FINAL EXAMINATION

Course name: **Game Theory I**
Subject name: **Economics**
Date: **26 November 2025**
Maximum marks: **100**
Duration: **3 hours**

1. Consider the following 3-player extensive game of complete but imperfect information (shown in Figure 1 below). Decision nodes in a common information set are connected with dashed lines. Payoffs are ordered with player 1's payoff listed first, player 2's second, and player 3's third. y is a number whose value is known to all players.

(a) Write out the set of pure strategies for each player. **(6 points)**

(b) Suppose that $y > 0$. Find all subgame perfect Nash equilibria of the game, both in which everyone uses pure strategies, and those in which one or more players use mixed strategies. Your answer may include the variable y as an argument. **(14 points)**

2. Consider the following 2-player bargaining game. In stage 0, player 1 proposes a division $x^0 \in [0, 1]$ of a cake of size 1. If player 2 accepts the proposal, player 1 receives a payofi of x^0 , and player 2 a payoff of $1 - x^0$. If player 2 rejects the proposal, the game moves to stage 1. In stage 1, the old cake is replaced by a new cake of size z. Player 2 proposes a division $x^1 \in [0, z]$ of the stage 1 cake. If player 1 accepts the proposal, player 1 receives a payoff of $\delta_1 x^1$, and player 2 a payoff of $\delta_2(z - x^1)$. If player 1 rejects the proposal, both players get a payoff of 0.

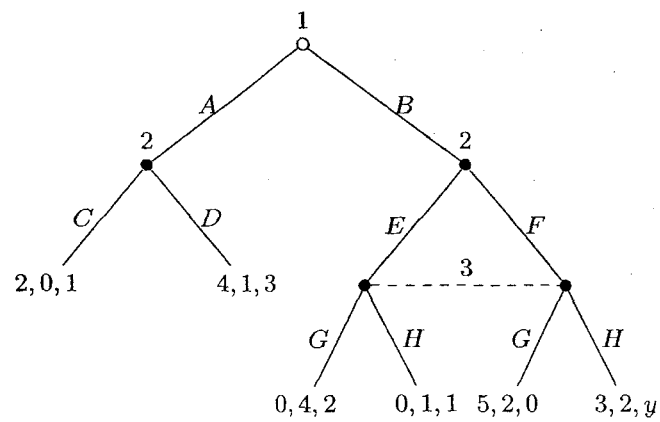

FIGURE 1. Game tree for problem #1

(a) Find an SPNE of the game when z is known by both players (there will be multiple cases to consider depending on the value of z)). **(10 points)**

(b) Assume that nature chooses z randomly from a uniform distribution on support $[0, \bar{z}]$, and that neither player can observe the realized z in stage 0, but that both observe the realized z at the beginning of stage 1. Find an SPNE of the game (there will be multiple cases to consider depending on the value of $\bar{z}$). **(10 points)**

(c) Assume again that nature chooses z randomly from a uniform distribution on support $[0, \bar{z}]$, and that player 1 cannot observe the realized z when he makes his offer in stage 0, but that player 2 can observe the realized z before she decides to accept or reject x^0. Find an SPNE of the game (there will be multiple cases to consider depending on the value of $\bar{z}$. **(10 points)**

3. Consider the following variation of the infinite-horizon Rubinstein bargaining game with two players. Instead of it being predetermined that player 1 makes all offers in even stages and player 2 makes all offers in odd stages, the player who makes an offer in a given stage is determined randomly at the beginning of that stage. That is, when the players bargain in stage t, they do not know who would make the offers in stages $\{t+1, t+2, t+3, ...\}$.

Otherwise, the rules of the game are the same: the players divide a cake of size 1; when one player makes an offer, the other may accept, ending the game, or reject, sending the game to the next stage; if the offer that is accepted in stage $t \in \{0, 1, 2, 3, ...\}$, results in a piece of cake of size x for player 1 and a piece of size $1 - x$ for player 2, player 1's payoff is $\delta_1^t x$ and player 2's payoff is $\delta_2^t(1 - x)$, with δ_1 and δ_2 lying in $(0, 1)$; and both players get payoffs of 0 if all offers are rejected.

