STATISTICAL MODELS FOR CONSUMER BEHAVIOUR IN INDIA

AMITA MAJUMDER

A dissertation submitted to the Indian Statistical Institute in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy

CALCUTTA
1984

Preface and Acknowledgements

This dissertation is concerned with the analysis of consumer behaviour based on time-series consumption data in the framework of a static complete demand system. On the methodological side, a new system of demand equations and an extension of a modified version of the Linear Expenditure System have been proposed. The empirical part of the dissertation presents comparison of alternative specifications of the proposed systems and some existing demand systems on the basis of Indian National Sample Survey data.

The dissertation is organised in six Chapters and five Appendices. Chapter I presents a survey of literature on the theory and methodology of demand analysis based mainly on time-series consumption data; Chapter II constitutes the methodological contribution to the present dissertation; Chapter III describes the nature and sources of consumer expenditure and price data used, and the estimation procedures employed in the empirical analyses. Chapters IV-VI report the results of empirical exercises based on the different demand systems. Appendices A, B and C are supplementary to Chapters II, III and IV, respectively; Appendix D is supplementary to Chapter VI; and, Appendix E reports results on some dynamised systems.

I am extremely indebted to *Dr. Dipankor Coondoo*, who guided me through every step in the preparation of this dissertation. Without his kind cooperation this work could never have been completed. I express my sincerest gratitude to him. I also thank him for allowing me to include, in this dissertation, some portions of the work done jointly by us. I convey my profound gratitude to *Professor Nikhilesh Bhattacharya* for his valuable suggestions and for taking time off his busy schedule to read an earlier draft of the entire manuscript and suggest substantial improvements in presentation. I am grateful to *Professor Angus Deaton* and to *Professor Robert A. Pollak* for their invaluable suggestions and comments on some portions of this work. I am especially thankful to *Professor Deaton* for providing the relevant computer program package which was extensively used in the empirical analyses presented in this dissertation. I sincerely thank *Dr. Partha P. Majumder* for his active help and support in completing this dissertation. I thank *Dr. R.L. Karandikar, Mrs. T. Rao, Dr. N. Sarkar, Mr. R. Mukhopadhyay, Mrs. P. Pal* and *Mr. T. Roy* for various kinds of help and encouragement throughout the period of my work. My sincere thanks are due to the *authorities of the Indian Statistical Institute, Calcutta,* for granting me the necessary facilities for work, and to the *members of the Economic Research Unit and the Computer Science Unit* for various types of help. Thanks are also due to *Mr. A.K. Basu* and *Mr. P. Swar* for doing an excellent typing job, and to *Md. Aslam* for cyclostyling the typed stencils. Finally, I thank *all members of my family* for constant moral support.

CONTENTS

Chapter 0

Statistical models for consumer behaviour in India: A summary

The present dissertation is about analysis of consumer behaviour based on time-series consumption data in the framework of a static complete demand system with special reference to India.

On the methodological side, the thesis proposes two static demand systems. One of them is based on a modification of the Simple Non-Additive Model (SNAM) of Deaton (1976) - a generalisation of Stone's Linear Expenditure System (LES) - that overcomes the limitations of the LES arising from the additivity of its underlying direct preference structure. The other system is based on the Price Independent Generalized Linearity (PIGL) of Muellbauer (1975). Both the systems contain far smaller number of parameters compared to those in flexible demand systems, e.g., the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a) or the Translog Systems of Christensen, Jorgenson and Lau (1975). However, unlike these flexible systems, the theoretical restrictions such as the adding-up, symmetry and homogeneity properties of the demand functions are built into the proposed systems.

The empirical part of the thesis is concerned with comparisons of the performances of the different variants of the proposed systems and the AIDS on the basis of Indian consumer expenditure data.

The dissertation is organised in six Chapters and five Appendices. In what follows, summary of the individual Chapters and Appendices are presented.

Chapter I presents a survey of literature on the theory and methodology of demand analysis based mainly on time-series consumption data. This Chapter has eight sections. Section 1.1 is introductory. Section 1.2 summarises the static theory of consumer behaviour and the implications of this theory for empirical demand analysis. Section 1.3 briefly presents the different results on separable preferences. Section 1.4 contains a brief description of the well-known static demand systems. Section 1.5 deals with the aggregation of demand equations across consumers. Section 1.6 presents a summary view of the different generalisations of the Linear Expenditure System. Section 1.7 surveys the investigations relating to the inclusion of demographic characteristics in complete demand systems. Finally, Section 1.8 presents a very brief account of the researches on dynamic demand analysis.

Chapter II constitutes the methodological contribution of the present dissertation. Two specifications of empirical demand systems have been presented here that provide the theoretical foundation of a major part of the empirical exercises reported in later Chapters. This Chapter has four sections. While Section 2.1 is introductory, Section 2.2 presents the proposed modification of Deaton's SNAM. The budget share equations corresponding to this modified SNAM is of the form

$$w_i = \frac{p_i \gamma_i (p)}{y} + \beta_i(p) \left(1 - \frac{\sum_k p_k \gamma_k(p)}{y}\right),\ i=1,2,\dots,n \quad \dots(0.1)$$

[with $\sum_i \beta_i(p) = 1$ in view of adding-up criterion]

with

$$\gamma_i(p) = c_i + a_i \log \left(\frac{p_i}{\pi_1}\right) \quad \dots(0.2)$$

and

$$\beta_i(p) = b_i + d_i \log \left(\frac{p_i}{\pi_2}\right), \quad \dots(0.3)$$

where

$$\pi_1 = \frac{\sum_k a_k p_k}{\sum_k a_k} \quad \dots(0.4)$$

$$\pi_2 = \{ \prod_k p_k^{d_k} \}^{\frac{1}{\sum_k d_k}} \quad \dots(0.5)$$

are two all-commodity price indices, and w_i : budget share for i-th item (there being n items), y: total expenditure, p_i : price of the i-th item, p: the vector of prices, and a_i, b_i, c_i, d_i are the 4n structural parameters of the system of which $4n - 1$ are independent in view of the adding-up restriction.

Section 2.3 presents the proposed system based on the PIGL, the budget share equations for which are of the form

$$w_i = \frac{p_i \gamma_i(p)}{\sum_k p_k \gamma_k(p)} + \beta_i(p) \log \left(\frac{y}{\sum_k p_k \gamma_k(p)}\right), \; i=1,2,\ldots,n \qquad \ldots(0.6)$$

[with $\sum_i \beta_i(p) = 0$ in view of adding-up criterion]

for which $\gamma_i(p)$ and $\beta_i(p)$ are the same as those defined in (0.2) - (0.5). It may be noted that for both (0.1) and (0.6) one can consider alternative variants by dropping either or both of the parameter sets a_i and d_i, $i=1,2,\ldots,n$. Thus, the full specifications (0.1) and (0.6) have been called the SNAM 2 and Variant 3 respectively. Those obtained by dropping d_i's have been called SNAM 1 and Variant 2 respectively. Finally, while (0.1) reduces to the LES when both a_i's and d_i's are dropped, the corresponding form of the proposed system (0.6) has been called here Variant 1. Sections 2.2 and 2.3 also discuss the theoretical properties of the proposed systems. Finally, Section 2.4 makes some concluding observations regarding the possible uses of these systems in empirical analysis.

Chapter III describes the nature and sources of consumer expenditure and price data used in the empirical analyses and also the general procedures employed in estimating the demand systems. This Chapter has four sections. The first section is introductory. Section 3.2 describes the nature of consumer expenditure data obtained from the different rounds of the Indian National Sample Survey that have been used in the analysis. Section 3.3 provides a brief description of the nature of wholesale price indices compiled by the Office of the Economic Adviser, Government of India, and also the procedure by which this itemwise wholesale price relatives have been combined to construct price indices for broad groups of items (using expenditure weights). Finally, Section 3.4 briefly outlines the Non-Linear Full-Information Maximum Likelihood (NLFIML) procedure employed in the estimation of the different variants of the modified SNAM and our proposed system.

The results of empirical exercises based on the different demand systems, viz., the LES, the AIDS, and the different specifications of the SNAM and our proposed system, are reported in Chapters IV - VI. These systems have been estimated separately for the rural and the urban sectors of India. Chapter IV presents a comparative study of the performances of the LES and the AIDS. In Chapter V the performances of the different variants of the SNAM and those of our proposed system have been compared to see the extent to which the results improve through increasing the price flexibility of these systems. Finally, Chapter VI presents comparison of the performances of the SNAM 2 and Variant 3 with that of the AIDS in view of the fact that the 4n - 1 independent parameter full specifications of the modified SNAM and of our proposed system were observed to give better results compared to the other specifications having smaller number of parameters.

Chapter IV contains four sections, of which Section 4.1 is introductory. Section 4.2 reports the empirical results of fitting the AIDS and the LES to the consumer expenditure data relating to rural and urban India. These systems have been estimated on two sets of data, viz., (i) the aggregate time-series data on the overall average of per capita consumer expenditure in rural or urban India and (ii) the disaggregated population-groupwise data, i.e., the average per capita consumer expenditure for each of three population groups - the poorest thirty per cent, the middle forty per cent and the richest thirty per cent of the population in the rural or the urban sector. Section 4.3 presents the itemspecific estimates of expenditure and own-price elasticities as obtained on the basis of the estimated AIDS and the LES. Finally, Section 4.4 makes some concluding observations. The results obtained in Chapter IV may be briefly summarised as follows:

(i) As is to be expected, judged by the goodness of fit measured by the itemspecific squared correlation coefficients between the observed and the estimated budget shares and those between the observed and estimated expenditures, the performance of the AIDS is generally much better both for aggregate and disaggregated data.

(ii) The itemspecific estimates of expenditure elasticities obtained from the aggregate and the disaggregated data show closer agreement for the LES than for the AIDS. However, for the disaggregated data the two systems tend to give similar estimates.

(iii) The non-compensated own-price elasticities estimated for the two systems are widely different for many items for both data sets, particularly for rural India.

(iv) The LES-based itemspecific own-price elasticities tend to show a broad proportionality relationship with the corresponding expenditure elasticities even in the present case where very broad itemgroups have been considered. This result is in conformity with Deaton's analytical conclusion regarding the interdependence of the own-price and expenditure elasticities for systems based on additive direct preferences (Deaton, 1974b).

Chapter V also has four sections, including an introductory section. Here, in Section 5.2 the estimation procedure has been briefly described. In Section 5.3, the empirical results relating to the different variants of the modified SNAM and our proposed system have been presented. These include the log-likelihood values, parameter estimates, measures of goodness of fit and the estimated elasticities corresponding to the different systems estimated separately for the rural and the urban sectors on the basis of disaggregated data. Finally, Section 5.4 gives a summary of the results obtained and some conclusions based on these results. Briefly, the results reported may be summarised as follows:

(i) For both the modified SNAM and our proposed system, the goodness of fit (as judged by the log-likelihood values and the item-specific squared correlation coefficients between observed and estimated budget shares) improve by allowing more and more price flexibility into the systems. In other words, SNAM 2 and Variant 3 are seen to perform better than their corresponding restricted versions. Also, on the whole, Variant 3 is seen to perform better than SNAM 2.

(ii) Comparison of the itemspecific expenditure elasticities and non-compensated price elasticities obtained from the different estimated systems suggests that the estimates of expenditure elasticity obtained from these systems are far more comparable than the corresponding estimates of own-price elasticity. A closer examination, however, revealed differences in the expenditure elasticities obtained from the modified SNAM set-up and the corresponding specifications of our proposed system.

Chapter VI presents an empirical comparison of the performances of SNAM 2 and Variant 3 - the full specifications of the modified SNAM and our proposed system respectively - with that of the AIDS. This Chapter has seven sections. Section 6.1 is introductory. Section 6.2 is a comparative study of the goodness of fit of these three systems based on the itemspecific squared correlations between the observed and the estimated budget shares computed for each system separately for the rural and the urban sectors. Section 6.3 reports some results on the empirical validity of the regularity conditions for individual systems. In Section 6.4, the estimated expenditure and own-price elasticities obtained from the three systems have been compared. Section 6.5 sets out some results on the test of significance of the auto-correlation of disturbances for the individual systems. Some results relating to a test of misspecification for the three systems have been presented in Section 6.6 . Finally, Section 6.7 gives a summary of results and makes some concluding observations. The salient findings of this Chapter may be summarised as follows:

(i) Judged by the goodness of fit, compared to the AIDS, Variant 3 of our proposed system performed reasonably well for most items in rural and urban India. This observation may be of some interest in view of the fact that Variant 3 involves a much smaller number of parameters compared to the AIDS. However, for some items in rural and urban India even the AIDS specification was found to be inadequate in terms of explanatory capacity.

(ii) Comparison of the itemspecific expenditure elasticities obtained from the three systems reveal that the estimates based on the AIDS and Variant 3 broadly agree, while those based on SNAM 2 are marginally different in many cases. In contrast, the estimates of own-price elasticity obtained from the different systems are found to be widely different. To check the plausibility of the estimates of the own-price elasticity, the pattern of variation of these elasticities over population groups was examined. For Variant 3, an inverse relationship between the price elasticity and the income level (strictly, expenditure level) was observed for most of the items in rural and urban India. The corresponding exercises for the AIDS and SNAM 2, however, failed to show such an intuitively plausible relationship.

(iii) For both SNAM 2 and Variant 3 the auto-correlation of the residuals was observed to be very high. For the AIDS, on the other hand, the problem of such auto-correlation was observed to be much less serious. Subsequent analysis suggested that the problem of auto-correlation observed for SNAM 2 and Variant 3 could be, to some extent, due to misspecification of the functional form. A test of misspecification of functional form revealed that SNAM 2 involves a larger misspecification compared to the other two systems.

There are five Appendices. Appendix A derives the cost functions underlying the modified SNAM (SNAM 2) and our proposed systems (Variant 1, Variant 2, and Variant 3) described in Chapter II. This Appendix also obtains the expressions for the expenditure and price elasticities for the systems.

Appendix B is concerned with the NLFIML estimation procedure described in Chapter III. At each stage of iteration, the NLFIML method requires the values of the first order partial derivatives of the budget share functions with respect to the parameters of the systems being estimated. This Appendix lists the expressions for these first order partial derivatives for alternative specifications of the SNAM 2 and Variant 3.

Appendix C is supplementary to Chapter IV. This has two parts. The first part presents the estimated non-compensated own and cross price elasticities for the LES and the AIDS, based on both aggregate and disaggregated data, separately for rural and urban India. The other part of this Appendix presents the results of some further exercises for the AIDS based on disaggregated data. Specifically, while estimating the AIDS, it was observed that a large number of parameters were statistically non-significant, thus indicating presence of considerable multicollinearity in the data. To examine whether the goodness of fit of the AIDS would be severely affected if some of the explanatory variables were dropped, stepwise regressions were done for the individual budget share equations of the AIDS. The results indicated that even with a much smaller number of price variables one would get the same order of goodness of fit as obtained for the corresponding full specification of the AIDS.

Appendix D reports some results of tests of homogeneity of demand systems across population groups. It may be pointed out that in Chapter VI, the results for the AIDS, SNAM 2 and Variant 3 fitted to the disaggregated population-groupwise data have been presented. Actually, data relating to all the three population groups have been used together to estimate a single system. This procedure implicitly assumes that the same set of parameter values would be applicable to all the three population groups. To examine the validity of such pooling, these tests of homogeneity were applied for the AIDS and also for the full specification of our proposed system, i.e., Variant 3. It turns out that the overall tests give an unfavourable verdict - they point to significant variation of the parameter values across the three population groups. However, on examination of the parameter estimates obtained from the three population groups separately, some of the parameters were observed to vary systematically, while the others showed small variation across population groups.

Finally, Appendix E reports on some dynamised systems fitted to the aggregate data. In the analysis based on aggregate data, the problem of auto-correlation of the itemspecific random disturbances was observed to be quite strong. In view of this, (a) the AIDS was re-estimated in its first-differenced form, and (b) the LES and the simplest version of our proposed system (Variant 1) were re-estimated by incorporating linear time trends in the budget shares, or the phenomenon of habit formation into the systems. For the residuals of the first-differenced version of the AIDS, auto-correlation was seen to be virtually absent. The dynamised versions of the LES and Variant 1 succeeded to some extent in reducing the degree of auto-correlation of the residuals observed in the corresponding static forms.

Chapter I

Survey of literature

1.1 Introduction

Ever since Ernst Engel (1857) formulated his famous law(s) for family budgets, interest in empirical studies on consumer behaviour has been a persistent phenomenon. Measurement of consumer response to changes in factors affecting the demand for consumer goods and empirical validation of theoretical postulates regarding consumer behaviour have been the central objectives of these studies.

According to the nature of the data used, these studies may be classified into three groups — analysis of cross-sectional household budget data, analysis of time series data on aggregate/average consumption of goods obtained from national accounts sources or from repetitive household budget surveys, and finally, analysis of continuous cross-sectional or panel data on household budgets.

The tradition of analysis of cross-sectional household budget data is the oldest. Typically, these studies are concerned primarily with the estimation of income responses, i.e., the Engel curves, through eliminating the effects of demographic and other socio-economic factors that are known to have considerable effects on household consumption decisions. Classical examples of such cross-sectional household budget studies are Allen and Bowley (1935), Wold and Jureen (1953), Prais and Houthakker (1955) and Friend and Jones (1960).

Analyses based on time-series or continuous cross-sectional data are required for measuring consumer responses to price changes - the price effects - and also for examining the dynamic aspects of consumer behaviour. There are two alternative approaches to time-series - based analyses. The first one may be called the single commodity/single equation approach in which consumer demand for a specific commodity is analysed in terms of factors governing the demand for and the supply of the commodity concerned. The complete demand system approach, on the other hand, recognizes the interrelatedness of consumer demand for different goods being consumed by the consumer and attempts to capture this interrelatedness assuming given supply conditions.[1/]

While the early works based on time-series data followed the single equation approach (e.g., Schultz, 1938; see also Stone, 1954a), interest in complete demand systems was stimulated by Stone (1954b) when he proposed the simplest version of the Linear Expenditure

---

1/ In principle, one can estimate the Engel curves underlying such systems on the basis of purely cross-sectional data. For some systems, the Engel curves may be interrelated in the sense that parameters relating to other items appear in the Engel curve for an item .

System - a system of commodity-specific linear demand equations incorporating several theoretical postulates of consumer behaviour as consistency requirements. Subsequently, research on consumer behaviour based on complete demand systems has proceeded in three directions, viz.,

(i) testing the empirical validity of the different theoretical restrictions on consumer behaviour in a 'pragmatic' framework of linear or linearlized system of demand equations (vide, Theil, 1965; Barten, 1966; and Byron, 1968, 1970a, 1970b);

(ii) development of complete demand systems based on specific forms of well-behaved preference structure (Houthakker, 1960); and finally,

(iii) development of demand systems corresponding to fairly general forms of preference structure that may be supposed to represent preferences through suitable parametric restrictions (vide, Christensen, Jorgenson and Lau, 1975; and Deaton and Muellbauer, 1980a).

It should be mentioned here that the above account relates only to studies done in the framework of static consumer behaviour. Analysis of the dynamic aspects of consumer behaviour has also made rapid strides since the 1960's in both their methodological and substantive aspects.

In what follows, the literature on studies on consumer behaviour, based on time-series data and carried out in the framework of complete demand systems, is briefly reviewed.[2/] The plan of exposition is as follows: Section 1.2 discusses the static theory of consumer behaviour and its implications for empirical analysis; section 1.3 defines the notion of separable preferences; section 1.4 describes some well-known static demand systems; section 1.5 presents a discussion on aggregation over consumers; section 1.6 discusses some generalisation of the IES based on the cost function/indirect utility function approach; section 1.7 is concerned with the treatment of household characteristics in complete demand systems; and finally, section 1.8 briefly discusses the dynamic considerations in demand analysis.

1.2 The static theory of consumer behaviour and its implications

The static theory of consumer behaviour is concerned with the problem of allocation of a given money income (in a given time-period) of a single consumer among different commodities, when all prices facing the consumer are assumed fixed. It also indicates the nature of the consumer's reaction to price(s) and income changes. This problem of allocation may also be posed equivalently as one of minimisation of the cost of enjoying a specified level of utility in a

2/ See Brown and Deaton (1972) for a detailed review on this topic. (See also Barten, 1977; and Phlips, 1983.) For a comprehensive discussion on the theory of consumer behaviour see Deaton and Muellbauer (1980b).

situation of given prices. This theory is a simplified exposition of reality and to some extent approximates the actual situation where all the commodities are non-durable in nature. However, real-life allocation problems are much more complicated because of the presence of rationing and subsidies in consumption of specific goods, durable goods and the phenomenon of habit formation, among other things. These would call for substantial modifications of the static theory including the introduction of dynamic consideration.

1.2.1 Utility maximisation and Marshallian demand functions

The standard problem of consumer's equilibrium in a static framework is essentially a constrained maximisation problem. Here the consumer is assumed to possess a smooth, twice-differentiable ordinal utility function $V(q_1, q_2, \ldots, q_n)$ defined over the quantities of consumption q_i $(i = 1, 2, \ldots, n)$ of n goods, and $V(q_1, q_2, \ldots, q_n)$ is maximised subject to the budget constraint $\sum_{i=1}^{n} p_i q_i = y$, p_i's being the fixed prices, and y the given level of money income during a specified period of time. The first order conditions of this maximisation problem are

$$\left.\begin{aligned} u &= \lambda p \\ p'q &= y \end{aligned}\right\} \qquad \ldots (1.2.1)$$

where $u = \left(\frac{\partial V}{\partial q_1}, \frac{\partial V}{\partial q_2}, \ldots, \frac{\partial V}{\partial q_n} \right)'$ is the vector of marginal utilities, p is the n-vector of prices $(p_1, \ldots, p_n)'$; q is the n-vector of quantities $(q_1, q_2, \ldots, q_n)'$ and λ the Lagrangean multiplier interpreted as the marginal utility of y. The second order condition of maximisation is that $U = \left[\frac{\partial^2 V}{\partial q_i \, \partial q_j} \right]$, the matrix of second order partial derivatives of V is negative semidefinite, i.e., $V(q_1, q_2, \ldots, q_n)$ is quasi-concave. The system of demand equations obtained by solving equation (1.2.1), viz.,

$$q_i = q_i \, (y, p_1, p_2, \ldots, p_n) \quad \text{for } i = 1, 2, \ldots, n \qquad \ldots (1.2.2)$$

are known as the Marshallian demand functions. These functions satisfy the adding-up criterion, viz.,

$$\sum_{i=1}^{n} p_i q_i = y \qquad \ldots (1.2.3)$$

and are homogeneous of degree zero in prices and income. The observable restrictions on the system (1.2.2), in view of its consistency with the utility maximisation behaviour, may be derived from the following system of differential equations obtained by total differentiation of equations (1.2.1), viz.,

$$\begin{bmatrix} U & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} \partial q \\ -\partial \lambda \end{bmatrix} = \begin{bmatrix} \lambda dp \\ dy - q' dp \end{bmatrix} \qquad \ldots (1.2.4)$$

The Slutsky decomposition of price response into income and substitution effects may be derived from equation (1.2.4) as

$$Q_p = [\frac{\partial q_i}{\partial p_j}] = \lambda U^{-1} - q_y' q_y \phi y - q_y q' \quad \ldots (1.2.5)$$

where

$$q_y = (\frac{\partial q_1}{\partial y}, \frac{\partial q_2}{\partial y}, \ldots, \frac{\partial q_n}{\partial y})'$$ is the vector of

of income effects, and $\phi = 1 / (\frac{\partial \log \lambda}{\partial \log y})$ is Frisch's money flexibility parameter. Equations (1.2.5) yield the income compensated price effects as

$$S = [s_{ij}] = [\lambda U^{-1} - q_y q_y' \phi y] = Q_p + q_y q' \quad \ldots (1.2.6)$$

This is obtained by adjusting income in such a way that after a price change the utility is left unaltered. Equations (1.2.6) are known as the Slutsky equations and form the pivot for all observable restrictions on the demand equation (1.2.2) that follow from the static theory. The restrictions and some of their implications are listed below:

(i) The adding-up property implies the following aggregation restrictions, viz.,

$$p'q_y = 1 \text{ and } p'S = 0 \quad \ldots (1.2.7)$$

which are known as the 'Engel aggregation' and 'Cournot aggregation' respectively. In other words, the reallocations of **the** budget due to income or price change must exhaust total income.

(ii) The demand functions are homogeneous of degree zero in prices and income when

$$Sp = 0 \qquad \ldots (1.2.8)$$

which suggests that proportional changes in all prices and income must leave the quantities demanded unaltered.

(iii) The symmetry restriction $U = U'$ is equivalent to

$$S = S' \qquad \ldots (1.2.9)$$

which asserts that the specific substitution effects are symmetric. Finally,

(iv) The negativity restriction, i.e., negative semidefiniteness of U or equivalently the quasi-concavity of the direct preference structure underlying (1.2.2) is equivalent to the negative semidefiniteness of S .

1.2.2 Cost-minimisation: Indirect utility function, cost-function and Hicksian demand functions

By considering the dual of the constrained utility maximisation problem, i.e., minimisation of the total cost $\sum_{i=1}^{n} p_i q_i$ subject to the utility constraint $\bar{V} = V(q_1, q_2, \ldots, q_n)$ where $\bar{V}$ is a specified level of utility, one obtains the Hicksian demand functions

$$q_i = q_i^* \ (p_1, p_2, \ldots, p_n, \bar{V}) \ ; \quad i = 1, 2, \ldots, n. \qquad \ldots (1.2.10)$$

On substituting the Marshallian demand functions in the direct utility function $V \ (q_1, \ldots, q_n)$ one gets an alternative representation of preferences in terms of prices and income. Thus, $V \ (q_1, q_2, \ldots, q_n) = V \ (q_i \ (y, p_1, p_2, \ldots, p_n); \ i = 1, 2, \ldots, n) \ ) = V^* \ (y, p_1, p_2, \ldots, p_n)$, which is known as the indirect utility function. Similarly, substituting the Hicksian demand functions into the budget expression $\sum_i p_i q_i$, the cost function $C \ (V^*, p_1, p_2, \ldots, p_n)$ is obtained. Note that, for given prices and income, $\bar{V} = V \ (q_1, q_2, \ldots, q_n) = V^* \ (y, p_1, p_2, \ldots, p_n)$, and $\sum_{i=1}^{n} p_i q_i = y = C \ (\bar{V}, p_1, p_2, \ldots, p_n)$, if q_i's are the optimal quantities, so that $C \ (\bar{V}, p_1, p_2, \ldots, p_n)$ is the minimum cost of attaining $\bar{V}$ at prices $p_1, p_2, \ldots, p_n$.

The concepts of the cost function and the indirect utility function are interrelated in the sense that when one specifies the cost function, the corresponding indirect utility function is, in principle, automatically specified. For well-behaved preferences the cost function $C \ (\bar{V}, p) = C \ (V^*, p)$, where p is the n-vector of prices and $\bar{V}$ is the reference utility level, exists and has the following properties:

(i) it is linear homogeneous in p ;

(ii) it is increasing in V^* and non-decreasing in p, and increasing for at least one price; and,

(iii) it is concave in p for any V^* .

The corresponding indirect utility function $V^*(y,p)$ has the following properties:

(i) it is strictly increasing in y if the preference structure is well-behaved and one can invert V^* to obtain the cost function, and

(ii) it is homogeneous of degree zero in prices and income.

1.2.3 Empirical implications of the static theory

The implications of the static theory of consumer's equilibrium summarised above for building models of consumer behaviour are as follows: One can explicitly specify a system of demand equations with an underlying preference structure in three alternative ways. First, a direct utility function may be specified and the corresponding Marshallian demand functions (1.2.2) obtained by solving the first order conditions (1.2.1). Next, one may specify an indirect utility function $V^*(y, p)$ explicitly and obtain the corresponding Marshallian demand functions invoking René Roy's identity, viz.,

$$q_i = -\frac{\partial V^* / \partial p_i}{\partial V^* / \partial y} \quad ; \quad i = 1, 2, \ldots, n. \qquad \ldots (1.2.11)$$

Note that (1.2.11) holds for optimal quantities in view of the adding-up restriction and the homogeneity of degree zero of V^* in p and y. Finally, one may specify explicitly a cost function $C(V^*, p)$ and obtain the

Hicksian demand functions using Shephard's equation, viz.,

$$\frac{\partial C(V^*, p)}{\partial p_i} = q_i \ ; \qquad i = 1, 2, \ldots, n. \qquad (1.2.12)$$

The corresponding Marshallian demand functions may then be derived by eliminating V^* from equation (1.2.12) through forcing $y = C\,(V^*, p)$.

The advantage of the approaches based on the cost function or the indirect utility function to the problem of specification of empirical demand systems may be brought out at this stage. To obtain the demand functions implied by a specific direct preference structure, one would solve explicitly for the optimal quantities,the system of equations (1.2.1) given by the first order utility maximisation conditions. An exact explicit solution of q_i's from (1.2.1) presupposes a fairly restrictive form of the direct preference structure. For the approaches based on the indirect utility function or the cost function, on the other hand, the demand functions are easily obtained through equations (1.2.11) and (1.2.12) respectively. Thus, one only has to start from C or V^* satisfying the conditions stated.

It should be noted that a system of demand equations satisfies the adding-up, homogeneity, symmetry and negativity restrictions if and only if the underlying preference structure, whether defined in terms of the direct utility function, the indirect utility function or the cost function, is well behaved in the sense that it represents consistent consumer's choice. The conditions needed for the demand functions to be consistent with preferences are known as the integrability conditions, and they play an important role in empirical analysis. It may be mentioned here that the symmetry condition is the fundamental integrability condition in the sense that given a demand function satisfying this condition, a utility function may be constructed (Samuelson, 1950). If in addition negativity holds, then the demand equations can be considered to be derived from the maximisation of the utility function. When one estimates a theoretically consistent demand system from actual data, it is implicitly assumed that the actual data correspond to a utility maximisation behaviour of the consumers concerned - an assumption that is likely to be highly simplifying in case of time-series data, particularly on aggregate/average consumption. This gives rise to the question of checking the empirical validity of the theoretical restrictions.

Two alternative approaches to testing the empirical validity of the theoretical restrictions are generally adopted. First, in the 'pragmatic' approach, on may specify, a priori, a form for the demand functions that would, through appropriate parametric restrictions be forced to be theoretically consistent (vide Theil, 1965; Barten, 1966). The other approach, known as the 'flexible functional form' approach, specifies a very general form for the preference function which, through proper restrictions on the parameters of the preference function itself would yield a consistent preference structure. The demand systems corresponding to the latter approach are known as flexible demand systems (vide Christensen, Jorgenson and Lau, 1975). In each of these approaches, both the general model and the model incorporating the parametric restrictions are estimated, and the validity of the restrictions are examined statistically.

1.3 Separable preferences

Commodity aggregation is one of the most important issues involved in the empirical implementation of the theory discussed above. The basis of the notion of commodity aggregation is purely empirical. While analysing consumer behaviour using actual data, a problem of manageability arises due to the presence of a large number of commodities which are actually consumed by consumers. The demand functions corresponding to these large number of commodities are often aggregated into functions corresponding to commodity-groups (e.g., 'cereals', 'animal food', 'clothing') but this can be theoretically justified only under specific conditions.

A set of conditions under which commodity aggregation is perfectly justified was first given by Hicks (1936) and Leontief (1936) in their "composite commodity theorem". The theorem states that commodities for which relative prices do not change over time can be treated as a single commodity for all analysis of consumer behaviour. In actual practice, however, commodity aggregation is mostly done by combining items of similar use into a single group. In so far as even closely related items show some variation in relative prices, such aggregation would at best be approximately consistent.

An alternative approach to dealing with the problem of commodity aggregation is through the notion of 'separable preferences'. The concept of separability was introduced independently be Sono (1945) and Leontief (1947). According to their notion of separability, in the utility function a group of commodities may be treated as separable from the remaining commodities when the marginal rates of substitution between commodities within the group are independent of the quantities of commodities outside the group.3/ Thus, for separable preferences, commodities may be partitioned into any number of groups, and preferences for commodities within each group can be described independently of the quantities of commodities in other groups.

The implications of separability in the context of consumer budgeting have been studied by Strotz (1957, 1959) and Gorman (1959),

---

3/ A slight variation of this notion of separability is due to Pearce (1964), and is known as 'Pearce separability'. Commodities conform to this specification if the marginal rate of substitution between two commodities within a group is independent of the consumption of any other commodity whether inside or outside the group.

among others. Separability provides a basis for 'the utility tree approach' and other approaches to decentralized budgeting, under which a consumer can first allocate his total expenditure (income) to each commodity group and then allocate each group-expenditure to individual commodities in the group.[4/] The notion of separability thus restricts the form of the preference function to some extent.

1.3.1 Weak separability

Among the various forms of separability considered in the literature, the least restrictive form is 'weak' separability. The type of separability discussed in the preceding paragraphs is in fact nothing but weak separability. In case of direct weak separability, the direct utility function is written as

$$V = V\ (\ V_1\ (q^1),\ V_2\ (q^2),\ \ldots,\ \ V_G\ (q^G)\ ) \qquad \ldots\ (1.3.1)$$

where V is an increasing function in its arguments, G is the number of commodity groups, and q^g is the g-th subvector of quantities consumed $(g = 1, 2, \ldots, G)$. In case of indirect weak separability, the indirect utility function is written as

$$V^* = V^*\ (\ V_1^*\ (\ \gamma^1),\ V_2^*\ (\ \gamma^2),\ \ \ldots\ ,\ \ V_G^*\ (\ \gamma^G)\ )\ \ \ldots\ (1.3.2)$$

4/ Several definitions and conditions of decentralizability have been proposed and analysed by Pollak (1969, 1970b), Lau (1969), Gorman (1970), Blackorby, Primont and Russell (1978). See also Deaton and Muellbauer (1980b) for a discussion on separability.

for some partition of commodities into G groups, where $\gamma^g = p^g/y^g$, p^g being the g-th subvector of prices, and y^g the total expenditure of the g-th group (g = 1, 2, ..., G), with $\Sigma\, y^g = y$.

A direct weak separable utility function of the form (1.3.1) implies and is implied by the following form of subgroup demand function for item i belonging to group g

$$q_{gi} = f_{gi}\ (\ y^g\ ,\ p^g\ ) \qquad \ldots\ (1.3.3)$$

where $y^g = \sum_k p_{gk}\, q_{gk}$ is the total expenditure on group g . For two-stage/decentralized budgeting, existence of subutility function in the context of weak separability means that one can define the group cost functions $C_g\ (\ V_g^*\ ,\ p^g\ )$, g = 1, 2, ..., G, having the requisite properties. The indirect utility function for group g may then be obtained as

$$V_g^* = \psi_g\ (\ y^g\ ,\ p^g\ ) \qquad g = 1, 2, \ldots, G. \qquad \ldots\ (1.3.4)$$

The total utility may then be written from (1.3.1) as

$$V = V\ (\ V_1^*\ ,\ V_2^*\ ,\ \ldots\ ,\ V_G^*\ ) \qquad \ldots\ (1.3.5)$$

For broad allocation among groups, (1.3.5) is maximised subject to

$$\sum_g C_g\ (\ V_g^*\ ,\ p^g\ ) = y \qquad \ldots\ (1.3.6)$$

The solution, however, requires knowledge of individual prices and quantities, which under certain conditions (vide Gorman, 1959) may be replaced by group price and quantity indices.

1.3.2 Implicit separability

Another form of separability due to Gorman (1970), viz., 'implicit' or 'quasi'-separability is defined as follows. Preferences are said to be implicitly separable if and only if the cost function can be written in the form

$$C\ (V^*, p) = C\ (V^*, C_1\ (V^*, p^1), C_2\ (V^*, p^2), \ldots, C_G\ (V^*, p^G)\ ) \quad \ldots (1.3.7)$$

where $C_g\ (V^*, p^g)$ is increasing in V^* and p^g $(g = 1, 2, \ldots, G)$. Here, in contrast to indirect weak separability, all the arguments contain total utility V^*. This form of separability provides a different and a more practical scheme for two-stage budgeting. The $C_g\ (V^*, p^g)$'s can be constructed suitably so that these may be treated as group price indices. The total cost function $C\ (V^*, p)$ can then be obtained and used in allocating expenditure to broad groups according to Shephard's equation (1.2.12).

1.3.3 Strong separability

Under the most restrictive form of separability, viz., strong separability, the ratio of marginal utilities of any pair of commodities belonging to distinct groups are independent of the quantities consumed of all other commodities. The direct utility function in this case is written as

$$V = f\ (\ V_1\ (q^1) + V_2\ (q^2) + \ldots + V_G\ (q^G)\ ) \qquad \ldots\ (1.3.8)$$

so that under suitable monotonic transformation (1.3.8) takes the explicitly additive form. This form of separability is alternatively known as 'block additivity'. A special case of 'block additivity', defined above, where each group consists of only one good, is referred to as 'direct additivity' or 'want independence'. Here the utility function takes the form

$$V = f\ (\ V_1\ (q_1) + V_2\ (q_2) + \ldots + V_n\ (q_n)\ ) \qquad \ldots\ (1.3.9)$$

n being the number of individual commodities. For the indirect utility function the forms corresponding to (1.3.8) and (1.3.9) are respectively

$$V^* = V^*\ (\ V_1^*\ (\ \gamma^1) + V_2^*\ (\ \gamma^2) + \ldots + V_G^*\ (\ \gamma^G)\ ) \qquad \ldots\ (1.3.10)$$

and

$$V^* = V^*\ (\ V_1^*\ (\ \gamma_1) + V_2^*\ (\ \gamma_2) + \ldots + V_n^*\ (\ \gamma_n)\ ) \qquad \ldots\ (1.3.11)$$

which are of the 'indirect block additive' and 'indirect additive' forms respectively.

The restrictiveness of the assumption of 'additivity' lies in the following implications.[5/] First, for the direct utility function, the expenditure elasticities and Frisch's money-flexibility parameter determine all the own- and cross-price elasticities (Frisch, 1959; Houthakker, 1960). Thus, the price elasticities of commodities are determined without the requirement of detailed price information. Second, direct additivity has been shown to impose an approximately proportional relationship between the expenditure and the own-price elasticities (Deaton, 1974b) and this distorts the measurement of both the expenditure and the price elasticities.

In case of indirect additivity, the cross-price elasticities for all commodities with respect to a particular price are equal (Houthakker, 1960). Finally, indirect additivity implies an approximately linear relationship between the expenditure and own-price elasticities (Deaton, 1974b).

In spite of all these restrictive consequences, the assumption of directly additive preferences has been very popular in empirical analysis, primarily because of the ease and simplicity it provides in deriving demand functions from well-behaved preferences.

5/ The empirical consequences of different separability assumptions are listed in Goldman and Uzawa (1964).

1.4 Some well-known systems of static demand equations

In this section we shall examine briefly some of the popular static demand systems that are widely used in empirical studies on consumer behaviour.

1.4.1 The Rotterdam models

These models are examples of the 'pragmatic' approach to demand analysis. In their work where these models were proposed, Theil (1965) and Barten (1966) suggested the following system of demand equations

$$w_i \operatorname{dlog} q_i = b_i \left(\operatorname{dlog} y - \sum_k w_k \operatorname{dlog} p_k \right) + \sum_j c_{ij} \operatorname{dlog} p_j \,, \qquad i = 1, 2, \ldots, n \quad \ldots (1.4.1)$$

in continuous form, where $w_i = \frac{p_i q_i}{y}$ is the budget share for the i-th item, b_i is the marginal propensity to consume item i , and $c_{ij} = \frac{p_i p_j s_{ij}}{y}$, s_{ij} being the compensated price effect for the i-th item with respect to the j-th price. The equation system (1.4.1) is obtained by considering a decomposition of the observed changes in demand into its real income and substitution components. The restrictions on consumer behaviour in the context of this system may be translated into restrictions on the parameters of the system, viz., b_i's and c_{ij}'s as follows:

(i) $\sum_k b_k = 1$ and $\sum_k c_{kj} = 0$ for all j in view of the adding-up restriction,

(ii) $\sum_j c_{kj} = 0$ for all k when the demand functions satisfy the homogeneity restriction,

(iii) $c_{ij} = c_{ji}$ for $i \neq j$ in view of the symmetry of the matrix $[s_{ij}]$, and

(iv) the matrix $[s_{ij}]$ is negative-semidefinite or equivalently the matrix $[c_{ij}]$ is negative-semidefinite in view of concavity of the underlying direct preference structure.

It may thus be noted that in the framework of the above model the empirical validity of the theoretical restriction may be tested conveniently through the corresponding tests of restriction on the parameters. The linearity of the system in the transformed variables facilitates such tests.

Theoretically, however, this model has been subjected to severe criticism (vide Mcfadden, 1964; Yoshihara, 1969; Philips, 1983). It has been shown that since b_i's and c_{ij}'s are treated as parameters, this model is consistent with a very simplistic preference structure that implies unitary expenditure and own-price elasticities for all the items and zero cross-price elasticities. Given this, the usefulness of the model as a framework for testing the empirical validity of the theoretical postulates of consumer theory is seriously questioned. However, as an empirical description of consumer behaviour, this model would still provide a convenient first order approximation to the underlying true demand system that is simple in appearance and easily estimable.

The system of equations (1.4.1) is popularly known as the 'absolute price variant' of the Rotterdam model. To provide a framework for testing the empirical validity of the hypothesis of directly additive preferences, a 'relative price variant' of (1.4.1) has also been proposed. This version of the Rotterdam model has the following form :

$$w_i \ d\log q_i = b_i \ d\log \bar{y} + \sum_j a_{ij} \left(d\log p_j - \sum_k b_k \ d\log p_k \right) \quad \ldots (1.4.2)$$

where

$$d\log \bar{y} = d\log y - \sum_j w_j \ d\log p_j$$

and

$a_{ij} = c_{ij} - \phi \ b_i \ b_j$, ϕ being the money flexibility parameter.

Additivity of preferences implies that $[a_{ij}]$ should be a diagonal matrix. Hence under additive preferences $a_{ij} = 0$ for $i \neq j$, and these restrictions may be tested empirically. A comparison of (1.4.1) with (1.4.2) shows that the former is linear and the latter non-linear in parameters. Thus, the absolute price variant of the Rotterdam model is much simpler to estimate. Under additivity or block additivity, however, the relative price version would have a smaller number of unknown parameters and thus it could be more convenient than the absolute price variant from the point of view of estimation.

The Rotterdam models have been popular systems for empirical applications - particularly for empirical verification of the theoretical restrictions on consumer behaviour. The important applications of the absolute price variant of this model include works by Barten and Turnovsky (1966), Barten (1967, 1969), Parks (1969), Lluch (1971), Goldman (1971), Deaton (1974a), Theil (1975) and Barten and Geyskens (1975). Barten (1968), Theil (1970) and also Theil (1975) applied the relative price variant of the model. These studies have been based on time series consumption data for countries like Britain, Holland, Sweden, Germany, Spain and Newzealand. Empirical tests of the various theoretical restrictions have not always been conclusive, nor have they given similar results in all the investigations. Some of the important findings of the tests based on this model are as follows:

Barten (1969) tested homogeneity and symmetry on the basis of Dutch data for the years 1923-1939 and 1950-1961 and found these hypotheses to be unacceptable. Barten and Geyskens (1975), however, observed that the theoretical restrictions of consumer theory are accepted on Dutch data for the years 1950–1969 and on German data for the years 1950–1968. Theil (1975) found support for the symmetry restriction on Dutch data for the years 1922–1963 (excluding the war years) and also for British data for the period 1900–1938 (excluding the war years). A similar conclusion was also reached by Parks (1969) on the basis of Swedish data for the period 1862–1955.

