INDIAN STATISTICAL INSTITUTE

End-Semestral Examination: 2025-26

Course: Master's in Quantitative Economics Year II

Subject: Public Economics

Date: 25/11/2025; Forenoon **Maximum Marks**: 60 **Duration:** 3 hours

The exam is closed book. No calculators are allowed. Answer all questions.

1. Consider a group with n members, $n \geq 2$. Let x_i be person i's consumption of a private good and g_i be her consumption of the group's public good produced through voluntary contributions by group members:

$$g_i = g = \frac{\sum_{i=1}^{n} s_i}{n^\beta}; \tag{1}$$

where $0 \leq \beta \leq 1$ and s_i is the contribution of person i. Preferences are given by:

$$u_i = (x_i{}^\rho + g^\rho)^{1/\rho}; \tag{2}$$

with $0 < \rho < 1$, and the individual budget constraint is:

$$x_i + s_i = w. \tag{3}$$

All individuals simultaneously choose their public good contributions s_i so as to maximize utility as in (2), subject to the public good technology as in (1) and the budget constraint as in (3).

(a) Find the equilibrium level of individual public good consumption, g. **(5 marks)**

(b) Prove that:

(i) if $(\beta + \rho - 1 \geq 0)$, then the equilibrium level of individual public good consumption declines monotonically as the population size n increases, with $\lim_{n \to \infty} g = 0$,

and

(ii) if $\beta = 0$, then the equilibrium level of individual public good consumption increases monotonically as the population size n increases, with $\lim_{n \to \infty} g = w$.

(8 marks + 2 marks)

(c) Let the total public good contribution be S; $S = gn^{\beta}$. Prove that, if $(\rho\beta + \rho - 1 \geq 0)$, then S is monotonically decreasing in n, with $\lim_{n \to \infty} S = 0$. **(5 marks)**

(d) Now find the efficient level of total public good expenditure, S^C. Prove that S^C is monotonically increasing in n, and, if $\beta < 1$, so is the efficient level of individual public good consumption (i.e., $\frac{S^C}{n^{\beta}}$). **(5 marks)**

2. Consider a market with an inverse demand function $p^d = A - \left(\frac{b}{2}\right) q$. Suppose the government imposes a (lump-sum) tax of t on each unit of output, the tax being paid by suppliers; $0 \leq t < A$.

(a) Suppose that the market is supplied competitively according to an inverse supply function $p^s = cq$; $c > 0$. Find the magnitude of price increase Δp^c due to the tax. **(5 marks)**

(b) Now suppose the market is supplied by a monopolist who produces according to the cost function $C = \left(\frac{c}{2}\right) q^2$; $c > 0$. Find the magnitude of price increase Δp^m due to the tax. **(5 marks)**

(c) Is Δp^c higher or lower than Δp^m? Explain how the ratio $\frac{\Delta p^m}{\Delta p^c}$ behaves as $\left(\frac{c}{b}\right)$ increases over $(0, \infty)$. **(5 marks)**

3. Suppose a consumer i has own income I_i; $i = 1,2$ and $I_1 = I + \delta, I_2 = I - \delta$, with $I > \delta > 0$. Preferences are given by:

$$u_i = x_i y_i,$$

where x_i and y_i are the amounts of the two goods, x and y respectively, consumed. Both prices are unity.

Suppose the state taxes each consumer's income by a proportion t, and returns the tax revenue entirely to the consumers collectively in the form of identical in-kind transfers, i.e., tI amount of the good y to each consumer, which cannot be resold for cash; $1 \geq t > 0$. Hence, under this scheme, each consumer maximizes her utility subject to the budget constraint:

$$x_i + y_i = (1 - t)I_i + tI;$$

and the non-conversion constraint:

$$y_i \geq tI.$$

(a) Prove that the utility of the poorer consumer, 2, is monotonically increasing in the tax rate t over $\left[0, \frac{1}{2}\right]$, and monotonically decreasing in t over $\left(\frac{1}{2}, 1\right]$. Explain your result **(15 marks)**

(b) Prove that there exists $\hat{t} \in (\frac{1}{2}, 1)$ such that the poorer consumer will be *worse off* under the balanced budget in-kind transfer scheme, compared to the status quo (i.e., $t = 0$), if and only if $t > \hat{t}$. Show that $\hat{t}$ monotonically approaches 1 as inequality increases (i.e., δ approaches I). **(5 marks)**