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A MONTE CARLO EVALUATION OF THE POWER OF SOME TESTS FOR HETEROSCEDASTICITY*

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Szroeter's asymptotically normal test outperforms the Goldfeld–Quandt test, the Breusch–Pagan Lagrange multiplier test and BAMSET, when it is possible to order the observations according to increasing variance. With no prior information on variance ordering, BAMSET is best. Some observations concerning degree of heteroscedasticity and model specification are made.

1. Introduction

It is well known that in the presence of heteroscedasticity of error variances, the least squares method has two major drawbacks: (i) inefficient parameter estimates and (ii) biased variance estimates which make standard hypothesis tests inappropriate. The importance of tests for heteroscedasticity is well recognized and a large number of tests have been proposed. There are test procedures for establishing a specific form of heteroscedasticity, and a wide range of tests for detecting only the presence or absence of heteroscedasticity. See, for example, Goldfeld and Quandt (1965), Rutemiller and Bowers (1968), Glejser (1969), Ramsey (1969), Theil (1971), Harvey and Phillips (1974), Harvey (1976), Bickel (1978), Szroeter (1978), Breusch and Pagan (1979), Harrison and McCabe (1979), White (1980), Carroll and Rupert (1981), King (1982), Barone-Adesi and Talwar (1983), Buse (1984), Ali and Giaccotto (1984), Evans and King (1985), and Judge et al. (1985, ch. 11). The various tests proposed by these authors have been well reviewed in the last three listed references, and so a further review will not be attempted here. We are concerned with evaluating a computationally simple asymptotic test which was proposed by Szroeter (1978) and which appears to have been overlooked in the

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studies listed above. This test, originally designed for structural and reduced form relations in dynamic simultaneous equation models can, under appropriate assumptions, also be used to test for heteroscedasticity in linear regression models. A *finite sample* version of the test was shown by King (1982) to be approximately locally best invariant under quite general conditions and its power has compared favourably with that of a number of other test procedures [Evans and King (1985)]. However, because the critical value of the finite sample version of the test depends on the set of regressors, this test is computationally demanding and unlikely to be computed routinely in standard econometric computer packages. In contrast, routine calculation of the asymptotic tests could easily take place, in much the same way as the Durbin-Watson statistic is routinely computed to test for autocorrelation. See Harrison (1980) for a discussion on the relative computational ease of a large number of tests for heteroscedasticity.

We have chosen to compare Szroeter's asymptotic test with three others – the Goldfeld–Quandt test [Goldfeld and Quandt (1965)], the Breusch–Pagan test [Breusch and Pagan (1979)], and BAMSET [Ramsey (1969)]. The Goldfeld–Quandt (G-Q) test has been chosen because it appears to be the most popular test in applied econometrics and its performance has been found to be satisfactory in many of the earlier studies. The Lagrange multiplier (LM) test proposed by Breusch and Pagan (B-P) is also popular and is simple to compute. It, too, has been found to be quite powerful in the presence of heteroscedasticity. The test BAMSET has been included following a suggestion from a referee that its performance is likely to be less sensitive to the assumption that, under the alternative hypothesis of heteroscedasticity, the observations can be ordered according to increasing variances.

Monte Carlo methods are used to examine the four test procedures for two different heteroscedastic variance structures with varying degrees of heteroscedasticity, and for small and large samples. Our results indicate that when it is possible to order the observations according to increasing variances, Szroeter's test is more powerful than the remaining three. When the observations have not been ordered the performances of Szroeter's test, BAMSET and the G-Q test all fall dramatically and, as hypothesized by the referee, BAMSET is better than the other two. The B-P test does not depend on whether or not the observations are ordered, but does depend on similarly strong prior information. Details of the variance specifications and the tests are given in section 2. The set-up of the Monte Carlo experiment and the results are presented in sections 3 and 4, respectively.