Lluch (1971) observed that the homogeneity hypothesis was not acceptable for the Spanish data for the years 1958–1964. He also observed that given the homogeneity restrictions, the hypothesis of symmetry could not be rejected. Similar results were also obtained by Deaton (1974a) for the British data for the period 1900–1970 (excluding the war years).

1.4.2 The Linear Expenditure System (LES) and some generalisations

The LES, proposed by Stone (1954b), is an example of a demand system consistent with a well-behaved preference structure. This system, unlike the Rotterdam models, has a smaller number of parameters and it provides a fairly simple description of consumer behaviour. The system is specified as

$$p_i q_i = p_i c_i + b_i \left(y - \sum_{j=1}^{n} p_j c_j\right) ; \quad i = 1, 2, \ldots, n \qquad \ldots (1.4.3)$$

with $y \geq \sum_{j=1}^{n} p_j c_j$, $1 > b_i > 0$ for all i, and $\sum_{j=1}^{n} b_j = 1$ for adding-up. The parameters of the system, viz., b_i, c_i $(i = 1, 2, \ldots, n)$ are usually given the following interpretations : c_i's are the committed quantities, to be consumed of the different commodities. That is, when

$$y = \sum_{j=1}^{n} p_j c_j \ , \ q_i = c_i \quad \text{and for} \quad y > \sum_{j=1}^{n} p_j c_j \ , \ q_i > c_i \ .$$ 6/

Following the interpretation of c_i's given above, $(y - \sum_j p_j c_j)$ is called the supernumerary income which is allocated among commodities in the proportions $b_1, b_2, \ldots, b_n$. Thus, b_i's are the marginal budget shares for different commodities.

The direct utility function underlying the LES is known as the Stone-Geary utility function. This is an additively separable function of the form.

$$V (q_1, q_2, \ldots, q_n) = \sum_{j=1}^{n} b_j \log (q_j - c_j) \qquad \ldots (1.4.4)$$

with $b_j > 0$, $q_j > c_j$ for $j=1, 2, \ldots, n$ which guarantees that the preference structure represented by (1.4.4) is well-behaved. The cost

---

6/ If the committed quantity interpretation of the c_i's is held, then $c_i > 0$ would be a reasonable restriction. However, with $c_i > 0$, $i = 1, 2, \ldots, n$, the LES is unable to incorporate price-elastic commodities.

function underlying the IES is

$$C(V^*, p) = \sum_{k=1}^{n} p_k c_k + \left(\prod_{k=1}^{n} p_k^{b_k} \right) V^* \qquad \ldots (1.4.5)$$

and the corresponding indirect utility function is

$$V^* (y, p) = \frac{y - \sum_{k=1}^{n} p_k c_k}{\prod_{k=1}^{n} p_k^{b_k}} \qquad \ldots (1.4.6)$$

The expenditure elasticity η_i of item i for the IES is given by

$$\eta_i = b_i / w_i \qquad \ldots (1.4.7)$$

where $w_i = \frac{p_i q_i}{y}$ is the budget share for commodity i. Given $b_i > 0$, $w_i > 0$ for all i, $\eta_i > 0$ for all i and this rules out the presence of inferior goods in the IES set-up.

The non-compensated price elasticities are given by

$$\mu_{ij} = \begin{cases} - b_i \dfrac{p_j c_j}{p_i q_i} & \text{for } i \neq j \qquad \ldots (1.4.8) \\[2ex] - 1 + (1 - b_i) \dfrac{c_i}{q_i} & \text{for } i = j \qquad \ldots (1.4.9) \end{cases}$$

It may be noted that if $c_i > 0$ given $c_i < q_i$ and $1 > b_i > 0$, $|\mu_{ii}| < 1$ — thus all commodities are price inelastic. The expressions for the compensated price elasticities are

$$\pi_{ij} = \begin{cases} \dfrac{b_i p_j}{p_i q_i} (q_j - c_j) & \text{for } i \neq j \quad \ldots (1.4.10) \\ (b_i - 1)\left(1 - \dfrac{c_i}{q_i}\right) & \text{for } i = j \quad \ldots (1.4.11) \end{cases}$$

Thus, given $1 > b_i > 0$, and $q_i > c_i$, $\pi_{ii} < 0$ for all i. Note, however, that $\pi_{ij} > 0$ for $i \neq j$ in equation (1.4.10) which means that there cannot be any complementarity relationship between any pair of goods in the LES set-up.

The LES has been most extensively used in empirical analyses of consumer behaviour. Ever since Stone (1954b) formulated and applied this system to describe the British consumer expenditure pattern, there have been numerous applications of this system to consumption data pertaining to a number of countries. Thus Paelinck (1964), Parks (1969), Pollak and Wales (1969), Yoshihara (1969), Leoni (1967), Dahlman and Klevmarken (1971), Deaton (1974a) and Theil (1975) used this system to analyse consumption data for countries like Belgium, Sweden, the U.S.A., Japan, Italy, Holland and Great Britain. There have also been a number of applications of the LES to the Indian data. While Rudra (1964), Rudra and Paul (1964), Bhattacharya (1967) and Joseph (1968) applied this system to the time series of average per capita consumer expenditure for India, Radhakrishna, Murthy and Shah (1979), and Radhakrishna and Murthy (1980) used this system to analyse the consumer expenditure patterns of disaggregated population groups.

The LES has also been employed in a number of international comparisons of consumer expenditure patterns. This includes studies by Baschet and Debreu (1971), Goldberger and Gamaletsos (1970), Solari (1971), Theil (1971), and Parks and Barten (1973). In these studies the consumption patterns of different European countries have been compared.

It has already been mentioned that the LES in its original formulation cannot incorporate inferior goods and complementarity between pairs of goods. Also, if the committed quantity parameters are assumed to be positive, the items become price inelastic. Apart from these, another important limitation of the LES is the restrictiveness of its non-compensated own-price elasticities resulting from the additivity of the direct utility function underlying the system (Deaton, 1974b). Yet, the LES has found most extensive use in empirical analyses because of the simplicity of its form and parametric parsimony. However, there have also been numerous attempts to generalise the LES to overcome its limitations. While some of these generalisations may be considered to be based on modifications of its underlying direct utility function, the others follow from generalisations of the underlying cost function/ indirect utility function. Some of the generalisations of the former category still retain the strong separability of the direct utility function. Here, some of the systems based on the modification of the direct utility function of the LES will be discussed.[7]

Two generalisations that retain the strong separability of the underlying direct utility function are due to Pollak (1971). The first one, known as the 'Generalised Bergson Family' of demand systems, is of the form

$$p_i q_i = p_i c_i + \phi_i(p) \{ y - \sum_{k=1}^{n} p_k c_k \} \quad , i = 1, 2, \ldots, n \quad \ldots(1.4.12)$$

where

$$\phi_i(p) = \frac{(p_i^{-\rho} b_i)^{\frac{1}{1-\rho}}}{\sum_k (p_k^{-\rho} b_k)^{\frac{1}{1-\rho}}} \quad , \quad b_i > 0 \ , i = 1, 2, \ldots, n .$$

and $1 > \rho > 0$. The direct utility function underlying (1.4.12) is

$$V(q_1, q_2, \ldots, q_n) = \sum_{k=1}^{n} b_k (q_k - c_k)^{\rho} \ , \ q_k > c_k \quad \ldots(1.4.13)$$

In the limiting case of $\rho \to 0$, equation (1.4.12) reduces to the LES.[8]

7/ The systems obtained by generalising the cost function/indirect utility function implied by the LES, will be discussed in section 1.6 below.

8/ Wales (1971) applied this system to post-war Canadian data and found that its performance was not significantly different from that of the LES.

The other system is a generalisation of Chipman's (1965) model and has the following form

$$p_i q_i = p_i \gamma_i(p) + \phi_i(p) \{ y - \sum_{k=1}^{n} p_k \gamma_k(p) \} , \quad i=1,2,\ldots, n \qquad \ldots(1.4.14)$$

where

$$\gamma_k(p) = c_k - b_k \log p_k$$

$$\phi_k(p) = \frac{b_k p_k}{\sum_j b_j p_j} \qquad k = 1, 2, \ldots, n .$$

and $b_k > 0$, $k = 1, 2, \ldots, n$. Equation (1.4.14) though formally similar to the IES, does not rest the IES. Equation (1.4.14) is generated by the following direct utility function

$$V(q_1, q_2, \ldots, q_n) = - \sum_{k=1}^{n} b_k \exp \left(\frac{c_k - q_k}{b_k}\right) \qquad \ldots(1.4.15)$$

with $b_k > 0$, $k = 1, 2, \ldots, n$. While equation (1.4.14) makes both the committed quantities and the marginal budget shares price flexible, the former system introduces price sensitivity into the marginal budget shares only. However, in view of Deaton's observation, the additivity of the direct utility functions (1.4.13) and (1.4.15) would rule out independent estimates of expenditure and own-price elasticities on the basis of either of these systems.

The multi-level generalisation of the LES in Brown and Heien (1972) is based on a block additive direct preference structure, called the S-branch utility tree, a concept developed in Strotz (1957, 1959). The specific form of the S-branch utility function used in this generalisation is

$$V(q_1, q_2, \ldots, q_n) = \left[\sum_{s=1}^{S} a_s \left\{ \sum_{i=1}^{n_s} b_{si} (q_{si} - c_{si})^{\rho_s} \right\}^{\frac{\rho}{\rho_s}} \right]^{\frac{1}{\rho}} \qquad \ldots(1.4.16)$$

where $b_{si} > 0$, $q_{si} > c_{si}$, $c_{si} \geq 0$, $a_s > 0$, $\rho < 1$, $\rho_s < 1$ and b_{si}, c_{si}, a_s, ρ, ρ_s are the parameters. Here the n-commodities are partitioned into S groups with n_s = the number of goods in group s (s = 1, 2, ..., S) and q_{si} is the quantity of the i-th item in the s-th group. The resulting system of demand equations is

$$q_{si} = c_{si} + \left(\frac{b_{si}}{p_{si}}\right)^{\sigma_s} \frac{\left[a_s^{\sigma} (x_s)^{\frac{\sigma - \sigma_s}{\sigma_s - 1}} \right]}{\left[\sum_{r=1}^{S} a_r^{\sigma} (x_r)^{\frac{\sigma - \sigma_r}{\sigma_r - 1}} \right]} \left(y - \sum_{r=1}^{S} \sum_{j=1}^{n_r} p_{rj} c_{rj}\right) \qquad \ldots (1.4.17)$$

Here, $\sigma = \frac{1}{1 - \rho}$, $\sigma_s = \frac{1}{1 - \rho_s}$ are the inter-group and intra-group elasticity of substitution parameters for pairs of goods, respectively;

p_{si} is the price of the i-th item in the s-th group, and

$$x_s = \sum_{j=1}^{n_s} \left(\frac{b_{sj}}{p_{sj}} \right)^{\sigma_s} p_{sj} , \qquad s = 1, 2, \ldots, S ,$$

$$y = \sum_{r=1}^{S} \sum_{j=1}^{n_r} p_{rj} \, q_{rj} \; .$$

The system allows for complementarity between pairs of goods and allows considerable flexibility for the cross-price effects. Even with $c_{si} > 0$ for all s and i, the own-price elasticities of this model can take any value between 0 to $-\infty$. In the special case where $n_s = 1$ for all s and $\sigma = 1$, the system collapses to the LES. The assumption of block-addivitity of the direct preference structure, however, does not eliminate fully the problem of independent estimation of the own-price and expenditure elasticities (Deaton, 1974b).

Carlevaro (1976) modified the LES by making the direct utility function underlying the LES non-additive, viz.,

$$V\,(q_1, q_2, \ldots, q_n) = G(f) + \sum_{i=1}^{n} b_i^{1} \log\,(q_i - c_i) \qquad \ldots (1.4.18)$$

where $f = f(q_1, q_2, \ldots, q_n)$ is the solution of the equation

$$\log f = \sum_{i=1}^{n} b_i^{o} \log\,(b_i^{o} + b_i^{1} \;\phi(f)\;) = \sum_{i=1}^{n} b_i^{o} \log\,(q_i - c_i) - \log K \; ,$$

K being a constant. ...(1.4.19)

The demand equations corresponding to (1.4.18) - (1.4.19) are

$$p_i q_i = p_i c_i + \{ b_i^{\circ} + b_i^{1} \phi (\frac{y - \Sigma p_j c_j}{P})\} \quad (y - \sum_j p_j c_j) \qquad \ldots(1.4.20)$$

with $\sum_{i=1}^{n} b_i^{\circ} = 1, \ \sum_{i=1}^{n} b_i^{1} = 0$ as the adding-up requirement, and

$P = K \prod_{i=1}^{n} p_i^{(b_i^1 + b_i^{\circ})}$. This system has non-linear Engel curves and the non-linearity of the Engel curves is controlled by the form of $\phi(.)$. In addition, the marginal budget shares here are dependent on prices as well as income. This system reduces to the LES when $b_i^{1} = 0$ for all i. One interesting special case of equation (1.4.20) is the quadratic case, i.e., when

$$\phi(\frac{y - \Sigma p_j c_j}{P}) = \frac{y - \sum_j p_j c_j}{P} . \quad 9/$$

1.4.3 The Indirect and Direct Addilog Systems

The Indirect Addilog System (IAS) of Houthakker (1960) is derived from an additive indirect utility function and thus is an example of the 'indirect utility function approach' to the specification of a system of demand equations. In spite of the well-known implications of the

---

9/ This case has been named the Quadratic Expenditure System (QES) in Pollak and Wales (1978). In section 1.6 we shall discuss some further QES type generalisations of the LES.

assumption of additivity for the preference structure, Houthakker favoured this assumption on two grounds, viz., (1) when one considers only broad groups of commodities, the limited substitution possibilities implied by the additivity of the preference structure may not be very implausible, and (2) this assumption considerably simplifies the use of the theory in empirical applications.

The indirect utility function underlying the IAS has the following functional form

$$V^*(y, p) = \sum_{i=1}^{n} a_i \left(\frac{y}{p_i}\right)^{b_i}, \quad b_i > -1 \qquad \ldots(1.4.21)$$

Using Roy's identity, the demand function for item i is obtained as

$$q_i = -\frac{\partial V^* / \partial p_i}{\partial V^* / \partial y} = \frac{a_i\, b_i\, y^{b_i}\, p_i^{-b_i - 1}}{\sum_{j=1}^{n} a_j\, b_j\, y^{b_j - 1}\, p_j^{-b_j}} \qquad \ldots(1.4.22)$$

It may be noted that (1.4.22) is what one would obtain if a system of simple double-logarithmic demand equations is forced to meet the adding-up restriction. Using (1.4.22), for any pair of commodities

$$\log q_i - \log q_j = \log a_i^* - \log a_j^* + (1+b_i) \log \left(\frac{y}{p_i}\right) - (1+b_j) \log \left(\frac{y}{p_j}\right) \qquad \ldots(1.4.23)$$

where $a_i^* = a_i b_i$. From the point of view of estimation, equation (1.4.23) in double-logarithmic form is preferable to equation (1.4.22), as the latter is non-linear.

The expenditure elasticities η_i $(i = 1, 2, \ldots, n)$ and non-compensated own and cross-price elasticities μ_{ij} $(i,j = 1, 2, \ldots, n)$ for the IAS are given by

$$\eta_i = 1 + b_i - \sum_{j=1}^{n} w_j b_j \qquad \ldots(1.4.24)$$

$$\mu_{ij} = w_i b_j - \delta_{ij} (1 + b_j) \qquad \ldots(1.4.25)$$

where δ_{ij} is Kronecker delta. It may be noted that the demand for an individual commodity is income elastic (income inelastic) if $b_i > \sum_j w_j b_j$ ($b_i < \sum_j w_j b_j$). Similarly, the own-price elasticity of demand for an individual commodity is controlled by (i.e., whether or not an item is price elastic) the sign of b_i. For this reason, the b_i's may be interpreted as 'reaction' parameters. Unlike the IES, IAS can incorporate inferior goods and complementary goods. Finally, the Engel curves underlying the system are non-linear.

The Direct Addilog System (DAS), also proposed by Houthakker (1960), is the direct analogue of the above system. For this system, the direct utility function is specified as

$$V(q_1, q_2, \ldots, q_n) = \sum_{i=1}^{n} a_i q_i^{b_i} \; ; \quad a_i > 0, \; b_i < 1 . \qquad \ldots(1.4.26)$$

With this form of a direct utility function, however, the form of the demand function cannot be explicitly determined through solving the first-order conditions for the constrained utility maximisation problem. As an alternative, the first-order conditions, viz., $\frac{\partial V}{\partial q_i} = \lambda p_i$ for $i=1, 2, \ldots, n$, may be expressed as

$$\log q_i = \frac{1}{b_i - 1} \left\{ (b_j - 1) \log q_j + \log \left(\frac{p_i}{p_j}\right) + \log \left(\frac{a_i^*}{a_j^*}\right) \right\} \qquad \ldots(1.4.27)$$

where $a_i^* = a_i b_i$. For (1.4.27), the ratio of expenditure elasticities is constant for any pair of commodities. It may be mentioned here that under indirect additivity the difference between expenditure elasticities is constant for any pair of commodities.

The empirical applications of the IAS have mostly been done in the context of comparison of alternative systems of demand equations. In some of the applications mentioned earlier of the LES and/or the Rotterdam model, viz., those of Parks (1969), Yoshihara (1969), Solari (1971), Baschet and Debreu (1971), Goldman (1971) and Theil (1975), the performance of the IAS has been compared to those of the LES and/or the Rotterdam model. Diverse results, however, emerged from these studies. In Parks' study (Parks, 1969), the Rotterdam model was found to perform best, followed by the LES and the IAS respectively. Yoshihara (1969) also found the LES to perform better than the IAS on Japanese data. Theil (1975), in his comparison of the performances of the IAS, the Rotterdam model and the LES on Dutch and British data, observed IAS to be inferior to both the LES and the Rotterdam model. However, in a prediction test on British data, LES was found to perform least satisfactorily. Solari (1971), and also Baschet and Debreu (1971), on the other hand, observed that the performances of the LES and IAS were more or less similar. In Goldman's study on the basis of overall performances, the Rotterdam model was seen to be the best, followed by the IAS and the LES respectively. However, for individual commodities, none of the systems could show any clear pattern of superiority (Goldman, 1971).

Apart from the comparative studies mentioned above, the IAS has been used by Somermeyer, Hilhorst and Wit (1962), Somermeyer and Langhout (1972) and also Somermeyer and Bannink (1973) in analysing the consumption pattern of the Dutch population. On Indian data, Radhakrishna and Murthy (1977) and also Radhakrishna, Murthy and Shah (1979) applied the IAS and they compared its performance with that of the LES. In their study, the IAS was generally found to perform better than the LES, although the parameter estimates suggested violation of the regularity conditions (negative semi-definiteness of the substitution matrix) for the IAS in some cases.

Compared to the IAS, the DAS has been less popular as an empirical model. Houthakker (1960) applied this system to a long time-series of consumption data for Sweden and Canada, and also to a set of short time-series data for 13 OECD countries. He observed that the DAS performed better than the IAS. In his application of this model to U.K. data (1900-1970), Deaton (1974a) found the performance of this system satisfactory when compared to other competing models, viz., the LES and the Rotterdam model. The elasticities corresponding to the DAS, however, turned out to be heavily constrained due to the additivity assumption.

1.4.4 The Transcendental logarithmic (Translog) models

The Transcendental logarithmic models were proposed by Christensen, Jorgenson and Lau (1975), who suggested the 'flexible functional form approach' to demand analysis. For the Direct Translog Model (DTM), the direct utility function is specified to be a quadratic form in logarithms of quantities consumed. Similarly, for the Indirect Translog Model (ITM), the underlying utility function is taken to be a similar function of the ratios of individual prices to income. Symbolically, the direct and indirect utility functions underlying the Translog Models are respectively

$$- \ln V = \alpha_o + \sum_i \alpha_i \ln q_i + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} \ln q_i \ln q_j \qquad \ldots(1.4.28)$$

and $$\ln V^* = \alpha_o + \sum_i \alpha_i \ln \left(\frac{p_i}{y}\right) + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} \ln \left(\frac{p_i}{y}\right) \ln \left(\frac{p_j}{y}\right) \qquad \ldots(1.4.29)$$

These forms of the utility functions are considered to be fairly flexible as they provide local second order approximation to the true utility function underlying a given set of consumption data. The demand functions in budget share form, derived from the above utility functions are

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \ln q_j}{\sum_k \alpha_k + \sum_k \sum_j \beta_{kj} \ln q_j}, \qquad i = 1, 2, \ldots, n \qquad \ldots(1.4.30)$$

corresponding to (1.4.28) i.e., the DTM, and

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \ln (p_j / y)}{\sum_k \alpha_k + \sum_k \sum_j \beta_{kj} \ln (p_j / y)}, \quad i = 1, 2, \ldots, n \quad \ldots(1.4.31)$$

for the ITM.

It may be noted that, as in the case of DAS described above, for the DTM the individual Marshallian demand functions cannot be explicitly solved from the first order conditions for constrained utility maximisation, and the form (1.4.30) has been obtained by rearranging these conditions. Several theoretical restrictions can be tested through restrictions on the parameters of the DTM. These are :

(1) Equality restriction: i.e., when the equations are consistent with utility maximisation,

$$\sum_k \beta_{ki} = \sum_l \beta_{lj} \quad \text{for all } i, j ;$$

(2) Symmetry restriction; $\beta_{ij} = \beta_{ji}$ for all i, j ;

(3) Additivity : $\beta_{ij} = 0$ for $i \neq j$;

(4) Homatheticity : $\sum_k \beta_{ki} = 0$ for all i .

For the ITM set-up, the theoretical restrictions may be tested through analogous set of restrictions on the parameters of the ITM. Some restrictions of the DTM have clear implications on the ITM set-up. For example, if the direct utility function is homothetic, the indirect utility function is homothetic. Also, if the direct utility function is additive and homothetic, the indirect utility function is additive and homothetic.

Christensen, Jorgenson and Lau (1975) applied the DTM and ITM to the time-series consumption for the U.S. for the period 1929-1972 to examine the empirical validity of these theoretical restrictions. Their results brought out clear evidence favouring the rejection of the theoretical restrictions implied by constrained utility maximising consumer behaviour.

1.5 Aggregation over consumers

The theory of consumer behaviour summarised above formalizes the behaviour of an optimizing rational consumer. The applicability of this theory to empirical analysis of observed aggregate consumption data, however, is not automatically guaranteed. Indeed, it is only under appropriate theoretical restrictions that a set of micro-demand functions would aggregate over consumers without giving rise to aggregation bias. Although the problem of aggregation is theoretical in nature, the conditions under which such aggregation is possible have important bearings on empirical analysis. In fact, such conditions impose specific restrictions on the form of the demand functions and the underlying preference structure.

In what follows, we discuss some results on aggregation of demand functions across the population of consumers, and their implications in empirical analysis. In subsection 1.5.1 we consider 'exact aggregation'; subsection 1.5.2 deals with Muellbauer's notion of consistent aggregation (Muellbauer, 1975); and finally subsection 1.5.3 presents the Almost Ideal Demand System

(AIDS) - a system proposed by Deaton and Muellbauer (1980a) based on the notion of consistent aggregation.

1.5.1 Exact aggregation

The results on aggregation of demand functions can be classified into two groups - those relating to exact aggregation and the others relating to consistent aggregation. Under exact aggregation, the form of the micro-demand functions should be such that these, when aggregated over consumers, would give an aggregate (or macro) demand function relating aggregate demand to aggregate income/total expenditure. Gorman (1953) and Green (1964) examined the conditions for this. However, the conditions turned out to be extremely stringent, viz., the micro-demand functions should all have linear Engel curves with identical slopes. This condition for exact linear aggregation is equivalent to the indirect utility function being of the form

$$V^{*}(y, p) = \frac{y - P(p)}{Q(p)} \qquad \ldots(1.5.1)$$

where $P(p)$ and $Q(p)$ are linear homogeneous price index functions. The cost function associated with (1.5.1) is the form

$$C(V^{*}, p) = P(p) + Q(p)\, V^{*} \qquad \ldots(1.5.2)$$

and is known as the 'Gorman polar form'. Any system of demand equations having linear Engel curves is a member of this Gorman family. In particular, for the LES, $P(p) = \sum_k p_k c_k$ and $Q(p) = \prod_k p_k^{b_k}$ with c_k and b_k as parameters and $\sum_k b_k = 1$.

1.5.2 Consistent aggregation, Generalised Linearity (GL) and Price Independent Generalised Linearity (PIGL)

Evidently, the requirements of exact aggregation are rather stringent. Muellbauer (1975) introduced the notion of 'consistent aggregation' to generate a much wider class of demand functions that would aggregate without resulting in any aggregation bias. This definition also covers exact linear aggregation as a special case.

Following Muellbauer, the system of budget share functions $w_i(y, p)$, $i = 1, 2, \ldots, n$, are defined to be consistently aggregated, if there exists a representative income level y^* ($= f(y_1, y_2, \ldots, y_H, p)$; y_h, $h = 1, 2, \ldots, H$, being the individual incomes) such that $\bar{w}_i = w_i(y^*, p)$, $i = 1, 2, \ldots, n$, where $\bar{w}_i = \dfrac{\sum_h w_{ih} y_h}{\sum_h y_h}$, w_{ih} being the h-th consumer's budget share for commodity i .

This definition of aggregation leads to a class of budget share functions of the form

$$w_i = a_i(p) + b_i(p)\, g(y, p) , \quad i = 1, 2, \ldots, n \qquad \ldots (1.5.3)$$

with $a_i(p)$, $b_i(p)$ such that $\sum_i a_i(p) = 1$, $\sum_i b_i(p) = 0$, in view of the adding-up criterion, and $g(y, p)$ is some function of y and p . The form (1.5.3) is known as 'Generalised Linearity' (GL) as this implies that any pair of budget share functions must be linearly related.

For (1.5.3), the representative income level y^* depends on the income distribution as also on the given prices. The special case of (1.5.3) for which y^* is independent of prices has been called 'Price Independent Generalised Linearity' (PIGL). PIGL has been shown to be equivalent to the budget share functions being of the form

$$w_i \begin{cases} = a_i(p) + b_i(p)\, y^{\varepsilon} \,, & \varepsilon \neq 0 \qquad \ldots(1.5.4) \\ = a_i(p) + b_i(p) \log y \,, & \varepsilon = 0 \qquad \ldots(1.5.5) \end{cases}$$

Accordingly, (1.5.4) and (1.5.5) have been called PIGL and Price Independent Generalised Log-Linear (PIGLOG) forms respectively.

The cost functions underlying (1.5.4) and (1.5.5) are

$$C^{-\varepsilon}(V^*, p) = [\{a(p)\}^{-\varepsilon} + \{b(p)\}^{-\varepsilon} V^*] \,, \quad \varepsilon \neq 0 \qquad \ldots(1.5.6)$$

$$C(V^*, p) = \{H(p)\}^{V^*} B(p) \,, \quad \varepsilon = 0 \qquad \ldots(1.5.7)$$

respectively, where $a(p)$, $b(p)$ and $B(p)$ are linear homogeneous functions and $H(p)$ is a homogeneous function of degree zero in prices.

It may be noted that the Gorman polar form is a special case of Muellbauer's more general form and is obtained as a special case when $\varepsilon = 1$.

Muellbauer's approach to consistent aggregation described above is based on the representative consumer approach where the representative consumer, with income y^*, is defined to be one whose budget shares are the average budget shares for different commodities assuming that such a consumer (or y^*) exists. In this approach, individual consumers are assumed to possess identical preferences. Recently, Lau (1977) and also Jorgenson, Lau and Stoker (1982) developed a theory of exact aggregation that can incorporate differences in individual preferences arising out of differences in individual attributes. An aggregate demand function, according to Lau's definition, is expressible as a function of prices and k independent index functions dependent on individuals' incomes and attributes. The results in Gorman (1953) and Muellbauer (1975) may be obtained as special cases of Lau's aggregation results.

1.5.3 Empirical implementation of the PIGL - The Almost Ideal Demand System (AIDS)

The PIGLOG budget share function (1.5.5) is a well-known form of Engel curve that had been advocated long ago by Working (1943) and later by Leser (1963) on various statistical considerations. The form

(1.5.4) allows more flexibility in respect of variation in y. In a sense, Muellbauer's work establishes the economic-theoretic justification for these forms of Engel curves.

In view of the plausibility of income responses displayed by Muellbauer's PIGLOG and PIGL forms, the corresponding cost functions (viz., (1.5.7) and (1.5.6) respectively) may be used to generate complete systems of demand equations capable of incorporating non-linear income effects. Indeed, Deaton and Muellbauer (1980a) proposed a fairly general system of demand equations - called the Almost Ideal Demand System (AIDS) - on this line. The explicit form of the cost function on which the AIDS is based is

$$\log c\ (V^*, p) = a_o + \sum_j a_j \log p_j + \tfrac{1}{2} \sum_j \sum_k c_{jk}^* \log p_j \log p_k + V^* b_o \prod_j p_j^{b_j} \qquad \ldots(1.5.8)$$

which is linear homogeneous in prices under the restrictions

$$\sum_j a_j = 1\ ,\ \sum_j c_{jk}^* = \sum_k c_{kj}^* = \sum_j b_j = 0 \qquad \ldots(1.5.9)$$

The resulting demand system in budget share equation form is given by

$$w_i = a_i + \sum_{j=1}^{n} c_{ij} \log p_j + b_i \log \left(\frac{y}{\bar{P}}\right),\ i=1,2,\ldots,n \qquad \ldots(1.5.10)$$

with

$$\log \bar{P} = a_o + \sum_j a_j \log p_j + \tfrac{1}{2} \sum_j \sum_k c_{jk} \log p_j \log p_k \qquad \ldots(1.5.11)$$

The c_{jk}'s in equations (1.5.10) and (1.5.11) are related to the c_{jk}^{*}'s in equation (1.5.8) through the following relation

$$c_{jk} = \frac{1}{2} (c_{jk}^{*} + c_{kj}^{*}) \qquad ...(1.5.12)$$

Equation (1.5.10) is a fairly flexible system in the sense that one can incorporate the theoretical restrictions, viz., adding-up properties, homogeneity and **symmetry** through incorporating suitable restrictions on the parameters of the system. Thus, adding-up property requires

$$\sum_j a_j = 1, \ \sum_j b_j = 0 \text{ and } \sum_j c_{jk} = 0 \text{ for } k = 1, 2, ..., n \qquad ...(1.5.13)$$

Homogeneity of the budget share equations is guaranteed by

$$\sum_j c'_{ij} = 0 \quad \text{for } i = 1, 2, ..., n \qquad ...(1.5.14)$$

Finally, symmetry of the Slutsky substitution matrix is ensured by

$$c_{ij} = c_{jk} \quad \text{for all } i, j = 1, 2, ..., n \qquad ...(1.5.15)$$

However, negative semi-definiteness of the substitution matrix of the AIDS cannot be ensured theoretically and has to be checked empirically. The generality of the AIDS is broadly comparable to those of Rotterdam model and the Translog models, since it may serve as a first order approximation to any demand system derived by utility maximisation.

The expenditure elasticity η_i for the AIDS is given by

$$\eta_i = 1 + b_i / w_i \ , \quad i = 1, 2, ..., n \qquad ...(1.5.16)$$

Assuming $w_i > 0$, commodities with $b_i > 0$ are luxury items, while commodities with $b_i < 0$ are necessary or inferior items. Given $b_i < 0$, a commodity is a necessity if $|b_i| \leq w_i$, and an inferior good if $|b_i| > w_i$. The expression for non-compensated price elasticities is

$$\mu_{ij} = \frac{1}{w_i} \{ c_{ij} - b_i (a_j + \sum_{k=1}^{n} c_{kj} \log p_k)\} - \delta_{ij} \quad \ldots(1.5.17)$$

$$i, j = 1, 2, \ldots, n$$

where δ_{ij} is Kronecker delta.

The corresponding expressions for the compensated price elasticity is

$$\pi_{ij} = \frac{1}{w_i} \{ c_{ij} + b_i b_j \log (\frac{y}{\bar{p}}) + w_i w_j \} - \delta_{ij} \quad \ldots(1.5.18)$$

$$i, j = 1, 2, \ldots, n .$$

Negative definiteness of the substitution matrix can be checked empirically by examining the eigen-values of the $[\pi_{ij}]$ matrix.

The AIDS was applied by Deaton and Muellbauer (1980a) to post-war British data (1954-1974) with eight commodity groups. They tested several restrictions on the system. On an equation by equation comparison, homogeneity was rejected in four cases. Symmetry was found to be rejected in general, irrespective of the maintained hypothesis containing or not containing homogeneity. The first application of the AIDS to Indian data was made by Ray (1980). He incorporated family-size into the original model and examined its performance based on NSS data (1952-1969) separately for rural and urban India. Later, Ray (1982) estimated the AIDS on the pooled cross-section NSS data (1952-1969) and compared the results with those based on the time-series data. The results in general turned out to be sensitive to (i) rural/urban data, (ii) time-series/cross-section data, and (iii) inclusion/exclusion of family-size.

1.6 Generalisations of LES based on cost and indirect utility functions

In this section we discuss some of the generalisations of the LES based on the cost function/indirect utility function approach. It has been mentioned earlier that the LES is consistent according to Gorman's notion of exact aggregation (Gorman, 1953), and thus possesses the following form of the indirect utility function

$$V^*(y, p) = \frac{y - P(p)}{Q(p)} \qquad \ldots(1.6.1)$$

with

$$P(p) = \sum_k p_k c_k , \quad Q(p) = \prod_k p_k^{b_k} \qquad \ldots(1.6.2)$$

with $\sum_k b_k = 1$.

One line of modification of the LES is to propose alternative forms of P(p) and Q(p), and the other is to specify an altogether different indirect utility-function/cost-function that would generate the LES as a special case. For these generalisations the explicit form of the corresponding direct utility functions cannot be known. However, in some cases, the additivity of the direct utility function underlying these generalisations can be checked by examining the relationship between the own-price elasticity and the expenditure elasticity for individual goods implied by these generalisations.

1.6.1 Generalisations involving many additional parameters

Stone's modification of the LES (Stone, 1964) is based on Q(p) as given in equation (1.6.2), and P(p) = $p'c$, where

$$= (\alpha' p)^{-1} D p \;, \quad c = (c_1, c_2, \ldots, c_n)' \qquad \ldots(1.6.3)$$

α and D being an n-vector of parameters and a symmetric matrix respectively, such that P(**p**) is linear homogeneous. This modification results in a complicated expression for the substitution matrix. Nasse (1970) suggested a similar modification where

$$C = \hat{\gamma}^{-1} D \gamma \qquad \ldots(1.6.4)$$

with $\hat{\gamma}$: a diagonal matrix, $\hat{\gamma}_{ii} = \sqrt{p_i}$; γ is an n-vector with $\sqrt{p_i}$ as the i-th element; and D is some symmetric matrix. Both equations (1.6.3) and (1.6.4) imply non-additive direct utility functions.

The Linear Translog System (LTS) of Lau and Mitchell (1971) has P(p) the same as that in equation (1.6.2), but Q(p) is taken to be

$$Q(p) = \{ \sqrt{\exp \left(\sum_{k=1}^{n} \sum_{j=1}^{n} a_{kj} \log p_k \log p_j \right)} \} \quad (\prod_k p_k^{b_k}) \quad \ldots(1.6.5)$$

The implication of this generalisation on the separability property of the underlying direct utility function cannot be known easily. The demand equations underlying the LTS are of the form

$$p_i q_i = p_i c_i + (b_i + \sum_{j=1}^{n} a_{ij} \log p_j) \, (y - \sum_{j=1}^{n} p_j c_j) \qquad \ldots(1.6.6)$$

which collapses to the LES if $a_{ij} = 0$ for all i,j .

Blackorby, Boyce and Russell (1978) proposed a system which is basically a generalisation of the Brown-Heien (1972) model. The system is based on a two-level Gorman-polar form cost function. The indirect utility function is thus given by (1.6.1), where

$$P(p) = \sum_{s=1}^{S} \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \gamma_{sij} \sqrt{p_{si} \, p_{sj}} \; ; \quad \gamma_{sij} = \gamma_{sji} \qquad \ldots(1.6.7)$$

and

$$Q(p) = \left(\sum_{s=1}^{S} \alpha_s^{\sigma} \; Q_s \, (p_s)^{1 - \sigma} \right)^{\frac{1}{1-\sigma}} \; ; \quad \sigma > 0 \qquad \ldots(1.6.8)$$

with $Q_s(p_s) = (\sum_{i=1}^{n_s} \beta_{si}^{\sigma_s} p_{si}^{1-\sigma_s})^{\frac{1}{1-\sigma_s}}$; $\sigma_s, \beta_{si} > 0$.

Here the n commodities are partitioned into S groups, the s-th group containing n_s commodities (s = 1, 2, ..., S). The resulting system collapses to the Brown-Heien system when $\gamma_{sij} = 0$ for every s and $i \neq j$. When the partitions of goods are single-member partitions, the system reduces to the LES.

The cost function underlying any system of demand equations having quadratic Engel curves is, in general, of the form

$$C(V^*, p) = A_1(p) + \{ \frac{B_1(p)}{B_2(p) + k V^*} \} \quad ...(1.6.7)$$

when $A_1(p)$, $B_1(p)$ and $B_2(p)$ are some functions of prices (Gorman, 1980). Taking $A_1(p) = \sum_{i=1}^{n} p_i c_i$ and various alternative forms of $B_1(p)$ and $B_2(p)$, one can generate numerous quadratic generalisations of the LES. For example, $B_1(p) = \prod_i p_i^{b_i}$ and $B_2(p) = \prod_k p_k^{-b_k} \sum_{i=1}^{n} a_i p_i$ lead to the generalised Quadratic Expenditure System of Howe, Pollak and Wales (1979).

Blundell and Ray (1982) have proposed a non-separable generalisation of the LES referred to as the Non-Linear Expenditure System (NLES) based on a cost function of the form

$$C^{\alpha} \quad (V^{*}, p) = P(p) + Q(p)\, V^{*} \qquad \ldots(1.6.10)$$

which is of the generalised Gorman-polar form. For equation (1.6.10) Blundell and Ray specified

$$P(p) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, (p_i p_j)^{\alpha/2} \qquad \ldots(1.6.11)$$

and $$Q(p) = \prod_{j} p_j^{\,b_j} \qquad \ldots(1.6.12)$$

where $c_{ij} = c_{ji}$ and $\sum_{j} b_j = 1$. The resulting demand equations take the form

$$p_i q_i = y^{1-\alpha} \sum_{j=1}^{n} c_{ij} \, (p_i p_j)^{\alpha/2} + b_i \{ y - y^{1-\alpha} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk} (p_j p_k)^{\alpha/2} \} \qquad \ldots(1.6.13)$$

which for $c_{ij} = 0 \;\; (i \neq j)$ and $\alpha = 1$ reduces to the LES.

1.6.2 SNAM - a parametrically parsimonious generalisation

It may be noted that many of the generalisations considered above represent attempts to introduce more flexibility in respect of price effects, in particular, own-price effects, into the LES by introducing a large number of additional parameters. Such systems may have the desired flexibility in respect of the price effects, but are likely to pose considerable computational problems in actual empirical exercises affecting particularly the reliability of the estimated parameters. In this context, a non-additive generalisation of the LES proposed by Deaton (1976) is very attractive. This system has a reasonably small number of parameters. Yet it has been found to give better results compared to the LES. The cost function underlying this generalisation is also of the Gorman polar form and has

$$P(p) = \sum_k p_k c_k + \sum_{k=1}^{n} a_k p_k \log \left(\frac{\sum_{j=1}^{n} a_j p_j}{p_k}\right) \qquad \ldots(1.6.14)$$

and

$$Q(p) = \left(\sum_{k=1}^{n} b_k p_k^{\rho}\right)^{1/\rho} . \qquad \ldots(1.6.15)$$

The form of the resulting demand equations is

$$p_i q_i = p_i c_i + a_i p_i \log \left(\frac{\sum_{k=1}^{n} a_k p_k}{p_i}\right) + \frac{b_i p_i^{\rho}}{\sum_k b_k p_k^{\rho}} \times$$

$$\left(y - \sum_k p_k c_k - \sum_k a_k p_k \log \left(\frac{\sum_j a_j p_j}{p_k}\right) \right), \quad i = 1,2,\ldots, n.$$

$$\ldots(1.6.16)$$

In the special case where $a_i = 0$ for all i and $\rho = 0$, equation (1.6.16) collapses to the LES. Deaton considered two other special cases of equation (1.6.16) in his empirical exercise. The first special case, called the Simple Non-Additive Model (SNAM) is obtained by taking $\rho = 0$ in equation (1.6.16), which gives

$$p_i q_i = p_i c_i + a_i p_i \log \left(\frac{\sum_k a_k p_k}{p_i} \right) + \frac{b_i}{\sum_k b_k} \left(y - \sum_k p_k c_k - \sum_k a_k p_k \log \left(\frac{\sum_j a_j p_j}{p_k} \right) \right) \quad \ldots(1.6.17)$$

and the other special case, called the Variant model, has $\rho=0$, $c_i=0$ for all i, giving

$$p_i q_i = a_i p_i \log \left(\frac{\sum_k a_k p_k}{p_i} \right) + \frac{b_i}{\sum_k b_k} \left(y - \sum_k a_k p_k \log \left(\frac{\sum_j a_j p_j}{p_k} \right) \right) \quad \ldots(1.6.18) .$$

In the empirical exercise, SNAM performed much better than the LES and also the Variant model both in terms of explanatory power, and the quality of the estimated expenditure and own-price elasticities.

It may be mentioned here that in Chapter II of the present thesis, a generalisation of the SNAM has been suggested. The budget share parameters are also made price sensitive in this model. This generalised SNAM is not a member of the family (1.6.16) and the functional form of the marginal budget shares in this model is different from that in the general model (1.6.16) of Deaton (1976).

1.7 Treatment of demographic characteristics in complete demand systems

Household characteristics such as size, age-sex composition, education level, occupation and other characteristics of household members are known to be important determinants of household consumption pattern. Usually, these factors are duly taken into account in the analysis of cross-sectional household budget data (*vide* Prais and Houthakker, 1955; Friend and Jones, 1960; Deaton, 1980). In the analyses of time-series data on aggregate or average consumption, on the other hand, these characteristics have often been relatively ignored. However, with the availability of large-scale computational facilities, analysis of time-series of ungrouped cross-sectional household budget data has become feasible. Such data are ideally suitable for the analysis of variation in consumption patterns arising out of variation in income and prices, and particularly in household characteristics. Indeed, the need for estimating complete demand systems incorporating household characteristics on the basis of time-series of cross-sectional data has often been recognised (e.g., Pollak and Wales, 1980).

