INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2024-2025

Image Processing - I

M.Tech.(Computer Science)

Date: 24.02.2025 **Full Marks: 40** **Time: 2 Hours**

Answer any **four** questions. All questions carry equal marks.

1. (a) Consider a 3-bit image of size 64×64 pixels. It has the intensity distribution as follows:

r_k	0	1	2	3	4	5	6	7
n_k	81	122	245	329	656	850	1023	790

where r_k denotes the k-th intensity level and n_k is the number of pixels that have intensity r_k.

(i) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(ii) Find out the intensity distribution of the histogram-equalized image of (i).

(b) Suppose a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass. [(5+2)+3=10]

2. (a) Assume continuous intensity values, and suppose that the histogram of an image can be approximated by the following probability density function (PDF):

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(ii) Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{for } 0 \le z \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Any local histogram processing method requires that a histogram be computed at each neighborhood location. Propose a method for updating the histogram from one neighborhood to the next, rather than computing a new histogram each time.

[(3+4)+3=10]

3. (a) Consider the following two continuous functions $f(x)$ and $h(x)$:

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the convolution function between $f(x)$ and $h(x)$.

(b) Prove that the differentiation of the output of a convolution of a signal with a filter, can be achieved by convolving the signal with the derivative of that filter.

(c) Show that the orientation of the edge computed using Sobel filters is equal to its true orientation.

[3+3+4=10]

4. (a) State and prove the convolution theorem.

(b) Prove that the Fourier transform is linear.

(c) Prove that the origin of the Fourier transform of a two-variable function $f(x, y)$ can be moved to the center of its corresponding $N \times N$ frequency square by multiplying $f(x, y)$ by $(-1)^{x+y}$.

[(2+3)+2+3=10]

5. (a) Suppose an image is filtered with the Laplacian kernel. Prove that the sum of the pixel values in the filtered image is 0.

(b) Prove that the Laplacian is invariant to rotation. You may assume continuous quantities.

(c) Obtain the Fourier transform of $\Delta f(x, y)$, the Laplacian of a two-variable function $f(x, y)$. Assume that x and y are continuous variables.

[2+5+3=10]

6. (a) Define accumulator of the Hough transform, considering normal representation of a line.

(b) What is the advantage of the normal representation of a line over its slope-intercept form for linking edgels lying on lines?

(c) Write down the steps of Hough transform for linking edgels.

(d) Analyze the computational complexity of (c).

[2+2+4+2=10]