2. Variance structures and tests for heteroscedasticity

Consider a linear regression model

$$y_t = X_t\beta + u_t, \qquad t = 1, 2, \ldots, T, \tag{1}$$

where $X_t = (1, x_t)$ contains the constant term and the tth observation on an explanatory variable, y_t is the tth observation on a dependent variable, β is a (2×1) vector of parameters, and the u_t are unobservable normal random errors with mean zero and variances as specified below. We restrict β and x_t to be of dimension 2 because this is in line with the design of our Monte Carlo experiment, not because it is necessary for carrying out the tests. Two types of variance structures are considered. The first specification is

$$V(u_t) = \sigma_t^2 = \exp(Z_t\delta) = kx_t^\gamma, \tag{2}$$

where

$$Z_t = (1, \log x_t) \quad \text{and} \quad \delta = (\log k, \gamma)'.$$

This model has been discussed by Geary (1966), Park (1966), Lancaster (1968), Kmenta (1971), and Harvey (1976), and, following Harvey, we shall refer to it as the *multiplicative heteroscedastic model*. The second variance assumption is

$$V(u_t) = \sigma_t^2 = (X_t\alpha)^2 = \alpha_0^2(1 + \lambda x_t)^2, \tag{3}$$

where

$$\alpha = (\alpha_0, \alpha_1)', \qquad \lambda = \alpha_1/\alpha_0, \qquad X_t = (1, x_t).$$

This model has been studied by Rutemiller and Bowers (1968), Glejser (1969) and Harvey (1974) and we will refer to it as the *additive heteroscedastic model*.

We shall now briefly describe the four tests.

2.1. Szroeter's test

To begin we assume that in model (1) the exact form of the variances (σ_t^2) may not be known but that all the observations can be arranged in such a way that $\sigma_{t-1}^2 \leqq \sigma_t^2$, $t = 2, 3, \ldots, T$. This can be done either with reference to certain exogenous variables appearing in our model (x_t in our case), or on the basis of predictive values of y_t based on OLS estimates.

If $e_t = y_t - X_t\hat{\beta}$, where $\hat{\beta}$ is any consistent estimator of β, the test statistic is defined by

$$Q = T(\tilde{h} - \bar{h}) \Big/ \left[2\sum_{t=1}^{T}(h_t - \bar{h})^2\right]^{\frac{1}{2}}, \tag{4}$$

where

$$\tilde{h} = \sum_t w_t h_t, \qquad w_t = e_t^2 \Big/ \sum_t e_t^2, \qquad \bar{h} = T^{-1}\sum_t h_t,$$

and h_t is a set of T non-stochastic scalars having the property that $h_t \geq h_s$ if $t > s$.

Under the null hypothesis H_0: $\sigma_t^2 = \sigma_{t-1}^2$, $t = 2, 3, \dots, T$, against the alternative hypothesis H_1: $\sigma_t^2 > \sigma_{t-1}^2$, Q follows a limiting normal distribution with mean zero and variance unity.

If we let $h_t = t$, $t = 1, 2, \dots, T$, the test statistic can be written as

$$Q = \left(\frac{6T}{T^2 - 1}\right)^{1/2}\left(\tilde{h} - \frac{T+1}{2}\right), \tag{5}$$

where

$$\tilde{h} = \sum_t te_t^2 \Big/ \sum_t e_t^2.$$

The test statistic (5) is very simple to compute, and, because it is asymptotic N(0, 1), the test is easy to apply. The observations for models (2) and (3) can be arranged according to increasing variances by arranging the x's in ascending order. Also, for $\tilde{h}$ we will use the residuals from the OLS estimator for β. Note that Q will be the same for both multiplicative and additive heteroscedasticity. We will refer to this test as the SZ test.

2.2. *Breusch–Pagan test*

This test is based on Silvey's (1959) LM test and was developed independently by Godfrey (1978) and Breusch and Pagan (1979). The test statistic for models (2) and (3) can be developed as follows.