To formulate complete demand systems suitable for this type of investigations, one has to bring the household characteristics into the analytical framework of the static utility-theoretic model discussed above. For household demographic characteristics, viz., size and age-sex composition, there have been a fair amount of progress in this direction. In what follows, these developments are discussed briefly.

1.7.1 Demographic scaling and translation

Barten (1964) proposed a theoretical framework for incorporating household composition into demand equations. In this framework, for each commodity, he defined effective household size that depends on the household composition vector, i.e., $m_i = m_i (a_1, a_2, \ldots, a_k)$, $i = 1, 2, \ldots, n$, where a_j : number of members of the j-th type (age-sex category, say) in the household, and i is the commodity subscript. The effective per capita consumption is then obtained by scaling the household consumption q_i by the corresponding effective household size m_i , viz., $q_i^* = \frac{q_i}{m_i}$, where q_i^* represents effective per capita consumption of commodity i .

The household's direct utility function is defined in terms of q_i^*'s as $V = V(q_1^*, q_2^*, \ldots, q_n^*)$. In view of these adjusted quantities the budget constraint is modified as $y = \sum_i p_i^* q_i^*$, where $p_i^* = p_i m_i$. Thus, in this set-up, the effect of variation in household composition is supposed to work exclusively through the prices, as changes in household composition result in an alteration in the relative price structure and thus affect the consumption basket. The resulting Marshallian demand functions corresponding to this demographic scaling are of the form

$$q_i^* = f_i \; (p_1^*, p_2^*, \ldots, p_n^* , y) . \qquad \ldots(1.7.1)$$

Gorman (1976) noted the limitation of the Barten model, where the household composition works through price effects exclusively. He proposed the following generalisation of the Barten model of demographic scaling. Consider the cost function underlying Barten's approach, viz., $C = C \; (p_1^*, p_2^*, \ldots, p_m^*, V^*)$ where C is the minimum cost of attaining a stipulated utility level V^*. Gorman suggested that a fixed cost component, exclusively dependent on prices and household composition should be added to the cost function. Thus, if $\sum_i p_i \alpha_i$ is the fixed cost component where α_i's are functions of the household composition vector $(a_1, a_2, \ldots, a_k)$, then the cost function would be

$$C = \sum_i p_i \; \alpha_i + C \; (p_1^*, p_2^*, \ldots, p_n^*, V^*) \; . \qquad \ldots(1.7.2)$$

Given a specification of the variable cost component $C \; (p_1^*, p_2^*, \ldots, p_n^*, V^*)$ of Gorman's cost function, one would obtain the corresponding system of demand equations.

The concept of fixed cost referred to above has been later called 'demographic translation'. Pollak and Wales (1978) demonstrated the feasibility of generating theoretically consistent systems of demand equations that incorporate household composition through demographic translation. In fact, they have shown that corresponding to any theoretically consistent demand system $q_i = f_i\ (p_1, p_2, \ldots, p_n, y)$ there exists a corresponding theoretically consistent system of the form

$$q_i = \alpha_i + f_i^*\ (p_1, p_2, \ldots, p_n, y - \Sigma\ p_k\ \alpha_k) \qquad \ldots(1.7.3)$$

where $\alpha_i = \alpha_i\ (a_1, a_2, \ldots, a_k)$, $i = 1, 2, \ldots, n$, are the demographic translation parameters.

1.7.2 Empirical applications of demand systems with demographic characteristics

There have been some applications of the types of model described above. Barten (1964) modified the Rotterdam model through demographic scaling in his empirical exercise. Using the same procedure Parks and Barten (1973) modified the LES, and Muellbauer (1977) reported an empirical exercise based on some systems belonging to the PIGL and PIGLOG class. Lau, Lin and Yotopolous (1978) applied demographic scaling to the Indirect Translog model in analysing consumption data for Taiwan.

Howe, Pollak and Wales (1979) used the demographically translated LES and Quadratic Expenditure System (QES) in their empirical analysis of U.K. data. Pollak and Wales (1980) incorporated both demographic scaling and translation into the QES, the Basic Translog Model (BTM) of Christensen, Jorgenson and Lau (1975), and a Generalised Translog Model (GTL). In all these studies, significant composition effects have been observed which suggests that the omission of the household demographic factors may considerably affect the estimates of pure price and income effects.

1.8 Dynamic considerations in demand analysis

The static demand systems discussed above may in some cases be inadequate as a framework for analysing time-series consumption data, as these systems fail to capture the dynamic elements that may be present in such data. Such an inadequacy gets reflected in the presence of serial correlation of the random disturbances of the stochastic versions of these systems actually used in estimation.

The sources of such dynamic factors may be inherent in the behaviour of a real-life consumer. There are broadly two reasons why a consumer would bring in dynamic considerations in his consumption decisions. First, if expenditures on durables and habit-forming non-durables are included in the consumer's budget, decisions about expenditure on such items may be directly and indirectly affected by the (physical or psychological) stocks of such goods possessed, the stream of income earned and prices in past periods, and also by expectations about future incomes and prices. Even when durable goods and the phenomenon of habit formation for non-durables are not considered, the consumer's decision problem would be a dynamic one, if the consumer takes into account his possible future well-being in deciding his current consumption pattern.

1.8.1 Approaches to the study of dynamic consumer behaviour

The types of possible dynamic factors mentioned above cannot always be captured through introducing time trends in an otherwise static system of demand equations. There have been broadly two approaches to studying the dynamics of consumer behaviour on the basis of time-series consumption data.

The first approach is essentially a single commodity approach in which consumption of a durable or a habit-forming non-durable commodity is modelled through bringing in explicitly the stock of such commodity and its change from period to period. The partial adjustment models of Chow (1957, 1960), Stone and Rowe (1957, 1960) and Houthakker and Taylor (1970; Ch. 1) are examples of this approach.

The other approach may be called the system approach in the sense that, under this, complete systems of dynamic demand equations are evolved as a comprehensive description of the dynamic aspects of consumer behaviour. These systems of dynamic demand equations can be interpreted in terms of utility maximising consumer behaviour and are thus utility-theoretic in nature. These models may further be classified into two groups. One group of models are called 'myopic' models as these models describe the dynamics of consumers' problem as one of maximisation of current period utility in the presence of dynamic factors such as stock-adjustment, habit formation etc., without any concern for the utility in future periods. The other models, viz., the 'intertemporal' or 'rational' models are those based on maximisation of a utility-stream over a given planning period or the life-cycle of the consumer. These models may be considered to provide a fuller description of the dynamic aspects of consumer behaviour.

1.8.2 Myopic models of dynamic consumer behaviour

Boulding (1950) and later Cramer (1957) attempted to construct a dynamic model of consumer behaviour in the utility-theoretic set-up. They considered a current utility function, defined over the physical stocks possessed of the different commodities, to be maximised subject to a suitable wealth constraint. Once the optimal stocks are worked out, the current purchases of different commodities are worked out through commodity-specific identities relating changes in stocks to current purchases and description.

Houthakker and Taylor (1970; Ch. 5) broadened the scope of Boulding-Cramer type models. They introduced state-variables measuring physical stocks for durables and psychological stocks for non-durables into their analysis and defined the current utility function in terms of both the state-variables and current purchases. The system of demand equations is then obtained maximising the current utility function subject to the budget constraint and the set of commodity-specific stock-adjustment identities. This model has been used in a number of empirical analyses relating to different countries (Houthakker and Taylor, 1970; Phlips, 1983; Taylor and Weiserbs, 1972) and has been found to give plausible results. In these applications the Houthakker-Taylor model based on a quadratic utility function has been used.[10/] Phlips (1972) considered the dynamised Stone-Geary utility function (by introducing stocks explicitly into the Stone-Geary utility function) to formulate a dynamic LES model.

Apart from the models mentioned above, there have been some attempts to dynamise an otherwise static system of demand equations,

---

[10/] A slightly different discrete model was estimated by Mattei (1971).

particularly the LES, by introducing habit-formation. Stone's (Stone, 1966; and also Stone, Brown and Rowe, 1964) own effort to dynamise the LES by introducing time trends in the parameters has been found to improve the empirical performance of the LES. However, such modification would not lead to a dynamic model in the proper sense of the term. Pollak (1970a) and Pollak and Wales (1969) introduced habit-formation into the LES through making the committed consumption parameters of the LES linear functions of lagged consumption defined in alternative ways. A similar modification of the QES has been done in Howe, Pollak and Wales (1979). On the whole, the empirical results corresponding to such dynamisation indicate significant improvements over those of the corresponding static versions.

1.8.3 Dynamic demand systems based on intertemporal utility maximisation

Even if the phenomena of habit-formation and durability of consumption goods are absent, the consumer may have to adjust his consumption decisions continuously over time, when current consumption decisions are subjected to planning stretched over more than one period. Indeed, the life-cycle hypothesis stipulates that a consumer considers the problem of maximisation of a utility function defined over the stream of consumption flows for a stipulated planning period, subject to constraints on corresponding anticipated flows of income and prices. Such an approach is generally known as the 'rational approach to dynamic consumer behaviour' and consequently the dynamic demand systems resulting from this are known as 'rational demand systems'.

Under the rational approach the consumer is assumed to maximise his utility functional, i.e., a discounted sum of the flow of utility over the planning period subject to a wealth constraint based on the flow of income. This involves a process of continuous period to period replanning (viz., strategy of consistent planning) or a once-for-all planning (viz., the strategy of pre-commitment). Strotz (1956) demonstrated that for consistent planning the discounting of the flow of utility should follow specific rules. The existence of the dynamic utility function implicit in the rational approach is also not always guaranteed. The conditions for existence of such a utility function was examined by Koopmans (1960) and Koopmans, Diamond and Williamson (1964).

A system of demand equations based on the rational approach is rather complicated in form and thus cannot be used conveniently in empirical analysis. A number of simplifying assumptions are generally made so that the resulting system could be estimable. For example, Phlips and Spinnewyn (1982), who applied such a model to the U.S. data, considered models based on decentralisation of the intertemporal decision problem. By considering decentralisation with unchanged tastes and decentralisation with changing tastes over time, they established an equivalence between the so-called 'myopic' models and the 'rational' ones.

Chapter II

A modification of SNAM and A new system of demand equations

2.1 Introduction

This chapter proposes two systems of demand equations that may be classified as theoretically consistent systems being based on well-defined preference structures. The first is based on a marginal modification of the Simple Non-Additive Model (SNAM) of Deaton (1976) that was proposed as a generalisation of Stone's Linear Expenditure System (LES) especially to overcome the additivity property of the direct utility function underlying the latter. The other is based on Muellbauer's notion of consistent aggregation (Muellbauer, 1975). To be precise, this latter system represents an attempt to build up a static system of demand equations within the framework of Price Independent Generalised Linearity (PIGL) by bringing in prices explicitly into the system of Leser-type budget share equations implied by the PIGL. The two proposed systems are somewhat competing in the sense that they involve the same number of parameters. However, on a priori considerations, the system based on the PIGL has an advantage over the modified SNAM in that the former can allow for non-linear Engel curves which the latter cannot.

The plan of this chapter is as follows: section 2.2 introduces the modified SNAM; section 2.3 proposes the new system based on PIGL; and finally, section 2.4 presents the summary and conclusions.

2.2. A suggested modification of the SNAM

As already discussed, the additivity of the direct utility function underlying the LES heavily constrains the non-compensated own-price elasticities implied by this system. In order to overcome this limitation of the LES, Deaton (1976) suggested a modification of the LES - the SNAM - by making the 'committed quantities' of the LES functions of prices in a specific manner. We propose here a further modification of the SNAM by making the 'marginal budget share' parameters functions of prices as well.

Our proposed modification of the SNAM takes the following form

$$p_i q_i = p_i \gamma_i(p) + \beta_i(p) \left\{ y - \sum_{j=1}^{n} p_j \gamma_j(p) \right\} \; ; \; i=1,2,\ldots, n \quad \ldots (2.2.1)$$

with $y \geq \sum_{j=1}^{n} p_j \gamma_j(p)$, and $\sum_{i=1}^{n} \beta_i(p) = 1$ in view of the adding up restriction, where

$$\gamma_j(p) = c_j + a_j \log \left(\frac{p_j}{\pi_1} \right) \quad \ldots . (2.2.2)$$

$$\pi_1 = \frac{\sum_{k=1}^{n} a_k p_k}{\sum_{k=1}^{n} a_k} \quad \ldots . (2.2.3)$$

$$\beta_j(p) = b_j + d_j \log \left(\frac{p_j}{\pi_2} \right) \quad \ldots . (2.2.4)$$

$$\text{and } \pi_2 = \prod_{k=1}^{n} (p_k)^{\frac{d_k}{\sum_j d_j}} \quad \ldots . (2.2.5)$$

Here, y, $(p_1, p_2, \ldots, p_n)$ and $(q_1, q_2, \ldots, q_n)$ stand as before for the total expenditure/income, the prices and the quantities of the n commodities, respectively. The system (2.2.1) - (2.2.5) is defined on the basis of 4n parameters, viz., a_j, b_j, c_j, d_j (j=1, 2, ... , n) of which 4n - 1 parameters are independent (one of the b_j's become dependent on the other b-parameters in view of the adding-up restriction).[1] It may be noted that this system reduces to Deaton's SNAM specification if $d_i=0$ for all i . In what follows we shall refer to the original SNAM, which is a special case of this system proposed here, as SNAM 1 and the full specification as SNAM-2 .

The cost function underlying SNAM-2 takes the form

$$C\ (V^*,\ p) = \sum_{k=1}^{n} p_k\, \gamma_k(p) + V^* \prod_{k=1}^{n} p_k^{\beta_k^*(p)} \qquad \ldots \quad (2.2.6)$$

where $\beta_k^*(p) = \tfrac{1}{2}\{\, b_k + \beta_k\ (p)\,\}$.[2]

The form of the corresponding indirect utility function is thus

$$V^* = \frac{y - \sum_{k=1}^{n} p_k\, \gamma_k\ (p)}{\prod_{k=1}^{n} p_k^{\beta_k^*\ (p)}}\ ,\quad \sum_{k=1}^{n} \beta_k^*\ (p) = 1 \qquad \ldots \quad (2.2.7)$$

---

[1] Note that the adding-up restriction $\sum_{j=1}^{n} \beta_j\ (p) = 1$ imposes restriction on $\sum_{j=1}^{n} b_j$ only, viz., $\sum_{j=1}^{n} b_j = 1$, since $\sum_{j=1}^{n} d_j \log\left(\frac{p_j}{\pi_2}\right)$ is equal to 0 by construction.

[2] Vide Appendix A, section A.2, for the derivation of the cost function (2.2.6).

and the cost function is clearly a member of the family of Gorman polar form [equation (1.5.2)] with

$$P(p) = \sum_{k=1}^{n} p_k \ \gamma_k \ (p) \quad \quad . . . \quad (2.2.8)$$

and

$$Q(p) = \prod_{k=1}^{n} p_k^{\ \beta_k^{*}(p)} \quad \quad . . . \quad (2.2.9)$$

Let us next consider the expressions for the various elasticities implied by the system.[3/] The Engel or expenditure elasticity is of the form

$$\eta_i = \frac{\beta_i \ (p)}{w_i} \ ; \ i=1, 2, \ldots , n . \quad \quad . . . \quad (2.2.10)$$

where w_i is the budget share for item i (i=1, 2, ... , n). Thus, given $w_i > 0$ in all income and price situations, item i is a luxury when $\beta_i(p) > w_i$, a necessary when $0 < \beta_i(p) < w_i$ and an inferior good when $\beta_i(p) < 0$.

The non-compensated price elasticities are given by

$$\mu_{ij} = - \frac{1}{p_i q_i} [\ \theta_i(p) \ \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k + p_j \ \beta_i \ (p) \ \gamma_j \ (p)$$

$$+ \{\beta_i(p) \ \delta_{ij} - d_j \ (\delta_{ij} - \frac{d_i}{\sum_k d_k})\} \ \{ y - \sum_j p_j \ \gamma_j \ (p)\}] \quad ... \ (2.2.11)$$

3/ Vide Appendix A, Section A.3, for derivation of the expressions for expenditure and price elasticities.

for any pair of items i and j , where $\theta_i(p) = \dfrac{a_i\, p_i}{\sum_{k=1}^{n} a_k\, p_k}$ and δ_{ij} is the Kronecker delta.

The corresponding expression for the compensated price elasticities are obtained from the relation $\pi_{ij} = \mu_{ij} + w_j\, \eta_i$.

Thus,

$$\pi_{ij} = -\frac{1}{p_i q_i} \Big[\theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k + \{ \beta_i(p) \, (\delta_{ij} - \beta_j(p)) - d_j \, (\delta_{ij} - \frac{d_i}{\sum_k d_k}) \} \{ y - \sum_j p_j \, \gamma_j(p) \} \Big] \quad \ldots (2.2.12)$$

for any pair of items i and j .

The system of demand equations (2.2.1) - (2.2.5) proposed above is theoretically consistent in the sense that, by construction, it meets the theoretical restrictions, viz., the adding-up property, symmetry and homogeneity. However, the concavity of the cost function (2.2.6) cannot be guaranteed through parametric restrictions and has to be checked empirically.

One may consider special cases of this modified SNAM for empirical applications. The special cases emerge from specific parametric restrictions. For example, when a_j and d_j (j=1, 2, ... , n) are dropped from the specification, the system reduces to the LES. If only

d_j (j=1, 2, ..., n) are dropped, one gets Deaton's SNAM-1 specification. Another generalisation of the IES which introduces price flexibility through marginal budget shares only may be obtained by dropping only a_j (j=1, 2, ..., n) from the full specification.

2.3 A new system of demand equations based on PIGL

In this section a new system of demand equations based on Muellbauer's notion of Price Independent Generalised Linearity (PIGL) is proposed (Muellbauer, 1975). As already mentioned, PIGL implies a specific form of budget share equations, viz.,

$$w_i = a_i\ (p) + b_i\ (p)\ y^{\varepsilon} \quad , \quad \varepsilon \neq 0 \ . \qquad \ldots \quad (2.3.1)$$

and the corresponding logarithmic limiting form called Price Independent Generalised Log linear (PIGLOG) form, i.e.,

$$w_i = a_i\ (p) + b_i\ (p) \log y \ , \quad \varepsilon = 0 \ . \qquad \ldots \quad (2.3.2)$$

where w_i : budget share for item i (i=1, 2, ..., n), y : income/total expenditure, a_i (p) and b_i (p) are independent functions of prices

$p = (p_1, p_2, \ldots, p_n)$ satisfying $\sum_{i=1}^{n} a_i\ (p) = 1$ and $\sum_{i=1}^{n} b_i\ (p) = 0$,

and ε is a parameter that controls the non-linearity of the income responses. The genesis of the PIGL and PIGLOG class of demand systems, viz., Muellbauer's notion of consistent aggregation is discussed briefly below.[4]

For a system of budget share equations $w_i(y, p)$, i=1, 2, ..., n, consistent aggregation is defined as the existence of a representative income level y^* such that $\bar{w}_i = w_i\ (y^*, p)$ for all i, $\bar{w}_i$ being the observed average budget share of item i . A necessary and sufficient condition for the existence of y^* is that each pair of budget share functions are linearly related - a phenomenon that has been called Generalised Linearity (GL). However, under GL, y^* will be a function of the income distribution as also of the prices, and forcing y^* to be independent of prices is equivalent to the budget share equations being of the PIGL or PIGLOG form. Muellbauer also characterised the cost function underlying the budget share equations (2.3.1) and (2.3.2) to be of the generalised Gorman polar form, viz.,

$$C\ (V^*, p) = [\{ a(p)\}^{-\varepsilon} + V^* \{b(p)\}^{-\varepsilon}]^{-\frac{1}{\varepsilon}}, \quad \varepsilon \neq 0 \quad .. \ (2.3.3)$$

and $$C\ (V^*, p) = \{ H(p) \}^{V^*} B(p), \quad \varepsilon = 0 \quad ... \ (2.3.4)$$

where a(p), b(p) and B(p) are homogeneous functions of degree one, H(p)

---

[4] Vide Chapter I, Section 1.5, for a fuller description of this.

is a homogeneous function of degree zero, and V^* represents the specified level of utility.

The forms (2.3.1) and (2.3.2) of budget share equations, having the requisite theoretical basis, have been taken as the starting point for building up empirical demand systems. For instance, the Almost Ideal Demand System (AIDS) proposed by Deaton and Muellbauer (1980a) is based on (2.3.2). Here, we propose the following algebraic forms for (2.3.1) and (2.3.2).

$$w_i = \frac{p_i\ \gamma_i(p)}{\alpha(p)} + \beta_i(p)\ \left[\frac{\left\{\frac{y}{\alpha(p)}\right\}^{\varepsilon} - 1}{\varepsilon}\right] \quad ; \quad \varepsilon \neq 0 \quad \ldots (2.3.5)$$

and $$w_i = \frac{p_i\ \gamma_i(p)}{\alpha(p)} + \beta_i(p) \log \left(\frac{y}{\alpha(p)}\right) ; \qquad \varepsilon = 0 \quad \ldots (2.3.6)$$

$$i=1, 2, \ldots, n.$$

where $\alpha(p) = \sum_{k=1}^{n} p_k\ \gamma_k(p)$ and $\gamma_i(p)$ and $\beta_i(p)$ are functions of prices such that $\sum_{k=1}^{n} \beta_k(p) = 0$ and $\sum_{k=1}^{n} p_k\ \gamma_k(p) > 0$ for all prices.

It may be noted that if $\gamma_k(p)$ and $\beta_k(p)$ for all k are assumed to be independent of p, then (2.3.5) and (2.3.6) specialize to systems that are very similar in appearance to the LES. However, $\gamma_k(p)$ and $\beta_k(p)$ should preferably be treated as functions of prices so that the systems could allow a reasonable degree of price flexibility. The specific functional forms for $\gamma_k(p)$ and $\beta_k(p)$ are proposed to be the same as those given in (2.2.2) - (2.2.5) above, viz.,

$$\gamma_j (p) = c_j + a_j \log \left(\frac{p_j}{\pi_1} \right),$$

$$\pi_1 = \frac{\sum_{k=1}^{n} a_k p_k}{\sum_{k=1}^{n} a_k},$$

$$\beta_j(p) = b_j + d_j \log \left(\frac{p_j}{\pi_2} \right)$$

and

$$\pi_2 = \prod_{k=1}^{n} (p_k)^{\frac{d_k}{\sum_j d_j}},$$

The system as described by (2.3.5), or (2.3.6) and (2.2.2) - (2.2.5) thus have $4n$ basic parameters, viz., a_k, b_k, c_k, d_k; $k=1, 2, \ldots, n$ (and also ε in case of (2.3.5)). In view of the adding-up requirement, $\sum_{k=1}^{n} b_k = 0$, and thus there are $4n-1$ ($4n$) independent parameters for the PIGLOG (PIGL) system.[5] Now, in so far as different items would need different values of ε, the parameter that controls the non-linearity of income responses, in the budget share equation, if one forced ε to be the same across different budget share equations, the resulting ε might be closer to the modal value of ε, possibly zero. In view of this, in what follows, we shall consider only the logarithmic limiting case of the proposed system as defined by (2.3.6) and (2.2.2) - (2.2.5).

The explicit form of the cost function underlying the logarithmic system is

$$C(V^*, p) = \alpha(p) \exp\left(V^* \prod_{j=1}^{n} p_j^{\beta_j^*(p)} \right) \qquad \ldots (2.3.7)$$

where

$$\alpha(p) = \sum_{j=1}^{n} p_j \gamma_j(p) \; ; \quad \beta_j^*(p) = \tfrac{1}{2}(b_j + \beta_j(p))$$ [6]

---

[5] The adding-up requirement for (2.3.5) and (2.3.6), viz., $\sum_{k=1}^{n} \beta_k(p) = 0$ is equivalent to the restriction $\sum_{k=1}^{n} b_k = 0$, since the remaining term in $\beta_k(p)$ adds up to zero automatically as pointed out in footnote 1.

[6] The cost function underlying the PIGL system (2.3.5) and (2.2.2) - (2.2.5) is $C(V^*, p) = \alpha(p)\left(1 - \varepsilon V^* \prod_{j=1}^{n} p_j^{\beta_j^*(p)}\right)$, $\varepsilon \neq 0$. For derivation of the cost functions see Appendix A, section A.2.

The indirect utility function corresponding to (2.3.7) is

$$V^* = \frac{\log Z}{\prod_{j=1}^{n} p_j^{\beta_j^*(p)}}, \quad Z = \frac{y}{\alpha(p)}, \quad \alpha(p) > 0, \sum_k \beta_k^*(p) = 0 \qquad \ldots (2.3.8)$$

Thus, given p, V^* is monotonically non-decreasing in y, if and only if $y > \alpha(p)$. Therefore $\alpha(p)$ may be interpreted as the cost of attaining a level of satisfaction for which the ordinal utility index is assigned zero value.

Turning to the expressions for the elasticities implied by the system, we have the expenditure/Engel elasticity given by

$$\eta_i = 1 + \frac{\beta_i(p)}{w_i} \qquad \ldots (2.3.9)$$

Thus, given $w_i > 0$, an item would be classified as a luxury item if $\beta_i(p) > 0$ and a necessary/inferior item if $\beta_i(p) < 0$ in a given price situation. Also, given the prices, η_i falls as y increases for necessary/inferior items.

The expression for compensated price effects for any pair of items i and j is given by

$$S_{ij} = -\frac{y}{p_i\, p_j} [\, \theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \, \Phi(p) + \alpha_i(p)\, \alpha_j(p) - w_i w_j$$

$$+ \{\delta_{ij}\, \beta_i(p) - d_i \,(\delta_{ij} - \frac{d_j}{\sum_k d_k}) - \beta_i(p)\, \beta_j(p) \} \log Z \,] \qquad \ldots (2.3.10)$$

which by construction of the system is symmetric.

Here

$$\left.\begin{aligned} \theta_i(p) &= \frac{a_i\, p_i}{\sum\limits_k a_k\, p_k} \\ \alpha_i(p) &= \frac{p_i\ \gamma_i\ (p)}{\sum\limits_j p_j\ \gamma_j\ (p)} \end{aligned}\right] \quad i = 1, 2, \ldots, n .$$

$$\Phi(p) = \frac{\sum\limits_j p_j\, a_j}{\sum\limits_j p_j\ \gamma_j\ (p)}$$

and δ_{ij} is Kronecker delta .

Next, the non-compensated and compensated price elasticities (denoted μ_{ij}, and π_{ij} , respectively) between any pair of items i and j are of the form

$$\mu_{ij} = -\frac{1}{w_i}\ [\ \theta_i(p)\ \{\theta_j(p) - \delta_{ij}\}\ \ \Phi(p) + \alpha_i(p)\,\alpha_j(p) + \beta_i(p)\ \alpha_j(p)$$

$$+ \{\delta_{ij}\ \beta_i(p) - d_i\ (\delta_{ij} - \frac{d_j}{\sum\limits_k d_k})\ \}\quad \log Z\ ] \qquad \ldots (2.3.11)$$

and

$$\pi_{ij} = \mu_{ij} + w_j\, \eta_i$$

$$= -\frac{1}{w_i}[\ \theta_i(p)\ \{\ \theta_j(p) - \delta_{ij}\ \}\quad \Phi(p) + \alpha_i(p)\quad \alpha_j(p) - w_i w_j$$

$$+ \{\delta_{ij}\ \ \beta_i(p) - d_i\ (\delta_{ij} - \frac{d_j}{\sum\limits_k d_k}) - \beta_i(p)\ \beta_j(p)\ \}\ \log Z\ ] \ldots (2.3.12)$$

By construction, the system described above satisfies the homogeneity, symmetry and adding-up restrictions. However, the concavity of the cost function, or equivalently, the negative semi-definiteness of the matrix of compensated price effects cannot be ensured through parametric restrictions and can only be checked empirically.

As in case of the modified SNAM set-up described in section 2.2, one may consider special cases of the system proposed above for empirical applications. The possible special cases of the system are as follows:

i) The a_j and d_j parameters are dropped altogether in which case the system is defined on 2n-1 independent parameters and comes close to the LES in appearance.

ii) Only the d_j parameters are dropped, in which case the income response coefficient is forced to be constant in all price situations. Thus this is a 3n-1 independent parameter specification.

iii) Another 3n-1 independent parameter specification is obtained when only the a_j parameters are dropped, in which case the $\gamma_i(p)$'s become constants. This specification thus rules out substitution possibilities for the poorest consumers.

In our subsequent empirical analysis we have used the full specification and the three special cases (i), (ii) and (iii) of the model in Chapter V. These specifications have been called Variant 3, Variant 1, Variant 2 and Variant 4 respectively.

2.4 Summary and conclusions

This chapter provides the theoretical foundation of some of the empirical demand analyses reported in the present dissertation. Some well-known models for consumer demand have been tried in these exercises, besides two new models introduced by the present author in this chapter. The first of these is based on a modification of Deaton's SNAM specification - referred to here as SNAM 2. The other specification proposed here (referred to as the Variant) is based on Muellbauer's notion of Price Independent Generalised Linearity/Loglinearity. The basic motivation behind this theoretical endeavour has been to evolve empirical demand systems that are reasonably flexible in respect of price and income responses, yet parsimonious in terms of number of parameters involved. These considerations have obvious relevance so far as empirical demand analysis based on time series data is concerned.

The modified SNAM set-up proposed here has the advantage that it can incorporate price flexibility in various degrees into the Linear Expenditure System - a system that is so popular for its structural simplicity. The Variant set-up (i.e., the other system proposed here), on the other hand, allows for non-linear Engel curves along with some amount of price flexibility. This is likely to be useful particularly in cases where one works with a time series of cross sectional consumption data. The Variant models, being based on PIGLOG, also admits consistent aggregation in Muellbauer's sense, and thus may be used to model macro demand functions with a consistent micro-economic theoretical foundation.

As already mentioned, the systems proposed here have relatively small number of parameters compared to other flexible demand systems like the Rotterdam model, the AIDS or the Translog systems, that have demonstrated superiority over other systems in respect of goodness of fit in actual empirical exercises. Actually, these flexible systems may at times turn out to be wasteful in terms of the number of degrees of freedom used up for estimation. This may affect the quality of the individual parameter estimates and consequently the quality of the elasticity estimates, particularly the price elasticities obtained. Indeed, there is a serious question of trade-off between the flexibility of a system and the quality of estimates of price elasticities - and in actual empirical applications the flexible systems may turn out to be more flexible than really needed for a reasonably good explanation of the given data. From this point of view, the systems proposed in this chapter may be of some relevance in applied demand analysis.

Chapter III

Data and estimation procedures

3.1 Introduction

This chapter briefly describes the sources and nature of the data used and the estimation procedures applied in the empirical analyses of consumer behaviour in India, the results of which are reported in the following chapters.

Data for the empirical analyses (i.e., estimation of some alternative forms of complete demand systems) have been compiled from two sources. The consumption data, viz., the time-series of consumer expenditure on broad groups of items as also on all-items have been compiled from the reports on consumer expenditure surveys published from time to time by the National Sample Survey Organisation (NSSO), Government of India, which conducted household consumer expenditure surveys on a continuing basis during 1950 to 1974 and thereafter at intervals of 4 or 5 years. The price data used in the analysis are basically the commodity-specific wholesale price indices compiled and published regularly by the Office of the Economic Adviser, Government of India - the authority responsible for regular collection and compilation of wholesale prices of a large number of commodities at all-India level. In the absence of appropriate retail/consumer prices or price indices, these commodity-specific wholesale price indices have been suitably combined by using expenditure weights based on consumer expenditure estimates thrown up by the NSSO to form item-group-specific price indices. These price indices have been used in the estimation of demand systems.

The alternative demand systems that have been estimated include the various specifications of the SNAM and Variant models (Variants 1, 2 and 3) described in the previous chapter, and also the AIDS. In the absence of cross-equation parametric restrictions (such as the symmetry restriction) AIDS can be estimated by single-equation OLS regression technique. All the other systems considered here are, however, non-linear in parameters, and therefore, have been estimated by a suitable Non-Linear Full Information Maximum Likelihood (NLFIML) method.

In what follows, the nature of consumption and price data, used and the estimation procedure employed in the present analysis are described. Section 3.2 and 3.3 briefly discuss the consumer expenditure data and price data, respectively, and section 3.4, the final section, presents an outline of the NLFIML estimation procedure.

3.2 NSS consumption data

The National Sample Survey (NSS) is a multipurpose socio-economic enquiry of all-India coverage carried out on a continuing basis in the form of successive rounds. The duration of a round has varied in the past from a few months to a complete year. The enquiry on consumer expenditure was a regular feature of the NSS beginning with the first round conducted during October-1950 - March 1951 upto the 28th round carried out during October 1973 - June 1974. Subsequently, however, it has been conducted at intervals of 4 to 5 years.

The data were collected by interview method from nationwide samples of households. Probability sampling was followed for selecting the households, and the sampling design was stratified, multistage (households being the final stage units) with provision for two or more interpenetrating subsamples, each of which throws up independent and equally valid estimates of the population characteristics. Rural and urban sectors were always distinguished and separate estimates were prepared for these two sectors; these estimates are available also for the rural and urban sectors of different states of India, at least for all rounds starting from the 13th round (September 1957 - May 1958).

Expenditure information collected from sample households related to a moving reference period of 30 days preceding the date of enquiry.[1/] To counteract the seasonal bias in the estimates that might result from the use of this moving reference period, a subround arrangement was followed and the interviews of sample households were equally distributed over these subrounds (seasons).

Estimates for the broad patterns of consumer expenditure are available in published form in NSS reports entitled 'Tables with notes on Consumer Expenditure' from the second round onwards separately by size classes of monthly per capita total consumer expenditure on all items (PCE) along with estimated percentages of population in each PCE class separately for the rural and the urban sectors of the country. Detailed tabulations covering long lists of items are also available for some of the rounds. Such tables are published separately for the rural and urban sectors of the individual states of India also. A typical NSS table on consumer expenditure is reproduced in Table 3.1 (this table is taken from the report for the 28th round enquiry).

The present analysis is based on consumption data thrown up by the NSS enquiries during 7th (October 1953 - March 1954) to 28th

---

1/ Actually, there had been some experimentation with other reference periods upto the 6th round, and two moving reference periods (last 30 days and last 365 days) were used for item-groups like 'clothings' in the rounds following the 28th.

Table 3.1 Consumer expenditure (Rs. 0.00) per person for a period of 30 days by broad groups of items and by monthly percapita expenditure classes

All India : rural

No. of Sample villages : 8680

Items	monthly percapita expenditure class in rupees														
	0-13	13-15	15-18	18-21	21-24	24-28	28-34	34-43	43-55	55-75	75-100	100-150	150-200	200 and above	all classes
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1. cereals	6.15	7.97	10.21	11.61	13.10	15.18	17.48	20.34	23.81	27.50	31.84	33.78	34.63	43.71	23.00
2. gram	0.04	-	0.00	0.07	0.05	0.08	0.17	0.17	0.22	0.28	0.51	0.64	0.66	0.82	0.25
3. cereal substitutes	0.39	0.80	0.31	0.61	0.50	0.41	0.39	0.34	0.40	0.33	0.31	0.35	0.31	0.47	0.38
4. pulse and products	0.16	0.29	0.37	0.52	0.62	0.90	1.15	1.55	1.93	2.55	3.32	3.80	4.79	6.45	1.99
5. milk and products	0.01	0.12	0.16	0.24	0.42	0.51	0.96	1.74	3.10	5.22	8.19	13.44	17.10	26.12	3.82
6. edible oil	0.21	0.38	0.57	0.60	0.70	0.93	1.11	1.54	1.91	2.51	3.13	4.11	5.16	7.17	1.99
7. meat, fish & egg	0.27	0.39	0.36	0.33	0.45	0.54	0.69	0.95	1.25	1.78	2.37	2.86	4.68	8.20	1.37
8. vegetables	0.55	0.68	0.70	0.82	0.94	1.10	1.31	1.63	1.57	2.44	3.04	3.51	3.98	6.53	2.00
9. fruits and nuts	0.17	0.10	0.08	0.16	0.18	0.19	0.22	0.34	0.45	0.71	1.20	2.08	3.01	5.23	0.61
10. sugar	0.10	0.20	0.27	0.30	0.42	0.59	0.74	1.14	1.54	2.25	2.97	4.41	5.40	8.41	1.67
11. salt	0.09	0.03	0.08	0.09	0.08	0.09	0.08	0.09	0.09	0.10	0.13	0.11	0.14	0.17	0.09
12. spices	0.41	0.42	0.56	0.73	0.08	0.81	0.97	1.14	1.36	1.58	1.83	2.30	2.90	4.50	1.35
13. beverages & refreshments	0.17	0.21	0.38	0.34	0.37	0.41	0.51	0.68	1.02	1.33	1.98	3.85	6.34	10.77	1.18
14. food : total	8.72	11.64	14.05	16.42	18.63	21.74	25.78	31.65	39.05	48.58	60.82	75.24	89.10	128.55	39.70
15. pan, tobacco & intoxicants	0.43	0.40	0.49	0.69	0.75	0.78	0.87	1.16	1.47	1.78	2.40	3.23	5.28	7.56	1.53
16. fuel and light	1.07	1.31	1.42	1.54	1.76	1.91	2.13	2.45	2.89	3.46	4.14	5.14	5.97	7.98	2.96
17. clothing	0.03	0.13	0.20	0.21	0.40	0.50	0.69	1.05	2.00	4.38	8.02	16.13	24.77	44.88	3.58
18. foot wear	-	-	0.01	0.01	0.01	0.02	0.04	0.05	0.12	0.27	-0.70	1.24	2.02	3.96	0.26
19. misc. goods & services	0.31	0.56	0.63	0.79	0.90	1.08	1.45	1.96	2.86	4.64	8.17	14.72	27.07	49.52	4.18
20. rents	-	-	-	0.01	0.00	0.00	0.00	0.00	0.03	0.08	0.15	0.30	0.45	1.19	0.05
21. taxes	-	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.13	0.03	0.16	0.01
22. durable goods	0.01	-	0.01	0.00	0.00	0.00	0.02	0.06	0.14	0.40	0.89	3.29	15.16	40.89	0.74
23. non food : total	1.85	2.40	2.76	3.25	3.82	4.29	5.20	6.73	9.51	14.98	24.49	44.18	80.75	156.14	13.31
24. total consumer expenditure	10.57	14.04	16.81	19.67	22.45	26.03	30.98	38.38	48.56	68.56	85.31	119.42	169.85	284.69	53.01
25. no. of sample households	55	49	145	272	412	749	1593	2760	3113	3168	1704	1025	255	167	15467

(October 1973 - June 1974) rounds (excluding the 26th round (June 1971 - June 1972) for which the estimates are not published.[2]

On the whole, the concepts, definitions and procedures followed in these different rounds of enquiry remained more or less uniform, so that the estimates of consumer expenditure from successive rounds of the NSS may be considered to form a time series of broadly comparable figures.[3]

Expenditure on nine broad groups of items have been considered in the present study. These nine broad groups of items, their commodity coverages and their shares in total consumer expenditure are presented in Table 3.2 . It may be pointed out that the observed budget shares for some of the broad item groups are indeed quite small.

2/ It may be mentioned that the moving reference period of last 30 days was uniformly followed for all items of the budget during these rounds.

3/ Some major procedural changes that have taken place over time may be noted however. These are (vide Chatterjee and Bhattacharya, 1975):

(1) Consumption out of home grown produce was imputed at local retail price upto 8th round, but from 9th round onwards, this imputation was made at ex-farm prices which was somewhat lower than the local retail prices, being exclusive of distributive margins; and

(2) A major change of questionnaire was made with effect from 19th round, when the earlier questionnaire became a part of the 'Integrated household survey' questionnaire which covered, in addition, the entire range of productive enterprises of the household, besides aspects like employment and unemployment. Also, during the 10th - 14th rounds, the 'consumer expenditure' questionnaire was extended in order to collect information on household receipts and disbursements as well.

Table 3.2 Commodity composition of nine broad item groups and their budget shares

All India : rural and urban

Broad item groups: Commodity number	Broad item groups: Title of commodity group	Commodities covered	budget share (%) (based on NSS 18th round estimates): all India rural	budget share (%) (based on NSS 18th round estimates): all India urban
(1)	(2)	(3)	(4)	(5)
1.	Cereals and cereal substitutes	Rice, wheat, jowar, bajra, maize, barley, small millets, ragi, tapioca, pea etc.	40.4	22.3
2.	Milk and milk products	Liquid milk, milk (condensed and powdered), ghee, butter , dahi, ghol, lassi and other milk products	7.2	9.4
3.	Edible oils	Vanaspati, mustard oil, coconut oil, gingelly oil, groundnut oil, linseed oil, refined oil, other edible oil and oil seeds	2.9	3.5
4.	Meat, fish, egg	Meat (goat meat, mutton, beef, pork, buffalo and other meat), egg, poultries, fish (fresh and dried), bird and others	2.7	3.3
5.	Sugar	Sugar (factory), khandsari sugar, gur (cane and others), candy, other sugar	3.3	3.3
6.	Other food	Tur, gram, moong, masoor, urad, salt, spices, fruits and nuts, beverages, refreshments, processed food, pickles, jams and jellies	13.5	17.8
7.	Clothing	Cotton (mill made, hand woven and khadi), wool, silk, rayon etc. including bedding and upholstery	8.1	6.3
8.	Fuel and light	Coke, coal, firewood, electricity, gas, dung cake, charcoal, kerosene, candle, matches and other lighting oil	6.6	6.3
9.	Other non food	Pan, tobacco and its products, drugs and intoxicants, amusements and sports, education, medicine, toilet articles, taxes, rents, sundry goods, furniture, services etc.	15.3	27.8

For example, items like 'edible oils', 'meat, fish, egg', and 'sugar', each account for roughly 3 per cent of the average consumer's budget in both rural and urban India. Yet, this grouping of items has been used presumably in consideration of expected differences in elasticities of demand. It may be mentioned that these elasticities, particularly the price elasticities for these items, are frequently needed in formulation of production and distributional policies in India.

We have constructed two sets of time series of estimated consumer expenditure for each broad group of items. First, a series of aggregate or average per capita expenditure (for all PCE levels taken together) have been built - we shall refer to this data set as the 'aggregate data'. The other set of time series data may be considered to be disaggregated in nature. This consists of three population-groupwise time-series of average per capita expenditures. The three population groups are the poorest 30 per cent, the middle 40 per cent and the richest 30 per cent of the population judged by the level of PCE. These population-groupwise estimates have been obtained through quadratic interpolation based on information given in tables like Table 3.1 .[4/] Thus, for example, average per capita

---

[4/] The percentage distribution of the population by PCE class is available from separate tables in the NSS reports.

expenditure on an item (x^*) for the bottom P^* per cent of population is calculated as follows. Let P_i be the estimated cumulative per cent population upto PCE class i , S_i the cumulative share of expenditure on the item upto PCE class i, and l the maximum of i such that $P_i \leq P^*$. A quadratic relation between S_i and P_i is fitted using the three observation-pairs (P_l, S_l), (P_{l+1}, S_{l+1}) and (P_{l+2}, S_{l+2}). The cumulative share of expenditure on the item S^* for the bottom P^* per cent population is thus obtained from the fitted quadratic relation. Finally, the average per capita expenditure on the item x^* for the bottom P^* per cent population is computed as $\frac{100\ S^*}{P^*} \cdot S$, S being the average expenditure on the item for all PCE classes taken together.

