INDIAN STATISTICAL INSTITUTE

<u>Semestral Examination</u>

M. Stat - I Year and M.Tech(CS) (Semester - II)

Optimization Technique

Date : 09/05/2025 Maximum Marks: 100 Duration : 3 Hours 30 Minutes

Note : You may answer any part of any question, but maximum you can score is 100.

1. Consider a general linear program in equational form: *maximize* $C^T x$ *subject to* $Ax = b$ *and* $x \geq 0$, where A is a $m \times n$ real matrix and b is a vector of size m of the real field. Prove that if there is at least one feasible solution and the objective function is bounded from above on the set of all feasible solutions, then there exists a basic feasible solution that is optimal. [18]

2. Justify whether we can claim that the simplex method is always finite if the entry variable in the pivot rule is the one with the smallest index among the eligible variables. [10]

3. Suppose that the objective function f_0 of a convex optimization problem is locally optimal at the point x in the domain of f_0.

   (i) Prove that the global minimum of the function f_0 will attain at point x.

   (ii) If the objective function f_0 is differentiable, then prove that f_0 is optimal at x if and only if x is feasible and $\nabla f_0(x)^T(y - x) \geq 0$ for all feasible y. [5+10=15]

4 (i) Explain the Branch and Bound algorithm to produce the optimal solution of Integer Linear Programming problem.

(ii)Solve the following Integer Linear Programming problem using the algorithm.

maximize $3x_1 + 5x_2$
subject to $2x_1 + 4x_2 \leq 25$,
$x_1 \leq 8$,
$x_2 \leq 5$,
$x_i \geq 0$, integer for $i = 1, 2$.

[5+13=18]

5. Consider an optimization problem in standard form:

   minimize $f_0(x)$
   subject to $f_i(x) \leq 0,\ i = 1, \ldots, m$
   $h_i(x) = 0,\ i = 1, \ldots, p$.

(i) Define Lagrange dual function of the above optimization problem and proof that this gives a lower bound on the optimal value $p*$ of the optimization problem.

(ii) State the condition for strong duality for the convex optimization problem. Justify that the proposed condition is sufficient. [(5+5)+(5+12)=27]

6. For a two-player zero sum game between players A and B having three strategies each with the following payoff matrix

	B1	B2	B3
A1	0	1	-2
A2	1	-2	3
A2	-2	3	-4

(a) Conclude with justification about the existence of a pure-strategy Nash equilibrium.

(b) Find with justification the mixed-strategy Nash equilibrium and their payoffs. [5+10=15]

7 Prove that bin packing problem does not have an approximation algorithm with approximation factor less than $\frac{3}{2}$ unless $P = NP$. [10]