been observed that the age at entry into the marriage market is very low and the pace of marriage very high for the Indian population with universal marriage for both sexes. The initial marriageability is slightly greater for women than for men. Among different models compared, Coale's three parameter curve gives a very satisfactory representation of Indian nuptiality.

RIASSUNTO

L'A., dopo aver passato in rassegna alcuni modelli di nuzialità, cerca di verificare l'applicabilità di alcuni di essi (modello Coale, modello Hernes e curve polinomiali) ai dati indiani.

Il metodo di Coale, che non funziona direttamente con i dati indiani, è stato corretto con un programma che utilizza gli scarti minimi assoluti. Questa correzione consente un buon adattamento del metodo ai dati provinciali. I modelli di Coale e di Hernes sono stati applicati ai dati dei censimenti indiani del 1971 e del 1981. E' stata inoltre fatta un'applicazione del modello di Hernes e della curva poli-

nomiale alla distribuzione della età al primo matrimonio (classi annuali) ricavata dall'esperienza del decennio 1961-71.

L'A. osserva che la popolazione indiana di entrambi i sessi entra nel mercato matrimoniale ad un'età molto giovane e che la nuzialità è molto elevata. La matrimonialità iniziale è lievemente più alta per le donne che per gli uomini. I tre parametri della curva di Coale forniscono una buona rappresentazione della nuzialità indiana.

RESUME

Dans cette étude on a passé en revue quelques modèles de nuptialité (modèle de Coale, modèle de Hernes et polynômes) et on les a appliqués à la situation de l'Inde. Comme la méthode de Coale ne fonctionne pas directement avec les données indiennes, un programme utilisant les moindres écarts absolus a été développé pour corriger le modèle original. Cette correction permet une bonne adaptation de la méthode. Les modèle de Coale et d'Hernes ont été appliqués aux données des recensements de l'Inde de 1971 et 1981. De plus on a appliqué le modèle d'Hernes et la curbe des polynômes à la distribution des âges au premier mariage (classement par années d'âge) qui se rapporte à la période 1961-71.

L'auteur remarque que l'âge d'entrée dans le marché matrimonial de la population indienne des deux sexes est très bas et que le taux brut de nuptialité est très élevé. Les taux spécifiques de nuptialité sont un peu plus élevés parmi les femmes aux âges plus bas. Les trois paramètres de la courbe de Coale représentent d'une façon satisfaisante la nuptialité indienne.