To conclude this section, some comments on the validity of the NSS-based consumption estimates are essential to help in judging their quality. There have been some discussions about the validity of NSS consumption estimates (Mukherjee, 1969a; Dandekar and Rath, 1971; Rudra, 1972; Mukherjee and Chatterjee, 1972; Srinivasan, Radhakrishnan and Vaidyanathan, 1974; Chatterjee and Bhattacharya, 1975; and Mukherjee and Saha, 1981). On the whole, the time series of NSS-based estimates of aggregate household consumer expenditure have been observed to agree fairly well with the corresponding aggregates derived from national income data, in spite of the fact that the two series have completely different data bases and are compiled by fully independent procedures. However, the pattern of household consumption expenditure as obtained from the NSS has been seen to be systematically different from that based on the product-flow estimates thrown up by the CSO, although the two aggregates of consumption expenditure tally reasonably. Some more specific complaints about the NSS budget data are :

(i) The NSS-based estimates of per capita consumption of cereals (one of the major heads of expenditure for the Indian households) are unduly high (vide Kansal, 1965; Chatterjee and Bhattacharya, 1975).

(ii) The NSS budget enquiries possibly under-estimate the consumption of rich households. (vide Rudra, 1972).

(iii) The use of a moving reference period of last 30 days for the collection of expenditure information from sample households may be responsible for biases due to seasonality of expenditure on such items as 'clothing' that are subject to considerable seasonal variations. (vide Chatterjee and Bhattacharya, 1975).

The examination of these possible deficiencies of NSS data would require systematic and continual examination, and their revision, if necessary, may be well-nigh impossible. However, the NSS consumption data have been extensively used in Engel curve analysis, and quite sensible estimates of Engel elasticity for various broad item-group have always been obtained. In the exercises made for fitting demand functions and estimating price elasticities, again, the results, at least the income and price elasticities, have generally been plausible (vide Ganguly et al. 1960; Iyenger, 1967; Bhattacharya and Maitra, 1970).

3.3 Price data

For estimation of empirical demand relationships, it is appropriate to use retail/consumer price data for the items considered. In the present case, however, suitable item-groupwise retail price data for the entire period covered by the analysis could not be obtained from any source and, therefore, the wholesale price data have been used.

In India a number of official agencies regularly collect retail prices faced by specific population groups and some of them also compile consumer price indices based on these price data. For example, the Directorate of Economics and Statistics, Government of India, collects retail prices of agricultural food products from selected important consuming centres spread over the country. The Labour Bureau, Government of India, collects retail prices of about 100 items from 50 industrial/mining/plantation centres of the country and compiles consumer price index numbers for industrial workers for individual centres and also for India as a whole. The NSSO collects rural retail prices on an all-India basis mainly from village markets and these prices are used by the Labour

Bureau in the compilation of Agricultural Labourers' Consumer Price Index (ALCPI) numbers for individual states and also for India as a whole. However, this wealth of valuable retail price information could not be used for the purpose of the present analysis mainly for two reasons. First, the available retail price data relate to specific population groups in the rural and the urban sectors, and cannot strictly be used for the entire population in rural and urban India. More importantly, from none of the sources mentioned above one would get details of retail prices/price-indices for the entire period covered in the present study enabling one to construct suitable price indices for the nine broad groups of items considered separately for the rural and the urban sectors of India.

As an alternative, we, therefore, turned to the available wholesale price data. The single most important source of wholesale price data in India is the Office of the Economic Adviser (OEA), Ministry of Industrial Development and Company affairs, Government of India. This agency has been compiling weekly wholesale price indices for groups of items and the overall wholesale price index for all-commodities for quite a long time. At present, these indices cover about 140 items belonging to three broad groups, viz., (i) primary articles, (ii) fuel, power, light and lubricant, and (iii) manufactured products. The average prices and the wholesale price indices based on these prices are regularly published by the OEA in its monthly publication entitled 'Wholesale Prices in India'.

The price indices for the nine broad groups of items that we have utilized in the empirical analyses, were actually constructed for similar analyses by Radhakrishna, Murthy and Shah (1979).5/ These indices have been compiled separately for the rural and the urban sectors of the country. The procedure of compilation is given below.

For each item in a broad group, the monthly wholesale price relatives (obtained from the publication of the OEA) have been averaged over the months covered by a NSS round to obtain the average price relative for that round. Using the expenditure weights for individual items in a broad group relative to the total expenditure on the broad group taken as a whole, the price index/relative for the broad group of items has been constructed. The expenditure weights have been worked out on the basis of estimated average consumer expenditure on individual items thrown up by NSS 13th round (September 1957 - May 1958) and separate weighting diagrams for the rural and the urban sectors have been used to construct the price index series for the rural and the urban population groups. Finally, as the published

---

5/ The price indices upto NSS 20th round (July 1965 - June 1966) have been reported in the reference cited here. Thanks are due to Professor R. Radhakrishna for providing the price indices for later rounds.

series of wholesale price relatives had undergone base changes (in 1961-62 and 1970-71), the computed series were converted to the original base (1952-53 = 100) through splicing.

The price index series described above have been used as approximations to the unknown corresponding series of indices for retail prices faced by rural and urban consumers during the period of our study. The use of such price indices may vitiate the estimated price responses to some extent and may therefore be questionable for two reasons. First, using the wholesale price relatives in place of the corresponding retail ones amounts to assuming that the retail trade and transport margins have always been a fixed proportion of the wholesale prices. However, market imperfections and the presence of excess-demand conditions in the economy from time to time would suggest non-parallel movements of wholesale and retail prices of items at times. Second, in our analysis, we have assumed that a single set of price indices is applicable to the entire population in rural or urban India, thus ruling out the presence of differentials in intertemporal price movements between richer and poorer consumers. As a matter of fact, there is some evidence that the price rise over time did, at times, affect the poorer section of the population more adversely than the rich (Mahalanobis, 1962; Iyengar and Bhattacharya, 1965; Mukherjee, 1969b).

3.4 Estimation procedure

Three types of static demand systems have been used in the empirical analysis. These are – the modified SNAM set-up, our proposed Variant systems and the AIDS. From the point of view of estimation, the AIDS is computationally simple under some approximation. However, the other two types of systems are non-linear in parameters and are thus computationally somewhat difficult. In what follows, we briefly describe the estimation procedures followed for these systems.

For the modified SNAM and our proposed models we adopted the following Non-Linear Full Information Maximum Likelihood (NLFIML) method and used the NLFIML computer package supplied by Professor Angus Deaton. The stochastic specification of a system is taken in general to be

$$w_{it} = f_i \ (p_t, y_t, \theta) + u_{it} \ ; \quad \begin{array}{l} i = 1, 2, \ldots, n \\ t = 1, 2, \ldots, T \end{array} \qquad \ldots(3.4.1)$$

where $p_t = (p_{1t}, p_{2t}, \ldots, p_{nt})$ is the vector of observations on n commodity prices at time t $(t = 1, 2, \ldots, T)$; y_t is the observed total expenditure (PCE) at time t ; w_{it} is the observed budget share for item i at time t , θ is the vector of parameters, and u_{it} is the additive random disturbance term in the i-th equation at time t .

The disturbance term u_{it} is assumed to have the following properties :

$$E(u_{it}) = 0 \quad \text{for all } i \text{ and } t \qquad \ldots(3.4.2)$$

and

$$E(u_{it}\, u_{js}) = \begin{cases} \sigma_{ij} & \text{for } t = s \\ 0 & \text{for } t \neq s \end{cases} \qquad \ldots(3.4.3)$$

where in view of the adding-up restriction, the matrix $\Sigma = [\sigma_{ij}]$ of contemporaneous covariances of the disturbances, has rank n − 1 . The NLFIML procedure used here takes care of this singularity of Σ . Writing (3.4.1) in vector form, we have

$$w = f(p, y, \theta) + u \qquad \ldots(3.4.4)$$

where u is assumed to follow a multivariate normal distribution $N(0, \Sigma \otimes I)$[6]. Here (X) stands for the Kronecker product. The log-likelihood function for u is

$$\log L(u) = \frac{-nT}{2} \log 2\pi - \tfrac{1}{2} \log \left| \Sigma \otimes I \right| - \tfrac{1}{2} V(\theta)' (\Sigma^{-1} \otimes I)\, V(\theta) \qquad \ldots(3.45)$$

where $V(\theta) = w - f(p, y, \theta)$.

---

[6] For items with very small observed budget shares, this assumption of normality in the estimation may be questioned. However, whether such an assumption would be reasonable would depend on the variance of the disturbance terms in the functions corresponding to these items. The order of inconsistency resulting from not assuming a truncated distribution for these disturbance terms, however, may be small.

The log-likelihood function for w is then

$$\text{Log } L(w) = -\frac{nT}{2}\log 2\pi - \frac{T}{2}\log|\Sigma| + T\log|A| - \tfrac{1}{2}V(\theta)'(\Sigma^{-1} \otimes I)V(\theta) \quad \ldots(3.4.6)$$

where $|A| = \left|\frac{\partial u}{\partial w}\right|$ is the Jacobian of transformation. Given this set-up, the NLFIML maximises log L(w) through iteration which works as follows. Starting with an initial value of ^, (3.4.6) for the linearised version of (3.4.4) is considered. The new set of parameters are worked out by modified Gauss-Newton method of Marquardt (1963). The general recursive expression for any stage of iteration, say the (k + 1)-th, is given by

$$\hat{\theta}_{k+1} = \hat{\theta}_k + [Z(\hat{\theta}_k)' \{\hat{\Sigma}(\hat{\theta}_k) \otimes I\} Z(\hat{\theta}_k) + \lambda_k I]^{-1} Z(\hat{\theta}_k)$$

$$\{\hat{\Sigma}(\hat{\theta}_k)^{-1} \otimes I\} V(\hat{\theta}_k) \quad \ldots(3.4.7)$$

where $\hat{\theta}_k$ and $\hat{\theta}_{k+1}$ are the parameter estimates for two successive stages of iteration k and k+1,

$$Z(\hat{\theta}_k) = \left.\frac{\partial f(p, y, \theta)}{\partial \theta}\right|_{\theta = \hat{\theta}_k}, \text{ and } \hat{\Sigma}(\hat{\theta}_k) = [\hat{\sigma}_{ij}(\hat{\theta}_k)]$$

with $\hat{\sigma}_{ij} = \frac{1}{T}\hat{u}_i(\hat{\theta}_k)'\hat{u}_j(\hat{\theta}_k)$; $i, j = 1, 2, \ldots, n$.

The λ_k in (3.4.7) represents the modification over the Gauss-Newton algorithm. For λ_k equal to zero this method reduces to the Gauss-Newton algorithm. By increasing λ_k the steepest descent method is approached. Since the Gauss-Newton method performs very well in the neighbourhood of the extremum, starting with a small λ, $\lambda_k > 0$ is decreased in each iteration unless this results in an unacceptable step. Note that here the step length is determined simultaneously with the step direction.

At each stage of iteration the value of the concentrated log-likelihood function, viz.,

$$\text{Log } L(w) = -\frac{nT}{2} \log 2\pi - \frac{T}{2} \log |\hat{\Sigma}(\hat{\theta}_k)| + T \log |A| - \frac{T}{2} \qquad \ldots(3.4.8)$$

is computed and compared with the value previously obtained. The process is continued till the convergence criteria are fulfilled (i.e., the parameter estimates as also the likelihood value stabilise) - in which case it gives the maximum value of Log L(w) .

For the estimation of the AIDS, we have considered the following stochastic specification of the system

$$w_{it} = a_i + b_i \log\left(\frac{y_t}{\bar{P}_t}\right) + \sum_{j=1}^{n} c_{ij} \log p_{jt} + u_{it} \qquad \ldots(3.4.9)$$

$$i = 1, 2, \ldots, n$$
$$t = 1, 2, \ldots, T$$

where $\bar{P}_t$ has been approximated by $\log \bar{P}_t = \sum_{k=1}^{n} w_{kt} \log p_{kt}$ for all t, following the suggestion in Deaton and Muellbauer (1980a). The random disturbances have been assumed to have

$$E(u_{it}) = 0 \quad \text{for all i and t} \qquad \ldots(3.4.10)$$

$$E(u_{it}\, u_{js}) = \begin{cases} \sigma_i^2 & \text{for } i = j \text{ and } t = s \\ 0 & \text{otherwise} \end{cases} \qquad \ldots(3.4.11)$$

The assumption of absence of contemporaneous correlation between cross-equation disturbances makes the application of single equation Ordinary Least Squares (OLS) technique to individual equation of the system possible. It may be noted that in the absence of cross-equation parametric restrictions (such as symmetry constraints), the use of OLS has also been suggested by Deaton and Muellbauer (1980a).

Given the time series nature of the data used, the assumption of absence of serial correlation is, a priori, rather restrictive. In fact, we have checked the presence of serial correlation (intragroup in case of disaggregated data) for the estimated systems, and have found fair amount of serial correlations in many cases for aggregate as also for the disaggregated data. This calls for reestimation of the systems taking into account the possible serial correlation in the stochastic disturbances.

It may be mentioned that for the disaggregated data, on subsequent analysis we found that the observed serial correlation may partly be due to misspecification of functional form. The treatment of serial correlation in the present case may, however, be somewhat difficult because of the 'combined time-series and cross-section' nature of the data. Typically, for such data, one should consider an error structure capable of incorporating both the intergroup correlation at any point of time and intragroup serial correlation over time. We have, however, not been able to estimate the systems taking into account these factors.

For the aggregate data, we have made an attempt to examine the extent to which the observed serial correlations in the item-specific disturbances could be due to omission of dynamic factors like changes in tastes and preferences over time. For this purpose we have estimated two dynamised versions of some of the systems, by incorporating (i) item-specific linear time trend in budget shares, and (ii) the phenomenon of habit formation in the original systems. As these results are not directly related to the other chapters of this dissertation, these have been reported in Appendix E.

Chapter IV

Linear Expenditure System and the Almost Ideal Demand System : An empirical comparison

4.1 Introduction

Statistical analyses of consumer expenditure pattern in India have mostly been confined to Engel curve analysis based on consumer expenditure data thrown up by different rounds of the National Sample Survey (*vide* Sinha, 1966; Singh, 1968; Gupta, 1968; Maitra, 1969; Bhattacharya and Maitra, 1970; Jain, 1972; Jain and Tendulkar, 1973; Coondoo, 1975; and, Planning Commission, 1979). However, there have also been some attempts to estimate complete demand systems based on time series data. These studies are, by and large, based on the Linear Expenditure System (LES) perhaps because this was the best known and also popular in view of its ability to formalise consumer behaviour in a simple manner.

Important applications of the LES to the Indian consumer expenditure data include studies by Rudra (1964), Rudra and Paul (1964), Bhattacharya (1967), Joseph (1968), Radhakrishna and Murthy (1973), Murthy (1977), Radhakrishna, Murthy and Shah (1979) and Radhakrishna and Murthy (1980).

The LES, as is well known, is derived from an additively separable direct utility function. As noted earlier, the empirical implication of additive separability of the direct utility function underlying the LES is rather disturbing.[1/] For any demand system based on an additively separable direct preference structure, the non-compensated own-price elasticities of commodities are approximately proportional to the corresponding expenditure elasticities (Deaton, 1974b). Given the high correlations between item expenditures and the total expenditure in most time series data, Deaton's result thus suggests that while the expenditure elasticities would be estimated more or less accurately by the LES, the corresponding estimates of the non-compensated own-price elasticities would tend to be proportional to the estimated expenditure elasticities - thus ruling out the possibility of getting independent (unbiased) estimates of the true own-price elasticities underlying the data.

It is, therefore, important to examine the extent to which the estimates of own-price elasticities based on the LES differ from those based on more flexible demand systems having no such prior restriction on the underlying direct preference structure.[2/]

1/ Vide Chapter I, section 1.3 .

2/ Deaton (1975) reports such a comparison of the own-price elasticities estimated from the LES and the constant elasticity demand equations.

Here we shall compare the results of the IES with those of the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a) that possesses a number of desirable properties, e.g., flexibility of the functional form, consistency in respect of aggregation over consumers, a very general form of underlying preference structure, and simplicity from the point of view of estimation. This comparison, among other things, should bring out the limitations, if any, of the IES based expenditure and price elasticities.

This chapter has three sections : section 4.2 presents the empirical results of fitting the IES and the AIDS to Indian consumer expenditure data separately for the rural and the urban sectors of the country; section 4.3 compares the expenditure and own-price elasticities estimated from the two systems, and finally section 4.4 draws some concluding observations.

4.2 Estimation and empirical results

The empirical exercises based on the IES and the AIDS reported in this chapter have been carried out on both the aggregate and the disaggregated population-groupwise consumer expenditure data, obtained through NSS 7th to 28th rounds separately for rural and urban India.

The IES has been estimated in its budget share form with the following stochastic specification

$$w_{it} = \frac{p_{it}\, q_{it}}{y_t} = \frac{p_{it}\, c_i}{y_t} + b_i \left(1 - \frac{\sum_j p_{jt}\, c_j}{y_t}\right) + u_{it} \qquad \ldots (4.2.1)$$

$$i = 1,2, \ldots, n$$
$$t = 1,2, \ldots, T$$

where

$$E(u_{it}) = 0 \text{ for all } i \text{ and } t$$

$$E(u_{it}\, u_{js}) = \begin{cases} \sigma_{ij} & \text{for } t = s \\ 0 & \text{for } t \neq s \,. \end{cases}$$

It may be noted that in view of the adding-up criterion, $\sum_i b_i = 1$ and the covariance matrix of the disturbance terms for any given t has rank $n-1$. In the estimation method used, viz., non-linear full information maximum likelihood method, this fact has been duly taken into account.[3/]

For the AIDS, the individual budget share equations have been estimated by single equation Ordinary Least Squares (OLS) method, assuming additive disturbances for all the individual budget share functions. The stochastic specification used for the AIDS is

---

3/ The expressions for the first order partial derivatives of the right-hand side of equation (4.2.1) with respect to the individual parameters, (derived for the maximum likelihood estimation) are listed in Appendix B.

thus

$$w_{it} = a_i + b_i \log \left(\frac{y_t}{\bar{P}_t} \right) + \sum_{j=1}^{n} c_{ij} \log p_{jt} + u_{it} \quad \ldots (4.2.2)$$

$$i = 1,2, \ldots, n$$
$$t = 1,2, \ldots, T$$

where

$$\log \bar{P}_t = a_o + \sum_{j=1}^{n} a_j \log p_{jt} + \tfrac{1}{2} \sum_j \sum_k c_{jk} \log p_{jt} \log p_{kt} \quad \ldots (4.2.3)$$

with

$$E(u_{it}) = 0 \text{ for all } i \text{ and } t$$

$$E(u_{it}\, u_{js}) = \begin{cases} \sigma_i^2 & \text{for } t = s,\ i = j \\ 0 & \text{otherwise} \end{cases}$$ [4/]

It should be pointed out here that $\bar{P}_t$ in equation (4.2.2) defined in equation (4.2.3) has been approximated by P_t^*, where $\log P_t^* = \sum_k w_{kt} \log p_{kt}$ $(t = 1,2, \ldots, T)$, following the suggestion in Deaton and Muellbauer (1980a). Use of this approximation enables one to estimate equation (4.2.2) by OLS ignoring the symmetery restriction.[5/]

4/ A more appropriate procedure would be to use Zellner's (1962) SURE method taking all equations together. For computational simplicity we have used the single equation OLS. The assumption that the covariance matrix is diagonal may, however, result in a loss of efficiency of the estimators.

5/ The homogeneity restriction, however, can be incorporated.

As already mentioned, the two systems, viz., the LES and the AIDS, have been estimated separately for rural and urban India on the basis of both aggregate and population-groupwise disaggregated consumer expenditure data. In what follows, we shall present a comparison of the empirical results of fitting the two systems to the aggregate and the disaggregated data.

4.2.1 Goodness of fit

Tables 4.1.1 and 4.1.2 present the itemwise squared correlation coefficients between observed and estimated expenditures (R_y^2) and between observed and estimated budget shares (R_w^2) respectively for the LES and the AIDS fitted to the two types of data separately for rural and urban India.

In terms of R_y^2 the AIDS appears to have performed somewhat better compared to the LES both on the aggregate and the disaggregated data. However, the superiority of the AIDS has been brought out more convincingly in Table 4.1.2 where R_w^2 may be seen to be higher in most cases and substantially higher in some cases for the AIDS; exception may be seen for 'meat, fish and egg', in urban India for disaggregated data. For the disaggregated data, the R_w^2 value for the AIDS ranges from 0.568 to 0.969 in the rural sector and from 0.407 to 0.995 in the urban sector. The LES, on the other hand, has R_w^2 value as

Table 4.1.1 Itemwise squared correlation coefficients between observed and estimated expenditures (R_y^2) for the LES and the AIDS

All India : rural and urban; aggregate and disaggregated data

Items	aggregate data				disaggregated data			
	All India rural		All India urban		All India rural		All India urban	
	LES	AIDS	LES	AIDS	LES	AIDS	LES	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Cereals and cereal substitutes	0.995	0.998	0.987	0.997	0.970	0.992	0.916	0.994
2. Milk and milk products	0.952	0.977	0.985	0.995	0.980	0.985	0.992	0.992
3. Edible oils	0.993	0.996	0.992	0.998	0.974	0.992	0.938	0.990
4. Meat, fish, egg	0.964	0.995	0.985	0.994	0.970	0.982	0.990	0.990
5. Sugar	0.990	0.997	0.988	0.997	0.947	0.983	0.935	0.971
6. Other food	0.987	0.995	0.976	0.995	0.969	0.989	0.948	0.986
7. Clothing	0.874	0.944	0.819	0.953	0.947	0.982	0.944	0.986
8. Fuel and light	0.990	0.994	0.995	0.998	0.986	0.989	0.988	0.993
9. Other non food	0.964	0.984	0.987	0.995	0.948	0.978	0.978	0.995

Table 4.1.2 Itemwise squared correlation coefficients between observed and estimated budget shares (R_w^2) for the LES and the AIDS

All India : rural and urban; aggregate and disaggregated data

Items	aggregate data				disaggregated data			
	All India rural		All India urban		All India rural		All India urban	
	LES	AIDS	LES	AIDS	LES	AIDS	LES	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Cereals and cereal substitutes	0.790	0.907	0.569	0.778	0.881	0.969	0.937	0.995
2. Milk and milk products	0.077	0.418	0.290	0.817	0.909	0.955	0.937	0.900
3. Edible oils	0.833	0.917	0.934	0.984	0.516	0.757	0.450	0.827
4. Meat, fish, egg	0.126	0.810	0.127	0.533	0.477	0.568	0.488	0.407
5. Sugar	0.700	0.953	0.449	0.933	0.671	0.890	0.035	0.511
6. Other food	0.069	0.891	0.597	0.948	0.009	0.639	0.108	0.766
7. Clothing	0.773	0.850	0.014	0.922	0.779	0.943	0.758	0.953
8. Fuel and light	0.462	0.823	0.669	0.803	0.909	0.958	0.894	0.972
9. Other non food	0.786	0.855	0.654	0.927	0.720	0.870	0.841	0.962

low as 0.009 for 'other food' in the rural sector, and 0.108 and 0.035 in the urban sector for 'other food' and 'sugar' respectively. On the aggregate data, the LES performed rather poorly for such items as 'milk and milk products', 'meat, fish and egg' for both rural and urban India, 'other food' for rural India, and 'clothing' for urban India. Coming to the comparison of R_W^2 values between the aggregate and the disaggregated data, LES shows a rise in R_W^2 values as one moves from aggregate to disaggregate data, for 'cereals and cereal substitutes', 'milk and milk products', 'meat, fish and egg', 'clothing' and 'fuel and light' for both rural and urban India and for 'other nonfood' for urban India. Correspondingly, the AIDS shows a rise in R_W^2 values for 'cereals and cereal substitutes', 'milk and milk products', 'clothing', 'fuel and light' and 'other nonfood' for both rural and urban India.

4.2.2 Estimates of parameters

The estimates of the parameters of the LES fitted to the aggregate and the disaggregated data have been presented in Table 4.2 along with their asymptotic standard errors. Judged by the standard errors, the estimated b and c parameters of the LES are significant for most of the items for both the sectors as also for the two types of data. In general, the b-parameters are more precisely estimated

Table 4.2 Estimates of the parameters of the LES*

All India : rural and urban; aggregate and disaggregated data

Items	aggregate data				disaggregated data			
	All India rural		All India urban		All India rural		All India urban	
	b	c	b	c	b	c	b	c
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Cereals and cereal substitutes	0.245 (0.027)	5.140 (0.272)	0.117 (0.016)	3.782 (0.275)	0.270 (0.010)	4.210 (0.121)	0.096 (0.008)	4.281 (0.104)
2. Milk and milk products	0.115 (0.021)	0.234 (0.215)	0.115 (0.014)	0.045 (0.266)	0.117 (0.003)	-0.099 (0.042)	0.127 (0.002)	-0.298 (0.058)
3. Edible oils	0.029 (0.003)	0.182 (0.014)	0.017 (0.002)	0.457 (0.019)	0.030 (0.001)	0.115 (0.008)	0.035 (0.001)	0.171 (0.015)
4. Meat, fish, egg	0.043 (0.003)	0.035 (0.016)	0.041 (0.003)	0.021 (0.015)	0.030 (0.001)	0.070 (0.008)	0.058 (0.001)	0.042 (0.013)
5. Sugar	0.027 (0.003)	0.235 (0.016)	0.020 (0.003)	0.321 (0.053)	0.038 (0.001)	0.052 (0.012)	0.028 (0.001)	0.141 (0.018)
6. Other food	0.136 (0.020)	0.867 (0.164)	0.151 (0.029)	1.115 (0.476)	0.125 (0.005)	0.695 (0.045)	0.179 (0.006)	0.457 (0.089)
7. Clothing	0.118 (0.023)	0.284 (0.227)	0.117 (0.016)	-0.844 (0.331)	0.124 (0.005)	-0.126 (0.053)	0.092 (0.003)	-0.417 (0.057)
8. Fuel and light	0.083 (0.010)	0.298 (0.105)	0.059 (0.006)	0.304 (0.108)	0.042 (0.001)	0.586 (0.017)	0.046 (0.001)	0.540 (0.022)
9. Other non food	0.204 (0.031)	0.794 (0.317)	0.363 (0.043)	-0.881 (1.439)	0.223 (0.009)	-0.095 (0.104)	0.358 (0.009)	-1.289 (0.226)

* Figures in parentheses are the asymptotic standard errors.

compared to the c - parameters. In some cases, viz., for 'clothing', and 'other non-food' in the urban sector for aggregate data, and for 'milk and milk products', 'clothing' and 'other non-food' in both rural and urban sectors for disaggregated data, the c - parameters turned out to be negative, most of them being significant.[6/]

The estimated parameters of the AIDS have been presented in Tables 4.3.1 - 4.3.4 . Tables 4.3.1 - 4.3.2 report the parameter estimates for the rural and urban sectors respectively based on the aggregate data, while Tables 4.3.3 - 4.3.4 present the corresponding estimates based on the disaggregated data.

The statistical significance of the estimated parameters, based on aggregate data, reported in Tables 4.3.1 - 4.3.2 has been summarised in Table 4.4.1 . It may be noted from this Table that while none of the marginal budget shares for the rural sector have t-ratios exceeding 2 in absolute value, for the urban sector only for two item groups (out of nine), viz., for 'clothing' and 'other non-food', the t-ratios for the marginal budget shares exceed 2 in absolute value. As regards the statistical significance of the c_{ii}'s, for the rural sector four out of nine such parameters have $|t|$ ratios greater than 2,

---

6/ A negative c implies that the corresponding item is price-elastic vide Chapter I, section 1.4 . Given the interpretation of c's as committed quantities, negative c's would be plausible for luxury items.

Table 4.3.1 Estimates of the parameters of the AIDS*

All India : rural; aggregate data

Items \ Parameters	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Cereals and cereal substitutes	0.585 (0.205)	-0.053 (0.071)	0.265 (0.068)	-0.153 (0.074)	-0.030 (0.048)	-0.024 (0.031)	-0.014 (0.021)	-0.006 (0.055)	0.043 (0.067)	-0.027 (0.060)	-0.031 (0.036)
2. Milk and milk products	0.149 (0.140)	-0.025 (0.049)	-0.049 (0.046)	0.066 (0.050)	-0.001 (0.033)	0.026 (0.021)	-0.009 (0.015)	-0.007 (0.038)	-0.048 (0.045)	0.000 (0.041)	-0.005 (0.025)
3. Edible oils	0.067 (0.031)	-0.014 (0.011)	-0.012 (0.010)	0.005 (0.011)	0.014 (0.007)	0.009 (0.005)	-0.005 (0.003)	-0.012 (0.008)	-0.011 (0.010)	0.005 (0.009)	0.001 (0.006)
4. Meat, fish, egg	0.054 (0.027)	-0.011 (0.009)	-0.039 (0.009)	0.021 (0.010)	0.015 (0.006)	0.007 (0.004)	0.003 (0.003)	-0.008 (0.007)	-0.012 (0.009)	0.009 (0.008)	-0.006 (0.005)
5. Sugar	0.052 (0.024)	-0.008 (0.008)	-0.007 (0.008)	0.014 (0.009)	-0.003 (0.006)	0.001 (0.004)	0.017 (0.002)	-0.010 (0.006)	-0.008 (0.008)	-0.004 (0.007)	0.007 (0.004)
6. Other food	0.141 (0.119)	-0.010 (0.041)	-0.091 (0.039)	0.058 (0.043)	0.082 (0.028)	0.021 (0.018)	0.015 (0.012)	-0.026 (0.032)	-0.102 (0.038)	0.011 (0.035)	-0.011 (0.021)
7. Clothing	-0.115 (0.150)	0.067 (0.052)	-0.036 (0.050)	-0.014 (0.054)	-0.009 (0.035)	-0.031 (0.023)	-0.001 (0.016)	0.026 (0.040)	0.096 (0.049)	0.016 (0.044)	-0.017 (0.027)
8. Fuel and light	0.018 (0.053)	-0.007 (0.018)	-0.033 (0.017)	-0.010 (0.019)	0.006 (0.012)	0.001 (0.008)	-0.001 (0.006)	0.012 (0.014)	-0.001 (0.017)	0.026 (0.016)	-0.012 (0.009)
9. Other non food	-0.013 (0.200)	0.061 (0.069)	0.002 (0.066)	0.015 (0.072)	-0.074 (0.047)	-0.010 (0.030)	-0.005 (0.021)	0.032 (0.054)	0.045 (0.065)	-0.035 (0.059)	0.074 (0.035)

* Figures in parentheses are the asymptotic standard errors.

Table 4.3.2 Estimates of the parameters of the AIDS*

All India : urban; aggregate data

Items \ Parameters	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Cereals and cereal substitutes	0.737 (0.270)	-0.152 (0.085)	0.114 (0.078)	-0.030 (0.086)	-0.050 (0.045)	-0.024 (0.029)	-0.012 (0.046)	-0.004 (0.034)	0.097 (0.072)	0.056 (0.097)	-0.117 (0.165)
2. Milk and milk products	0.018 (0.104)	0.026 (0.033)	-0.042 (0.030)	0.054 (0.033)	-0.008 (0.017)	0.045 (0.011)	-0.002 (0.018)	0.000 (0.013)	-0.060 (0.028)	0.004 (0.038)	-0.044 (0.064)
3. Edible oils	0.059 (0.036)	-0.008 (0.012)	0.014 (0.011)	-0.020 (0.012)	0.010 (0.006)	0.006 (0.004)	-0.016 (0.006)	-0.004 (0.005)	0.005 (0.010)	-0.022 (0.013)	0.045 (0.022)
4. Meat, fish, egg	0.039 (0.049)	-0.002 (0.016)	-0.031 (0.014)	0.016 (0.016)	0.015 (0.008)	0.007 (0.005)	0.012 (0.008)	-0.003 (0.006)	-0.023 (0.013)	0.038 (0.018)	-0.061 (0.030)
5. Sugar	0.088 (0.034)	-0.017 (0.011)	0.010 (0.010)	-0.001 (0.011)	-0.017 (0.006)	-0.004 (0.004)	0.005 (0.006)	-0.015 (0.004)	0.030 (0.009)	-0.036 (0.012)	0.061 (0.021)
6. Other food	0.509 (0.237)	-0.109 (0.075)	-0.081 (0.069)	0.094 (0.075)	0.070 (0.040)	0.004 (0.026)	-0.009 (0.040)	-0.026 (0.030)	-0.135 (0.063)	-0.155 (0.085)	0.275 (0.145)
7. Clothing	-0.251 (0.142)	0.097 (0.045)	-0.070 (0.041)	0.004 (0.045)	0.040 (0.024)	0.015 (0.015)	0.070 (0.024)	-0.002 (0.018)	-0.031 (0.038)	0.176 (0.051)	-0.319 (0.087)
8. Fuel and light	0.115 (0.046)	-0.016 (0.015)	-0.011 (0.013)	0.006 (0.015)	-0.007 (0.008)	0.006 (0.005)	-0.007 (0.008)	-0.008 (0.006)	0.004 (0.012)	0.006 (0.017)	0.008 (0.028)
9. Other non food	-0.314 (0.257)	0.182 (0.081)	0.096 (0.074)	-0.124 (0.081)	-0.052 (0.043)	-0.053 (0.028)	-0.042 (0.044)	0.061 (0.032)	0.114 (0.069)	-0.068 (0.092)	0.158 (0.157)

* Figures in parentheses are the asymptotic standard errors.

Table 4.3.3 Estimates of the parameters of the AIDS*

All India : rural; disaggregated data

Items \ Parameters	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Cereals and cereal substitutes	1.001 (0.022)	-0.192 (0.005)	0.208 (0.075)	-0.086 (0.080)	-0.018 (0.054)	-0.030 (0.033)	0.011 (0.025)	-0.048 (0.055)	-0.047 (0.076)	0.078 (0.065)	0.018 (0.043)
2. Milk and milk products	-0.082 (0.007)	0.053 (0.002)	-0.031 (0.025)	0.024 (0.027)	-0.007 (0.018)	0.016 (0.011)	-0.011 (0.008)	0.016 (0.018)	-0.026 (0.026)	0.023 (0.022)	-0.016 (0.014)
3. Edible oils	0.027 (0.003)	0.000 (0.001)	-0.010 (0.008)	0.004 (0.009)	0.013 (0.006)	0.011 (0.004)	-0.007 (0.003)	-0.009 (0.006)	-0.012 (0.009)	0.008 (0.007)	-0.003 (0.005)
4. Meat, fish, egg	0.010 (0.003)	0.004 (0.001)	-0.026 (0.010)	0.017 (0.011)	0.008 (0.007)	0.005 (0.004)	0.001 (0.003)	-0.002 (0.007)	-0.008 (0.010)	0.003 (0.009)	-0.001 (0.006)
5. Sugar	-0.005 (0.003)	0.011 (0.001)	-0.004 (0.010)	0.005 (0.010)	-0.001 (0.007)	0.002 (0.004)	0.016 (0.003)	-0.012 (0.007)	-0.001 (0.010)	-0.002 (0.008)	0.002 (0.005)
6. Other food	0.143 (0.012)	-0.010 (0.003)	-0.093 (0.042)	-0.049 (0.045)	0.070 (0.030)	-0.017 (0.018)	0.042 (0.014)	-0.043 (0.031)	-0.029 (0.043)	0.120 (0.037)	0.007 (0.024)
7. Clothing	-0.094 (0.010)	0.059 (0.002)	-0.026 (0.033)	-0.009 (0.036)	-0.012 (0.024)	-0.023 (0.014)	-0.008 (0.011)	0.025 (0.024)	0.071 (0.034)	0.024 (0.029)	-0.026 (0.019)
8. Fuel and light	0.135 (0.004)	-0.024 (0.001)	-0.040 (0.012)	-0.003 (0.013)	0.007 (0.009)	0.007 (0.005)	-0.002 (0.004)	0.003 (0.009)	-0.007 (0.012)	0.027 (0.010)	-0.014 (0.007)
9. Other non food	-0.137 (0.025)	0.100 (0.006)	0.020 (0.085)	0.098 (0.092)	-0.061 (0.062)	0.030 (0.037)	-0.042 (0.028)	0.071 (0.063)	-0.035 (0.087)	-0.126 (0.075)	0.034 (0.049)

* Figures in parentheses are the asymptotic standard errors.

Table 4.3.4 Estimates of the parameters of the AIDS*

All India : urban; disaggregated data

Parameters / Items	a_i	b_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}	c_{i7}	c_{i8}	c_{i9}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Cereals and cereal substitutes	0.906 (0.010)	-0.196 (0.002)	0.104 (0.043)	-0.032 (0.047)	-0.047 (0.027)	-0.036 (0.018)	-0.017 (0.028)	0.011 (0.020)	0.087 (0.045)	0.115 (0.058)	-0.175 (0.099)
2. Milk and milk products	-0.022 (0.009)	0.037 (0.002)	-0.007 (0.038)	-0.005 (0.042)	-0.026 (0.024)	0.026 (0.016)	-0.016 (0.025)	-0.013 (0.018)	-0.001 (0.041)	-0.016 (0.052)	0.066 (0.089)
3. Edible oils	0.054 (0.004)	-0.006 (0.001)	0.013 (0.016)	-0.023 (0.018)	0.009 (0.010)	0.007 (0.007)	-0.019 (0.011)	-0.005 (0.008)	0.002 (0.017)	-0.018 (0.022)	0.053 (0.038)
4. Meat, fish, egg	0.020 (0.004)	0.004 (0.001)	-0.024 (0.015)	0.015 (0.017)	0.008 (0.010)	0.007 (0.006)	0.005 (0.010)	-0.001 (0.007)	-0.016 (0.016)	0.020 (0.021)	-0.031 (0.036)
5. Sugar	0.042 (0.004)	-0.003 (0.001)	0.009 (0.019)	-0.005 (0.021)	-0.011 (0.012)	-0.002 (0.008)	0.008 (0.013)	-0.018 (0.009)	0.024 (0.020)	-0.044 (0.026)	0.073 (0.045)
6. Other food	0.130 (0.012)	0.009 (0.003)	-0.052 (0.054)	0.064 (0.060)	0.071 (0.035)	-0.003 (0.023)	0.004 (0.035)	-0.011 (0.026)	-0.123 (0.057)	-0.169 (0.074)	0.255 (0.126)
7. Clothing	-0.066 (0.006)	0.038 (0.001)	-0.067 (0.028)	0.021 (0.031)	0.028 (0.018)	0.017 (0.012)	0.046 (0.018)	-0.008 (0.013)	-0.021 (0.030)	0.139 (0.038)	-0.256 (0.065)
8. Fuel and light	0.132 (0.003)	-0.021 (0.001)	-0.019 (0.011)	0.003 (0.012)	-0.006 (0.007)	0.005 (0.005)	-0.002 (0.007)	-0.009 (0.005)	0.006 (0.011)	0.015 (0.015)	0.002 (0.025)
9. Other non food	-0.200 (0.019)	0.138 (0.004)	0.042 (0.084)	-0.040 (0.092)	-0.025 (0.053)	-0.020 (0.035)	-0.009 (0.055)	0.055 (0.040)	0.042 (0.089)	-0.043 (0.114)	0.014 (0.195)

* Figures in parentheses are the asymptotic standard errors.

Table 4.4.1 Frequency distribution of estimated parameters of the AIDS according to absolute value of their t-ratios

All India : rural and urban; aggregate data

Sector	range of t-ratio	Parameters of AIDS a_i	b_i	c_{ii}	c_{ij} $(i \neq j)$
(1)	(2)	(3)	(4)	(5)	(6)
All India rural	$\|t\| \geq 2$	4	0	4	6
	$1 \leq \|t\| < 2$	2	4	4	25
	$\|t\| < 1$	3	5	1	41
All India urban	$\|t\| \geq 2$	5	2	0	15
	$1 \leq \|t\| < 2$	2	4	4	26
	$\|t\| < 1$	2	3	5	31

Table 4.4.2 Frequency distribution of estimated parameters of the AIDS according to absolute value of their t-ratios

All India : rural and urban; disaggregated data

Sector	range of t-ratio	Parameters of AIDS a_i	b_i	c_{ii}	c_{ij} $(i \neq j)$
(1)	(2)	(3)	(4)	(5)	(6)
All India rural	$\|t\| \geq 2$	8	8	5	11
	$1 \leq \|t\| < 2$	1	0	2	22
	$\|t\| < 1$	0	1	2	39
All India urban	$\|t\| \geq 2$	9	9	1	10
	$1 \leq \|t\| < 2$	0	0	2	20
	$\|t\| < 1$	0	0	6	42

while for the urban sector all the $|t|$ ratios are less than 2. The c_{ij}'s $(i \neq j)$ have $|t|$ ratios less than 2 in most of the cases for both sectors.

Table 4.4.2 summarises the results of the statistical significance of the estimated parameters set out in Tables 4.3.3 - 4.3.4 based on the disaggregated data. It may be seen from this Table that in contrast to the results based on the aggregate data, all the marginal budget-shares (except that for 'edible oil' for the rural sector), have t-ratios greater than or equal to 2 in absolute value. The number of statistically significant c_{ij}'s is, however, comparable for the two sets of data. Out of nine c_{ii}'s five for the rural sector and one for the urban sector have absolute values of t-ratios greater than or equal to 2; and most of the c_{ij}'s $(i \neq j)$ have t-ratios less than 2 in absolute value.

4.3 Estimates of elasticities

In this section we shall examine the itemwise expenditure elasticities and the own price elasticities based on the IES and the AIDS for rural and urban India. These elasticities have been estimated at the sample average of prices and total expenditures.[7] In what follows,

[7] It may be noted, however, that the elasticity based on the aggregate and the disaggregated data sets have been estimated at their respective sample averages of prices and total expenditures.

we shall first compare the estimated expenditure elasticities based on the aggregate and the disaggregated data given by the two systems. Next, we shall compare the corresponding non-compensated own-price elasticities.

4.3.1 Expenditure elasticities

Table 4.5 presents the itemwise expenditure elasticities for the LES and the AIDS separately for rural and urban India based on the aggregate and the disaggregated data.