Let $\hat{u}_t$ be the OLS residual for the tth observation for the model (1), let the estimated residual variance be $\hat{\sigma}^2 = T^{-1}\sum_t \hat{u}_t^2$, and define r as a vector with typical element $r_t = (\hat{u}_t^2/\hat{\sigma}^2 - 1)$. Then, the LM statistic is

$$LM = \tfrac{1}{2} r'Z(Z'Z)^{-1}Z'r, \tag{6}$$

where Z is a $(T \times 2)$ matrix with tth row given by $Z_t = (1, \log x_t)$ for the multiplicative model and $Z_t = (1, x_t)$ for the additive model.

Under the null hypothesis of homoscedasticity, LM in (6) is distributed as $\chi^2_{(1)}$. (If Z_t was of dimension p, then LM would follow a $\chi^2_{(p-1)}$ distribution.) Because the test statistic for the multiplicative model is based on $Z_t = (1, \log x_t)$ and for the additive model on $Z_t = (1, x_t)$ we will have two LM tests. The first we refer to as the Breusch–Pagan multiplicative [B-P(M)] test, and the other will simply be referred to as the Breusch–Pagan (B-P) test.

2.3. The Goldfeld–Quandt test

The test proposed by Goldfeld and Quandt (1965) is carried out as follows.

(i) Arrange all the observations according to increasing variances as in Szroeter's test.
(ii) Discard 'c' central observations and fit two separate regressions to each of the remaining $(T-c)/2$ observations.
(iii) Obtain S_1 and S_2, the residual sums of squares from the regessions fitted to the first and last $(T-c)/2$ observations.
(iv) Under the null hypothesis of homoscedasticity the statistic $R = S_2/S_1$ has an F-distribution with $[(T-c-4)/2, (T-c-4)/2]$ degrees of freedom.

In our experiments where we used sample sizes of $T = 20$ and 50, we set $c = 4$ and 10, respectively.

2.4. BAMSET

The version of the test BAMSET [Ramsey (1969)] which we employed is given by

$$BS = (T-2)\log\hat{\sigma} - \frac{T-2}{3}\sum_{i=1}^{3}\log s_i^2, \tag{7}$$

where

$$\hat{\sigma}^2 = (T-2)^{-1}\sum_{t=1}^{T}\hat{u}_t^2, \qquad s_i^2 = (3/(T-2))\sum_{t\in S_i}\hat{u}_t^2,$$

and the $\hat{u}_t$ are the OLS residuals. Note that, for our settings of T, $(T-2)/3$ is an integer. The sets of observations used to define the three residual groups are as follows. For $T=20$, $S_1 = \{1,2,\ldots,7\}$, $S_2 = \{8,9,\ldots,13\}$, $S_3 = \{14,15,\ldots,20\}$. For $T=50$, $S_1 = \{1,2,\ldots,17\}$, $S_2 = \{18,19,\ldots,33\}$, $S_3 = \{34,35,\ldots,50\}$. Under the null hypothesis of homoscedasticity BS is treated as an asymptotic $\chi^2_{(2)}$ random variable.

Other versions of this test can be constructed depending on the number of the groupings and whether alternative sets of residuals such as BLUS [Theil (1971)] or recursive [Harvey and Phillips (1974)] are used in place of OLS residuals. Our choice of three groups was based on Ramsey's recommendation; OLS residuals were chosen because of their computational ease and their apparent superiority in experiments conducted by Ramsey and Gilbert (1972) and Ali and Giaccotto (1984).

An important characteristic of each test is the amount of prior information required concerning the type of heteroscedasticity under the alternative hypothesis. The SZ test assumes the observations can be ordered according to increasing variances. The G-Q test makes a similar assumption, or at least that the observations can be placed into two groups – one containing observations with potentially high variances and the other containing observations with potentially low variances. The B-P test requires knowledge of the explanatory variables upon which the variances depend. With one explanatory variable such prior information is similar to that required by the SZ and G-Q tests. With more than one explanatory variable the B-P test requires relatively less prior information because a decision about the relative importance of different explanatory variables is not needed.

