Indian Statistical Institute

End-Semester Examination

M. Stat Second Year 2024-25

Second Semester

Theory of Games and Statistical Decisions

Time: 2 hour 30 minutes **Date**: April 30, 2025 **Full Marks**: 60

Attempt all questions!!

1. (a) For a $2 \times 2 \times 2$ non-cooperative game, discuss with reasons how to get the set of all equilibrium situations, in mixed extension.

   (b) Describe a real life example of a $2 \times 2 \times 2$ non-cooperative game with sets of strategies and payoff functions for each player. Apply the method considered in part (a) to solve the game. (8+8=16)

2. (a) For a two person zero sum non-cooperative game (finite or infinite), show that there exists A_0 and B_0 such that (x^*, y^*) is an equilibrium situation iff $(x^*, y^*) \in A_0 \times B_0$.

   (b) Consider the mixed extension of the matrix game denoted by the matrix $C_{m\times n}$ where $m, n > 2$. Let $C_{\cdot j}$ denote the j^{th} column of C. Let (X^*, Y^*) be a saddle point of the mixed extension. Let $Y^* = (y_1^*, \ldots, y_n^*)$. Show that for all j_0 with $y_{j_0}^* > 0$, $X^* C_{\cdot j_0}$ is constant. (5+5=10)

3. (a) Consider mixed extension of a matrix game with matrix C having all entries positive. Formulate how you will solve the game in terms of primal and dual of some linear programming problem (LPP). Explain how to get optimal strategies of the players and value of the game from the LPP.

   (b) If some of the entries of the matrix C are negative, discuss how to modify the method of solution. (9+3=12)

4. Consider the matrix $C = \begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix}$. Discuss how to solve using graphical methods the mixed extension of the 2×2 matrix game. Find (numerical) optimal strategies for each player and value of the game. (9)

5. Consider the following statistical game. Let the set of states of nature be $\{\theta_1, \theta_2\}$. Consider the following risk set

$$S = \{(r_1, r_2) : (r_1 - 5)^2 + 3(r_2 - 3)^2 \leq 28\}$$

   (a) Discuss how to solve the game and get the minimax point.

   (b) Find (numerical) value of the game.

   (c) Derive the LFC. (7+2+4=13)