A comparison of the expenditure elasticities based on the aggregate and the disaggregated data shows that for the LES, the two sets of elasticities are more or less in agreement, with some discrepancies for 'meat, fish and egg', 'sugar' and 'fuel and light' in the rural sector and for 'edible oil', 'sugar' and 'clothing' in the urban sector. For the AIDS, the two sets of elasticities tend to have larger discrepancies on the whole compared to those for the LES. This may be so for the following reason : the parameters determining the value of the expenditure elasticities are b's for the LES, the b's for the AIDS. The b-parameters of the LES are very precisely estimated from both the aggregate and the disaggregated data, and are close in magnitude for both the rural and the urban sectors. This agreement is, however, expected in view of the linearity of the underlying Engel curve. For AIDS, on the other hand, the b-parameters estimated from the disaggregated data

Table 4.5 Estimates of itemwise expenditure elasticities calculated at sample average of prices and total expenditures for the LES and the AIDS

All India : rural and urban; aggregate and disaggregated data

Items	aggregate data				disaggregated data			
	All India rural		All India urban		All India rural		All India urban	
	LES	AIDS	LES	AIDS	LES	AIDS	LES	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Cereals and cereal substitutes	0.579	0.873	0.481	0.363	0.640	0.562	0.387	0.269
2. Milk and milk products	1.563	0.679	1.217	1.261	1.576	1.758	1.331	1.413
3. Edible oils	0.931	0.566	0.423	0.814	0.996	1.006	0.888	0.860
4. Meat, fish, egg	1.707	0.589	1.199	0.939	1.156	1.165	1.104	1.106
5. Sugar	0.883	0.734	0.658	0.396	1.293	1.375	0.942	0.887
6. Other food	1.042	0.929	0.857	0.408	0.945	0.925	1.031	1.049
7. Clothing	1.528	1.944	1.906	2.477	1.595	1.884	1.489	1.642
8. Fuel and light	1.382	0.882	0.983	0.736	0.690	0.621	0.756	0.675
9. Other non food	1.366	1.432	1.400	1.742	1.516	1.736	1.397	1.589

have small standard errors, while those estimated from the aggregate data are mostly non-significant for both sectors. Thus the magnitudes of b-parameters estimated from the two sets of data are not comparable. Moreover, the underlying Engel curve in this case being non-linear, the estimated b's based on aggregate data perhaps involve some aggregation bias.

Coming to the comparison of expenditure elasticities obtained from the two systems, the following observations may be made on the basis of the results reported in Table 4.5 .

(i) For either sector, the expenditure elasticities from the two systems show a closer agreement in case of disaggregated data. This is, however, expected in view of the above mentioned fact that the b-parameters of the LES and the b-parameters of the AIDS are precisely estimated from disaggregated data.

(ii) In all the cases, for the disaggregated data, the AIDS tends to give a larger (smaller) elasticity for luxury (necessary) items when compared to the LES.[8/]

4.3.2 Price elasticities

Table 4.6 presents the non-compensated own price elasticities for the LES and the AIDS for both the rural and the urban sectors.

[8/] For aggregate data, the classification of items (into luxury/necessary items) according to the LES does not tally with that according to the AIDS.

Table 4.6 Estimates of itemwise non compensated own price elasticities calculated at sample average of prices and total expenditures for the LES and the AIDS*

All India : rural and urban; aggregate and disaggregated data

Items	LES		AIDS	
	All India rural	All India urban	All India rural	All India urban
(1)	(2)	(3)	(4)	(5)
1. Cereals and cereal substitutes	-0.474	-0.460	-0.034	-0.164
	-0.589	-0.392	-0.098	0.041
2. Milk and milk products	-0.838	-0.983	-0.104	-0.468
	-1.067	-1.111	-0.590	-1.025
3. Edible oils	-0.502	-0.352	-0.526	-0.752
	-0.690	-0.760	-0.604	-0.783
4. Meat, fish, egg	-0.897	-0.968	-0.715	-0.812
	-0.795	-0.938	-0.809	-0.806
5. Sugar	-0.476	-0.539	-0.439	-0.792
	-0.887	-0.803	-0.462	-0.736
6. Other food	-0.607	-0.738	-1.178	-0.902
	-0.689	-0.895	-1.299	-1.069
7. Clothing	-0.823	-1.473	0.425	-1.224
	-1.077	-1.237	0.162	-1.305
8. Fuel and light	-0.746	-0.804	-0.566	-0.886
	-0.493	-0.657	-0.529	-0.729
9. Other non food	-0.772	-1.081	-0.494	-0.300
	-1.027	-1.118	-0.648	-0.844

* In each cell the upper figure is based on aggregate data, and the lower on disaggregated data.

Estimates based on disaggregated data have been reported along with those based on aggregate data in the same table.[9/]

The price elasticities estimated from the aggregate and the disaggregated data may be compared in terms of the sign and the magnitude of a specific elasticity obtained from the two sets of data. In many cases, the cross price elasticities have opposite sign for aggregate and disaggregated data, but the differences are quite small. As for the own-price elasticities, in only one case (for 'cereals and cereal substitutes' for AIDS on urban data) the estimates of own-price elasticities have opposite signs for the two types of data. For IES, in the rural sector for 'milk and milk products', 'clothing' and 'other nonfood' and in the urban sector for 'milk and milk products', while the elasticities based on aggregate data are less than 1, those based on disaggregated data turn out to be greater than 1.[10/] For AIDS, such occurrences may be found for 'milk and milk products' and 'other food' in the urban sector.

Comparison of the magnitudes of the price elasticities obtained from the two sets of data turns out to be much more difficult. So far

---

9/ The estimates of corresponding non-compensated cross price elasticities have been reported in Tables C.1-C.4 in Appendix C.

10/ It may be noted that in cases where the own-price elasticity values have (numerically) exceeded 1, the corresponding c-parameters are negative and these items have turned out to be luxury items according to Table 4.5.

as the estimates of own-price elasticities are concerned, no clear pattern of difference of the estimates based on the two systems could be discerned for either sector. On the whole, except for some cases, the estimates of own-price elasticities based on the aggregate data are not widely different from the corresponding estimates based on the disaggregated data. For the LES, considerable difference may be observed for 'sugar', and smaller but appreciable differences are seen for 'milk and milk products', 'fuel and light', 'clothing', and 'other non-food' for the rural sector, and for 'edible oils', 'sugar' and 'clothing' in case of the urban sector. For the AIDS differences in estimates of own-price elasticities from the two sets of data may be seen to be large for 'milk and milk products' and 'clothing' for the rural sector, and 'milk and milk products' and 'other non-food' for the urban sector.[11/]

The estimates of non-compensated own-price elasticities based on the LES and the AIDS are strikingly different for many items for the rural sector; for the urban sector the number of such large differences appears to be smaller. For aggregate data, large discrepancies between the LES-based and the AIDS-based elasticity values may be observed for

---

11/ The estimates of own-price elasticity for 'clothing' based on the AIDS are positive for both sets of data. Bhattacharya and Maitra (1970) also obtained a positive own-price elasticity for this item in their analysis of the Log-log-inverse demand functions based on NSS 7th-22nd round data.

all items except for 'edible oils', 'meat, fish, egg' and 'sugar' in the rural sector, and for all items except for 'meat, fish, egg', 'clothing' and 'fuel and light' in the urban sector. For disaggregated data, such differences may be observed for all items, except for 'edible oils' and 'meat, fish, egg' in the rural sector, and for only 'cereals and cereal substitutes', and 'other non-food' in the urban sector. For the rural sector, for some items, e.g., for 'milk and milk products', 'clothing', and 'other non-food', the elasticity values are greater than 1 by LES, while they are less than 1 in absolute value by the AIDS for disaggregated data; the order is reserved in case of 'other food' for both aggregate and disaggregated data. For the urban sector, for 'other non-food', the LES-based elasticity is greater than 1, while the AIDS based value is less than 1 in absolute value, and for 'other food' the reverse is observed for disaggregated data.

Table 4.7 presents the itemwise ratios of non-compensated own-price elasticity and the expenditure elasticity for the LES and the AIDS separately for the rural and the urban sectors. It may be observed that the estimates of the non-compensated own-price elasticities based on the LES tend to bear a fixed proportional relationship with the corresponding estimates of the expenditure elasticities. In eight out of nine cases, the constant of proportionality turns out to be approximately equal to -0.5 for aggregate data, and -0.7 for disaggregated data for the rural

Table 4.7 Itemwise ratio of the estimated non-compensated own-price elasticity and the expenditure elasticity for the LES and the AIDS

All India : rural and urban; aggregate and disaggregated data

Items	LES				AIDS			
	All India rural		All India urban		All India rural		All India urban	
	aggregate data	disaggregated data	aggregate data	disaggregated data	aggregate data	disaggregated data	aggregate data	disaggregated data
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Cereals and cereal substitutes	-0.819	-0.920	-0.956	-1.013	-0.039	-0.174	-0.452	0.152
2. Milk and milk products	-0.536	-0.677	-0.808	-0.835	-0.153	-0.336	-0.371	-0.725
3. Edible oils	-0.539	-0.692	-0.832	-0.856	-0.929	-0.600	-0.924	-0.910
4. Meat, fish, egg	-0.586	-0.688	-0.807	-0.850	-1.214	-0.694	-0.864	-0.729
5. Sugar	-0.539	-0.686	-0.819	-0.852	-0.598	-0.336	-2.000	-0.830
6. Other food	-0.583	-0.729	-0.861	-0.868	-1.268	-1.404	-2.211	-1.205
7. Clothing	-0.539	-0.675	-0.773	-0.831	0.219	0.086	-0.494	-0.795
8. Fuel and light	-0.540	-0.714	-0.818	-0.869	-0.642	-0.852	-1.204	-1.080
9. Other non food	-0.565	-0.677	-0.772	-0.800	-0.345	-0.373	-0.172	-0.530

sector. For the urban sector the constant is approximately equal to -0.8 for both types of data. It may be noted that in all cases, 'cereals and cereal substitutes' turns out to be the only exception.[12/] For AIDS, on the other hand, the ratios are totally different and are highly variable across items. In some cases there are large differences between ratios based on aggregate and disaggregated data for the same item. For the rural sector, the ratio ranges from -0.039 to -1.268 for aggregate data, and from 0.086 to -1.404 for the disaggregated data. For the urban sector, the corresponding ranges are from -0.172 to -2.211 and from 0.152 to -1.205 . Figures 4.1 and 4.2 illustrate the relationship between non-compensated own-price elasticities and corresponding expenditure elasticities for aggregate and disaggregated data for the two systems, respectively for the rural sector and the urban sector. Clearly, while the points for the LES lie more or less on a straight line, those for the AIDS are fairly scattered for both sectors as also for both types of data.

12/ The proportionality relationship as derived in Deaton (1974b) is an approximate one and is based on the assumption that the cross-price elasticities are negligible compared to the own-price elasticities. It may, however, be observed from Tables C.1-C.4 in Appendix C, that for 'cereals and cereal substitutes' this assumption is empirically not quite supported.

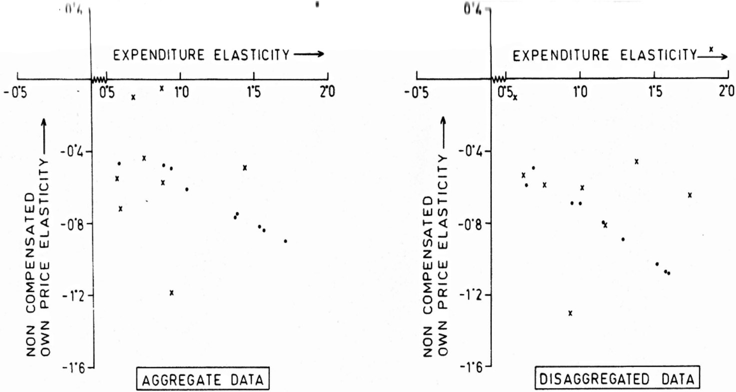
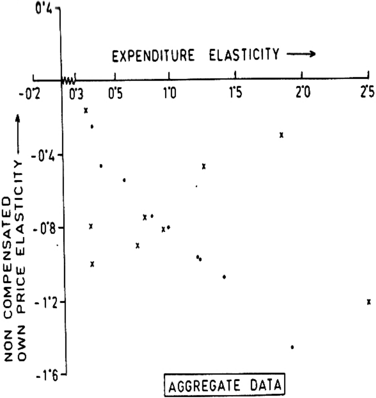


Figure 4.1 Scatter diagram showing the relationship between expenditure elasticities and non-compensated own-price elasticities for the LES (∘) and the AIDS (×)

All India: rural; aggregate and disaggregated data



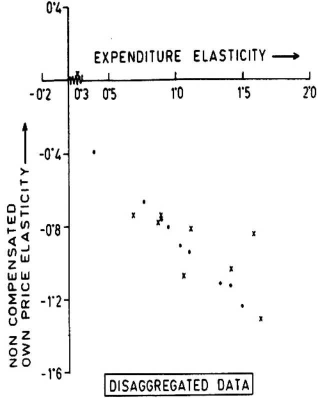


Figure 4.2 Scatter diagram showing the relationship between expenditure elasticities and non-compensated own-price elasticities for the LES (•) and the AIDS (×)

All India: urban; aggregate and disaggregated data

4.4 Summary and conclusions

In this chapter the empirical performances of the IES and the AIDS have been compared on the basis of aggregate and disaggregated Indian consumer expenditure data separately for the rural and the urban sectors. On the whole, the AIDS appears to have performed better than the IES. However, this is only to be expected as the AIDS, by construction, has larger explanatory power.[13] Specific observations on the performances of the two systems are as follows.

As regards the measure of goodness of fit, both in terms of item expenditures and budget shares, the AIDS appears to perform better than the IES for both aggregate and disaggregated data for both sectors.

13/ To have some idea as to whether AIDS is really worth all the parameters that it requires, and whether one is getting good value per parameter, a stepwise regression procedure has been applied as a rough guide. The results are reported in Appendix C. A more meaningful approach would be to estimate the homogeneity and symmetry restricted versions of the AIDS and check for the validity of these restrictions. We have estimated the homogeneous version of the AIDS on both the aggregate and disaggregated sets of data. The homogeneity hypothesis is rejected in no case. The symmetry restricted version of the AIDS could not, however, be estimated because of the non-availability of appropriate computational facilities for the non-linear full information maximum likelihood estimation of the AIDS with the present number of parameters.

The itemwise expenditure elasticities estimated from the aggregate and the disaggregated data are in close parity for the LES, while those for the AIDS show somewhat larger discrepancies. A comparison of the expenditure elasticities based on the two models reveals that these elasticities show closer agreement for the disaggregated data than for the aggregate data.

Finally, the non-compensated own-price elasticities based on the LES and the AIDS are strikingly different for many items for either set of data especially for the rural sector. While the LES-based elasticities show a broad proportionality relationship with the corresponding expenditure elasticities, the AIDS-based elasticities are relatively unconstrained.

Chapter V

Empirical performances of the alternative specifications of modified SNAM and the proposed system

5.1 Introduction

In chapter II we have suggested a generalisation of the LES, called SNAM 2, which is basically a modified version of Deaton's (1976) Simple Non-Additive Model (referred to as SNAM 1 here). We have also proposed a set of new systems of demand equations, which have been called the 'Variant' models, based on Muellbauer's (1975) notion of consistent aggregation.

In the present chapter, on the one hand, the performances of the LES, SNAM 1 and SNAM 2, and, on the other hand, those of the four alternative specifications of the 'Variant' models, viz., Variant 1, Variant 2, Variant 3 and Variant 4 have been compared empirically. The two sets of nested models have been analysed based on population groupwise disaggregated consumer expenditure data on nine broad groups of items for rural and urban India.

The plan of this chapter is as follows: section 5.2 describes the procedure of fitting the six models to the data; section 5.3 presents the empirical results on log-likelihood values, parameter estimates, goodness of fit and elasticitics; and finally, section 5.4 gives the summary and conclusions.

5.2 Estimation

The seven systems, viz., the LES, SNAM 1, SNAM 2, Variant 1, Variant 2, Variant 3 and Variant 4 have been estimated in budget share forms by Non-Linear Full Information Maximum Likelihood (NLFIML) method. The data used here are the disaggregated consumer expenditure data for the rural and the urban sectors of India over the period covered by NSS 7th to 28th rounds.[1/]

For the purpose of estimation, an additive disturbance term has been introduced in the functional form of each of the models. Thus, the stochastic specification of the systems are:

$$\text{LES} : \quad w_{it} = \frac{p_{it}\, c_i}{y_t} + b_i \left(1 - \frac{\sum_{j=1}^{n} p_{jt}\, c_j}{y_t} \right) + u_{it} \qquad \ldots (5.2.1)$$

$$\text{SNAM 1} : \quad w_{it} = \frac{p_{it}\, \gamma_i\,(p_t)}{y_t} + b_i \left(1 - \frac{\sum_{j=1}^{n} p_{jt}\, \gamma_j\,(p_t)}{y_t} \right) + u_{it} \qquad \ldots (5.2.2)$$

1/ The LES has been fitted on the same data in Chapter IV. Some of the results for the LES have, however, been reported in this chapter also to facilitate comparison with those for the other systems.

$$\text{SNAM 2}: w_{it} = \frac{p_{it}\ \gamma_i\ (p_t)}{y_t} + \beta_i(p_t)\ (1 - \frac{\sum_{j=1}^{n} p_{jt}\ \gamma_j\ (p_t)}{y_t}) + u_{it} \quad \ldots (5.2.3)$$

$$\text{Variant 1}: w_{it} = \frac{p_{it}\ c_i}{\sum_{j=1}^{n} p_{jt}\ c_j} + b_i \log\ (\frac{y_t}{\sum_{j=1}^{n} p_{jt}\ c_j}) + u_{it} \quad \ldots (5.2.4)$$

$$\text{Variant 2}: w_{it} = \frac{p_{it}\ \gamma_i\ (p_t)}{\sum_{j=1}^{n} p_{jt}\ \gamma_j\ (p_t)} + b_i \log\ (\frac{y_t}{\sum_{j=1}^{n} p_{jt}\ \gamma_j\ (p_t)}) + u_{it} \quad \ldots (5.2.5)$$

$$\text{Variant 3}: w_{it} = \frac{p_{it}\ \gamma_i\ (p_t)}{\sum_{j=1}^{n} p_{jt}\ \gamma_j\ (p_t)} + \beta_i(p_t) \log\ (\frac{y_t}{\sum_{j=1}^{n} p_{jt}\ \gamma_j(p_t)}) + u_{it} \quad \ldots (5.2.6)$$

and

$$\text{Variant 4}: w_{it} = \frac{p_{it}\ c_i}{\sum_{j=1}^{n} p_{jt}\ c_j} + \beta_i\ (p_t) \log\ (\frac{y_t}{\sum_{j=1}^{n} p_{jt}\ c_j}) \quad \ldots (5.2.7)$$

for $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, T$ with

$$\gamma_i\ (p_t) = c_i + a_i \log \frac{p_{it}}{\pi_{1t}}\ ; \quad \pi_{1t} = \frac{\sum_{k=1}^{n} a_k\ p_{kt}}{\sum_{k=1}^{n} a_k} \quad \ldots (5.2.8)$$

$$\text{and} \quad \beta_i\ (p_t) = b_i + d_i \log \frac{p_{it}}{\pi_{2t}}\ ; \quad \pi_{2t} = \prod_{k=1}^{n} p_{kt}^{\frac{d_k}{\sum_j d_j}} \quad \ldots (5.2.9)$$

For all the six systems, the disturbance term u_{it} has been assumed to have the following properties.[2]

$$E\,(u_{it}) = 0 \quad \text{for all i and t} \qquad \ldots (5.2.10)$$

$$E\,(u_{it}\,u_{js}) = \begin{cases} \sigma_{ij} & \text{for } t = s \\ 0 & \text{for } t \neq s \end{cases} \qquad \ldots (5.2.11)$$

At this point we may mention the problems faced in the estimation of parameters of SNAM 1 (urban data) and SNAM 2 (rural and urban data).[3] In these three estimations, after a number of iterations, the parameter a_n was found to diverge, while the other parameters were almost stabilizing.[4] In these three cases, we have (i) allowed a_n to continue to be large till the program came to an end normally, and (ii) restricted a_n at the value after which a sudden jump to a large value was found to occur, while the other parameters were left unrestricted.

---

[2] The expressions for the first order partial derivatives of the right hand side of equations (5.2.1) - (5.2.7) with respect to the parameters, have been given in Appendix B.

[3] In Deaton (1976), the estimation of the parameters of SNAM 1 on British data with eight broad groups of items posed an identification problem of the following type. It was shown that if p_i's are close to π_1, which may very well be the case with only eight broad groups, a_i's and c_i's cannot be precisely estimated.

[4] If at any stage a_j is such that $\pi_1 \approx p_j$, so that from our formulation, $\frac{\partial w_{it}}{\partial a_j} \approx 0$ for all i=1, 2, ..., n, and t=1, 2, ..., T, the elements in the direction matrix corresponding to a_j will be approximately zero. Consequently in the next stage a_j will tend to be very large, while other parameters will change marginally.

No such problem of estimation, however, occurred in case of the alternative Variant specifications.

5.3 Comparison of results

In this section we present the parameter estimates of the models fitted to the rural and urban data, and compare the performances of the LES-SNAM 1 - SNAM 2 and of Variant 1 - Variant 2 - Variant 3 - Variant 4 on the basis of log-likelihood values, measures of goodness of fit (R^2_W) and expenditure and own-price elasticities.[5/]

5.3.1 Log-likelihood values

Table 5.1.A presents the 2 × log likelihood values for the seven models, viz., LES, SNAM 1, SNAM 2, Variant 1, Variant 2, Variant 3 and Variant 4, along with the corresponding number of parameters. For each of SNAM 1 and SNAM 2, two values have been presented where necessary - one for unrestricted a_n and the other for restricted a_n .

5/ R^2_W and elasticities of Variant 4 have not been reported here.

Table 5.1.A : Log likelihood values for the alternative specification of the 'SNAM' and 'Variant' models

All India: rural and urban; disaggregated data

System	Number of parameters	2 × log likelihood	
		All India rural	All India urban
(1)	(2)	(3)	(4)
LES	18	3523.6	3534.7
SNAM 1	27	3568.9	3598.9* (3607.9)
SNAM 2	36	3691.7* (3690.2)	3670.7* (3686.3)
Variant 1	18	3511.6	3465.9
Variant 2	27	3688.0	3726.1
Variant 3	36	3758.2	3778.2
Variant 4	27	3614.4	3678.7

* restricted estimates; figures in parentheses are the unrestricted estimates.

It may be seen from the table that introduction of prices through $\gamma_j(p)$ and $\beta_j(p)$ improves the performance of both types of models as one moves from the simple versions to the extended versions.[6] To assess the

[6] If one is interested in bringing in prices in the model in a manner so that the empirical results improve considerably, Variant 2 would seem to be preferable to Variant 4, as Variant 4 does not allow any substitution possibility for the poorest consumer. From Table 5.1.A, between Variant 2 and Variant 4, the former turns out to be a superior specification for both rural and urban data. Variant 4 is, therefore, excluded from the subsequent analysis.

statistical significance of the improvements, the formal likelihood ratio test was applied to each of the two sets of nested models. Table 5.1.B below summarises the results.[7/] The test based on this table indicates

Table 5.1.B : Difference of log-likelihood values for alternative nested specifications of the 'SNAM' and 'Variant' models

All India: rural and urban; disaggregated data,

Improvement due to generalisation		$(2 \times \log \text{likelihood}) = \chi^2(9)$*	
From	To	rural India	urban India
(1)	(2)	(3)	(4)
LES	SNAM 1	45.3	64.2
SNAM 1	SNAM 2	122.8	71.8
Variant 1	Variant 2	176.4	260.2
Variant 2	Variant 3	70.2	52.1

* Critical value of $\chi^2(9)$ at 5% level of significance is 16.92 .

significant improvements as one passes from LES through SNAM 2 and from Variant 1 through Variant 3 .

As the two sets of specifications are non-nested, the log-likelihood values for the similar specifications of the SNAM and the

7/ The results of SNAM 1 (rural) and SNAM 2 (rural and urban) to be reported henceforth will be based on restricted a_n .

Variant models are not strictly comparable. However, one may observe that whereas LES performs better than Variant 1, the order of performance is reversed in the extended specification. This is seen for both rural and urban sectors of the country.

5.3.2 Parameter estimates

The full set of estimated parameters and the corresponding asymptotic standard errors for the LES, SNAM 1 and SNAM 2 have been presented in Tables 5.2.1 and 5.2.2 for the rural and the urban sectors of India respectively. Tables 5.3.1 and 5.3.2 present the parameter estimates and the corresponding asymptotic standard errors for Variant 1, Variant 2 and Variant 3 for rural and urban India respectively. We shall discuss the parameter estimates obtained from the different systems below.[8]

For SNAM 1, all the b-parameters and most of the c-parameters for both sectors are significant (exceptions are 'milk and milk products', 'sugar', 'clothing' and 'other non-food' for the rural sector, and 'milk and milk products', 'meat, fish, egg', and 'other food' for the urban sector). The standard errors for all the b-parameters being very small, the b-parameters are very precisely estimated. The a-parameters for 'cereals and cereal substitutes', 'milk and milk products', 'meat, fish, egg',

[8] The discussion on the LES parameters has been reported in Chapter IV.

Table 5.2.1 Estimates of parameters for the LES, SNAM 1 and SNAM 2*

All India : rural; disaggregated data

Items	LES		SNAM 1			SNAM 2**			
	c	b	c	b	a	c	b	a	d
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	4.210 (.121)	0.270 (.010)	4.374 (.216)	0.268 (.011)	-1.014 (.181)	2.946 (.288)	0.267 (.011)	-0.059 (.215)	0.017 (.020)
2. Milk and milk products	-0.099 (.042)	0.117 (.003)	-0.061 (.096)	0.118 (.003)	0.228 (.110)	-0.675 (.128)	0.121 (.003)	-0.024 (.109)	0.057 (.018)
3. Edible oils	0.115 (.008)	0.030 (.001)	0.121 (.021)	0.031 (.001)	0.006 (.014)	0.003 (.025)	0.027 (.001)	0.012 (.012)	0.015 (.002)
4. Meat, fish, egg	0.070 (.008)	0.030 (.001)	0.097 (.022)	0.029 (.001)	-0.041 (.014)	-0.046 (.028)	0.029 (.001)	0.020 (.015)	0.004 (.002)
5. Sugar	0.052 (.012)	0.038 (.001)	0.050 (.030)	0.039 (.001)	0.045 (.019)	-0.120 (.036)	0.036 (.001)	0.011 (.014)	0.030 (.002)
6. Other food	0.695 (.045)	0.125 (.005)	0.698 (.090)	0.127 (.005)	0.146 (.133)	0.135 (.117)	0.117 (.005)	0.149 (.131)	0.078 (.021)
7. Clothing	-0.126 (.053)	0.124 (.005)	-0.069 (.107)	0.124 (.004)	0.298 (.160)	-0.700 (.142)	0.134 (.004)	-0.235 (.246)	0.238 (.047)
8. Fuel and light	0.586 (.017)	0.042 (.001)	0.614 (.033)	0.042 (.001)	-0.192 (.066)	0.391 (.044)	0.039 (.001)	0.070 (.058)	-0.041 (.007)
9. Other non food	-0.095 (.104)	0.223 (.009)	0.076 (.191)	0.222 (.010)	-5.949 (7.753)	-1.166 (.276)	0.231 (.009)	-5.940 (-)	0.102 (.032)

* Figures in parentheses are the asymptotic standard errors.

** These estimates are based on the restriction $a_9 = -5.940$.

Table 5.2.2 . Estimates of parameters for the LES, SNAM 1 and SNAM 2*

All India : urban; disaggregated data

Items	LES		SNAM 1**			SNAM 2***			
	c	b	c	b	a	c	b	a	d
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	4.281 (.104)	0.096 (.008)	4.302 (.154)	0.093 (.007)	-1.343 (.230)	4.847 (.136)	0.089 (.008)	-2.208 (.247)	0.118 (.043)
2. Milk and milk products	-0.298 (.058)	0.127 (.002)	-0.313 (.196)	0.128 (.002)	0.233 (.138)	0.475 (.170)	0.119 (.003)	-0.650 (.228)	0.109 (.035)
3. Edible oils	0.171 (.015)	0.035 (.001)	0.157 (.052)	0.036 (.002)	0.008 (.036)	0.378 (.038)	0.022 (.002)	-0.168 (.017)	0.040 (.004)
4. Meat, fish, egg	0.042 (.013)	0.038 (.001)	0.046 (.057)	0.038 (.001)	-0.018 (.039)	0.273 (.051)	0.038 (.001)	-0.164 (.038)	-0.002 (.003)
5. Sugar	0.141 (.018)	0.028 (.001)	0.109 (.047)	0.029 (.001)	0.190 (.061)	0.299 (.041)	0.028 (.002)	-0.035 (.061)	0.002 (.006)
6. Other food	0.457 (.090)	0.179 (.006)	0.500 (.260)	0.183 (.007)	-0.372 (.320)	1.445 (.275)	0.213 (.012)	-0.591 (.515)	-0.113 (.027)
7. Clothing	-0.417 (.057)	0.092 (.003)	-0.413 (.155)	0.090 (.003)	0.572 (.238)	0.152 (.124)	0.087 (.003)	-0.168 (.171)	0.054 (.013)
8. Fuel and light	0.540 (.022)	0.046 (.001)	0.569 (.067)	0.046 (.001)	-0.572 (.129)	0.844 (.058)	0.044 (.001)	-0.983 (.128)	0.013 (.008)
9. Other non food	-1.259 (.226)	0.358 (.009)	-1.274 (.544)	0.356 (.010)	-5.592 (-)	0.822 (.475)	0.357 (.010)	-6.062 (-)	0.093 (.038)

* Figures in parentheses are the asymptotic standard errors.

** These estimates are based on the restriction $a_9 = -5.592$.

*** " " " " " " $a_9 = -6.062$.

'sugar', 'fuel and light' in the rural sector, and for 'cereals and cereal substitutes', 'sugar', 'clothing', and 'fuel and light' in the urban sector are statistically significant, while for the other items the a-parameters are non-significant. It may be mentioned here that the a-parameter for 'other non-food' (urban data) does not have any standard error, since the parameter has been fixed at that value.

For SNAM 2, all the b-parameters for both sectors, and the c-parameters (except for 'edible oil', 'meat, fish, egg' and 'other food' for the rural sector, and 'clothing' and 'other non-food' for the urban sector) are significant. None of the a-parameters for the rural sector is significant. The a-parameters for the urban sector are all negative and many of them are significant, with the exception of 'sugar', 'other food', 'clothing' and 'other non-food'.[9/] Finally, most of the d-parameters (except for 'cereals and cereal substitutes' and 'meat, fish, egg' for the rural sector, and 'meat, fish, egg', 'sugar' and 'fuel and light' for the urban sector) are significant and are mostly positive for both the sectors.[10/] However, significant negative value of d is found for 'fuel and light' in the rural sector, and for 'other food' in the urban sector.

9/ The negative sign of a-parameters is reasonable as it indicates substitution possibilities at very low levels of living.

10/ A positive value of d means that with rising relative price of a commodity, its income elasticity goes up, which confirms Cramer's conjecture (Cramer, 1970).

Table 5.3.1 Estimates of parameters for the three Variant models*

All India : rural; disaggregated data

Items	Variant 1		Variant 2			Variant 3			
	c	b	c	b	a	c	b	a	d
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	4.600 (.264)	-0.185 (.005)	12.401 (.793)	-0.191 (.005)	-0.795 (1.978)	0.774 (.206)	-0.182 (.005)	-0.711 (.210)	0.100 (.036)
2. Milk and milk products	0.182 (.048)	0.052 (.002)	6.671 (3.052)	0.052 (.002)	-3.645 (2.146)	-0.072 (.011)	0.053 (.002)	0.055 (.010)	0.009 (.007)
3. Edible oils	0.155 (.014)	0.001 (.001)	1.286 (.422)	0.000 (.001)	-0.452 (.163)	0.021 (.006)	-0.002 (.001)	-0.015 (.004)	0.006 (.001)
4. Meat, fish, egg	0.115 (.013)	0.006 (.001)	1.478 (.548)	0.000 (.001)	-0.897 (.331)	0.008 (.004)	0.004 (.001)	-0.006 (.003)	0.001 (.001)
5. Sugar	0.124 (.016)	0.011 (.001)	1.963 (.815)	0.011 (.001)	-0.838 (.402)	-0.002 (.002)	0.010 (.001)	-0.014 (.006)	0.015 (.002)
6. Other food	0.911 (.075)	-0.012 (.005)	5.511 (1.669)	-0.012 (.004)	-0.676 (.863)	0.129 (.038)	-0.017 (.004)	-0.156 (.038)	0.063 (.014)
7. Clothing	0.169 (.050)	0.057 (.002)	7.235 (3.337)	0.059 (.002)	-6.115 (3.193)	-0.081 (.014)	0.059 (.002)	0.033 (.013)	0.050 (.011)
8. Fuel and light	0.648 (.039)	-0.024 (.001)	1.992 (.242)	-0.024 (.001)	-2.403 (.501)	0.106 (.028)	-0.025 (.001)	-0.075 (.016)	-0.014 (.004)
9. Other non food	0.451 (.101)	0.093 (.005)	13.107 (5.913)	0.101 (.005)	-16.668 (8.315)	-0.131 (.024)	0.100 (.005)	0.106 (.022)	0.024 (.009)

* Figures in parentheses are the asymptotic standard errors.

Table 5.3.2 Estimates of parameters for the three Variant models*

All India : urban; disaggregated data

Items	Variant 1		Variant 2			Variant 3			
	c	b	c	b	a	c	b	a	d
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	1.540 (.062)	-0.242 (.004)	5.301 (.555)	-0.195 (.002)	-4.238 (.524)	3.335 (.394)	-0.195 (.004)	-2.550 (.390)	-0.000 (.002)
2. Milk and milk products	-0.047 (.006)	0.044 (.002)	0.960 (.322)	0.037 (.002)	-1.017 (.316)	0.227 (.085)	0.033 (.002)	-0.319 (.131)	-0.003 (.002)
3. Edible oils	0.027 (.002)	0.006 (.001)	0.514 (.095)	-0.006 (.001)	-0.196 (.035)	0.252 (.041)	-0.011 (.001)	-0.119 (.021)	-0.005 (.002)
4. Meat, fish, egg	0.003 (.002)	0.011 (.001)	0.432 (.104)	0.004 (.001)	-0.311 (.075)	0.169 (.036)	0.002 (.001)	-0.126 (.027)	-0.002 (.001)
5. Sugar	0.021 (.003)	0.006 (.001)	0.439 (.086)	-0.003 (.001)	-0.112 (.060)	0.202 (.036)	-0.006 (.001)	-0.032 (.028)	-0.002 (.001)
6. Other food	0.012 (.014)	0.060 (.004)	2.761 (.642)	0.008 (.004)	-5.189 (1.642)	1.087 (.226)	-0.007 (.005)	-1.528 (.421)	-0.013 (.006)
7. Clothing	-0.013 (.006)	0.023 (.002)	0.518 (.231)	0.038 (.002)	0.063 (.132)	0.021 (.049)	0.044 (.003)	0.056 (.080)	0.004 (.002)
8. Fuel and light	0.128 (.008)	-0.007 (.001)	1.046 (.167)	-0.020 (.001)	-0.603 (.109)	0.555 (.080)	-0.020 (.001)	-0.442 (.064)	0.000 (.000)
9. Other non food	-0.060 (.011)	0.099 (.003)	2.196 (.890)	0.137 (.004)	-3.049 (1.067)	0.215 (.191)	0.161 (.006)	-1.465 (.506)	0.017 (.007)

* Figures in parentheses are the asymptotic standard errors.

We now discuss the results presented in Tables 5.3.1 - 5.3.2 . For Variant 1, all the b and c values are significantly different from zero for both the sectors. Exceptions are: c-values for 'meat, fish, egg', and 'other food' for the urban sector, and b-values for 'edible oil' for the rural sector. The b-values for 'cereals and cereal substitutes' and 'fuel and light' are negative for both sectors, and the b-value for 'other food' is negative for the rural sector.

For Variant 2, all the estimated b and c parameters for both the sectors are significant, the only exception being the b-value for 'edible oil' for rural India. The a-parameters except for 'cereals and cereal substitutes', 'milk and milk products', 'other food' and 'clothing' in the rural sector, and 'sugar' and 'clothing' in the urban sector, are significant and are mostly negative for both sectors.

For Variant 3, all the estimated b-parameters are significant for both sectors (exceptions are: 'meat, fish, egg' and 'other food' for the urban sector). Same is the case with the estimates of c-parameters except for 'sugar' for the rural sector, and 'clothing' and 'other non-food' for the urban sector. The estimates of a-parameters are by and large significant (with the exception of 'sugar' and 'clothing' in urban India) and are mostly negative. Positive and significant a-values are seen for 'milk and milk products', 'clothing' and 'other non-food' in the rural sector. The pattern of the estimates of the d-parameters obtained from the two sectors are somewhat different. Most of the estimated d's turn out to be significantly different from zero. They are relatively small in magnitude for the urban sector. While the d's for the rural sector are mostly positive, those for the urban sector are mostly negative. However, given the relatively small magnitudes of d's for the urban sector, the results perhaps indicate that while for the rural consumers income elasticities for most items react positively to rise in relative prices of the items, for urban consumers the income elasticities are comparatively insensitive to relative price variations.

5.3.3 Goodness of fit

To compare the performances of the six demand systems and to examine their goodness of fit at the item level, the itemwise squared correlation coefficients between observed and predicted budget shares (R_w^2) were calculated for each of the estimated systems.[11] Table 5.4 presents the R_w^2 values by items for each system fitted for the rural and the urban sectors. The results may be summarised as follows:

The performances of the IES and Variant 1 are more or less similar, Variant 1 performing slightly better for the rural sector and somewhat worse

---

11/ As all the systems were estimated in budget share equation form, R_w^2's were used to compare the goodness of fit.

Table 5.4 Itemwise squared correlation coefficients between observed and predicted budget shares (R_W^2) for different systems. All India : rural and urban; disaggregated data

Items	All India Rural						All India Urban					
	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	0.881	0.867	0.878	0.955	0.963	0.963	0.937	0.949	0.949	0.977	0.990	0.991
2. Milk and milk products	0.909	0.911	0.914	0.930	0.935	0.937	0.937	0.942	0.945	0.875	0.878	0.876
3. Edible oils	0.516	0.532	0.650	0.646	0.640	0.626	0.450	0.483	0.676	0.019	0.766	0.719
4. Meat, fish, egg	0.477	0.542	0.529	0.274	0.474	0.522	0.488	0.479	0.475	0.363	0.335	0.312
5. Sugar	0.671	0.742	0.898	0.768	0.825	0.907	0.035	0.165	0.101	0.107	0.193	0.177
6. Other food	0.009	0.024	0.231	0.013	0.168	0.295	0.108	0.046	0.012	0.014	0.279	0.673
7. Clothing	0.779	0.802	0.845	0.914	0.919	0.912	0.758	0.811	0.869	0.823	0.915	0.910
8. Fuel and light	0.909	0.924	0.921	0.892	0.914	0.934	0.894	0.900	0.899	0.395	0.949	0.967
9. Other non food	0.720	0.667	0.729	0.850	0.856	0.862	0.841	0.815	0.831	0.949	0.955	0.959

for the urban sector than the LES. The Variant 1 fit is noticeably inferior for 'edible oils' and 'fuel and light' in the urban sector.

Introduction of prices through $\gamma_j(p)$ and $\beta_j(p)$ generally improves the R_W^2 - values, the improvements, however, being small in general. The improvement is quite clear for the Variant set-up in the urban sector, while elsewhere it is not so clear. As one moves from the simple versions to the extended versions, noticeable improvements of R_W^2 value may be observed for 'other food' in the SNAM set-up and for 'other food' and 'meat, fish, egg' in the Variant set up for the rural sector. For the urban sector, considerable improvement may be observed for 'edible oils' in the SNAM set-up and marked improvements may be observed for 'edible oils', 'other food' and 'fuel and light' in the Variant set-up. However, in some cases, (e.g., for the urban sector, for 'meat, fish, egg' and 'other food' in the SNAM set-up, and for 'meat, fish, egg' in the Variant set-up) a slightly declining trend may be observed.

Variant 3 in general, performs better than SNAM 2 in both sectors taking all items together. The R_W^2 values for 'other food' in the rural sector for both the models, for 'sugar' and 'other food' for SNAM 2 and for 'meat, fish, egg' and 'sugar' for Variant 3 in the urban sector are rather small. The R_W^2 value for 'other food' based on Variant 3 is markedly better than that based on SNAM 2 for urban India.

5.3.4 Expenditure and own-price elasticities

Tables 5.5 and 5.6 give the itemwise expenditure and own-price elasticities estimated for different systems at the sample average of prices and total expenditure for the two sectors.

As regards the expenditure elasticities, the estimates for alternative SNAM specifications are close to one another, but differ to some extent from the other estimates. Variant 1, Variant 2 and Variant 3 give very similar estimates for the rural sector. For the urban sector, however, the agreement is not as much; considerable discrepancies may be observed for almost all items except for 'milk and milk products'. On the whole, for both sectors, the estimated elasticities based on the two sets of models are comparable. The estimates based on SNAM 2 and Variant 3 are fairly close for both sectors (exceptions are: 'milk and milk products', 'clothing' and 'other non-food' for the rural sector, and 'clothing' and 'other non-food' for the urban sector).

The estimates of itemwise own-price elasticities show much greater variation across the systems. Within the SNAM set-up there are large discrepancies in elasticity values estimated from the three systems especially for 'sugar', 'clothing', and 'fuel and light' for the rural sector and for 'milk and milk products', 'edible oils', 'other food' and 'clothing' for the urban sector. For the Variant set-up, marked discrepancies may be observed for almost all items except for 'sugar' for

Table 5.5 Estimates of itemwise expenditure elasticity from different systems at sample average of prices and total expenditure. All India : rural and urban; disaggregated data

Items	All India Rural						All India Urban					
	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	0.640	0.640	0.635	0.586	0.568	0.578	0.387	0.381	0.375	0.123	0.286	0.279
2. Milk and milk products	1.576	1.582	1.584	1.765	1.769	1.768	1.331	1.332	1.334	1.484	1.415	1.424
3. Edible oils	0.996	1.003	1.026	1.031	0.999	1.024	0.888	0.907	0.931	1.144	0.860	0.896
4. Meat, fish, egg	1.156	1.148	1.173	1.239	1.155	1.169	1.104	1.109	1.087	1.336	1.108	1.120
5. Sugar	1.293	1.317	1.316	1.378	1.368	1.378	0.942	0.959	0.950	1.191	0.892	0.929
6. Other food	0.945	0.959	0.959	0.910	0.913	0.933	1.031	1.047	1.052	1.344	1.044	1.073
7. Clothing	1.595	1.596	1.592	1.823	1.849	1.802	1.489	1.486	1.488	1.415	1.711	1.692
8. Fuel and light	0.690	0.686	0.671	0.624	0.621	0.616	0.756	0.761	0.752	0.883	0.682	0.676
9. Other non food	1.516	1.482	1.512	1.694	1.746	1.733	1.397	1.378	1.382	1.417	1.578	1.556

Table 5.6 Estimates of itemwise own-price elasticity from different systems at sample average of prices and total expenditure. All India : rural and urban; disaggregated data

Items	All India Rural						All India Urban					
	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	-0.589	-0.688	-0.681	-0.213	-0.529	-0.489	-0.392	-0.556	-0.396	0.027	-0.626	-0.612
2. Milk and milk products	-1.067	-0.852	-0.832	-0.672	-0.384	-0.939	-1.111	-1.008	-0.637	-1.330	-1.036	-1.164
3. Edible oils	-0.690	-0.650	-0.504	-0.057	-0.435	-0.353	-0.760	-0.764	-0.292	-0.415	-0.419	-0.481
4. Meat, fish, egg	-0.795	-0.382	-0.917	-0.223	-0.727	-0.818	-0.938	-0.969	-0.960	-0.927	-0.879	-0.973
5. Sugar	-0.887	-0.764	-0.361	-0.339	-0.300	-0.395	-0.808	-0.539	-0.587	-0.585	-0.242	-0.145
6. Other food	-0.689	-0.598	-0.383	-0.065	-0.305	-0.232	-0.895	-1.003	-1.380	-0.951	-1.642	-1.158
7. Clothing	-1.077	-0.820	-0.022	-0.718	-0.745	-0.132	-1.237	-0.841	-0.561	-1.142	-0.343	-0.555
8. Fuel and light	-0.493	-0.641	-1.284	0.238	-1.071	-1.097	-0.657	-1.009	-0.956	0.223	-0.524	-0.708
9. Other non food	-1.027	-1.151	-0.816	-0.647	-0.928	-0.791	-1.118	-1.274	-1.160	-1.139	-1.179	-0.962

the rural sector, and for 'cereals and cereal substitutes', 'sugar', 'other food', 'clothing' and 'fuel and light' for the urban sector.