Ideally, the observations should also be ordered for BAMSET. However, relative to SZ and G-Q, we would expect the decline in power of BAMSET to be less sensitive to an inappropriate ordering. This hypothesis was tested by estimating the power of SZ, G-Q and BAMSET for both ordered and unordered observations. The performance of B-P does not, of course, depend on the order of the observations.

3. Set-up of the Monte Carlo experiment

Throughout the experiment β was set at $\beta = (\beta_0, \beta_1)' = (10, 1)'$. Two different sample sizes $T = 20$ and $T = 50$ were considered; and the x_t's were initially generated from two different distributions, uniform and lognormal, and were then held fixed in repeated samples. The lower and upper parameters of the uniform distribution were, respectively, 20 and 100; in the lognormal case, the x_t's were found from $x_t = e^{q_t}$, where the q_t's were generated from $N(3.8, 0.4^2)$. Five thousand replications were generated for each combination of sample size, regressor type and error variance model. The u_t's were drawn from a normal distribution with mean zero and variances given by (2) and (3) for the multiplicative and additive models, respectively. The severity of the heteroscedasticity was controlled by varying the parameters γ and λ; we considered 18 values of λ between 0 and 2.0, 55 values of γ between -7 and 7, and an additional 6 values of λ equal to 10, 50, 100, 500, 1000 and 5000. For negative values of γ the observations were ordered according to decreasing rather than increasing values of x_t. The powers of all five tests (SZ, B-P, B-P(M), G-Q, and BAMSET) were estimated by calculating the proportion of rejections in 5000 replications at a 5% level of significance.

4. Results

A convenient measure of the degree of heteroscedasticity which was suggested by Surekha (1980), and later used by Evans and King (1985), is the

Table 1

Powers of tests for selected models and parameter values.

Model[a]	Parameter	C of V	Tests				
			SZ	G–Q	B–P	B–P(M)	BAMSET
AL	$\lambda = 0.04$	0.410	0.534	0.464	0.416	0.400	0.275
ML	$\gamma = 1.35$	0.416	0.557	0.489	0.429	0.421	0.294
AU	$\lambda = 0.03$	0.412	0.597	0.541	0.470	0.444	0.345
MU	$\gamma = 1.25$	0.406	0.591	0.536	0.461	0.446	0.343
AL	$\lambda = 500$	0.619	0.819	0.764	0.703	0.718	0.568
ML	$\gamma = 2$	0.619	0.819	0.764	0.703	0.718	0.568
AU	$\lambda = 500$	0.608	0.887	0.847	0.800	0.812	0.692
MU	$\gamma = 2$	0.608	0.887	0.847	0.800	0.812	0.692

[a] The code for model type is : M = multiplicative, A = additive, U = uniform and L = lognormal.

coefficient of variation of the variances (*C of V*). This measure is invariant with respect to the units of measurement of the variables and, for a given x-vector (lognormal or uniform), it turns out to be the major determinant of the powers of the tests. In table 1 we have presented the estimated powers of the tests for some selected values of λ and γ which lead to similar values of the *C of V*. Whether or not the heteroscedasticity is multiplicative or additive has little bearing on the power of each test when the x-vector is the same and the *C of V*'s are similar. In fact, in the last four rows of the table the powers are identical for identical *C of V*'s. This result generally held throughout, although there were some instances where there were slight differences in the powers for identical *C of V*'s.

Another result illustrated by table 1 is that, for a given *C of V*, the power of each test is greater for uniform x than for lognormal x. This result also held throughout, except for a few cases when the *C of V* was less than 0.3.

Considering negative values of γ for the multiplicative model is equivalent to considering two more types of x for that model – one which is the inverse of uniformally distributed x and one which is the inverse of lognormally distributed x. A general comparison of these results with those for $\gamma > 0$ showed that the powers of the tests for $\gamma > 0$ tended to be greater than the powers of the corresponding tests for $\gamma < 0$ and similar *C of V*'s.

