

Indian Statistical Institute
M Tech (QROR) I Year: 2025-26
Semester I
Back Paper Examination
Operations Research I

Date: 19/12/2025

Time: 3 hours

Full Marks: 100

Note: (i) Answer all the questions.

(ii) Calculators are allowed.

1. Write short notes on:

(a) Kendall's notation

(b) Cycling

(c) PERT

(d) Stochastic optimization

[5+5+5+5]=20

2. (a) Calculate the dimension for a set of symmetric $n \times n$ matrices.

(b) Find dual from primal conversion

$$\text{Max } Z = x_1 - 2x_2 + 3x_3 + 4x_4 + 5x_5 + 10x_6$$

Subject to

$$-2x_1 + x_2 + 3x_3 + 4x_4 - 3x_6 = 29$$

$$2x_1 + 3x_2 + 4x_3 + 7x_4 + 4x_6 = 15$$

$$3x_1 + 9x_2 - 3x_3 + 9x_4 + 4x_5 \leq 20$$

$$2x_1 + 2x_2 - 3x_4 + 5x_6 \leq 30$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

(c) Solve the Linear Programming Problem using the Graphical Method

$$\text{Max } Z = 2x_1 + 3x_2$$

Subject to

$$2x_1 + x_2 = 30$$

$$x_1 + 4x_2 \leq 70$$

$$4x_1 + 5x_2 \leq 100$$

$$x_1, x_2 \geq 0$$

(d) Prove that the parabolic region $\{(x_1, x_2) \in R^2 | x_2^2 \leq 4ax_1\}$ is convex. but $\{(x_1, x_2) \in R^2 | x_2^2 > 4ax_1\}$ is not convex.

(e) Let $W = \{(a_1, a_2, a_3, a_4, a_5) \in R^5 : a_1 + a_3 + a_5 = 0, a_2 = a_4\}$. Find the basis for the subspace W and hence state the dimension of W . **[3+4+5+5+3]=20**

3. (a) In a transportation problem with m sources and n destinations, prove that any basic feasible solution will have exactly $(m + n - 1)$ positive allocations.

(b) Three factories S_1, S_2, S_3 supply goods to three warehouses D_1, D_2 and D_3 . The supply, demand, and transportation costs (per unit) are given below:

Source / Destination	D_1	D_2	D_3	Supply
S_1	8	6	10	20
S_2	9	7	4	30
S_3	3	4	2	25
Demand	25	25	25	

(i) Find an initial feasible solution using the Least Cost Method (LCM).

(ii) Improve it to the optimal solution using the MODI Method. **[10+5+5]=20**

4. (a) Show that a basic feasible solution to a transportation problem is optimal if and only if all the reduced costs for the non-basic cells are greater than or equal to zero.

(b) A company has four workers (W_1, W_2, W_3, W_4) and four jobs (J_1, J_2, J_3, J_4). Each worker can perform each job, but the time required (in hours) is different for each worker-job combination, as shown in the table below:

Workers / Jobs	J_1	J_2	J_3	J_4
W_1	9	2	7	8
W_2	6	4	3	7
W_3	5	8	1	8
W_4	7	6	9	4

i) Formulate the problem as an Assignment Model.

ii) Use the Hungarian Method to find the optimal assignment of workers to jobs such that the total time is minimized.

iii) State the optimal assignment and the minimum total time.

iv) Comment on whether multiple optimal solutions exist. **[10+2+4+2+2]=20**

5. (a) State the assumptions of Economic order quantity (EOQ) model. Derive the EOQ model with planned shortages.

(b) A company sells different brands of electronic items. Past sales for a particular item have resulted in the following probability distribution for demand:

Demand per week	0	15	30	45	60
Probability	0.25	0.27	0.17	0.18	0.13

The lead time, in weeks, is described by the following distribution:

Lead time (weeks)	1	2	3
Probability	0.32	0.41	0.27

Based on cost considerations as well as storage space, the company has decided to order 150 of them each time an order is placed whenever there are only 18 units left at the end of the week. The carrying cost is Rs. 35 per week for each unit that is left in the inventory at the end of the week. The stockout cost is set at Rs. 325 per stockout.

Use the linear congruential method to generate a sequence of random numbers for demand with $X_0 = 15$, $a = 12$, $c = 14$, and $m = 40$ and a sequence of random numbers for lead time with $X_0 = 16$, $a = 19$, $c = 11$, and $m = 50$.

Simulate 8 weeks of operation for the company with currently 25 units in inventory. What would be the weekly carrying cost and the total stockout cost under this situation? **[6+14]=20**