One of the purposes of introducing price flexibility into the parameters of the LES is to overcome the problem created by the additivity of the direct utility function underlying the LES, so that the approximate proportionality relationship between the expenditure and non-compensated own-price elasticities is no longer valid. To check the improvement in this respect empirically for the SNAM set-up, the ratio between the expenditure elasticity and non-compensated own-price elasticity for each item was calculated for the LES, SNAM 1 and SNAM 2 . Table 5.7 reports these ratios for rural and urban data. The ratios between the expenditure and non-compensated own-price elasticities for the Variant models have also been calculated for rural and urban India and reported in Table 5.7 . Clearly, while the LES shows an approximate proportionality relationship between the expenditure elasticities and non-compensated own-price elasticities for almost all items for both sectors, (only 'cereals and cereal substitutes' stands out from the rest) the other systems do not.[12] For the other systems the ratios are variable across items. The ratios, in general, tend to be less than or near 1 in absolute value, with the exception of 'fuel and light' for SNAM 2, Variant 2 and Variant 3 in the rural sector, and 'cereals and cereal substitutes' for SNAM 1, Variant 2

---

[12] The fact that 'cereals and cereal substitutes' stands out as an exception has been reported in Chapter IV.

Table 5.7 Itemwise ratio of the estimated non-compensated own-price elasticities and the expenditure elasticity for the SNAM and Variant models
All India : rural and urban; disaggregated data

Items	All India Rural						All India Urban					
	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3	LES	SNAM 1	SNAM 2	Variant 1	Variant 2	Variant 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	-0.920	-1.075	-1.072	-0.364	-0.931	-0.846	-1.013	-1.459	-1.056	0.219	-2.189	-2.194
2. Milk and milk products	-0.677	-0.539	-0.525	-0.381	-0.217	-0.531	-0.835	-0.757	-0.478	-0.896	-0.732	-0.817
3. Edible oils	-0.692	-0.648	-0.496	-0.055	-0.435	-0.345	-0.856	-0.842	-0.314	-0.363	-0.487	-0.537
4. Meat, fish, egg	-0.688	-0.768	-0.782	-0.180	-0.629	-0.700	-0.850	-0.874	-0.883	-0.694	-0.793	-0.869
5. Sugar	-0.686	-0.580	-0.274	-0.246	-0.219	-0.287	-0.852	-0.562	-0.618	-0.491	-0.271	-0.156
6. Other food	-0.729	-0.624	-0.399	-0.071	-0.334	-0.249	-0.868	-0.958	-1.312	-0.708	-1.573	-1.079
7. Clothing	-0.675	-0.514	-0.014	-0.394	-0.403	-0.073	-0.831	-0.566	-0.377	-0.807	-0.200	-0.328
8. Fuel and light	-0.714	-0.939	-1.913	-0.381	-1.725	-1.781	-0.869	-1.326	-1.271	0.026	-0.768	-1.047
9. Other non food	-0.677	-0.777	-0.540	-0.382	-0.532	-0.456	-0.800	-0.924	-0.839	-0.804	-0.113	-0.618

and Variant 3, 'other food' for SNAM 2, Variant 2 and 'fuel and light' for SNAM 1 and SNAM 2 in the urban sector.

5.4 Summary and conclusions

In this chapter, empirical performances of alternative forms (that allow for different degrees of flexibility of price responses) of the SNAM models and the Variant models have been examined on Indian consumer expenditure data. In general, the performances of the systems improve by increasing the flexibility of price responses. On the whole, Variant 3 performs better than the comparable SNAM of the SNAM set-up.

The itemwise expenditure and non-compensated own-price elasticities obtained from different systems are compared. The expenditure elasticities estimated for all the systems are broadly comparable, elasticities based on the SNAM specifications being marginally different from those based on the Variant models. The price elasticities are far less comparable. The IES-based expenditure and non-compensated own-price elasticities show a near-proportional relationship for almost all items for rural and urban India, while the corresponding estimates based on its extended versions and the other systems are relatively free.

Chapter VI

SNAM 2, Variant 3 and AIDS : An empirical comparison

6.1 Introduction

From the empirical analyses reported in the previous chapter, we have seen that on the whole, SNAM 2 has performed significantly better than the other two specifications of the LES - SNAM 1 - SNAM 2 set-up. Correspondingly, Variant 3 has performed best in the Variant 1 - Variant 2 - Variant 3 set-up. In the present chapter, these two best fitting models, viz., SNAM 2 and Variant 3 have been compared empirically with the AIDS on the basis of the population groupwise disaggregated Indian consumer expenditure data. This comparison has been based on several criteria, viz., goodness of fit, ability to meet the regularity conditions, plausibility of the estimated elasticities, and properties of residuals revealed by tests of misspecification.

Section 6.2 reports the comparative goodness of fit of the estimated models; results on empirical validity of the regularity conditions are presented in section 6.3; the estimates of elasticities based on these models are examined in section 6.4; sections 6.5 and 6.6 report on tests of serial correlation of the residuals and misspecification of functional forms respectively - for the three models; and finally, section 6.7 draws some concluding observations.

6.2 Comparative goodness of fit

To examine the relative goodness of fit of SNAM 2, Variant 3 and AIDS at the item level, the itemwise squared correlation coefficients between observed and predicted budget shares (R_W^2) have been calculated for each item. Table 6.1 lists the R_W^2 values by items for each system fitted to the rural and urban data. As could be expected, AIDS performs best, on the whole, taking all items together. One may note in particular that the R_W^2 values for 'other food' in the rural sector and for 'sugar' in the urban sector are much too low for SNAM 2 and Variant 3 compared to the corresponding AIDS values.

From Table 6.1, Variant 3 appears to be somewhat closer to the AIDS than the SNAM 2 . Thus, for 'cereals and cereal substitutes', 'milk and milk products', 'other food', 'clothing', 'fuel and light', and 'other non-food' in the rural sector and for all items except 'meat, fish, egg' in the urban sector, the R_W^2 values for Variant 3 are closer (than those of SNAM 2) to the R_W^2 values based on the AIDS . In the urban sector, the R_W^2 value based on SNAM 2 for 'other food' is strikingly lower than those for Variant 3 and AIDS .

Table 6.1 Itemwise squared correlation coefficients between observed and predicted budget shares (R_W^2)

All India : rural and urban; disaggregated data

Items	All India Rural			All India Urban		
	SNAM 2	Variant 3	AIDS	SNAM 2	Variant 3	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	0.878	0.963	0.969	0.949	0.991	0.995
2. Milk and milk products	0.914	0.937	0.955	0.945	0.876	0.900
3. Edible oils	0.650	0.626	0.757	0.676	0.719	0.827
4. Meat, fish, egg	0.529	0.522	0.568	0.475	0.312	0.407
5. Sugar	0.898	0.907	0.890	0.101	0.177	0.511
6. Other food	0.231	0.295	0.639	0.012	0.673	0.766
7. Clothing	0.845	0.912	0.943	0.869	0.910	0.953
8. Fuel and light	0.921	0.934	0.958	0.899	0.967	0.972
9. Other non food	0.729	0.862	0.870	0.831	0.959	0.962

6.3 Empirical validity of regularity conditions

The regularity conditions, i.e., the concavity of the cost functions underlying each of the three systems have been checked empirically. For this, the negative semi-definiteness of the matrix of compensated price effects calculated at each sample point has been checked. For each of the three population groups in either sector, we thus have 20 such checks for every system. Table 6.2 below summarises these results.

Table 6.2 : Frequency distribution of the number of positive characteristic roots of the 9 × 9 matrix of compensated price effects obtained from different systems and population groups.

All India: rural and urban; disaggregated data

System	Population group*	Number of positive characteristic roots						
		All India rural			All India urban			
		2	3	4	0	1	2	3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
SNAM 2	1	20	–	–	–	20	–	–
	2	20	–	–	7	13	–	–
	3	20	–	–	5	15	–	–
Variant 3	1	16	4	–	–	19	1	–
	2	18	2	–	–	7	13	–
	3	–	18	2	–	20	–	–
AIDS	1	–	6	14	–	–	8	12
	2	–	18	2	–	–	20	–
	3	–	20	–	–	–	18	2

* Population groups 1, 2 and 3 stand respectively for the lowest 30 per cent, middle 40 per cent and the top 30 per cent of the population based on ranking by the level of per capita total expenditure.

It may be observed that only for the SNAM 2 specification the regularity conditions are satisfied for the urban population groups 2 and 3, in 7 and 5 cases respectively, out of a total of 20 cases. In all other cases, the regularity conditions are violated for each of three systems. The significance of this phenomenon in empirical demand analysis is, however, somewhat controversial. As Deaton and Muellbauer (1980a) note, 'if the cost function doesn't exist, why worry about its concavity?' In any case, violation of regularity conditions does not become a hindrance to further analysis with the system. Judging by the number of positive roots, SNAM 2 performs the best and the AIDS performs least satisfactorily.

6.4 Expenditure and own-price elasticities

Tables 6.3 and 6.4 give the itemwise expenditure and own-price elasticities estimated for SNAM 2, Variant 3 and AIDS, at the sample average of prices and total expenditure separately for each of the two sectors.

The three models give more or less similar and highly sensible estimates for the expenditure elasticities, the estimates based on SNAM 2 being a little different from those based on Variant 3 and AIDS which show excellent agreement. This is due to the fact that SNAM 2 has a linear

Table 6.3 Estimates of itemwise expenditure elasticity from different systems at sample average values of prices and total expenditure

All India : rural and urban; disaggregated data

Items	All India Rural			All India Urban		
	SNAM 2	Variant 3	AIDS	SNAM 2	Variant 3	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	0.635	0.578	0.562	0.375	0.279	0.269
2. Milk and milk products	1.584	1.768	1.758	1.334	1.424	1.414
3. Edible oils	1.026	1.024	1.006	0.931	0.896	0.860
4. Meat, fish, egg	1.173	1.169	1.165	1.087	1.120	1.106
5. Sugar	1.316	1.378	1.375	0.950	0.929	0.887
6. Other food	0.959	0.933	0.925	1.052	1.073	1.049
7. Clothing	1.592	1.802	1.884	1.488	1.692	1.642
8. Fuel and light	0.671	0.616	0.621	0.752	0.676	0.675
9. Other non food	1.512	1.733	1.736	1.382	1.556	1.590

Table 6.4 Estimates of itemwise own-price elasticity from different systems at sample average values of prices and total expenditure

All India : rural and urban; disaggregated data

Items	All India Rural			All India Urban		
	SNAM 2	Variant 3	AIDS	SNAM 2	Variant 3	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	-0.681	-0.489	-0.098	-0.396	-0.612	0.041
2. Milk and milk products	-0.832	-0.939	-0.590	-0.637	-1.164	-1.025
3. Edible oils	-0.509	-0.353	-0.604	-0.292	-0.481	-0.783
4. Meat, fish, egg	-0.917	-0.818	-0.809	-0.960	-0.973	-0.806
5. Sugar	-0.361	-0.395	-0.462	-0.587	-0.145	-0.736
6. Other food	-0.383	-0.232	-1.299	-1.380	-1.158	-1.069
7. Clothing	-0.022	-0.132	0.162	-0.561	-0.555	-1.305
8. Fuel and light	-1.284	-1.097	-0.529	-0.956	-0.708	-0.729
9. Other non food	-0.816	-0.791	-0.648	-1.160	-0.962	-0.844

Engel curve, whereas the AIDS and Variant 3 have similar non-linear Engel curves.

The estimates of itemwise own-price elasticities for the three systems show much less agreement. The relative magnitudes of the elasticities estimated from the different systems do not reveal any clear pattern in general. However, for rural India, the SNAM 2 estimates tend to be higher than the corresponding Variant 3 estimates. The elasticity for 'clothing' for AIDS in the rural sector is positive.[1/] We may note that in the rural sector, almost all items (except for 'fuel and light' for SNAM 2 and Variant 3, and for 'other food' for AIDS) are price inelastic. The elasticities based on urban data tend to be more sensible in this respect. For SNAM 2, 'other food' and 'other non-food', for Variant 3, 'milk and milk products' and 'other food', and for the AIDS, 'milk and milk products', 'other food' and 'clothing' have price elasticities greater than 1 in absolute value.

To judge the plausibility of the estimates of own-price elasticities from the three models, it was examined whether the estimates of itemwise own-price elasticities for the three population groups show any reasonable pattern. Intuitively, it is reasonable to expect that the absolute value of the own-price elasticities would

---

1/ This has been reported in Chapter IV of this dissertation.

decline as real income level increases. Table 6.5 presents the estimated itemwise elasticities for the three population groups at prices and total expenditures of the 28th round of the NSS.[2/] The following patterns may be observed from Table 6.5 . In the rural sector, the AIDS estimates declined as income level increased for only three items (cereals and cereal substitutes', 'clothing' and 'fuel and light'), and almost no trends were observed for 'edible oils', 'meat, fish, egg' and 'other food'. For SNAM 2, this declining pattern was observed for only three items ('milk and milk products', 'sugar' and 'other non-food') and 'meat, fish, egg' showed no trend. In case of Variant 3, except for 'fuel and light', declining trend was observed for all other (items except for 'meat, fish, egg', which showed almost no trend).

For the urban sector, however, the declining trend in elasticities was observed for all the models for many of the items. Increasing trends were observed for 'sugar' and 'other food' for SNAM 2, 'sugar' and 'clothing' for Variant 3 and 'cereals and cereal substitutes' and 'other non-food' for the AIDS. Almost no trends were observed for 'edible oils' for Variant 3, and for 'edible oils', 'meat, fish, egg', 'sugar' and 'other food' for AIDS.

2/ It may be mentioned that the pattern and extent of variation of the elasticities over population groups does not change much over the rounds.

Table 6.5 Estimates of itemwise own price elasticity (calculated at prices and total expenditures) of NSS 28th round by population groups for SNAM 2, Variant 3 and AIDS

All India : rural and urban; disaggregated data

Items	SNAM 2 population group*			Variant 3 population group*			AIDS population group*		
	1	2	3	1	2	3	1	2	3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
All India Rural									
1. Cereals and cereal substitutes	-0.545	-0.628	-0.744	-0.683	-0.577	-0.316	-0.318	-0.197	0.138
2. Milk and milk products	-1.948	-0.985	-0.725	-1.233	-1.057	-0.964	-0.162	-0.506	-0.706
3. Edible oils	-0.491	-0.540	-0.581	-0.706	-0.636	-0.536	-0.672	-0.673	-0.674
4. Meat, fish, egg	-0.883	-0.884	-0.886	-1.061	-1.040	-1.014	-0.792	-0.807	-0.829
5. Sugar	-0.467	-0.395	-0.358	-0.383	-0.260	-0.134	-0.352	-0.463	-0.582
6. Other food	-0.303	-0.381	-0.453	-0.454	-0.295	-0.032	-1.259	-1.267	-1.281
7. Clothing	-0.062	0.079	0.115	0.322	0.093	-0.021	4.181	0.861	-0.098
8. Fuel and light	-0.791	-1.095	-1.522	-0.890	-0.975	-1.192	-0.586	-0.513	-0.310
9. Other non food	-1.169	-0.812	-0.682	-1.155	-0.998	-0.907	-0.045	-0.496	-0.718
All India Urban									
1. Cereals and cereal substitutes	-0.463	-0.433	-0.383	-0.746	-0.665	-0.400	-0.360	-0.161	0.534
2. Milk and milk products	-1.326	-0.764	-0.564	-1.254	-1.189	-1.140	-1.838	-1.029	-1.022
3. Edible oils	-0.646	-0.501	-0.395	-0.680	-0.670	-0.655	-0.844	-0.835	-0.821
4. Meat, fish, egg	-1.242	-1.161	-1.109	-1.619	-1.574	-1.524	-0.783	-0.796	-0.812
5. Sugar	-0.221	-0.498	-0.710	-0.225	-0.256	-0.296	-0.789	-0.779	-0.762
6. Other food	-0.762	-1.238	-1.546	-1.277	-1.157	-1.022	-1.070	-1.068	-1.066
7. Clothing	-0.848	-0.565	-0.497	-0.219	-0.447	-0.548	-2.258	-1.519	-1.279
8. Fuel and light	-1.223	-1.073	-0.920	-0.780	-0.742	-0.664	-0.774	-0.737	-0.653
9. Other non food	-1.801	-1.220	-0.970	-1.212	-0.871	-0.631	-0.649	-0.792	-0.869

* Population group 1, 2 and 3 stand respectively for the lowest 30 percent middle 40 percent and top 30 percent of population based on ranking by the level of percapita total expenditure.

6.5 Tests of serial correlation of SNAM 2, Variant 3 and AIDS

Given the nature of the data used, viz., a time series of population-groupwise consumption data, the problem of serial correlation is likely to be present in the estimated residual of the systems discussed above. To detect this for the three estimated systems, viz., SNAM 2, Variant 3 and AIDS, we calculated the simple first order autocorrelation coefficient r_{ij} between the estimated residuals e_{ijt} and e_{ijt-1}, when e_{ijt} denotes the estimated residual for the i-th item (i = 1, 2, ..., 9), j-th population group (j = 1, 2, 3) and t-th time point (t = 1, 2, ..., 20).[3] The estimated correlation coefficientsare given in Table 6.6 . Presence of positive serial correlation is tested by standard test for linear correlation.[4] The results of these tests have been summarised in Table 6.7 .

---

[3] The rounds are not of equal duration and there are occasional gaps between two consecutive rounds. Hence, strictly speaking, taking t = 1, 2, ..., 20 for the 20 rounds covered here is an over-simplification.

[4] There are obvious objections against such a procedure, and therefore we used it only for rough guidance. It may also be mentioned here that because of the nature of the data, standard procedure of calculating equation - specific Durbin-Watson statistics could not be used.

Table 6.6 Estimates of first order serial correl tion coefficients of residuals for different systems and population groups

All India : rural and urban; disaggregated data

Items	SNAM 2 population group*			Variant 3 population group*			AIDS population group*		
	1	2	3	1	2	3	1	2	3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
All India Rural									
1. Cereals and cereal substitutes	0.705	0.384	0.610	0.618	0.510	0.574	0.544	0.139	0.287
2. Milk and milk products	0.585	0.167	0.479	0.416	0.144	0.656	0.078	-0.342	0.353
3. Edible oils	0.539	0.248	0.420	0.602	0.314	0.541	0.577	0.034	0.195
4. Meat, fish, egg	-0.232	0.254	0.104	-0.179	0.178	0.070	-0.015	-0.099	-0.119
5. Sugar	0.131	0.155	-0.094	-0.158	0.075	0.275	0.041	0.038	0.137
6. Other food	0.179	0.320	0.571	0.066	0.278	0.511	0.135	0.131	0.061
7. Clothing	0.846	0.378	0.179	0.792	0.533	0.238	0.121	-0.179	-0.010
8. Fuel and light	0.417	0.577	0.406	0.568	0.593	0.301	0.389	0.241	0.012
9. Other non food	0.452	0.372	0.590	0.246	0.357	0.302	0.351	0.452	0.495
All India Urban									
1. Cereals and cereal substitutes	0.574	-0.036	0.599	0.335	-0.225	0.699	0.322	-0.188	0.138
2. Milk and milk products	0.296	0.307	0.002	0.402	0.173	0.041	0.405	-0.249	0.263
3. Edible oils	0.467	0.559	-0.013	0.177	0.480	0.340	0.130	0.103	-0.410
4. Meat, fish, egg	0.348	-0.349	0.049	0.258	-0.112	-0.011	0.131	-0.256	-0.291
5. Sugar	0.844	0.780	0.231	0.829	0.607	0.571	0.662	0.084	0.792
6. Other food	0.897	0.832	0.818	0.438	0.193	0.569	0.680	0.447	0.654
7. Clothing	0.748	0.514	-0.056	0.650	0.425	-0.060	0.139	0.083	-0.185
8. Fuel and light	0.102	0.578	0.022	0.261	0.595	-0.068	0.324	0.540	0.227
9. Other non food	0.796	0.405	0.582	0.349	-0.045	-0.057	0.556	0.057	0.343

* Population group 1, 2 and 3 stand respectively for the lowest 30 percent, middle 40 percent and top 30 percent of population based on ranking by the level of percapita total expenditure.

Table 6.7 : Results of test for the presence of first order serial correlation of residuals for a given population group : Number of rejections (at 5% level of significance) of the hypothesis of no autocorrelation for different systems and population groups

All India: rural and urban; disaggregated data

System	Number of rejections out of a total of 9 cases					
	all India rural			all India urban		
	population group 1 (lowest 30%)	population group 2 (middle 40%)	population group 3 (upper 30%)	population group 1 (lowest 30%)	population group 2 (middle 40%)	population group 3 (upper 30%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
SNAM 2	6	1	6	6	6	3
Variant 3	5	3	4	4	4	3
AIDS	2	1	1	4	2	2

Judged by the number of cases of rejection of the null hypothesis of zero serial correlation, the AIDS turns out to be the best. The results for the AIDS suggest that at least for the data relating to the rural sector, serial correlation is not a serious problem. For the other two systems, however, the number of cases of rejection of the null hypothesis of zero serial correlation is generally higher compared to the AIDS for each of the two sectors. For SNAM 2 and Variant 3 the serial correlation for population groups 1 (lowest 30 per cent population) and 3 (top 30 per cent population) of the rural sector are found to be significant for many items, while those for group 2 (middle 40 per cent population) are not. For the urban sector, population group 3 has a smaller number of significant cases compared to the other two population groups. The phenomenon of serial correlation of the residuals is also reflected to some extent by the number of positive and negative residuals of the three models presented in Tables 6.8.1 and 6.8.2 for rural and urban India respectively. To have some idea about the order of magnitudes of the residuals, for each system the sum of absolute values of the residuals have been calculated for each item for either sector. The values are presented in Table 6.9 . For SNAM 2, the middle population group shows higher values compared to the lowest and top population groups for many items for both sectors, with the exceptions of 'meat, fish, egg' and 'fuel and light' in the rural sector, and 'milk and milk products' and 'other food' in the urban sector. For Variant 3, except for 'milk and milk products', 'meat, fish, egg', 'clothing' and 'fuel and light' in the rural sector, and 'other food', 'clothing' and 'fuel and light' in the urban sector, the same pattern is observed for all other items.

In case of AIDS, the middle percentile group shows higher values for 'cereals and cereal substitutes', 'edible oils', 'clothing' and 'other non-food' in the rural sector, while in the urban sector the exceptions to such a pattern are 'other food', 'clothing' and 'fuel and light'. It may also be observed that for either sector, the order of magnitudes of the sums of absolute values of the residuals is generally lowest for the AIDS. These results taken as a whole suggest that the

Table 6.8.1 Number of positive and negative residuals for different systems and population groups

All India : rural; disaggregated data

Items	SNAM 2 population group*						Variant 3 population group*						AIDS population group*					
	1		2		3		1		2		3		1		2		3	
	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1. Cereals and cereal substitutes	5	15	20	0	3	17	6	14	18	2	6	14	6	14	18	2	7	13
2. Milk and milk products	10	10	4	16	14	6	6	14	9	11	10	10	8	12	9	11	8	12
3. Edible oils	11	9	15	5	7	13	9	11	15	5	7	13	8	12	17	3	6	14
4. Meat, fish, egg	10	10	15	5	7	13	5	15	18	2	7	13	7	13	20	0	7	13
5. Sugar	14	6	8	12	9	11	9	11	14	6	8	12	10	10	16	4	7	13
6. Other food	9	11	14	6	8	12	8	12	13	7	10	10	9	11	14	6	9	11
7. Clothing	9	11	1	19	17	3	9	11	4	16	7	13	13	7	3	17	11	9
8. Fuel and light	7	13	16	4	8	12	10	10	9	11	13	7	9	11	7	13	12	8
9. Other non food	17	3	0	20	16	4	18	2	0	20	13	7	18	2	0	20	11	9

* Population group 1, 2 and 3 stand respectively for the lowest 30 percent, middle 40 percent and top 30 percent of population based on ranking by the level of percapita total expenditure.

Table 6.8.2 Number of positive and negative residuals for different systems and population groups

All India : urban; disaggregated data

Items	SNAM 2						Variant 3						AIDS					
	population group*						population group*						population group*					
	1		2		3		1		2		3		1		2		3	
	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative	positive	negative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1. Cereals and cereal substitutes	6	14	20	0	0	20	10	10	10	10	13	7	13	7	9	11	8	12
2. Milk and milk products	9	11	14	6	7	13	2	18	19	1	5	15	3	17	20	0	3	17
3. Edible oils	7	13	19	1	4	16	4	16	18	2	8	12	5	15	20	0	3	17
4. Meat, fish, egg	8	12	16	4	6	14	5	15	16	4	5	15	5	15	17	3	4	16
5. Sugar	9	11	18	2	4	16	8	12	18	2	8	12	5	15	20	0	8	12
6. Other food	8	12	9	11	8	12	9	11	10	10	10	10	9	11	6	14	11	9
7. Clothing	9	11	5	15	14	6	7	13	8	12	10	10	10	10	8	12	9	11
8. Fuel and light	5	15	20	0	2	18	7	13	16	4	6	14	7	13	16	4	10	10
9. Other non food	16	4	0	20	16	4	19	1	0	20	15	5	18	2	0	20	13	7

* Population group 1, 2 and 3 stand respectively for the lowest 30 percent, middle 40 percent and top 30 percent of population based on ranking by the level of percapita total expenditure.

Table 6.9 Sum of absolute values of the residuals for different systems and population groups

All India : rural and urban; disaggregated data

Items	All India Rural									All India Urban								
	SNAM 2 population group*			Variant 3 population group*			AIDS population group*			SNAM 2 population group*			Variant 3 population group*			AIDS population group*		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1. Cereals and cereal substitutes	0.424	0.891	0.608	0.294	0.420	0.267	0.288	0.385	0.267	0.291	0.642	0.411	0.163	0.191	0.164	0.103	0.184	0.096
2. Milk and milk products	0.106	0.155	0.146	0.086	0.110	0.167	0.071	0.086	0.134	0.078	0.087	0.115	0.116	0.182	0.148	0.111	0.169	0.122
3. Edible oils	0.040	0.049	0.030	0.046	0.048	0.030	0.035	0.038	0.028	0.058	0.106	0.057	0.054	0.093	0.061	0.049	0.081	0.029
4. Meat, fish, egg	0.028	0.033	0.058	0.033	0.039	0.050	0.038	0.037	0.043	0.034	0.062	0.042	0.042	0.070	0.050	0.038	0.072	0.045
5. Sugar	0.034	0.042	0.028	0.026	0.038	0.034	0.038	0.037	0.038	0.095	0.107	0.057	0.096	0.098	0.064	0.071	0.076	0.050
6. Other food	0.216	0.232	0.180	0.198	0.215	0.185	0.170	0.164	0.117	0.363	0.293	0.500	0.136	0.139	0.327	0.126	0.106	0.281
7. Clothing	0.175	0.443	0.207	0.155	0.151	0.156	0.118	0.135	0.122	0.108	0.201	0.113	0.076	0.135	0.140	0.063	0.092	0.106
8. Fuel and light	0.062	0.055	0.074	0.068	0.042	0.068	0.043	0.030	0.055	0.051	0.100	0.063	0.042	0.033	0.037	0.038	0.030	0.042
9. Other non food	0.351	0.687	0.498	0.346	0.426	0.349	0.294	0.422	0.338	0.401	0.849	0.530	0.232	0.362	0.297	0.227	0.345	0.281

* Population group 1, 2 and 3 stand respectively for the lowest 30 percent, middle 40 percent and top 30 percent of population based on ranking by the level of percapita total expenditure.

phenomenon of serial correlation of residuals may partly be due to misspecification of the functional form rather than omission of dynamic elements. In the next section an attempt has been made to test for possible misspecification of the functional form.

Apparently the observed serial correlations of the residuals for all the systems noted above would call for reestimation of the systems taking into account the possible autocorrelation of the stochastic disturbances. In case of misspecification of the functional form, the other standard assumptions of the model, viz., that the expectation of the error is zero and that the error is independent of the regressors, are likely to be violated. Thus, usual methods of reestimation may not be very helpful in such a situation (vide Maddala, 1977; Judge et al., 1980). Ideally, given this type of data, one should consider a suitable error structure to take into account the intergroup correlation at a point of time and/or the intragroup serial correlation over time. An approach to this problem would be to define an error components model for the equation disturbances (e.g., Tiao and Ali, 1971; Darrough et al., 1983). Alternatively, one may assume a suitable covariance structure for the composite error terms that can incorporate these two types of correlations mentioned above (e.g., Parks, 1967). However, owing to limited time at our disposal, these refinements have not been incorporated in the empirical analyses.

6.6 Test of misspecification of functional form of SNAM 2, Variant 3 and AIDS

In view of the results obtained in the previous section, we now test for the possible misspecification of functional forms of the above mentioned systems.

Since the disaggregated data consist of average expenditures for three population groups in each round, it is possible that the specific form of Engel curves underlying Variant 3 , AIDS (i.e., Leser-type semilog form) and SNAM 2 (linear form) could not capture fully the non-linearity of income responses implicit in the data at least for some items. To check this, we applied an empirical test of misspecification of functional form, very similar to one proposed by Ramsey (1969).

To test the hypothesis that there is no misspecification (i.e., residuals from a regression have zero means) against the alternative that means of residuals can be approximated by some function of the included independent variables, Ramsey proposed two tests, called the Regression Specific Error Test (RESET) and the Rank Specific Error Test (RASET). The test procedure we followed here is similar to RESET, where the residual is regressed on functions of the estimates of the dependent variable, and an F-test is applied for the joint significance of the coefficient in the regression equation.

For each item, we calculated the estimated budget share $\hat{w}_{ijt}$ (where i, j, t stand respectively for item, population group and NSS rounds) and the corresponding residuals $e_{ijt} = w_{ijt} - \hat{w}_{ijt}$ (w_{ijt} being the observed budget share). We then fitted a quadratic relationship between $\hat{w}_{ijt}$ and e_{ijt} with e_{ijt} as the dependent variable, by Ordinary Least Squares (OLS).5/

Algebraically, the regression equation is given by

$$e_{ijt} = a_i^* + b_i^* \hat{w}_{ijt} + c_i^* \hat{w}_{ijt}^2 \quad , \quad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots, n; \quad j = 1, 2, 3 \qquad \ldots (6.6.1)$$

where

$$(1)\ \hat{w}_{ijt} = \frac{p_{it}\,\gamma_i\,(p_t)}{y_{jt}} + b_i\,\left(1 - \frac{\sum_k p_{kt}\,\gamma_k\,(p_t)}{y_{jt}}\right), \text{ for SNAM 2} \qquad \ldots (6.6.2)$$

$$(2)\ \hat{w}_{ijt} = \frac{p_{it}\,\gamma_i\,(p_t)}{\sum_k p_{kt}\,\gamma_k\,(p_t)} + \beta_i\,(p_t)\,\log\left\{\frac{y_{jt}}{\sum_k p_{kt}\,\gamma_k\,(p_t)}\right\}, \text{ for Variant 3} \qquad \ldots (6.6.3)$$

with

$$\gamma_i\,(p_t) = c_i + a_i \log p_{it} - a_i \log\left(\frac{\sum_k a_k\,p_{kt}}{\sum_k a_k}\right)$$

and

$$\beta_i\,(p_t) = b_i + d_i \log p_{it} - d_i\left(\frac{\sum_k d_k \log p_{kt}}{\sum_k d_k}\right)$$

5/ This procedure and application of the RESET type test, however, may not be strictly valid for the nonlinear set-up of SNAM 2 and Variant 3. It has been used only as an approximation.

and

$$(3)\ \hat{w}_{ijt} = a_i + b_i \log\left(\frac{y_{jt}}{\bar{P}_t}\right) + \sum_k c_{ik} \log p_{kt} \quad \text{for AIDS}, \qquad \ldots (6.6.4)$$

with

$$\log \bar{P}_t = \sum_k w_{kt} \log p_{kt} \ .$$

$p_t = (p_{1t}, p_{2t}, \ldots, p_{nt})$ is the vector of prices at time t and y_{jt} is the total expenditure of j-th population group in t-th period.

The estimates of the parameters a_i^*, b_i^* and c_i^* $(i = 1,2, \ldots, n)$ along with the corresponding standard errors, and also the itemwise coefficient of determination R^2 $\left(= 1 - \frac{\text{Residual sum of squares}}{\text{Total sum of squares}} \right)$ have been presented in Tables 6.10.1 and 6.10.2 respectively for rural and urban sectors. Judging subjectively, in the rural sector, the misspecification of functional form of the demand equations appears to be largest for 'cereals and cereal substitutes' and 'other non-food', for which the R^2 values for all the three models are fairly high. In the urban sector, the R^2 values for 'cereals and cereal substitutes' and 'fuel and light' in AIDS and Variant 3 are fairly low, while in SNAM 2 it is high, for 'other non-food', the R^2 values for AIDS and SNAM 2 are fairly high, while for Variant 3 it is relatively low, and for 'milk and milk products' the R^2 values for AIDS and Variant 3 are higher compared to that for SNAM 2.

One can assess the significance of the estimated coefficients b^* and c^* in Tables 6.10.1 and 6.10.2 . To test the overall significance of the regression statistically, we calculated the F - values

Table 6.10.1 Estimates of parameters of equation (6.2.1) with corresponding standard errors+ and coefficients of determination for different systems

All India : rural; disaggregated data

Items	SNAM 2				Variant 3				AIDS			
	a*	b*	c*	R^2	a*	b*	c*	R^2	a*	b*	c*	R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	-1.015 (.070)	4.436 (.304)	-4.636 (.317)	0.790	-0.233 (.055)	1.085 (.250)	-1.195 (.275)	0.249	-0.303 (.042)	1.400 (.191)	-1.534 (.209)	0.487
2. Milk and milk products	0.025 (.005)	-1.110 (.183)	9.524 (1.545)	0.400	-0.006 (.006)	0.216 (.213)	-1.663 (1.632)	0.018	-0.002 (.005)	0.082 (.175)	-0.635 (1.344)	0.004
3. Edible oils	0.040 (.028)	-2.662 (1.869)	44.013 (30.391)	0.038	0.031 (.028)	-2.101 (1.828)	35.489 (29.854)	0.031	-0.043 (.020)	2.777 (1.280)	-43.950 (20.221)	0.076
4. Meat, fish, egg	-0.067 (.033)	5.621 (2.808)	-117.333 (58.759)	0.065	-0.059 (.036)	4.808 (2.971)	-96.635 (60.913)	0.051	0.009 (.022)	-0.776 (1.845)	16.064 (37.967)	0.003
5. Sugar	0.003 (.004)	-0.200 (.261)	2.881 (4.530)	0.019	-0.002 (.004)	0.124 (.310)	-1.878 (5.292)	0.004	0.008 (.005)	-0.651 (.346)	11.826 (6.223)	0.060
6. Other food	-0.661 (.493)	9.781 (7.276)	-36.045 (26.773)	0.031	-0.021 (.271)	0.337 (4.056)	-1.342 (15.146)	0.000	0.147 (.125)	-2.187 (1.851)	8.030 (6.783)	0.024
7. Clothing	0.023 (.007)	-0.980 (.244)	7.913 (1.891)	0.237	0.001 (.006)	-0.031 (.201)	0.189 (1.394)	0.001	0.009 (.004)	-0.370 (.141)	2.768 (1.031)	0.112
8. Fuel and light	-0.013 (.014)	0.370 (.412)	-2.607 (2.905)	0.014	0.035 (.011)	-1.086 (.341)	7.973 (2.510)	0.152	0.034 (.008)	-1.047 (.241)	7.760 (1.776)	0.251
9. Other non food	0.166 (.014)	-3.395 (.265)	14.316 (1.099)	0.749	0.099 (.013)	-1.830 (.223)	7.014 (.838)	0.552	0.094 (.011)	-1.754 (.200)	6.749 (.760)	0.581

+ The figures in parentheses are the standard errors.

Table 6.10.2 Estimates of parameters of equation (6.2.1) with corresponding standard errors+ and coefficient of determination for different systems

All India : urban; disaggregated data

Items	SNAM 2 a^*	SNAM 2 b^*	SNAM 2 c^*	SNAM 2 R^2	Variant 3 a^*	Variant 3 b^*	Variant 3 c^*	Variant 3 R^2	AIDS a^*	AIDS b^*	AIDS c^*	AIDS R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. Cereals and cereal substitutes	-0.199 (.018)	1.445 (.127)	- 2.261 (.198)	0.695	0.012 (.011)	-0.099 (.087)	0.170 (.147)	0.023	0.009 (.008)	-0.073 (.066)	0.125 (.111)	0.022
2. Milk and milk products	-0.008 (.010)	0.251 (.271)	-1.622 (1.662)	0.019	-0.110 (.016)	2.736 (.390)	-15.865 (2.248)	0.466	-0.082 (.014)	2.033 (.337)	-11.779 (1.942)	0.392
3. Edible oils	0.003 (.027)	-0.304 (1.388)	5.663 (17.376)	0.035	0.016 (.015)	-0.768 (.740)	8.981 (9.215)	0.024	-0.020 (.012)	1.007 (.597)	-12.066 (7.143)	0.048
4. Meat, fish, egg	-0.072 (.046)	4.537 (2.841)	-70.671 (43.966)	0.044	-0.170 (.057)	10.330 (3.427)	-156.022 (50.850)	0.158	-0.141 (.062)	8.520 (3.714)	-127.897 (55.753)	0.084
5. Sugar	-0.037 (.122)	2.527 (7.646)	-42.191 (119.510)	0.007	0.024 (.039)	-1.258 (2.524)	15.301 (41.021)	0.048	0.024 (.024)	-1.555 (1.549)	24.970 (24.869)	0.017
6. Other food	0.477 (.795)	-4.840 (9.323)	12.026 (27.278)	0.087	-0.201 (.156)	2.061 (1.742)	-5.181 (4.835)	0.072	0.435 (.108)	-5.049 (1.245)	14.474 (3.563)	0.224
7. Clothing	0.018 (.005)	-1.046 (.236)	11.436 (2.419)	0.292	-0.008 (.005)	0.358 (.213)	-3.276 (1.936)	0.048	0.001 (.003)	-0.053 (.135)	0.513 (1.258)	0.003
8. Fuel and light	-0.120 (.016)	3.546 (.470)	-25.440 (3.384)	0.500	-0.010 (.011)	0.298 (.331)	-2.204 (2.486)	0.014	0.000 (.009)	0.003 (.293)	-0.041 (2.206)	0.000
9. Other non food	0.255 (.016)	-2.902 (.175)	7.049 (.420)	0.832	0.084 (.016)	-0.843 (.158)	1.832 (.340)	0.338	0.120 (.011)	-1.238 (.109)	2.740 (.239)	0.698

+ The figures in parentheses are the standard errors.

$\left(F = \dfrac{R^2 / (k-1)}{(1 - R^2) / (T - k)}\right.$ where k is the number of parameters estimated) for each item, for the rural as well as the urban sector. These values have been reported in Table 6.11 . In almost all cases, the F-values exceeded the critical value at 5 per cent level of significance for items where the individual regression coefficients b^* and c^* turned out to be significantly different from zero.

In the rural sector, the calculated F-value exceeds the critical value at 0.5 per cent level of significance for 'cereals and cereal substitutes' and 'other non-food' in all the three models, for 'milk and milk products' and 'clothing' in SNAM 2, for 'fuel and light' in AIDS. The F-value exceeds the critical value at 1 per cent level of significance for 'fuel and light' in Variant 3, and it exceeds the critical value only at 5 percent level of significance for 'clothing' in AIDS. In the urban sector, the calculated F-value is significant at 0.5 per cent level for 'cereals and cereal substitutes', 'clothing', 'fuel and light' and 'other food' in SNAM 2, for 'milk and milk products' and 'other non-food' in Variant 3 and AIDS, and for 'other food' in AIDS. The F-value exceeds the critical value at 1 per cent level of significance for 'meat, fish, egg' in Variant 3. In three cases, viz., in AIDS for 'edible oils' in the rural sector and 'meat, fish, egg' in the urban sector, and in SNAM 2 for 'meat, fish, egg' in the rural sector, however, the individual regression parameters b^* and c^*

Table 6.11 Itemwise F-values for different systems

All India : rural and urban; disaggregated data

Items	All India Rural			All India Urban		
	SNAM 2	Variant 3	AIDS	SNAM 2	Variant 3	AIDS
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	106.892***	9.470***	27.012***	64.973***	0.677	0.635
2. Milk and milk products	18.992***	0.519	0.112	0.549	24.921***	18.398***
3. Edible oils	1.132	0.909	2.361	1.031	0.689	1.431
4. Meat, fish, egg	2.004	1.522	0.089	1.309	5.368**	2.631
5. Sugar	0.558	0.129	1.806	0.198	1.421	0.505
6. Other food	0.909	0.011	0.701	2.719	2.215	8.250***
7. Clothing	8.838***	0.014	3.602*	11.749***	1.437	0.083
8. Fuel and light	0.405	5.089**	9.551***	28.500***	0.413	0.006
9. Other non food	84.955***	35.059***	39.470***	140.840***	14.520***	65.710***

* significant at 5% level exceeding critical value $F_{2, 57} = 3.16$

** significant at 1% level exceeding critical value $F_{2, 57} = 5.01$

*** significant at 0.5% level exceeding critical value $F_{2, 57} = 5.83$

were significantly different from zero at 5 per cent level of significance while jointly the effect turned out to be non-significant at the same level of significance.