When λ was increased from 50 to 5000 the *C of V*'s and the powers of the tests did not change, suggesting that there is an upper bound to the degree of heteroscedasticity (*C of V*) which can be modelled using the additive specification. Investigating this matter further, we considered the square of the *C of V* defined by

$$(C\ of\ V)^2 = \frac{1}{T-1}\left\{\sum\sigma_i^4 - \frac{\left(\sum\sigma_i^2\right)^2}{T}\right\}\Bigg/\left\{\frac{1}{T}\sum\sigma_i^2\right\}^2. \tag{8}$$

Table 2

Maximum coefficients of variation of the variances.

	Model				
	AL($\lambda \geq 50$)	AU($\lambda \geq 50$)	M(max)	ML($\gamma = 7$)	MU($\gamma = 7$)
$T = 20$	0.70	0.67	4.47	2.25	1.69
$T = 50$	0.62	0.61	7.07	2.06	1.54

Substituting for σ_t^2 from (3) and taking limits yielded

$$UB_A = \lim_{\lambda \to \infty} (C\ of\ V)^2 = \frac{T^2}{T-1}\left\{\sum x_t^4 / \left(\sum x_t^2\right)^2 - \frac{1}{T}\right\}. \quad (9)$$

Thus, there is indeed an upper bound to the degree of heteroscedasticity which can be modelled with an additive heteroscedastic specification. A similar exercise applied to the multiplicative specification yielded an upper bound of

$$UB_M = \frac{T^2}{T-1}\left(1 - \frac{1}{T}\right) = T. \quad (10)$$

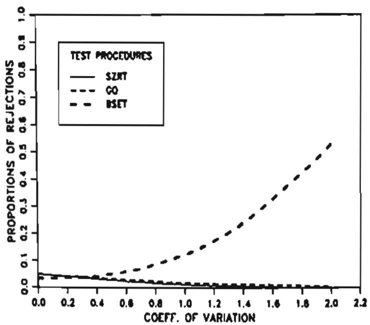

Fig. 1. Power of tests: Unordered observations, $T = 20$.

Thus, there is also a limit to the degree of heteroscedasticity which can be modelled using a multiplicative specification, but in our experiment we did not consider values of γ sufficiently large for the results to show any evidence of such a limit. Table 2 indicates the maximum *C of V*'s [square roots of (9) and (10)] obtainable for the data in our experiment, as well as the largest *C of V*'s that we considered for the multiplicative model ($\gamma > 0$). Note that the upper bound for the multiplicative model does not depend upon whether or not the x's are uniform or lognormal. It is evident, both from table 2 and a comparison of (9) and (10), that the maximum degree of heteroscedasticity which can be modelled using the additive specification is much less than that achievable using the multiplicative specification. With the additive specification the maximum degree of heteroscedasticty is not sufficient, even when $T = 50$, for the power of any of the tests to become unity. See table 1. In contrast, with the multiplicative specification and $T = 50$, the powers of all tests have reached unity for *C of V* = 1.81 with lognormal x, and for *C of V* = 1.24 with uniform x; these values are well below the upper bound of 7.07. These findings clearly have general implications for heteroscedastic error model specification.

We turn now to a discussion of the sizes and relative powers of the five tests. With respect to the sizes, we first note that, for a proportion of 0.05 and 5000 replications, the standard error of an estimated proportion is 0.0031. With $T = 50$ and the homoscedastic case ($\lambda = \gamma = 0$), all estimated proportions were within the range 0.05 ± 0.0062, although the B-P and B-P(M) tests, with sizes of 0.047 and 0.044 for uniform x and 0.044 and 0.045 for lognormal x, were at

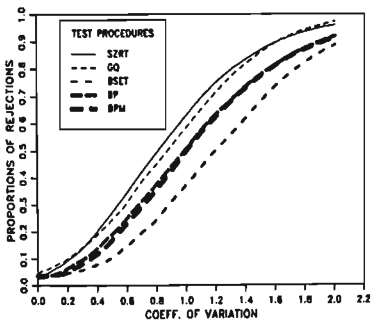

Fig. 2. Power of tests: Ordered observations, $T = 20$.

