Design of Experiments

Program: B.Stat Third Year
Academic Year: 2024-25, Second Semester
Mid-Semester Exam, Date: 26 February, 2025
Duration: 2 hours, Total Points: 80

This is a closed book, closed notes exam. Calculators without communication features are allowed. Unless otherwise mentioned, follow the notations used in class. Unless specifically asked to prove, you may use any result that was proved or stated in class. In that case, you must clearly state the result that you are using. Statistical Tables will be provided. Answer all questions. However, the maximum you can score is 80.

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1. [$10 \times 2 = 20$] Consider a block design with v treatments and b blocks.

   (a) Write down the underlying linear model in vector-matrix notations. Derive the normal equations, and the reduced normal equations for adjusted block totals.

   (b) Construct the ANOVA table for testing equality of the block effects.

2. [$6 \times 2 = 12$] Below is the incidence matrix of a block design:

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

   (a) Investigate if the design is connected.

   (b) Investigate if the design is balanced.

3. [$7 \times 3 = 21$] Consider a completely randomized design with five treatments, four units per treatment, and treatment means:

$$3.2892, \quad 10.256, \quad 8.1157, \quad 8.1825, \quad 7.5622$$

   The mean squared error is 4.012.

   (a) Construct an ANOVA table for this experiment and test the null hypothesis that all treatments have the same mean.

   (b) Test the null hypothesis that the average response in treatments 1 and 2 is the same as the average response in treatments 3, 4, and 5.

   (c) Execute the SNK procedure to compare the means of the five treatments. Draw the underline diagram.

4. [$6 \times 5 = 30$] Consider a block design with v treatments and b blocks, and let n be the total number of experimental units in the design. Prove or disprove the following statements:

   (a) If the design is connected, $n \geq b + v - 1$.

   (b) If $n \geq b + v - 1$, the design must be connected.

   (c) It is possible to find a vector $\mathbf{p}$ of length v satisfying $\mathbf{p}^T \mathbf{1}_v \neq 0$ such that $\mathbf{p}^T \boldsymbol{\tau}$ is estimable under the block design.

   (d) Even if the incidence matrix is a multiple of $\mathbf{J}_{vb}$, the design may still not be orthogonal.

   (e) Even if an element of the incidence matrix is zero, the design can still be orthogonal.