The above results can be summarised as follows: (1) On the whole, SNAM 2 involves a larger degree of misspecification compared to AIDS and Variant 3 for both sectors, and (2) Variant 3 performs reasonably well compared to AIDS in both sectors.

At this point it may be interesting to compare the results of sections 6.3 and 6.4. We may note that many of the high values of the serial correlations in Table 6.6, and of the sums of absolute values of residuals in Table 6.9 correspond to or are associated with significant F-values in Table 6.11 .

It may thus be concluded that for the given set of disaggregated data, there is evidence of possible misspecification. It, however, cannot be judged whether or not the absence of dynamic elements is the source of positive serial correlation without extensive work involving fitting of dynamic models.

6.7 Summary and conclusions

In this chapter, the empirical performances of SNAM 2, Variant 3 and AIDS have been compared.[6/] On the whole, Variant 3

6/ To check whether the assumption that the three groups of population have the same parameter values is reasonable, we have applied pooling tests for Variant 3 and AIDS. These results are reported in Appendix D.

performed better than SNAM 2. Compared to the AIDS, Variant 3 performed reasonably well for most of the items in rural and urban India despite the fact that the number of parameters involved in the AIDS is far larger than in Variant 3. For some items, however, even the AIDS specification seemed inadequate.

The itemwise expenditure and own-price elasticities obtained from different systems were compared. The expenditure elasticities estimated for all the systems were of the same order of magnitude, elasticities from SNAM 2 being marginally different from those of AIDS and Variant 3. The price elasticities from the three models were often quite different. The non-compensated own-price elasticities from Variant 3 showed an inverse relationship with income level for most of the items in rural and urban India. The elasticities from SNAM 2 and AIDS, however, failed relatively often to show such an intuitively plausible pattern for rural India.

The presence of serial correlation in the residuals of the three models was examined separately for the three population groups. The results suggested that misspecification of functional form could be the cause of significant serial correlation values. Performance of Variant 3 was generally more satisfactory than that of SNAM 2 .

A Ramsey type test of misspecification of functional forms was carried out to detect whether the functional forms of the models

SNAM 2, Variant 3 and AIDS were adequate for the given data. The results clearly indicated that even the AIDS specification was inadequate for some items in rural and urban India. Comparative results for the three systems showed that SNAM 2 involved more serious misspecification than Variant 3, which performed reasonably well compared to the AIDS.

Appendix A

Cost functions and elasticity expressions for the SNAM and Variant systems

A.1. Introduction

In this Appendix we shall derive the explicit form of cost functions underlying the SNAM and Variant systems discussed in Chapter II, and also obtain the expressions for the various elasticities for these systems. Sections A.2 and A.3 present the derivation of these cost functions and the expressions for various elasticities respectively for these two families of demand systems.

A.2. Cost functions

Let us first consider SNAM 2, the general form of the SNAM family. The budget share equations for this system are

$$w_i = \frac{p_i q_i}{y} = \frac{p_i \gamma_i(p)}{y} + \beta_i(p) \left\{ 1 - \frac{\sum_j p_j \gamma_j(p)}{y} \right\}, \quad i = 1, 2, \ldots, n \qquad \ldots(A.2.1)$$

with

$$\gamma_i(p) = c_i + a_i \log p_i - a_i \log \left(\frac{\sum_k a_k p_k}{\sum_k a_k} \right) \qquad \ldots(A.2.2)$$

and

$$\beta_i(p) = b_i + d_i \log p_i - d_i \left(\frac{\sum_k d_k \log p_k}{\sum_k d_k}\right) \qquad \ldots(A.2.3)$$

where the symbols have the same meaning as in Chapter II. To obtain the cost function underlying (A.2.1) - (A.2.3), we assume $C(V^*, p) = y$ where $C(V^*, p)$ is the cost function, V^* being the level of utility, and use $w_i = \frac{\partial \log C(V^*, p)}{\partial \log p_i}$. Thus, (A.2.1) may be written as

$$\frac{\partial \log C}{\partial \log p_i} = \frac{p_i \gamma_i(p)}{C} + \beta_i(p) \left\{ 1 - \frac{\sum_j p_j \gamma_j(p)}{C} \right\}, \quad i=1,2,\ldots,n \qquad \ldots(A.2.4)$$

or equivalently

$$\frac{\partial C}{\partial p_i} = \gamma_i(p) + \frac{\beta_i(p)}{p_i} \left\{ C - \sum_j p_j \gamma_j(p) \right\}, \quad i=1,2,\ldots, n \qquad \ldots(A.2.5)$$

where C stands for $C(V^*, p)$. (A.2.5) is, thus, a system of n partial differential equations. Define

$$z = C - \sum_j p_j \gamma_j(p), \qquad \ldots(A.2.6)$$

to have

$$\frac{\partial z}{\partial p_i} = \frac{\partial C}{\partial p_i} - \frac{\partial}{\partial p_i} \sum_j p_j \gamma_j(p), \quad i = 1, 2, \ldots, n. \qquad \ldots(A.2.7)$$

Now, given the definition of $\gamma_j(p)$ in (A.2.2) above, we have

$$\frac{\partial}{\partial p_i} \sum_j p_j \gamma_j(p) = c_i + a_i \log p_i + a_i - a_i \log \frac{\sum_k a_k p_k}{\sum_k a_k}$$

$$- \left(\sum_j a_j p_j \right) \frac{\sum_k a_k}{\sum_k a_k p_k} \cdot \frac{a_i}{\sum_k a_k}$$

$$= \gamma_i (p) , \quad i = 1, 2, \ldots, n \qquad \ldots(A.2.8)$$

Thus, (A.2.7) becomes

$$\frac{\partial z}{\partial p_i} = \frac{\partial C}{\partial p_i} - \gamma_i (p) , \quad i = 1, 2, \ldots, n \qquad (A.2.9)$$

using (A.2.5) and (A.2.6) in (A.2.9) yields

$$\frac{\partial z}{\partial p_i} = \frac{\beta_i(p)}{p_i} z , \quad i = 1, 2, \ldots, n . \qquad (A.2.10)$$

Given the definition of $\beta_i(p)$ in (A.2.3), separating the variables in (A.2.10) and integrating on both sides, we have

$$\log z = b_i \log p_i + \frac{d_i}{2} (\log p_i)^2 - \frac{1}{\sum_k d_k} \frac{\left(\sum_k d_k \log p_k\right)^2}{2}$$

$$+ \log K_i \; (V^*, p_j, j = 1,2,\ldots, n, \; j \neq i) \quad \ldots(A.2.11)$$

where $\log K_i \; (V^*, p_j, j = 1, 2, \ldots, n, j \neq i)$ is the constant of integration. Rewriting (A.2.11) in exponential form, we have

$$z = p_i^{\,b_i + \frac{d_i}{2}\log p_i} \; \prod_k p_k^{\,-\frac{d_k}{2}\frac{\sum_j d_j \log p_j}{\sum_j d_j}} \; K_i\,(V^*, p^i) \qquad \ldots(A.2.12)$$

where $p^i = (p_j,\ j = 1, 2, \ldots, n,\ j \neq i)$. Since the l.h.s. of (A.2.12) is independent of i, $K_i(V^*, p^i)$ should be such that

$$p_s^{\,b_s + \frac{d_s}{2}\log p_s} \; K_s(V^*, p^s) = p_r^{\,b_r + \frac{d_r}{2}\log p_r} \; K_r(V^*, p^r)$$

$$\ldots(A.2.13)$$

for any pair of $r, s = 1, 2, \ldots, n,\ r \neq s$. Dividing (A.2.13) by $\prod_k p_k^{\,b_k + \frac{d_k}{2}\log p_k}$ on both sides

$$\frac{K_s(V^*, p^s)}{\prod_{k\neq s} p_k^{\,b_k + \frac{d_k}{2}\log p_k}} = \frac{K_r(V^*, p^r)}{\prod_{k\neq r} p_k^{\,b_k + \frac{d_k}{2}\log p_k}} \quad \text{for any } r, s, r \neq s \qquad \ldots(A.2.14)$$

which suggests that $K_s(V^*, p^s)$ is of the form

$$K_s(V^*, p^s) = V^* \prod_{k\neq s} p_k^{\,b_k + \frac{d_k}{2}\log p_k} \qquad \ldots(A.2.15)$$

Thus, for any i, (A.2.12) is of the form

$$z = p_i^{\,b_i + \frac{d_i}{2}\log p_i} \left(\prod_k p_k^{\,-\frac{d_k}{2}\frac{\sum_j d_j \log p_j}{\sum_j d_j}} \right) V^* \prod_{k \neq i} p_k^{\,b_k + \frac{d_k}{2}\log p_k}$$

$$= V^* \prod_k p_k^{\,b_k + \frac{d_k}{2}\log p_k - \frac{d_k}{2 \sum_j d_j} \sum_j d_j \log p_j}$$

$$= V^* \prod_k p_k^{\,\frac{1}{2}(b_k + \beta_k(p))}$$

$$= V^* \prod_k p_k^{\,\beta_k^*(p)} \quad , \quad \text{say.} \qquad \ldots(A.2.16)$$

Substituting (A.2.16) in (A.2.6), we have the explicit form of cost function underlying (A.2.1) - (A.2.3) as

$$C(V^*, p) = \sum_j p_j \gamma_j(p) + V^* \prod_j p_j^{\,\beta_j^*(p)} \qquad \ldots(A.2.17)$$

where

$$\beta_j^*(p) = \tfrac{1}{2}(b_j + \beta_j(p)) .$$

The cost function underlying Variant 3, the general form of the Variant family, may be derived in a similar manner. For Variant 3 the individual budget share equations are of the form

$$w_i = \frac{p_i \, \gamma_i(p)}{\sum_j p_j \, \gamma_j(p)} + \beta_i(p) \log \left(\frac{y}{\sum_j p_j \, \gamma_j(p)}\right) , \quad i = 1, 2, \ldots, n \qquad \ldots (A.2.18)$$

where $\gamma_i(p)$ and $\beta_i(p)$ are as given in (A.2.2) and (A.2.3) respectively. As before, assume $y = C(V^*, p)$ and use $w_i = \frac{\partial \log C}{\partial \log p_i}$ to write (A.2.18) as

$$\frac{\partial \log C}{\partial p_i} = \frac{\gamma_i(p)}{\sum_j p_j \, \gamma_j(p)} + \frac{\beta_i(p)}{p_i} \log \left(\frac{C}{\sum_j p_j \, \gamma_j(p)}\right) , \quad i=1,2,\ldots, n \qquad \ldots (A.2.19)$$

Next, define

$$z = \frac{C}{\sum_j p_j \, \gamma_j(p)} \qquad \ldots (A.2.20)$$

so that

$$\frac{\partial \log z}{\partial p_i} = \frac{\partial \log C}{\partial p_i} - \frac{1}{\sum_j p_j \, \gamma_j(p)} \; \frac{\partial}{\partial p_i} \sum_j p_j \, \gamma_j(p) \; , \quad i = 1,2,\ldots, n \qquad \ldots (A.2.21)$$

Given $\frac{\partial}{\partial p_i} \sum p_j \, \gamma_j(p) = \gamma_i(p)$, using (A.2.19) and (A.2.20), (A.2.21) simplifies to

$$\frac{\partial \log z}{\partial p_i} = \frac{\beta_i(p)}{p_i} \log z \; , \quad i = 1, 2, \ldots, n \qquad \ldots (A.2.22)$$

Note that (A.2.22) is formally similar to (A.2.10), so that the system of partial differential equations (A.2.22) may be solved in an analogous manner, and the solution is of the form

$$\log z = V^* \prod_k p_k^{\beta_k^*(p)} \qquad \ldots(A.2.23)$$

Thus, by substituting (A.2.20) in (A.2.23) we obtain the explicit form of the cost function underlying Variant 3 to be

$$C(V^*, p) = \sum_j p_j \gamma_j(p) \ \exp \{ \ V^* \prod_j p_j^{\beta_j^*(p)} \} \qquad \ldots(A.2.24)$$

where

$$\beta_j^*(p) = \tfrac{1}{2} (b_j + \beta_j(p)) .$$

The cost function corresponding to the general PIGL case of our proposed system, i.e., for the system

$$w_i = \frac{p_i \gamma_i(p)}{\sum_j p_j \gamma_j(p)} + \beta_i(p) \frac{\{ \frac{y}{\sum_j p_j \gamma_j(p)} \}^{\varepsilon} - 1}{\varepsilon}, \quad i=1,2,\ldots, n \qquad \ldots(A.2.25)$$

may be derived in the following manner.

Assuming $y = C(V^*, p)$, using $w_i = \frac{\partial \log C}{\partial \log p_i}$ as before, and defining z as in (A.2.20), (A.2.25) reduces to

$$z^{-(\epsilon+1)} \frac{\partial z}{\partial p_i} + \frac{\beta_i(p)}{\epsilon p_i} z^{-\epsilon} = \frac{\beta_i(p)}{\epsilon p_i} \; ; \quad i = 1, 2, \ldots, n \qquad \ldots(A.2.26)$$

Further, defining $z^{-\epsilon} = x$, (A.2.26) can be written as

$$\frac{\partial x}{\partial p_i} - \frac{\beta_i(p)}{p_i} x = - \frac{\beta_i(p)}{p_i} \; ; \quad i = 1, 2, \ldots, n \qquad \ldots(A.2.27)$$

The solution of the above system of differential equations is given by

$$x = e^{\beta_i^1(p)} \left\{ - \int \frac{\beta_i(p)}{p_i} e^{\beta_i^1(p)} \partial p_i + K_i(\epsilon, V^*, p^i) \right\}$$

$$i = 1, 2, \ldots, n . \qquad \ldots(A.2.28)$$

where p^i has the same meaning as before,

and

$$\beta_i^1(p) = (b_i + \frac{d_i}{2} \log p_i) \log p_i - \frac{(\sum_k d_k \log p_k)^2}{2 \sum_k d_k} \qquad \ldots(A.2.29)$$

(A.2.28) further simplifies to

$$x = 1 + K_i(\epsilon, V^*, p^i) e^{\beta_i^1(p)} \; ; \quad i = 1, 2, \ldots, n . \qquad \ldots(A.2.30)$$

Now, substitution of equation (A.2.29) in (A.2.30) yields

$$(x - 1) = K_i(\varepsilon, V^*, p^i)\ p_i^{\,b_i + \frac{d_i}{2} \log p_i}\ \prod_k p_k^{\frac{d_k \sum_j d_j \log p_j}{2 \sum_k d_k}} \qquad \ldots(A.2.31)$$

Note that (A.2.31) is formally similar to (A.2.12), so that

$$(x - 1) = f(\varepsilon)\ V^* \prod_k p_k^{\beta_k^*(p)} \qquad \ldots(A.2.32)$$

where

$$\beta_k^*(p) = \tfrac{1}{2}\,(b_k + \beta_k(p))$$

substituting the values of x and z in equation (A.2.32), the explicit form of the cost function is obtained as

$$C(V^*, p)^{-\varepsilon} = \{\Sigma\, p_j\, \gamma_j(p)\}^{-\varepsilon}\ \{1 - \varepsilon\, V^* \prod_k p_k^{\beta_k^*(p)}\}^{1/} \qquad \ldots(A.2.33)$$

with

$$\beta_k^*(p) = \tfrac{1}{2}\,(b_k + \beta_k(p))\ .$$

A.3. Elasticities

In this section we shall obtain the expression for various elasticities of the SNAM 2 and Variant 3 of our proposed model as specified

---

1/ In order that $\frac{\partial V^*}{\partial y} > 0$, $f(\varepsilon)$ and ε should have opposite signs. Thus, $f(\varepsilon) = -\varepsilon$ has been chosen.

in (A.2.1) - (A.2.3), and (A.2.18) and (A.2.2) - (A.2.3) respectively. It may be noted that the corresponding elasticity expressions for the special cases of these systems, viz., the LES and the SNAM 1 in case of the SNAM family and Variant 1, Variant 2 and Variant 4 in case of our proposed system, can be obtained by dropping the parameters (that do not appear in the corresponding specific systems) from the elasticity expressions derived here.

In deriving the expressions for the various elasticities we shall frequently need the expressions for the partial derivatives of $\gamma_i(p)$, $\beta_i(p)$ and $\sum_k p_k \gamma_k(p)$ where $\gamma_i(p)$ and $\beta_i(p)$ are as defined in (A.2.2) and (A.2.3) above. These derivatives are

$$\frac{\partial \gamma_i(p)}{\partial p_j} = \delta_{ij} \frac{a_i}{p_j} - \frac{a_i a_j}{\sum_k a_k p_k}$$

$$= \frac{\theta_i(p)}{p_i p_j} \{ \delta_{ij} - \theta_j(p) \} \sum_k a_k p_k \qquad \ldots(A.3.1)$$

$$\frac{\partial \beta_i(p)}{\partial p_j} = \delta_{ij} \frac{d_i}{p_j} - \frac{d_i d_j}{p_j \sum_k d_k} = \frac{d_j}{p_j} (\delta_{ij} - \frac{d_i}{\sum_k d_k}) \qquad \ldots(A.3.2)$$

and

$$\frac{\partial \sum_k p_k \gamma_k(p)}{\partial p_j} = \gamma_j(p) \qquad \ldots(A.3.3)$$

where

$$\theta_i(p) = \frac{a_i p_i}{\sum_k a_k p_k} \qquad \ldots(A.3.4)$$

and δ_{ij} is the Kronecker delta.

The expenditure elasticity η_i for SNAM 2 is given by

$$\eta_i = \frac{\beta_i(p)}{w_i} \qquad \ldots(A.3.5)$$

The expression for the non-compensated price elasticities μ_{ij} for any pair of goods i and j is

$$\mu_{ij} = \frac{\partial \log q_i}{\partial \log p_j} = \frac{p_j}{q_i} \{ \frac{\partial \gamma_i(p)}{\partial p_j} - \delta_{ij} \frac{\beta_i(p)}{p_i^2} (y - \sum_k p_k \gamma_k(p))$$

$$+ \frac{(y - \sum_k p_k \gamma_k(p))}{p_i} \frac{\partial \beta_i(p)}{\partial p_j} - \frac{\beta_i(p)}{p_i} \frac{\partial \sum_k p_k \gamma_k(p)}{\partial p_j} \}$$

which on substitution of (A.3.1) - (A.3.3) becomes

$$\mu_{ij} = - \frac{1}{p_i q_i} [\theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k + p_j \beta_i(p) \gamma_j(p)$$

$$+ \{\beta_i(p) \delta_{ij} - d_j (\delta_{ij} - \frac{d_i}{\sum_k d_k}) \} (y - \sum_k p_k \gamma_k(p))] \qquad \ldots(A.3.6)$$

The corresponding compensated price elasticities

π_{ij} ($= \mu_{ij} + w_j \eta_i$) are of the form

$$\pi_{ij} = - \frac{1}{p_i q_i} [\theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k +$$

$$\{ \beta_i(p) (\delta_{ij} - \beta_j(p)) - d_j (\delta_{ij} - \frac{d_i}{\sum_k d_k}) \} (y - \sum_k p_k \gamma_k(p))] \qquad \ldots(A.3.7)$$

The elasticity expressions for the SNAM 1 and the LES can be obtained by dropping the irrelevant parameters from (A.3.5) - (A.3.7). Thus, for the SNAM 1

$$\eta_i = \frac{b_i}{w_i} \qquad \ldots(A.3.8)$$

$$\mu_{ij} = - \frac{1}{p_i q_i} [\theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k + p_j b_i \gamma_j(p)$$

$$+ b_i \delta_{ij} (y - \sum_k p_k \gamma_k(p))] \qquad \ldots(A.3.9)$$

and

$$\pi_{ij} = - \frac{1}{p_i q_i} [\theta_i(p) \{ \theta_j(p) - \delta_{ij} \} \sum_k a_k p_k +$$

$$b_i (\delta_{ij} - b_j) (y - \sum_k p_k \gamma_k(p))] . \qquad \ldots(A.3.10)$$

Similarly, for the LES

$$\eta_i = \frac{b_i}{w_i} \qquad \ldots(A.3.11)$$

$$\mu_{ij} = - \frac{1}{p_i q_i} [p_j c_j b_i + b_i \delta_{ij} (y - \sum_k p_k c_k)]$$

$$= - b_i \frac{p_j c_j}{p_i q_i} \delta_{ij} (1 - \frac{c_i}{q_i}) \qquad \ldots(A.3.12)$$

and

$$\pi_{ij} = - \frac{1}{p_i q_i} \{ b_i \ (\delta_{ij} - b_j) \ (y - \sum_k p_k c_k) \} = (b_j - \delta_{ij})(1 - \frac{c_i}{q_i}) \qquad \ldots(A.3.13)$$

Let us now obtain the expressions of various elasticities for Variant 3 of our proposed system. For this system, the expenditure elasticity is

$$\eta_i = 1 + \frac{\beta_i(p)}{w_i} \qquad \ldots(A.3.14)$$

The non-compensated price elasticity for any pair of goods i and j takes the form

$$\mu_{ij} = \frac{\partial \log q_i}{\partial \log p_j} = \frac{p_j}{q_i} [\frac{y}{\sum_k p_k \gamma_k(p)} \frac{\partial \gamma_i(p)}{\partial p_j} - \frac{\gamma_i(p) \ y}{(\sum_k p_k \gamma_k(p))^2} \frac{\partial \sum_k p_k \gamma_k(p)}{\partial p_j}$$

$$+ (\frac{y}{p_i} \frac{\partial \beta_i(p)}{\partial p_j} - \frac{\delta_{ij} \beta_i(p)}{p_i^2}) \log (\frac{y}{\sum_k p_k \gamma_k(p)})$$

$$- \frac{\beta_i(p)}{p_i} \frac{y}{\sum_k p_k \gamma_k(p)} \frac{\partial}{\partial p_j} \sum_k p_k \gamma_k(p)]$$

which on simplification becomes

$$\mu_{ij} = - \frac{1}{w_i} [\theta_i(p) \ \{ \theta_j(p) - \delta_{ij} \} \quad \Phi(p) + \alpha_i(p) \ \alpha_j(p)$$

$$+ \beta_i(p) \ \alpha_j(p) + \{ \delta_{ij} \beta_i(p) - d_j \ (\delta_{ij} - \frac{d_i}{\sum_k d_k}) \} \log Z] \qquad \ldots(A.3.15)$$

where

$$\Phi(p) = \frac{\sum_k p_k a_k}{\sum_k p_k \gamma_k(p)}$$

$$\alpha_i(p) = \frac{p_i \gamma_i(p)}{\sum_k p_k \gamma_k(p)}$$

and

$$Z = \frac{y}{\sum_k p_k \gamma_k(p)} \quad .$$

The corresponding compensated price elasticities are of the form

$$\pi_{ij} = -\frac{1}{w_i} \; [\, \theta_i(p) \, \{ \, \theta_j(p) - \delta_{ij} \, \} \, \Phi(p) + \alpha_i(p) \, \alpha_j(p) - w_i w_j$$

$$+ \{ \delta_{ij} \, \beta_i(p) - d_j \, (\delta_{ij} - \frac{d_i}{\sum_k d_k}) - \beta_i(p) \, \beta_j(p) \, \} \, \log Z \,] \quad \ldots(A.3.16)$$

The elasticity expressions for Variant 2, Variant 1 and Variant 4 can be obtained by dropping the irrelevant parameters from (A.3.14) - (A.3.16). Thus, for Variant 2

$$\eta_i = 1 + \frac{b_i}{w_i} \quad \ldots(A.3.17)$$

$$\mu_{ij} = -\frac{1}{w_i} \; [\, \theta_i(p) \, \{ \, \theta_j(p) - \delta_{ij} \, \} \; \Phi(p) + \alpha_i(p) \, \alpha_j(p)$$

$$+ b_i \alpha_j(p) + \; \delta_{ij} \, b_i \, \log Z \,] \quad \ldots(A.3.18)$$

and

$$\pi_{ij} = -\frac{1}{w_i} [\ \theta_i(p) \{\ \theta_j(p) - \delta_{ij}\ \}\ \ \Phi(p) + \alpha_i(p)\ \alpha_j(p) - w_i w_j$$

$$+ \{\delta_{ij}\ b_i - b_i\ b_j\}\ \log Z\ ] \qquad \ldots(A.3.19)$$

Similarly, for Variant 1,

$$\eta_i = 1 + \frac{b_i}{w_i} \qquad \ldots(A.3.20)$$

$$\mu_{ij} = -\frac{1}{w_i} [\ \frac{p_i p_j c_i c_j}{(\sum_k p_k c_k)^2} + b_i \frac{p_j c_j}{\sum_k p_k c_k} + \delta_{ij}\ b_i \log Z\ ]$$

$$= -\frac{1}{w_i} [\ \frac{p_j c_j}{\sum_k p_k c_k} \{\ \frac{p_i c_i}{\sum_k p_k c_k} + b_i\} + \delta_{ij}\ b_i \log Z\ ] \qquad \ldots(A.3.21)$$

and

$$\pi_{ij} = -\frac{1}{w_i} [\ \frac{p_i p_j c_i c_j}{(\sum_k p_k c_k)^2} - w_i w_j + \{\ \delta_{ij}\ b_i - b_i b_j\}\ \ \log Z\ ]$$

$$\ldots(A.3.22)$$

and finally, for Variant 4,

$$\eta_i = 1 + \frac{\beta_i(p)}{w_i} \qquad \ldots(A.3.23)$$

$$\mu_{ij} = -\frac{1}{w_i}\left[\frac{p_j c_j}{\sum_k p_k c_k}\left\{\frac{p_i c_i}{\sum_k p_k c_k} + \beta_i(p)\right\} + \delta_{ij}\,\beta_i(p)\log z\right]$$

$$\ldots(A.3.24)$$

and

$$\pi_{ij} = -\frac{1}{w_i}\left[\frac{p_i p_j c_i c_j}{(\sum_k p_k c_k)^2} - w_i w_j + \{\delta_{ij}\,\beta_i(p) - \beta_i(p)\,\beta_j(p)\}\log z\right]$$

$$\ldots(A.3.25)$$

Appendix B

Derivatives of demand functions with respect to their parameters

It has been mentioned in Chapter III that the demand systems have been estimated by Non-Linear Full-Information Maximum Likelihood (NLFIML) method using the NLFIML package. The NLFIML package uses the modified Gauss-Newton method for maximisation of the log-likelihood. Thus, for the system of budget share equations in stochastic form

$$w = f\ (p,\ y,\ \theta) + u \qquad \ldots(B.1)$$

where w : vector of observations on all the budget shares, y : the vector of observations on total expenditures, p : the matrix of observations on prices, θ : the vector of parameters of the system, and u : the vector of random disturbances, estimation of θ is done through iteration where the parameter estimates for the (k + 1)th stage $\hat{\theta}_{k+1}$ are related to the parameter estimates $\hat{\theta}_k$ of the previous stage through the relation

$$\hat{\theta}_{k+1} + 1 = \hat{\theta}_k + \{ Z(\hat{\theta}_k)' \ (\hat{\Sigma}\ (\hat{\theta}_k)\ (X)\ I\ )\ Z(\hat{\theta}_k) + \lambda_k\ I\ \}^{-1}$$

$$\{ Z(\hat{\theta}_k)\ (\hat{\Sigma}\ (\hat{\theta}_k)\ (X)\ I\ )\ V(\hat{\theta}_k)\} \qquad \ldots(B.2)$$

where

$$Z(\hat{\theta}_k) = \left. \frac{\partial f(p, y, \theta)}{\partial \theta} \right|_{\theta = \hat{\theta}_k} ,$$

$$V(\hat{\theta}_k) = w - f\ (p, y, \hat{\theta}_k) \ ,$$

$\hat{\Sigma}(\hat{\theta}_k)$ is the contemporaneous covariance matrix of the equation disturbances estimated at the kth stage of iteration, and (X) stands for Kronecker product. This estimation procedure thus needs the first-order partial derivates of the individual budget share functions with respect to the individual parameters of the system. In what follows, we shall list the expressions for these first-order partial derivates for the SNAM 2 and Variant 3 of our proposed system with respect to their parameters. It may be noted that the expressions for the first-order partial derivatives of the special cases of the SNAM 2 and Variant 3, viz., the SNAM 1 and the LES and Variant 2, Variant 1 and Variant 4, can be obtained by dropping the parameters, from the expressions for the SNAM 2 / Variant 3, that do not appear in a specific special case.

For the SNAM 2 system the budget share equations are of the form

$$w_i = \frac{p_i \gamma_i(p)}{y} + \beta_i(p)\ (1 - \frac{\alpha(p)}{y}) \ , \quad i = 1,2,\dots, n \qquad \dots(B.3)$$

with

$$\gamma_i(p) = c_i + a_i \log p_i - a_i \log \left(\frac{\sum_k a_k p_k}{\Sigma a_k}\right) \qquad \ldots(B.4)$$

$$\beta_i(p) = b_i + d_i \log p_i - d_i \left(\frac{\sum_k d_k \log p_k}{\sum_k d_k}\right) \qquad \ldots(B.5)$$

and

$$\alpha(p) = \sum_k p_k \gamma_k(p) \qquad \ldots(B.6)$$

The parameters of this system are a_i, b_i, c_i and d_i , $i=1,2,\ldots, n$.

The first-order partial derivatives of the R.H.S. of (B.3) with respect to these parameters may be worked out to be as follows:

$$\frac{\partial w_i}{\partial b_j} = \delta_{ij} \left(1 - \frac{\alpha(p)}{y}\right) \qquad \ldots(B.7)$$

$$\frac{\partial w_i}{\partial c_j} = \frac{p_i}{y} \left(\delta_{ij} - \beta_i(p) \right) \qquad \ldots(B.8)$$

$$\frac{\partial w_i}{\partial a_j} = \frac{1}{y} \left[p_i \left\{ \delta_{ij} \log \left(\frac{p_j}{\pi_1}\right) - \frac{a_i}{\sum_k a_k} \left(\frac{p_j}{\pi_1} - 1\right) \right\} \right.$$

$$\left. - \beta_i(p)\, p_j \left(\log\left(\frac{p_j}{\pi_1}\right) + \frac{\pi_1}{p_j} - 1\right) \right] \qquad \ldots(B.9)$$

and

$$\frac{\partial w_i}{\partial d_j} = \left(\delta_{ij} - \frac{d_i}{\sum_k d_k} \right) \log \left(\frac{p_i}{\pi_2}\right) \left(1 - \frac{\alpha(p)}{y}\right) \qquad \ldots(B.10)$$

where

$$\pi_1 = \frac{\sum_k a_k p_k}{\sum_k a_k}\ , \quad \pi_2 = \prod_k p_k^{\frac{d_k}{\sum_j d_j}} \qquad \ldots(B.11)$$

and δ_{ij} : Kronecker delta .

For Variant 3 of our proposed system, viz.,

$$w_i = \frac{p_i \gamma_i(p)}{\alpha(p)} + \beta_i(p) \log \left(\frac{y}{\alpha(p)}\right) \qquad \ldots(B.12)$$

where $\gamma_i(p)$ and $\beta_i(p)$ are same as in (B.4) and (B.5) respectively, the parameters are again a_i, b_i, c_i and d_i , $i = 1, 2, \ldots, n$. The partial derivatives of the R.H.S. of (B.12) with respect to these parameters may be seen to be as follows :

$$\frac{\partial w_i}{\partial b_j} = \delta_{ij} \log \left(\frac{y}{\alpha(p)}\right) \qquad \ldots(B.13)$$

$$\frac{\partial w_i}{\partial c_j} = \frac{p_j}{\alpha(p)} \left[\delta_{ij} - \frac{p_i \gamma_i(p)}{\alpha(p)} - \beta_i(p)\right] \qquad \ldots(B.14)$$

$$\frac{\partial w_i}{\partial a_j} = \frac{1}{\alpha(p)} \left[p_i \left\{ \delta_{ij} \log\left(\frac{p_j}{\pi_1}\right) - \frac{a_i}{\sum_k a_k} \left(\frac{p_j}{\pi_1} - 1\right)\right\} \right.$$

$$\left. - p_j \left\{ \frac{p_i \gamma_i(p)}{\alpha(p)} + \beta_i(p) \right\} \left\{ \log\left(\frac{p_j}{\pi_1} + \frac{\pi_1}{p_j} - 1\right) \right\} \right] \qquad \ldots(B.15)$$

and

$$\frac{\partial w_i}{\partial d_j} = \left(\delta_{ij} - \frac{d_i}{\sum_k d_k} \right) \log\left(\frac{p_j}{\pi_2}\right) \log\left(\frac{y}{\alpha(p)}\right) . \qquad \ldots(B.16)$$

Appendix C

The LES and the AIDS : Additional empirical results

C.1 Introduction

This Appendix has two parts. The first part presented in Section C.2 is supplementary to Chapter IV. In this section the estimates of the non-compensated own and cross price elasticities for the LES and the AIDS fitted to the aggregate and also the disaggregated data for rural and urban India have been reported. The second part, given in Section C.3, presents the results of applying stepwise regression to the itemspecific budget share equations of the AIDS on the basis of disaggregated data. This exercise was undertaken in order to examine whether one could dispense with some of the price variables in individual budget share equations. Such an exercise was felt necessary in view of the multicollinearity problem observed to be present.

C.2 Non-compensated price elasticities for the LES and the AIDS

Table C.1 - C.4 present the estimates of non-compensated own and cross price elasticities for the LES and the AIDS. Tables C.1 - C.2 present the elasticities for rural India estimated on the basis of

Table C.1 Estimates of itemwise non-compensated own and cross price elasticities calculated at sample average of prices and total expenditure for LES

All India : rural; aggregate and disaggregated data*

Items \ Item no.	1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	-0.474 -0.589	-0.008 0.004	-0.009 -0.006	-0.002 -0.003	-0.010 -0.002	-0.034 -0.030	-0.009 0.004	-0.009 -0.021	-0.025 0.003
2. Milk and milk products	-0.460 -0.374	-0.838 -1.067	-0.025 -0.015	-0.004 -0.009	-0.026 -0.005	-0.093 -0.074	-0.024 -0.010	-0.026 -0.051	-0.067 0.008
3. Edible oils	-0.274 -0.236	-0.013 0.006	-0.502 -0.690	-0.003 -0.005	-0.015 -0.003	-0.055 -0.046	-0.014 0.007	-0.016 -0.032	-0.040 0.005
4. Meat, fish, egg	-0.503 -0.274	-0.023 0.007	-0.027 -0.011	-0.897 -0.795	-0.028 -0.004	-0.101 -0.054	-0.026 0.008	-0.029 -0.038	-0.073 0.006
5. Sugar	-0.280 -0.307	-0.012 0.007	-0.014 -0.013	-0.002 -0.007	-0.476 -0.887	-0.052 -0.061	-0.014 0.009	-0.015 -0.042	-0.038 0.007
6. Other food	-0.307 -0.224	-0.014 0.005	-0.016 -0.009	-0.003 -0.005	-0.017 -0.003	-0.607 -0.689	-0.016 0.006	-0.017 -0.031	-0.044 0.005
7. Clothing	-0.450 -0.378	-0.021 0.009	-0.024 -0.015	-0.004 -0.009	-0.025 -0.006	-0.091 -0.075	-0.823 -1.077	-0.026 -0.052	-0.065 0.008
8. Fuel and light	-0.407 -0.164	-0.019 0.004	-0.022 -0.007	-0.004 -0.004	-0.023 -0.002	-0.082 -0.032	-0.021 0.005	-0.746 -0.493	-0.059 0.003
9. Other non food	-0.402 -0.359	-0.018 0.009	-0.022 -0.015	-0.004 -0.008	-0.022 -0.005	-0.081 -0.071	-0.021 0.010	-0.023 -0.049	-0.772 -1.027

* In each cell the upper figure is based on aggregated data and the lower on disaggregated data.

Table C.2 Estimates of itemwise non compensated own and cross price elasticities calculated at sample average of prices and total expenditure for AIDS

All India : rural; aggregate and disaggregated data*

Items \ Item no.	1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	-0.034 -0.098	-0.348 -0.233	-0.063 -0.023	-0.050 -0.064	-0.027 0.025	0.002 -0.044	0.093 0.063	-0.054 -0.111	-0.069 -0.022
2. Milk and milk products	-0.472 -1.176	-0.104 -0.590	0.004 -0.124	0.349 0.212	-0.104 -0.161	-0.042 0.113	-0.650 -1.298	0.025 0.215	-0.053 -0.117
3. Edible oils	-0.139 -0.309	0.208 0.116	-0.526 -0.604	0.312 0.328	-0.126 -0.215	-0.322 -0.288	-0.385 -0.380	0.196 0.252	0.053 -0.090
4. Meat, fish, egg	-1.267 -1.200	0.833 0.663	0.580 0.300	-0.715 -0.809	0.118 0.055	-0.242 -0.103	-0.496 -0.286	0.368 0.077	-0.196 -0.028
5. Sugar	-0.086 -0.486	0.469 0.212	-0.071 -0.053	0.039 0.063	-0.439 -0.462	-0.298 -0.468	-0.288 -0.005	-0.117 -0.112	0.218 0.118
6. Other food	-0.613 -0.596	0.424 -0.362	0.586 0.512	0.150 -0.121	0.109 0.306	-1.178 -1.299	-0.732 -0.215	0.082 0.881	-0.078 0.036
7. Clothing	-0.983 -1.255	-0.328 -0.068	-0.192 -0.221	-0.482 -0.356	-0.059 -0.136	0.238 0.241	0.425 0.162	0.147 0.234	-0.272 -0.265
8. Fuel and light	-0.480 -0.246	-0.153 -0.078	0.111 0.127	0.025 0.117	-0.005 -0.024	0.204 0.094	-0.023 -0.154	-0.566 -0.529	-0.194 -0.265
9. Other non food	-0.208 -0.571	0.047 0.782	-0.548 -0.478	-0.094 0.210	-0.056 -0.308	0.167 0.418	0.345 -0.179	-0.282 -1.037	-0.494 -0.648

* In each cell the upper figure is based on aggregate data and the lower on disaggregated data.

Table 6.3 Estimates of itemwise non compensated own and cross price elasticities calculated at sample average of prices and total expenditure for LES

All India : ~~rural~~ urban; aggregate and disaggregated data*

Items \ Item no.	1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	-0.460 -0.392	-0.001 0.004	-0.013 -0.004	-0.001 -0.001	-0.007 -0.002	-0.026 -0.009	0.016 0.006	-0.006 -0.008	0.016 0.018
2. Milk and milk products	-0.182 -0.221	-0.983 -1.111	-0.033 -0.013	-0.001 -0.003	-0.017 -0.008	-0.066 -0.030	0.040 0.021	-0.015 -0.029	0.040 0.062
3. Edible oils	-0.063 -0.148	-0.001 0.011	-0.352 -0.760	0.000 -0.002	-0.006 -0.005	-0.023 -0.020	0.014 0.014	-0.005 -0.019	0.014 0.042
4. Meat, fish, egg	-0.179 -0.183	-0.002 0.013	-0.032 -0.011	-0.968 -0.938	-0.017 -0.007	-0.065 -0.024	0.039 0.018	-0.015 -0.024	0.040 0.052
5. Sugar	-0.098 -0.156	-0.001 0.011	-0.018 -0.009	-0.001 -0.002	-0.539 -0.803	-0.036 -0.021	0.022 0.015	-0.008 -0.021	0.022 0.044
6. Other food	-0.128 -0.171	-0.002 0.012	-0.023 -0.010	-0.001 -0.002	-0.012 -0.006	-0.738 -0.895	0.028 0.017	-0.011 -0.023	0.028 0.048
7. Clothing	-0.284 -0.247	-0.004 0.018	-0.051 -0.015	-0.002 -0.003	-0.027 -0.009	-0.104 -0.033	-1.473 -1.237	-0.024 -0.032	0.063 0.070
8. Fuel and light	-0.147 -0.126	-0.002 0.009	-0.026 -0.007	-0.001 -0.002	-0.014 -0.005	-0.053 -0.017	0.032 0.012	-0.804 -0.657	0.033 0.035
9. Other non food	-0.209 -0.232	-0.003 0.017	-0.038 -0.014	-0.002 -0.003	-0.020 -0.009	-0.076 -0.031	0.046 0.022	-0.018 -0.031	-1.081 -1.118

* In each cell the upper figure is based on aggregate data and the lower on disaggregated data.

Table C.4 Estimates of itemwise non compensated own and cross price elasticities calculated at sample average of prices and total expenditure for AIDS

All India : urban; aggregate and disaggregated data*

Items \ Item no.	1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Cereals and cereal substitutes	-0.164 0.041	-0.190 -0.134	-0.168 -0.127	-0.074 -0.116	0.009 -0.035	0.242 0.140	0.298 0.266	0.287 0.507	-0.559 -0.771
2. Milk and milk products	-0.568 -0.443	-0.468 -1.025	-0.099 -0.320	0.443 0.279	-0.033 -0.194	-0.108 -0.204	-0.558 0.026	0.019 -0.217	-0.417 0.802
3. Edible oils	0.458 0.433	-0.470 -0.556	-0.752 -0.783	0.153 0.168	-0.375 -0.455	-0.033 -0.099	0.084 0.048	-0.520 -0.428	1.084 1.278
4. Meat, fish, egg	-0.821 -0.765	0.441 0.436	0.419 0.221	-0.812 -0.806	0.342 0.133	-0.047 -0.046	-0.639 -0.460	1.057 0.557	-1.682 -0.871
5. Sugar	0.680 0.401	0.007 -0.163	-0.550 -0.372	-0.109 -0.079	-0.792 -0.736	-0.270 -0.602	0.929 0.817	-1.207 -1.472	2.043 2.455
6. Other food	-0.104 -0.334	0.544 0.357	0.417 0.391	0.044 -0.020	-0.008 0.021	-0.902 -1.069	-0.832 -0.678	-0.789 -0.945	1.426 1.423
7. Clothing	-0.901 -1.641	-0.018 0.365	0.510 0.426	0.160 0.266	0.973 0.741	-0.625 -0.222	-1.224 -1.305	2.573 2.271	-4.714 -4.202
8. Fuel and light	-0.036 -0.013	0.108 0.036	-0.090 -0.073	0.111 0.092	-0.090 -0.014	-0.021 -0.097	0.014 0.065	-0.886 -0.729	0.104 -0.018
9. Other non food	-0.026 -0.345	-0.544 -0.161	-0.261 -0.146	-0.250 -0.101	-0.221 -0.060	-0.050 0.153	0.587 0.225	-0.336 -0.248	-0.300 -0.844

* In each cell the upper figure is based on aggregate data and the lower on disaggregated data.

the aggregate and the disaggregated data for the LES and the AIDS respectively. Table C.3-C.4 present the corresponding estimates for urban India.

As we have already discussed the own-price elasticities in Chapter IV, here we shall discuss only the cross-price elasticities. It may be observed in Tables C.1 and C.3, that for the LES, for both the sectors and the two types of data, the magnitudes of cross-price elasticities are mostly negligible compared to the estimates of the corresponding own-price elasticities. There is, however, an important exception to this general pattern, that is, the cross-price elasticities for different itemgroups with respect to the price of 'cereals and cereal substitutes' for both the sectors and both sets of data are often larger. No such systematic pattern is observed for the estimated cross-price elasticities based on the AIDS. The relatively large magnitude of the LES based estimates of the cross-price elasticities with respect to the price of 'cereals and cereal substitutes' would explain why the proportionately relationship between the LES based estimates of expenditure and own-price elasticity was not observed for 'cereals and cereal substitutes' in Chapter IV.

