Indian Statistical Institute
M. Tech (QR & OR) II Year - 2025-2026
Semester-I
End-semester examination
Subject: Reliability II

Examination Date: 24th November 2025 | Duration: 3 hours

Full Marks: 100

*Note:* Answer all questions. | Scientific calculators are allowed.

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1. The following table provides data on lifetimes ($t$) of of 6 items along with failure status $\delta$ ($\delta = 1$ if failed and $\delta = 0$ if censored) and a covariate $z$:

| Item $i$ | $t$ | $\delta$ | $z$ |
|---|---|---|---|
| 1 | 6 | 1 | 2 |
| 2 | 6 | 1 | 4 |
| 3 | 9 | 0 | 3 |
| 4 | 10 | 1 | 1 |
| 5 | 12 | 0 | 2 |
| 6 | 18 | 1 | 7 |

Assume that the lifetime of each individual follows a Cox Proportional Hazards (PH) model of the form

$$h(t \mid z) = h_0(t)\,\exp(\beta z),$$

where the baseline hazard $h_0(t)$ is unspecified and $\beta$ is the regression coefficient.

(a) Find partial likelihood function $L_p(\beta)$, for $\beta$, based on the given data. Discuss briefly how the model parameter $\beta$ can be estimated from $L_p(\beta)$.

(b) Suppose the estimate of $\beta$ is $\hat{\beta} = -0.5$. Find the Breslow estimator of the baseline cumulative hazard function $H_0(t)$. Using the Breslow estimate of $H_0(t)$, estimate the survival probability of a subject with covariate value $z = 10$ at time $t = 2$.

[(8+2)+(8+2)=20]

2. (a) Describe Free Replacement (FRW) and Pro-Rata Warranty (PRW) policies.

(b) Consider a product that exhibits a constant failure rate, with a mean time to failure (MTTF) of 50 months. A combined FRW/PRW policy is to be applied, where $w_1 = 5$ months (free replacement period) and $w_2 = 10$ months (end of warranty period). The cost of replacement is \$120. Finc the expected warranty cost under this policy.

(c) Discuss in detail the procedure to determine the optimum warranty period for combined FRW/PRW policy by considering the economic benefit function.

[((2+2)+ 10 +6 = 20]

3. (a) Define decreasing mean residual life (DMRL) distribution.

(b) Show that $IFR \Rightarrow DMRL$.

(c) If each of the independent components of a coherent system has an IFRA life distribution, then show that the system itself has an IFRA distribution. You can use the result $h(\mathbf{p}^\alpha) \geq h^\alpha(\mathbf{p})$, where $h(\mathbf{p})$ is the reliability function of the system and $\mathbf{p}^\alpha = (p_1^\alpha, \cdots, p_n^\alpha)$.

[2+5+8=15]

4. (a) Suppose that failures of a repairable system are modeled by a renewal process with inter-arrival time distribution $F$. Let $N(t)$ be the number of failures by time $t$ and $\Lambda(t) = E(N(t))$. Show that $\Lambda(t)$ satisfies the equation

$$\Lambda(t) = F(t) + \int_0^t \Lambda(t-x)\, dF(x).$$

(b) Suppose that failures of a repairable system are modelled by an HPP with rate 0.001 per day. Find the expected number of failures in the interval $[0, 5]$ days, given that 20 failures occurred in $[0, 20]$ days.

[6 + 7 = 13]

5. A repairable system is observed for 10 failures and the failure times (in days) are

$$9, 20, 45, 58, 72, 85, 92, 105, 112, 130.$$

The failure process of the system is modeled by an NHPP with intensity function

$$\lambda(t) = \alpha\lambda(\lambda t)^{\alpha-1}, \alpha > 0, \lambda > 0, t > 0.$$

(a) Derive the maximum likelihood estimates of $\alpha$ and $\lambda$.

(b) Find the estimate of expected number of failures of the system in $(130, 200]$ days.

[10 + 5 = 15]

6. The failure rate of a system is given by $\lambda(t) = 0.002\sqrt{t}$; $t$ in hours. If the design life of the system is 5000 hours, determine the following:

(a) Reliability of the unmaintained system at the end of the design life.

(b) Reliability of the system assuming perfect maintenance after every 1000 hours of operation.

(c) Reliability of the system assuming imperfect maintenance after every 1000 hours of operations with a 2 percent probability of failure of the system immediately after the maintenance.

[5 + 6 + 6 = 17]