Indian Statistical Institute
Semester 1 (2025-2026)
M. Stat 1st Year
Backpaper
Stochastic Processes

Date: 18-12-2025 **Time:** 3 hours Total Points: 100

Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or result proved in class state it explicitly.

1. (a) Students log in at a terminal in the manner of a Poisson (λ) process and start downloading files whose size follows a distribution G. Find the total expected download size by students who arrive by time t. (b) From the beginning of a day customers arrive at an ATM in the manner of a renewal process with law F and withdraw a random amount $Y \sim G$ of cash. Write an expression for the sum of money disbursed by time t and determine the limit $\lim_{t\to\infty} E(\text{total amount disbursed by time } t)/t$. 10 + 10

2. (a) A continuous time MC has two states labelled 0 and 1. The waiting time in state 0 is $Exp(\lambda)$ and the waiting time in state 1 is $Exp(\mu)$. Show that (assuming $X_0 = 0$, i.e. $P_{00}(0) = 1, P_{01}(0) = 0$)
$$P_{00}(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}.$$
(b) Consider a linear growth birth and death process (i.e. with n particles the birth rate is $n\lambda$ and the death rate is $n\mu$) starting from $X(0) = 1$. Show that $P(k \text{ births before the first death}) = (\mu/(\lambda+\mu))(\lambda/(\lambda+\mu))^k$.
10 + 10

3. Birth and death with immigration: In a birth and death process the birth and death rates depend on the population size as λ_n, μ_n, respectively. One sometimes also has an immigration process with rate γ which adds to the population following an independent $Poi(\gamma)$ process. Find the new birth and death rates for a birth and death process with Immigration. 10

4. For a renewal process with distribution $F(x)$ compute $p(t) = P(\text{the number of renewals in } (0, t] \text{ is odd})$ in two ways: (a) using direct counting, (b) by deriving a renewal type equation for $p(t)$ and showing that formally it leads to the same solution. 10 + 10

5. In a branching chain the number of offspring per individual has a Bin(2, p) distribution. (a) Starting with one individual find the extinction probability, and the probability that the population becomes extinct for the first time in the third generation. (b) Suppose that instead of starting with one individual, the initial number of individuals is Poi(λ). Show that in this case if $p > 1/2$, the extinction probability is given by $\exp\{\lambda(1-2p)/p^2\}$. 10 + 10

6. In a branching "process" (as opposed to a chain) starting with one particle, after a random time with density f, this particle itself dies and gives rise to a random number of particles with generating function $h(z), 0 < z < 1$. Show that the generating function $G(z, t)$ of the number of particles in the branching process at time t satisfies $G(z, t) = [1 - F(t)]z + \int_0^t h(G(z, t-u))f(u)du$. (Here the f may be of nonexponential type leading to non-Markov branching processes.) 10