INDIAN STATISTICAL INSTITUTE

Semestral Examination : 2025-26

M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY

Note : Answer any <u>FIVE</u> questions

[Symbols have their usual meaning]
(Calculator is allowed)

Date: 17.11.25 | Full marks:100 | Time: 3 hr

1. a) State and prove Markov's inequality.
   b) State and prove Chebyschev's inequality. What does it provide?
   c) Let $X \sim$ gamma distribution with scale parameter $\alpha = 1$ and p. Show that $P(X \geq 2p) \leq 1/p$ (State the results you have used).

   [6+(6+2)+6 = 20]

2. a) Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ and the correlation coefficient between X and Y be ρ. Derive the joint distribution of X and Y.
   b) Let $X_i \sim$ iid $N(0,1)$, i = 1,2. Let $Y = \frac{X_1}{X_2}$. Find the distribution of Y. Find a measure of central tendency of Y.

   [10+(6+4) = 20]

3. a) Let ($x_1, x_2, \ldots, x_n$) be a sample of size n drawn from $N(\mu, \sigma^2)$. Find the distribution of sample mean and sample variance and show that they are independent.
   b) Let X and Y be two random variables with E(X) = 5, Var (X) = 2, E(Y) = 10, Var (Y) = 3, and correlation coefficient between X and Y be 0.4.
   Let Z_1 and Z_2 be two random variables defined as follows
   $Z_1 = 3X + 5Y$, and $Z_2 = 2X - 3Y$.
   Find $E(Z_1)$, $Var(Z_1)$, $E(Z_2)$, $Var(Z_2)$ and correlation coefficient between Z_1 and Z_2.
   (state the results you have used).

   [(8+2)+10 = 20]

4. a) Let $X_i \sim$iid exp (λ) , i = 1(1) n .
   Let $X_{(1)} =$ Min $(X_1, \ldots, X_n)$
   and $X_{(n)} =$ Max $(X_1, \ldots, X_n)$
   Find the joint distribution of $X_{(1)}$ and $X_{(n)}$.
   b) Let $X_i \sim$ independent negative binomial distribution with parameters r_i and p respectively, i =1,2.
   Let $Y = X_1 + X_2$. Find the distribution of Y.
   c) Let $X \sim N(\mu, \sigma^2)$ and $Y = X^2$. Find the distribution of Y.

   [10+6+4 = 20]

5. a) The quality assurance manager of a bolt manufacturing company knows that as per contract, his client will tolerate a maximum of 2% rejection with respect to diameter of the pipes. He has taken a set of 100 observations on diameter of the selected pipes and estimated the average as 30 cm and the standard deviation as 3 cm. Where should he set his upper specification limit of the diameter of the pipes so that he can satisfy his client (assume that the distribution of the diameter follows normal distribution and specification for diameter is having only upper specification limit)?

b) In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices, only one answer being correct. The probability that he makes the guess is 1/3, the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copies it is 1/8. Find the probability that he knew the answer to the question given that he has answered it correctly.

[10+10 = 20]

6. a) Let $A_1, A_2, \ldots\ldots, A_r$ be r events not necessarily mutually exclusive. Find the probability of occurrence of exactly m events ($m < r$).

b)) Let $X \sim Bin(n, p)$. Prove that $P(X \leq k)$ $\frac{1}{B(n-k, k+1)} \int_0^q z^{n-k-1}(1-z)^k dz.$
where, $q = 1-p$ and integer valued k is such that $0 < k < n$.

[12+8 = 20]