(a) Suppose that the probability that player 1 makes the offer in a given stage is independent of who has made offers in all prior stages, and that this probability is a number $\pi \in (0, 1)$. Find an SPNE of the game with the property that player 1 always makes the same offer x^1 when he is chosen to make an offer, and player 2 always makes the same offer x^2 when he is chosen to make an offer. **(10 points)**

(b) Suppose now that the probability that player 1 makes the offer in a given stage t depends on who made the offer in the prior stage $t-1$. Specifically, if player 1 made the offer in stage $t - 1$, the probability that player 1 makes the offer in stage t is π_1. If player 2 made the offer in stage $t - 1$, the probability that player 1 makes the offer in stage t is π_2. Find an SPNE of the game with the property that player 1 always makes the same offer x^1 when he is chosen to make an offer, and player 2 always makes the same offer x^2 when he is chosen to make an offer. **(10 points)**

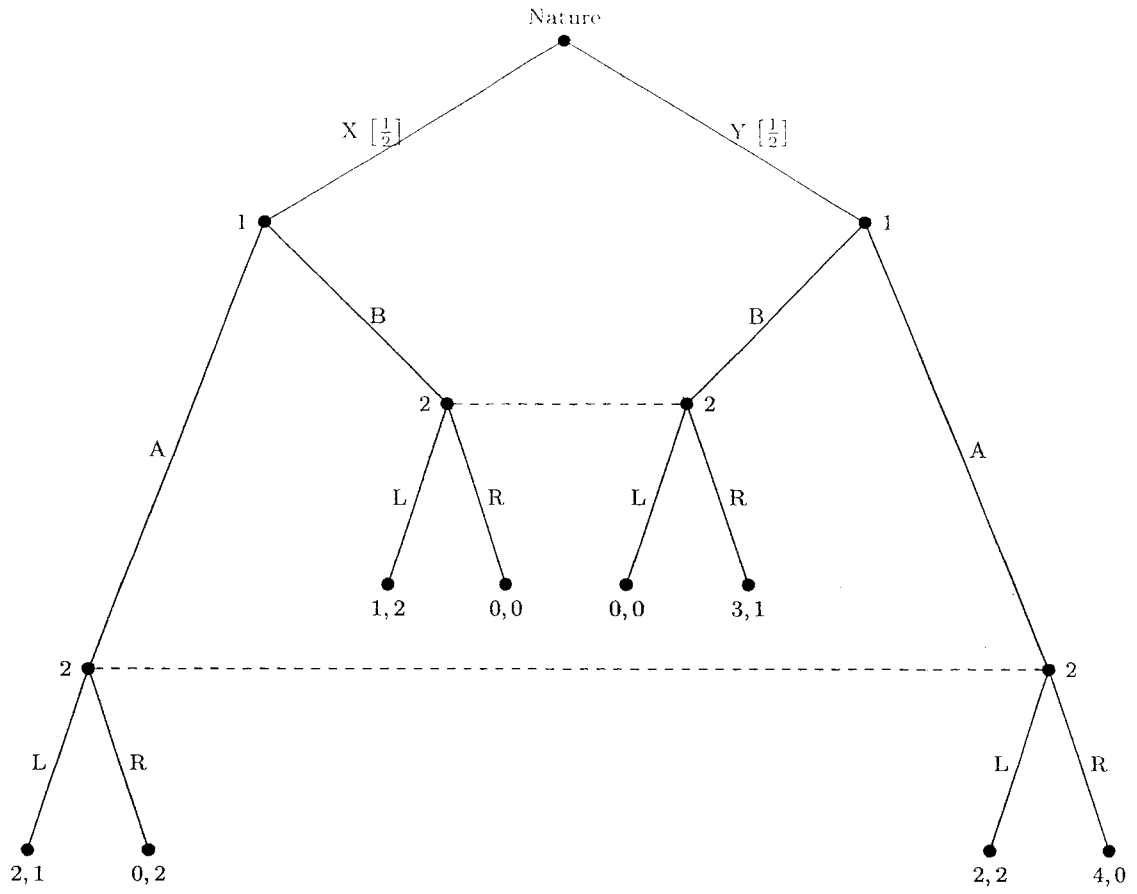

FIGURE 2. Game tree for problem #4

4. Consider the above 2-player game, written in extensive form (shown in Figure 2 above). Nature first chooses a type for player 1, either X or Y, with probability of each shown in [].

(a) Find player 2's sequentially rational actions at each of his information sets, as a function of his belief μ that nature chose X. **(10 points)**

(b) Are there sequential equilibria in pure strategies in which player 1 chooses the same action whether nature chooses X or Y? If yes, provide such equilibria in their entirety. If no, prove your answer. **(10 points)**

(c) Are there sequential equilibria in pure strategies in which player 1 chooses an action after nature chooses X that is different from the action that player 1 chooses after nature chooses Y? If yes, provide such equilibria in their entirety. If no, prove your answer. **(10 points)**