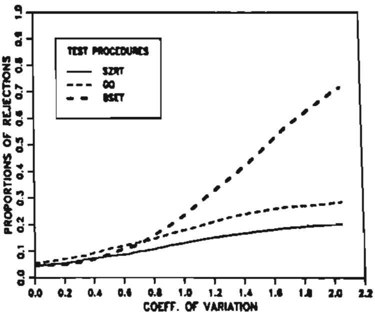

Fig. 3. Power of tests: Unordered observations, $T = 50$.

the low end of this range. In contrast, with $T = 20$, only the finite sample G-Q test had an estimated size within the specified confidence interval. All the asymptotic tests (SZ, B-P, B-P(M) and BAMSET) had finite sample sizes well below the specified 5% significance level. These sizes ranged from 0.031 for SZ and BP with lognormal x to 0.038 for BAMSET with uniform x. Similar results for the B-P test were reported by Godfrey (1978) and Breusch and Pagan (1979).

As mentioned earlier, the relative powers of the various tests are likely to depend heavily on whether or not the observations have been ordered according to increasing variances. In figs. 1 to 4 we have graphed the powers for both ordered and unordered observations for the multiplicative model with lognormal x and $\gamma > 0$. These powers are graphed against the *C of V* since it is a reasonable measure of the degree of heteroscedasticity, and the results for additive heteroscedasticity (and lognormal x) are essentially identical, except that they do not extend beyond *C of V*'s greater than 0.70 ($T = 20$) or 0.62 ($T = 50$). We have not presented the results for $\gamma < 0$ and uniform x because these cases led to identical conclusions about the *relative powers* of the tests.

From figs. 2 and 4 we observe that, when the observations have been ordered according to increasing variances, the SZ test is most powerful, followed by the G-Q test, the B-P and B-P(M) tests (which are very similar), and BAMSET. For $T = 20$, the G-Q test is better than SZ for low and high *C of V*'s, although the difference in performance at the low end seems to occur because SZ has incorrect finite sample size.

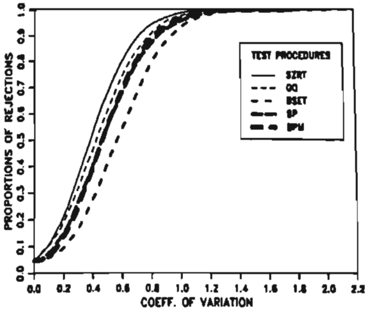

Fig. 4. Power of tests: Ordered observations, $T = 50$.

Figs. 1 and 3 show the powers of the SZ, G-Q and BAMSET tests when the observations have not been ordered according to increasing variances. The powers are very similar, and very poor, for *C of V*'s up to 0.6. After this point BAMSET is clearly best since its power function begins to rise more steeply while the other power functions only gradually increase ($T = 50$) or *fall* ($T = 20$). With uniform x (which we have not graphed), the performance of BAMSET was similar, but the power functions of SZ and G-Q gradually increased for $T = 20$ and fell for $T = 50$. These results, and an examination of the x-vectors, showed that whether the powers of the SZ and G-Q tests increased or remained below their size did not depend on sample size but rather on the distribution of the unordered x's.

5. Conclusions

If it is impossible to order the observations according to increasing variances, and there is insufficient prior information to relate the variances to some explanatory variable(s), then from the tests that we have considered, only BAMSET is viable. However, its power is extremely poor when compared with that which can be achieved, by any of the tests, if the observations are appropriately ordered. Such an ordering leads to a clear prescription for Szroeter's test. Given the ease with which both Szroeter's test and BAMSET can be computed, and given their respective superior performances under circumstances of some and no prior information about variances, we feel that serious consideration should be given to the routine calculation of both these statistics.

The maximum degree of heteroscedasticity which can be modelled using an additive heteroscedastic specification is much less than that which can be achieved using a multiplicative heteroscedastic specification. Due consideration should be given to this fact when choosing a variance model for estimation purposes.

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