As regards the comparability of the estimates of cross-price elasticities obtained from the two types of data, the following observations may be made. For either system the signs of the elasticities estimated from the two types of data generally agree -

for both the sectors. For the rural sector, the AIDS shows a much better agreement. The comparison of the magnitude of the elasticities estimated from the two types of data is, however, much more difficult and no clear conclusion can be drawn in this respect.

C.3 Stepwise regressions and the AIDS

It may be observed in Tables 4.3.1 - 4.3.4 of Chapter IV that a large number of estimated c_{ij} parameters have high standard errors. This perhaps indicates a fair amount of multicollinearity in the logarithms of prices used as explanatory variables in the itemspecific budget share equations of the AIDS. It may be noted that the AIDS, as proposed, is a flexible system of budget share equations each having a large number of parameters that would provide first-order approximation to any true unknown budget share equations underlying a given set of consumption data. However, given that in most time-series data the prices tend to be correlated, it is worthwhile to see whether the AIDS is really worth all the parameters it has. To examine this for the present set of data, the stepwise regression procedure was applied to estimate the individual budget share equations based on the disaggregated set of data for rural and urban India separately. The BMDP package has been used for the stepwise regressions.

Table C.5 Itemwise R^2 and $\bar{R}^2$ values for the step-wise regressions and the corresponding $\bar{R}^2$ values for the full-specification AIDS/All India : rural and urban; disaggregated data

		all India rural				all India urban			
		step-wise regression			AIDS	step-wise regression			AIDS
Sl. no.	item group	logarithms of variables included as regressors	R^2	$\bar{R}^2$	$\bar{R}^2$	logarithms of variables included as regressors	R^2	$\bar{R}^2$	$\bar{R}^2$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.	Cereals and cereal substitutes	income	0.946	0.945	0.963	income, price of milk etc, price of other non-food	0.993	0.992	0.994
2.	Milk and milk products	income	0.937	0.936	0.946	income	0.876	0.874	0.880
3.	Edible oils	price of cereals etc. price of edible oils	0.677	0.666	0.707	income, price of milk etc, price of edible oils	0.804	0.794	0.792
4.	Meat, fish, egg	income	0.389	0.379	0.480	income	0.339	0.328	0.286
5.	Sugar	income, price of sugar, price of other food	0.886	0.880	0.868	income, price of sugar, price of other food	0.453	0.424	0.411
6.	Other food	income, price of milk etc.	0.548	0.524	0.565	income, price of edible oils, price of other non-food	0.713	0.692	0.718
7.	Clothing	income, price of cereals etc., price of clothing	0.937	0.933	0.931	income, price of cereals etc.	0.930	0.928	0.943
8.	Fuel and light	income, price of cereals etc., price of other non-food	0.954	0.951	0.949	income, price of cereals	0.968	0.966	0.966
9.	Other non food	income, price of sugar	0.856	0.851	0.843	income, price of other non-food	0.960	0.959	0.954

Table C.5 presents a summary of the stepwise regression results. This table gives the names of explanatory variables that enter into the regression equations for itemspecific budget shares at the first few steps and the corresponding R^2 and $\bar{R}^2$ values.[1/] To facilitate comparison, the $\bar{R}^2$ values corresponding to the full-specification of the AIDS have also been presented here. It may be observed that with small number of explanatory variables the R^2 and $\bar{R}^2$ values of the truncated regressions are generally quite high. Also, the $\bar{R}^2$'s for the truncated regressions are quite close to (and in some cases even higher than) the ones for the full specification of the AIDS. These results thus clearly demonstrate how the large number of price parameters contained in the AIDS could virtually turn out to be superfluous in the presence of high correlation amongst the price variables.

[1/] The 'income' variable in Table C.5 is actually the real income defined as $y/\bar{P}$, with $\bar{P} = \sum_{i=1}^{n} w_i \log p_i$; where y, p_i $(i=1,2,\ldots,n)$ and w_i $(i=1,2,\ldots,n)$ have the same meanings as before.

Appendix D

The AIDS and Variant 3 : Homogeneity across population groups

In the empirical analyses based on population-groupwise disaggregated consumption data reported in Chapters IV, V and VI, demand systems have been estimated under the assumption that a single system would be applicable to all the three population groups. Thus, it may be of interest to examine if this assumption of homogeneity of demand systems across population groups is empirically plausible. In what follows, we report some results on tests of homogeneity of demand systems across population groups, applied to the two best fitting systems, viz., the AIDS and Variant 3.

For the AIDS, homogeneity across population groups has been tested for individual itemgroups through a single equation procedure described in Deaton (1980). This procedure takes into account the heterogeneity of disturbance variance across population groups. Following this procedure, population-group specific observations are first standardised by dividing by the respective estimated standard deviations obtained from the group-specific regression estimates. A pooled weighted regression equation is then estimated on the basis of these standardised observations. Thus, let RSS_j : residual sum of squares for the regression equation for population-group j (= 1, 2, 3) based on the standardised observations for that group, and RSS_p : residual sum of squares for the pooled regression equation mentioned above. The statistic for the pooling test is then defined as

$$F = \frac{(RSS_p - \sum_{j=1}^{3} RSS_j) / 2K}{\sum_{j=1}^{3} RSS_j / (T - 3K)} \qquad \ldots (D.1)$$

K being the number of parameters in the equation and T being the total number of observations in all the three groups together. In view of the fact that RSS_j's relate to regression equations estimated on standardised data, (D.1) simplifies to

$$F = \frac{RSS_p - T + 3K}{2K} \qquad \ldots (D.2)$$

which under the null-hypothesis of intergroup homogeneity is distributed as F (2K, T-3K).

Table D.1 : Itemwise F-values for pooling test based on AIDS

All India: rural and urban; disaggregated data

Items	All India rural	All India urban
(1)	(2)	(3)
1. Cereals and cereal substitutes	12.034*	0.529
2. Milk and milk products	15.219*	2.338*
3. Edible oils	5.455*	5.795*
4. Meat, fish, egg	2.429*	18.569*
5. Sugar	1.556	7.616*
6. Other food	12.444*	1.735
7. Clothing	5.243*	0.111
8. Fuel and light	11.210*	0.164
9. Other non-food	11.001*	2.263*

* Significant at 5 per cent level exceeding critical value $F_{22, 27} = 1.96$

Table D.1 presents the itemwise computed F-values separately for rural and urban India. It may be seen from this table that with (22, 27) degrees of freedom for most of the itemgroups in rural and urban India, the homogeneity is rejected at 5 per cent level of significance. Exceptions are 'sugar' for rural India and 'cereals and cereal substitutes', 'other food', 'clothing' and 'fuel and light' for urban India.

For Variant 3 an approximate pooling test has been done on the basis of the likelihood values.[1] In this case, the system has been estimated separately for each of the three population groups and the sum of twice-log-likelihood values obtained for each population group (called L_S here) has been compared with the corresponding twice-log-likelihood value for the system estimated on the basis of pooled data (called L_P here). The difference $L_S - L_P$ is asypototically distributed as $\chi^2_{n_S - n_P}$, where $n_S = 3n_P$, n_P being the number of parameters in the system. The estimated $L_S - L_P$ values are presented in Table D.2 below, along with the corresponding group specific and pooled twice-log-likelihood values.

Table D.2 : Difference of log-likelihood values for alternative estimation of Variant 3 from pooled and non-pooled data

All India: rural and urban; disaggregated data.

Sector	Twice-log-likelihood value				$L_S - L_P$ *
	Population group 1	Population group 2	Population group 3	Pooled	
(1)	(2)	(3)	(4)	(5)	(6)
All-India rural	1477.3	1836.7	1476.7	3758.2	1032.5
All-India urban	1457.0	1415.6	1416.1	3778.2	510.5

* The critical value of χ^2_{72} at 5 per cent level of significance is 92.8 .

1/ This test procedure followed here is an approximate one, as it does not take into account the heterogeneity of the variances of the itemspecific random disturbances across population groups.

The results presented in Table D.2 clearly indicate the heterogeneity of consumption patterns as described by Variant 3 across population groups.

In view of the above results, the validity of using pooled data in the empirical analysis presented in this dissertation may be questioned. It may, however, be mentioned that the results presented above pertain to testing of equality of all parameters across population groups simultaneously. It is possible that systematic or large variation of some, but not all, of the parameters across population groups leads to significant values of the test statistics. To get a rough idea of this, the parameter estimates obtained by fitting the AIDS and Variant 3 to the three population groups separately were examined. It turned out that for Variant 3, while the variation of other parameters were small, the c-parameters for the rural sector and the a-parameters for the urban sector varied systematically across population groups, the magnitude of variation being larger in case of the former. As for the AIDS, no such general pattern could be observed. For each equation, however, many of the parameters showed little variation across population groups. As this study of heterogeneity of consumption behaviour across population groups was undertaken very late in the course of the present study, these results could not be taken into account for re-doing the results based on the assumption of homogeneity.

Appendix E

Estimates of some dynamised systems based on aggregate data

E.1 Introduction

In Chapter IV, we presented estimates of the AIDS and the LES fitted to aggregate and disaggregated consumption data for rural and urban India. Serial correlation of item-specific random disturbances is commonly encountered in analyses of aggregate time series consumption data on the basis of static demand systems. Presence of serial correlation in these cases are generally ascribed to the inadequacy of static demand systems in capturing dynamic elements, e.g., shifts in tastes and preferences over time, habit formation etc., implicit in aggregate time series consumption data.

To cope with this, dynamic versions of static demand systems are sometimes tried out. Two approaches to dynamisation of a static demand system are popular, viz., the introduction of time trends in the individual demand equations or in the relevant parameters of a static system, and the introduction of the phenomenon of habit formation into the system by making some of the relevant parameters dependent on past consumption levels (Pollak and Wales, 1969; Pollak, 1970a).

In our exercise based on aggregate data, the problem of serial correlation of the item-specific random disturbances was observed to be quite strong. In view of this, some further exercises based on the aggregate data were undertaken. Here, we report the results of these exercises. In section E.2, the results based on the AIDS in its first-differenced form are discussed. The results relating to the dynamised LES and also the dynamised Variant 1 of our proposed system are presented in section E.3 .

E.2 The first-differenced AIDS

For the AIDS fitted to aggregate consumption data for rural and urban India, the equation specific Durbin-Watson (D-W) statistics were all observed to be very low - clearly indicating the presence of positive serial correlation in the item-specific random disturbances. The AIDS in its first-differenced forms were therefore estimated. As is well-known, the first-differenced AIDS with item-specific intercept terms is useful in examining the presence of linear time trends in the observed budget shares. We have estimated the first-differenced AIDS once with the intercept terms and again without the intercepts in order to detect the presence of such linear time trends in the observed budget shares. These systems have been estimated separately for the rural and the urban sectors.

Table E.1 presents the itemwise adjusted coefficients of determination ($\bar{R}^2$) for the two versions of the AIDS, estimated by

Table E.1 Itemwise adjusted coefficient of determination ($\bar{R}^2$) for AIDS : 1st differenced, with and without intercepts

All India : rural and urban; aggregate data

Items	All India rural		All India urban	
	with intercept	without intercept	with intercept	without intercept
(1)	(2)	(3)	(4)	(5)
1. Cereals and cereal substitutes	0.919	0.918	0.742	0.689
2. Milk & milk products	-0.208	-0.109	0.700	0.652
3. Edible oils	0.546	0.584	0.684	0.709
4. Meat, fish, egg	0.293	0.351	0.223	0.289
5. Sugar	0.868	0.873	0.803	0.819
6. Other food	0.323	-0.150	0.016	-0.270
7. Clothing	0.336	0.245	0.410	0.452
8. Fuel & light	0.102	0.170	0.734	0.611
9. Other non food	0.308	0.369	0.542	0.581

single-equation ordinary least squares method for the two sectors. A comparison of the $\bar{R}^2$ values for the two specifications of the first-differenced AIDS suggests that the introduction of the intercept terms does not, in general, improve the results. Indeed, the intercept terms turned out to be significant only in a few cases, viz., for 'other food' in case of the rural sector, and for 'fuel and light' in case of the urban sector. However, the problem of multicollinearity was observed to be very strong, so that the above observation regarding the presence of time trends in budget shares may not be conclusive.

To examine the extent to which first-differencing the AIDS could eliminate the observed serial correlations in the item-specific random disturbances, we calculated the equation-specific D-W statistics for the two versions of the first-differenced AIDS. Table E.2 presents the computed equation-specific D-W statistics for the originally estimated AIDS and its first-differenced versions separately for the two sectors. It may be observed from this table that for the first-differenced version of the AIDS the problem of positive serial correlation is virtually absent. It may thus be concluded that the aggregate time series consumption data for the rural and the urban sector are auto-regressive in nature.

Table E.2 Item specific Durbin-Watson (D-W) statistics for AIDS, 1st differenced AIDS without intercept and 1st differenced AIDS with intercept

All India : rural and urban; aggregate data

Items	D-W statistics for rural India			D-W statistics for urban India		
	AIDS	1st differenced AIDS without intercept	1st differenced AIDS with intercept	AIDS	1st differenced AIDS without intercept	1st differenced AIDS with intercept
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	0.494	2.554	2.561	0.342	1.891	1.860
2. Milk & milk products	1.450	2.462	2.466	0.583	1.830	1.853
3. Edible oils	0.535	2.719	2.715	0.327	1.907	1.904
4. Meat, fish, egg	0.470	2.627	2.625	0.304	1.842	1.839
5. Sugar	0.508	2.272	2.176	0.310	1.840	1.837
6. Other food	0.529	2.623	2.625	0.328	1.960	2.789
7. Clothing	0.817	2.586	2.904	0.308	1.879	1.878
8. Fuel & light	0.544	2.684	2.686	0.393	2.303	2.030
9. Other non food	0.653	2.638	2.638	0.359	1.836	1.833

E.3 Dynamised LES and Variant 1

To capture the auto-regressive and time-dependent elements observed to be present in the aggregate consumption data, we estimated two types of dynamised versions of the LES and Variant 1 of our proposed system. The first type incorporates linear time trend in the individual budget shares. Thus, the LES with linear time trend takes the form

$$w_{it} = a_i\ t + \frac{p_{it}\ c_i}{y_t} + b_i\ (1 - \frac{\sum_j p_{jt}\ c_j}{y_t}) + u_{it} \qquad \ldots(E.1)$$

for i = 1, 2, ..., n , where $a_i t$ represents the time trend in w_{it} and the other notations have their usual meaning. Similarly, Variant 1 with linear time trend is of the form

$$w_{it} = a_i\ t + \frac{p_{it}\ c_i}{\sum_j p_{jt}\ c_j} + b_i\ \log\ (\frac{y_t}{\sum_j p_{jt}\ c_j}) + u_{it} \qquad \ldots(E.2)$$

for i = 1, 2, ..., n .

The other type of dynamisation is based on the introduction of habit formation phenomenon. Here, we have considered the case of the committed quantities being linearly dependent on the previous level of quantity consumption, i.e., $c_i(q_{it-1}) = c_i + a_i\ q_{it-1}$, where q_{it-1}: consumption of item i in period t-1 . Thus, the "habit-formation" version of the LES is of the form

$$w_{it} = \frac{p_{it}\ (c_i + a_i\ q_{it-1})}{y_t} + b_i\ (1 - \frac{\sum_j p_{jt}\ (c_j + a_j q_{jt-1})}{y_t}) + u_{it} \qquad \ldots(E.3)$$

for i = 1, 2, ..., n . The corresponding Variant 1 system is given by

$$w_{it} = \frac{p_{it}\ (c_i + a_i q_{it-1})}{\sum_j p_{jt}\ (c_j + a_j q_{jt-1})} + b_i\ \log\ (\frac{y_t}{\sum_j p_{jt}\ (c_j + a_j q_{jt-1})}) + u_{it} \qquad \ldots(E.4)$$

for i = 1, 2, ..., n .

The systems represented by (E.1), (E.2), (E.3) and (E.4) were estimated for the rural and the urban sectors on aggregate time series consumption data. Under the assumptions $E(u_{it}) = 0$ for all i and t, and $E(u_{it}\ u_{js}) = \sigma_{ij}$ for t = s and $E(u_{it}\ u_{js}) = 0$ for $t \neq s$, these systems have been estimated by Non-Linear Full-Information Maximum Likelihood method using the NLFIML package.

Table E.3 presents the twice log-likelihood values for these systems separately for rural and urban India. To facilitate comparison, the corresponding log-likelihood values of the static versions have also been reported in this table. It may be observed that for rural India, introduction of a linear time trend results in a

Table E.3 Comparative values of twice log-likelihood

All India : rural and urban; aggregate data

Models / Sectors	LES			Variant 1		
	constant taste	with time trend	habit formation	constant taste	with time trend	habit formation
(1)	(2)	(3)	(4)	(5)	(6)	(7)
All India rural	1443.7	1507.0	1435.0	1443.0	1503.5	1470.1
All India urban	1471.6	1560.6	1747.1	1454.1	1535.1	1455.3
Number of parameters	18	27	27	18	27	27

much larger rise in likelihood values than the introduction of habit-formation for both the LES and Variant 1 . Actually, introduction of habit-formation in the LES leads to a fall in the likelihood value. For urban India, the results for the two systems are somewhat different in nature. For this sector, introduction of habit formation in the LES results in a larger rise in the likelihood value compared to the introduction of linear time trend. For Variant 1, on the other hand, introduction of linear time trend signifiaantly improves the likelihood value, but the introduction of habit formation fails to do so.

To examine whether the introduction of linear time trend or habit formation could improve the situation regarding serial correlation of the item-specific random disturbances, we computed the D-W statistics for the two dynamised versions of the LES and Variant 1. Tables E.4 and E.5 present these D-W statistics for the rural and the urban sectors for the alternative versions of the LES and Variant 1 respectively.

From Table E.4 it may be observed that in the rural sector, the LES with time trend shows some improvements in the picture of serial correlations. But the habit-formation version of the LES, in general, suffers from more serious serial correlation problem compared to the original LES. In the urban sector, on the other hand, both the versions of the LES improve the picture of serial correlation - LES with time trend doing somewhat better than the habit-formation version.

Table E.4 Item specific Durbin-Watson (D-W) statistics for the LES, LES with linear time trend and LES with habit formation

All India : rural and urban; aggregate data

Items	All India rural			All India urban		
	LES	LES with time trend	LES with habit formation	LES	LES with time trend	LES with habit formation
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	0.614	0.622	0.791	0.805	0.928	1.116
2. Milk and milk products	0.859	0.927	0.882	0.973	0.996	1.786
3. Edible oils	0.753	1.215	0.991	1.169	2.450	1.191
4. Meat, fish, egg	1.812	1.777	1.569	2.643	2.149	2.094
5. Sugar	0.461	2.088	0.460	0.707	1.445	2.109
6. Other food	0.187	0.945	0.198	0.106	0.897	0.703
7. Clothing	0.655	1.551	0.682	0.562	2.282	1.214
8. Fuel and light	1.264	1.836	0.842	1.566	2.216	1.789
9. Other non food	1.248	1.732	0.754	0.649	2.083	0.879

Table E.5 Item specific Durbin-Watson (D-W) statistics for Variant 1, Variant 1 with linear time trend, and Variant 1 with habit formation

All India : rural and urban; aggregate data

Items	All India rural			All India urban		
	Variant 1	Variant 1 with time trend	Variant 1 with habit formation	Variant 1	Variant 1 with time trend	Variant 1 with habit formation
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. Cereals and cereal substitutes	0.679	0.794	0.578	1.129	1.238	1.623
2. Milk and milk products	0.865	1.189	1.998	0.998	0.919	1.648
3. Edible oils	1.203	1.066	1.174	0.454	1.050	1.476
4. Meat, fish, egg	1.814	1.859	1.110	2.618	1.726	1.923
5. Sugar	0.329	1.745	0.590	0.405	0.942	1.786
6. Other food	0.157	0.865	0.301	0.220	0.986	2.036
7. Clothing	1.212	1.606	2.013	0.535	1.886	1.639
8. Fuel and light	1.237	1.919	1.328	1.800	1.568	2.087
9. Other non food	1.253	1.737	1.397	0.651	2.018	1.260

The results for Variant 1 in Table E.5 are broadly similar in nature to those in Table E.4. Actually, the relative superiority of the time trend version for rural India and of the habit-formation version for urban India in accounting for the observed serial correlations (at least partly) is brought out more clearly here.

Before concluding we shall summarise the results regarding the presence of time trend and habit-formation as reflected through the estimates of the dynamised versions of the LES and Variant 1. As regards the presence of time trend, for the rural sector, both the systems indicated significant time trend for 'sugar', 'other food', 'clothing', 'fuel and light' and 'other non-food' items. For the urban sector, the estimates based on Variant 1 indicated significant time trend for 'edible oils', 'meat, fish, egg', 'sugar', 'other food', 'clothing' and 'other non-food', and the estimated LES indicated significant trend for 'cereals and cereal substitutes', in addition. So far as habit-formation is concerned, the results based on the two systems showed some differences. While the LES-based estimates indicated significant habit-formation for all the items in the urban sector, the estimated Variant 1 did not indicate significant habit-formation for 'edible oils', 'clothing' and 'fuel and light' for this sector. For the rural sector, the estimated LES indicated significant habit-formation for 'cereals and cereal substitutes', 'edible oils', 'other food' and 'fuel and light' only, while the results based on Variant 1 indicated significant habit-formation in respect of all the nine item groups.

To conclude, it may be pointed out that the dynamised versions of the LES and Variant 1 succeeded to some extent in accounting for the presence of positive serial correlations that were observed to be present when the static version of these systems were estimated on aggregate time series consumption data. However, these dynamised versions cannot be accepted as satisfactory specifications for the aggregate data analysed here because for such important items as 'cereals and cereal substitutes', 'edible oils', 'clothing' etc., the problem of serial correlation continues to be present.

REFERENCES

Allen, R.G.D. and A. Bowley (1935) : Family Expenditure. London: P.S. King and Son.

Barten, A.P. (1964) : Family composition, prices and expenditure patterns, in P.E. Hart, G. Mills, and J.K. Whitaker (eds.), Econometric Analysis for National Economic Planning, London: Butterworth.

Barten, A.P. (1966) : Theorie en empiric van een volledig stelsel van vraegvergelijkingen, Doctoral dissertation, Rotterdam: University of Rotterdam.

Barten, A.P. (1967) : Evidence on the Slutsky conditions for demand equations, Review of Economics and Statistics, 49: 77-84.

Barten, A.P. (1968) : Estimating demand equations, Econometrica, 36: 213-251.

Barten, A.P. (1969) : Maximum likelihood estimation of a complete system of demand equations, European Economic Review, 1: 7-73.

Barten, A.P. (1977) : The systemsof consumer demand functions approach: a review, Econometrica, 45: 23-51.

Barten, A.P. and E. Geyskens (1975) : The negativity condition in consumer demand, European Economic Review, 6: 227-260.

Barten, A.P. and S.J. Turnovsky (1966) : Some aspects of the aggregation problem for composite demand equations, International Economic Review, 7: 231-259.

Baschet, J. and P. Debreu (1971) : Systemes de lois de demande: une comparaison internationale, Annales de l'I.N.S.E.E. (Institut national de la statistique et des études économiques), No. 6.

Bhattacharya, N. (1967) : Consumer behaviour in India: An application of the Linear Expenditure System, Economic and Political Weekly, 47 : 2093-2098.

Bhattacharya, N. and T. Maitra (1970) : An analysis of Engel curves based on NSS household budget data for 7th to 22nd rounds, Indian Statistical Institute Tech. Report No. Econ/9/70 (mimeographed).

Blackorby, C., R. Boyce and R.R. Russel (1978) : Estimation of demand systems generated by Gorman Polar form: A generalisation of the S-branch utility tree, Econometrica, 46: 345-363.

Blackorby, C., D. Primont and R.R. Russell (1978) : Duality, Separability and Functional Structure: Theory and Economic Applications. Amsterdam: North-Holland.

Blundell, R.W. and R. Ray (1982) : A non-separable generalisation of the Linear Expenditure System allowing non-linear Engel curves, Economics Letters, 9: 349-354.

Boulding, K.E. (1950) : A Reconstruction of Economics. New York: John Wiley.

Brown, A. and A. Deaton (1972) : Surveys in applied economics: Models of consumer behaviour, Economic Journal, 82: 1145-1236.

Brown, M. and D. Heien (1972) : The S-branch utility tree: A generalisation of the Linear Expenditure System, Econometrica, 40: 737-747.

Byron, R.P. (1968) : Methods for estimating demand equations using prior information: A series of experiments with Australian data, Australian Economic Papers, 7: 227-248.

Byron, R.P. (1970a) : A simple method for estimating demand systems under separable utility assumptions, Review of Economic Studies, 37: 261-274.

Byron, R.P. (1970b) : The restricted Aitken estimation of sets of demand relations, Econometrica, 38: 816-830.

Carlevaro, F. (1976) : A generalisation of the Linear Expenditure System, in L. Solari and J.N. Du Pasquier (eds.), Private and Enlarged Consumption, 73-92. New York: North-Holland for ASEPELT.

Chatterjee G.S. and N. Bhattacharya (1975) : Some observations on NSS household budget data , in V.M. Dandekar and P. Venkataramaiah (eds.) Data Base of Indian Economy - II, 97-109.

Chipman, J.S. (1965) : A survey of the theory of International Trade; Part II, The Neo-Classical Theory, Econometrica, 33: 685-760.

Chow, G. (1957) : Demand for Automobiles in the U.S.: A study in consumer durables. Amsterdam: North-Holland.

Chow, G. (1960) : Statistical demand functions for automobiles and their use for forecasting, in A.C. Harberger (ed.), The Demand for Durable Goods, 149-178. Chicago, University of Chicago Press.

Christensen, L.R., D.W. Jorgenson and L.J. Lau (1975) : Transcendental logarithmic utility functions, American Economic Review, 65: 367-383.

Coondoo, D. (1975) : Contributionsto the Analysis of Consumer Behaviour, Ph.D. Thesis. Indian Statistical Institute.

Cramer, J.S. (1957) : A dynamic approach to the theory of consumer demand, Review of Economic Studies, 24: 73-86.

Cramer, J.S. (1970) : Interaction of price and income in consumer demand, European Economic Review, 1: 428-436.

Dahlman, C.J. and A. Klevmarken (1971) : Den Privata Konsumtionen 1931-1975. Uppsala.

Dandekar, V.M. and N. Rath (1971) : Poverty in India, Economic and Political Weekly, 6(1&2).

Darrough, M.N., R.A. Pollak and T.J. Wales (1983) : Dynamic and stochastic structure: An analysis of three time series of household budget studies, The Review of Economics and Statistics, 65: 274-281.

Deaton, A.S. (1974a) : The analysis of consumer demand in the United Kingdom, 1900-1970, Econometrica, 42: 341-367.

Deaton, A.S. (1974b) : A reconsideration of the empirical implications of additive preferences, Economic Journal, 84: 338-348.

Deaton, A.S. (1975) : The measurement of income and price elasticities, European Economic Review, 6: 261-274.

Deaton, A.S. (1976) : A Simple Non-Additive Model, in L. Solari and J.N. Du Pasquier (eds.), Private and Enlarged Consumption, 55-72, New York: North Holland for ASEPELT.

Deaton, A.S. (1980) : Analysing the food-shares in a household survey, Living Standards Measurement Study, World Bank.

Deaton, A.S. and J. Muellbauer (1980a) : An Almost Ideal Demand System, American Economic Review, 70: 312-326.

Deaton, A.S. and J. Muellbauer (1980b) : Economics and Consumer Behaviour. Cambridge University Press.

Engel, E. (1857) : Die Produktions - und Konsumtionsverhältnisse des Königreichs Sächsen. Zeitschrift des Statistischen Bureaus des Königlichen Sächsischen Ministerium des Innern. Nos 8 and 9. Reprinted in Bulletin de l'Institut International de Statistique 9 (1895).

Friend, I. and R. Jones (eds.) (1960) : Proceedings of the Conference on Income, Consumption and Savings, Vol. I: Consumption. University of Pennsylvania Press.

Frisch, R. (1959) : A complete scheme for computing all direct and cross demand elasticities in a model with many sectors, Econometrica, 27: 177-196.

Ganguly, A. et al. (1960) : Studies on Consumer Behaviour. Bombay Asia Publishing House.

Goldberger, A.S. and T. Gamaletsos (1970) : A cross-country comparison of consumer expenditure patterns, European Economic Review, 1: 357-400.

Goldman, S.M. (1971) : An empirical comparison of alternative functional forms of demand systems, Unpublished doctoral dissertation, The University of Chicago.

Goldman, S.M. and H. Uzawa (1964) : A note on separability in demand analysis, Econometrica, 32: 387-398.

Gorman, W.M. (1953) : Community preference fields, Econometrica, 21: 63-80.

Gorman, W.M. (1959) : Separable utility and aggregation, Econometrica, 27: 469-481.

Gorman, W.M. (1970) : Quasi separable preferences, costs and technologies, Chapel Hill: University of North Carolina, (Mimeo.).

Gorman, W.M. (1976) : Tricks with utility functions, in M. Artis and R. Nobay (eds.), Essays in Economic Analysis. Cambridge University Press.

Gorman, W.M. (1980) : Some Engel curves, in A.S. Deaton (ed.), Essays in Theory and Measurement of Consumer Behaviour. Cambridge University Press.

Green, H.A.J. (1964) : Aggregation in Economic Analysis. Princeton: Princeton University Press.

Gupta, D.B. (1968) : A comparison of consumption patterns in Uttar Pradesh and Madras: A study of inter-regional variations with reference to India, Indian Economic Review, 3 (new series): 129-144.

Hicks, J.R. (1936) : Value and Capital. Oxford: Oxford University Press.

Houthakker, H.S. (1960) : Additive preferences, Econometrica, 28: 244-257.

Houthakker, H.S. and L.D. Taylor (1970) : Consumer Demand in the United States 1929-1970: Analysis and Projections. Harvard University Press.

Howe, H., R.A. Pollak and T.J. Wales (1979) : Theory and time series estimation of the Quadratic Expenditure System. Econometrica, 47: 1231-1247.

Iyengar, N.S. (1967) : Some estimates of Engel elasticities based on National Sample Survey data, Journal of the Royal Statistical Society, Series A, 130: 84-101.

Iyengar, N.S. and N. Bhattacharya (1965) : On the effect of differentials in consumer price index on measures of inequality, Sankhyā, Series B, 27: 47-56.

Jain, L.R. (1972) : An empirical evaluation of the system of addilog Engel curves, Presented at the 12th Indian Econometric Conference, Kanpur.

Jain, L.R. and S. Tendulkar (1973) : Analysis of occupational differences in consumer expenditure patterns in India, Sankhyā, Series B, 35: 239-267.

Jorgenson, D.W., L.J. Lau and T. Stoker (1982): The Transcendental Logarithmic Model of aggregate consumer behaviour, in R. Basmann and G. Rhodes (eds.), Advances in Econometrics. JAI Press.

Judge, G.G., W.E. Griffiths, R.C. Hill and T.C. Lee (1980): The Theory and Practice of Econometrics, New York: John Wiley and Sons.

Kansal, S.M. (1965) : Preliminary estimates of total consumption expenditure in India 1950-51 to 1963-64, Indian Statistical Institute Planning Unit Discussion Paper no. 5, September (mimeographed).

Koopmans, T.C. (1960) : Stationary utility and impatience, Econometrica, 28: 287-309.

Koopmans, T.C., P.A. Diamond and R.E. Williamson (1964) : Stationary utility and time perspective, Econometrica, 32: 82-100.

Lau, L.J. (1969) : Budgeting and decentralization of allocation divisions, Memorandum No. 89, Centre for Research in Economic Growth, Stanford University.

Lau, L.J. (1977) : Existence conditions for aggregate demand functions, Technical Report 248, Institute for Mathematical Studies in Social Sciences, Stanford University.

Lau, L.J., W.T. Lin and P.A. Yotopolous (1978) : The linear logarithmic expenditure system: An application to consumption-leisure choice, Econometrica, 46: 843-868.

Lau, L.J. and B.M. Mitchell (1971) : A linear logarithmic expenditure system: An application to U.S. data, Econometrica, 39: 87-88.

Leoni, R. (1967) : An analysis of expenditure on private consumption, Revista di Politica Economica (selected papers).

Leontief, W. (1936) : Composite commodities and the problem of index numbers, Econometrica, 4: 39-59.

Leontief, W. (1947) : Introduction to a theory of the internal structure of functional relationships, Econometrica, 15: 361-373.

Leser, C.E.V. (1963) : Forms of Engel functions, Econometrica, 31: 694-703.

Lluch, C. (1971) : Consumer demand functions, Spain, 1958-64, European Economic Review, 2: 277-302.

Maddala, G.S. (1977): Econometrics. New York: McGraw-Hill.

Mahalanobis, P.C. (1962) : A preliminary note on the consumption of cereals in India, Bulletin of the International Statistical Institute, 39: 53-76.

Maitra, T. (1969) : On regional and temporal variation in Engel curves in rural India, Indian Statistical Institute Technical Report No. Econ/2/69 (mimeo.)

Marquardt, D.W. (1963) : An Algorithm for Least-squares estimation of nonlinear parameters, Journal of the Society of Industrial and Applied Mathematics, 11: 431-441.

Mattei, A. (1971) : A complete system of dynamic demand equations, European Economic Review, 2: 251-276.

Mcfadden, D. (1964) : Existence conditions for Theil-type preferences, Department of Economics, University of California, Berkeley (mimeographed).

Muellbauer, J. (1975) : Aggregation, income distribution and consumer demand, Review of Economic Studies, 42: 525-543.

Muellbauer, J. (1977) : Testing the Barten model of household composition effects and the cost of children, Economic Journal, 87: 460-487.

Mukherjee, M. (1969a) : National Income of India: Trends and Structure. Calcutta, Statistical Publishing Society.

Mukherjee, M. (1969b) : Size and areal distributions of the level of living in India, Sankhya, Series B, 459-478.

Mukherjee, M. and G.S. Chatterjee (1972) : On the validity of NSS estimates of consumption expenditure, Arthavijnana, 14: 113-121.

Mukherjee, M. and S. Saha (1981) : Reliability of national income and allied estimates, The Journal of Income and Wealth, 5: 131-136.

Murthy, K.N. (1977) : On a non-linear expenditure system, Anvesak, Vol.2.

Nasse, P. (1970) : Analyse des effets de substitution dans un systeme complet de fonctions de demande, Annales de l'inseé (Institut national de la statistique et des études économiques), No. 5.

Paelinck, J. (1964) : Fonctions de consommation pour la Belgique 1948-59, Facultés. Universitaires N-D de la Paix, Namur.

Parks, R.W. (1967): Efficient estimation of a system of regression equations when disturbances are both serially and contemporaneously correlated, Journal of the American Statistical Association, 62: 500-509.

Parks, R.W. (1969) : Systems of demand equations: An empirical comparison of alternative functional forms, Econometrica, 37: 629-650.

Parks, R.W. and A.P. Barten (1973) : A cross-country comparison of the effects of prices, income and population composition on consumption patterns, Economic Journal, 83: 834-852.

Pearce, I.F. (1964) : A Contribution to Demand Analysis. Oxford: Oxford University Press.

Phlips, L. (1972) : A dynamic version of the linear expenditure model, Review of Economics and Statistics, 54: 450-458.

Phlips, L. (1983) : Applied Consumption Analysis. Amsterdam: North-Holland (Revised edition).

Phlips, L. and F. Spinnewyn (1982) : Rationality versus Myopia in Dynamic Demand Systems, Centre for Operation Research & Econometrics (Core) Reprint No. 468, Universite Catholique de Louvain.

Planning Commission (1979) : Task Force on Projections of Minimum Needs and Effective Consumption Demand.

Pollak, R.A. (1969) : Conditional demand functions and consumption theory, Quarterly Journal of Economics, 83: 70-78.

Pollak, R.A. (1970a) : Habit formation and dynamic demand functions, Journal of Political Economy, 78: 745-763.

Pollak, R.A. (1970b) : Budgeting and decentralization, Discussion Paper 157, Department of Economics, University of Pennsylvania.

Pollak, R.A. (1971) : Additive utility functions and linear Engel curves, Review of Economic Studies, 38: 401-413.

Pollak, R.A. and T.J. Wales (1969) : Estimation of the Linear Expenditure System, Econometrica, 37: 611-628.

Pollak, R.A. and T.J. Wales (1978) : Estimation of complete demand systems from household budget data: The Linear and Quadratic expenditure systems, American Economic Review, 68: 348-359.

Pollak, R.A. and T.J. Wales (1980) : Comparison of the Quadratic Expenditure System and Translog Demand Systems with alternative specifications of Demographic effects, Econometrica, 48: 595-611.

Prais, S.J. and H.S. Houthakker (1955) : The Analysis of Family Budgets. Cambridge: Cambridge University Press. (Second ed. 1971)

Radhakrishna, R. and G.V.S.N. Murthy (1973) : Some results on the estimation of alternative demand systems for India, 13th Indian Econometric Conference (mimeo.).

Radhakrishna, R., G.V.S.N. Murthy and N.C. Saha (1979): Models of consumer behaviour for Indian Economy, Sardar Patel Institute of Economic and Social Research, Ahmedabad.

Radhakrishna, R. and K.N. Murthy (1977): An application of Indirect Addilog System to consumer behaviour in India, 17th Indian Econometric Conference, Trivandrum, India.

Radhakrishna, R. and K.N. Murthy (1980): Models of complete expenditure systems for India, Working Paper 80-98, International Institute for Applied Systems Analysis.

Ramsey, J.B. (1969) : Tests of specification errors in classical linear least squares regression analysis, Journal of the Royal Statistical Society, B, 31: 350-371.

Ray, R. (1980) : Analysis of a time series of household expenditure surveys for India, Review of Economics and Statistics, 62: 595-602.

Ray, R. (1982) : The testing and estimation of complete demand systems on household budget surveys: An application of AIDS, European Economic Review, 17: 349-369.

Ray, R. (1983) : A dynamic analysis of expenditure patterns in Rural India, U.K. : Department of Econometrics, University of Manchester.

Rudra, A. (1964) : Relative rates of growth: Agriculture and industry, The Economic Weekly, November 7, 1173-1187.

Rudra, A. (1972) : Savings, investment and consumption, in C.R. Rao (ed.), Data Base of Indian Economy - I, 106-150.

Rudra, A. and P. Paul (1964) : Demand elasticity for foodgrains, The Economic Weekly, November 28, 1893-1894.

Samuelson, P.A. (1950) : The problem of integrability in utility theory, Economica , 17: 355-385.

Schultz, H. (1938) : The Theory and Measurement of Demand. Chicago: Chicago University Press.

Singh, B. (1968) : Role of occupational factors in household consumption, Indian Economic Review, 3 (new series): 85-110.

Sinha, R.P. (1966) : An analysis of food expenditure in India, Journal of Farm Economics, 48: 113-123.

Solari, L. (1971) : Théorie des choix et fonctions de consommation semi - agrégées Modeles Statiques, Geneva: Librarie Droz.

Somermeyer, W.H. and R. Bannink (1973) : A consumption-savings model and its applications. Amsterdam : North Holland.

Somermeyer, W.H., G.M. Hilhorst and J.W.W.A. Wit (1962) : A method of estimating price and income elasticities from time series and its application to consumers' expenditures in the Netherlands, 1949-1959, Statistical Studies n° 13, Central Bureau of Statistics, The Hague.

Somermeyer, W.H. and A. Langhout (1972) : Shapes of Engel curves and demand curves; implications of the expenditure allocation model applied to Dutch data, European Economic Review, 3: 351-386.

Sono, M. (1945) : The effect of price changes on the demand and supply of separable goods (in Japanese), Kokumin Keisai Zasshi, 74: 1-51. (Translated English version in International Economic Review, 2: 239-271, 1960).

Srinivasan, T.N., P.N. Radhakrishnan and A. Vaidyanathan (1974) : Data on distribution of consumption expenditure in India: An evaluation, in T.N. Srinivasan and P.K. Bardhan (eds.), Poverty and Income Distribution in India, pp. 148-162.

Stone, J.R.N. (1954a) : The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom, 1920-1938, Vol. I, Cambridge: Cambridge University Press.

Stone, J.R.N. (1954b) : Linear expenditure systems and demand analysis: An application to the pattern of British demand, Economic Journal, 64: 511-527.

Stone, J.R.N. (1964) : Models for demand projections, in C.R. Rao (ed.), Essays on Econometrics and Planning. Oxford: Pergamon Press.

Stone, J.R.N. (1966) : The changing pattern of consumption, in Mathematics in the Social Sciences and Other Essays. Cambridge, Mass.: M.I.T. Press.

Stone, J.R.N., J.A.C. Brown and D.A. Rowe (1964) : Demand analysis and projections for Britain 1900-1970: A study in method, in J. Sandee (ed.), Europe's Future Consumption. Amsterdam: North Holland.

Stone, J.R.N., and D.A. Rowe (1957) : The market demand for durable goods, Econometrica, 25: 423-443.

Stone, J.R.N., and D.A. Rowe (1960) : The durability of consumers' durable goods, Econometrica, 28: 407-416.

Strotz, R.H. (1956) : Myopia and inconsistency in dynamic utility maximization, Review of Economic Studies, 23: 165-180.

Strotz, R.H. (1957) : The empirical implications of a utility tree, Econometrica, 25: 269-280.

Strotz, R.H. (1959) : The utility tree - a correction and further appraisal, Econometrica, 27: 482-488.

Taylor, L.D. and D. Weiserbs (1972) : On the estimation of dynamic demand functions, Review of Economics and Statistics, 54: 459-465.

Theil, H. (1965) : The information approach to demand analysis, Econometrica, 33: 67-87.

Theil, H. (1970) : Value share transitions in consumer demand theory, Econometrica, 38: 118-127.

Theil, H. (1971) : Introduction to demand and index number theory, Report 7126, Centre for Mathematical Studies in Business and Economics, University of Chicago (mimeographed).

Theil, H. (1975): Theory and Measurement of Consumer Demand, Vol. 1. Amsterdam: North Holland.

Tiao, G.C. and M.M. Ali (1971): Analysis of correlated random effects: Linear model with two random components, Biometrika, 58: 37-51.

Wales, T.J. (1971) : A generalised linear expenditure model of the demand for non-durable goods in Canada, Canadian Journal of Economics, 4: 471-484.

Wold, H. and L. Jureen (1953) : Demand Analysis. Uppsala: Almqvist and Wiksells.

Working, E.J. (1943) : Statistical laws of family expenditure, Journal of the American Statistical Association, 38: 43-56.

Yoshihara, K. (1969) : Demand functions: An application to the Japanese expenditure pattern, Econometrica, 37: 257-274.

Zellner, A. (1962) : An efficient method of estimating seemingly unrelated regressions and test for aggregation bias, Journal of the American Statistical Association, 57: 348-368.