INDIAN STATISTICAL INSTITUTE

First Semestral Examination : (2024-2025)

PGDBA 1st Year

Stochastic Processes and Applications

PART B

Date: 17. 12. 24 Maximum marks: 50 Time: 90 minutes

Note: Answer as much as you can. Maximum you can score combining Parts A and B is 100. You may use a scientific calculator and any result proved in class after clearly stating the result. **Proofs will not be considered complete unless explanations are given for every step.**

1. Consider a discrete time Markov chain with the state space $\mathcal{S} = \{0, 1, 2, 3, 4, 5\}$, and the transition matrix

$$P = \begin{bmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \end{bmatrix}$$

   (a) Determine which states are recurrent and which states are transient.

   (b) Find the period of each state.

   (c) Find the mean time spent in each of the transient states.

   (d) Find the absorption probabilities to the closed sets from each of the transient states.

   (e) Find a stationary distribution of the process, if it exists. Is it unique?

   (f) Find the matrix which is the limit of P^n as $n \to \infty$. [3+6+5+5+5+6=30]

2. Consider a Markov Chain on non-negative integer having transition matrix P such that $p_{i,(i+1)} = p$ and $p_{i0} = (1 - p)$, for $i = 0, 1, \ldots$.

   (a) Verify whether the Markov Chain is irreducible or not.

   (b) Is the Markov chain apreiodic? Justify your answer.

   (c) Verify whether the state 0 is recurrent or transient.

   (d) Find a stationary distribution, if it exists. Otherwise justify why it won't exist.

   [2+2+4+5=13]

3. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ.

   (a) Find its covariance function

$$C_N(t_1, t_2) = Cov(N(t_1), N(t_2)) \quad \text{for} \quad t_1, t_2 \geq 0$$

   (b) Find $P[N(2) \geq 3 | N(5) = 10]$.

   (c) Let S_1 be its first arrival time. Find the conditional distribution of S_1, given $N(t) = 1$.

   [4+4+4]