Indian Statistical Institute

MSTAT II - Computational Finance

Final Exam. (Lab) / Semester II 2024-25

Date - May 5, 2025 / Time - 2 and 1/2 hours

Maximum Score - 25

NOTE : ***THIS IS AN OPEN NOTE LAB-BASED EXAM. COPIES OF THE 7 PRESENTATIONS DONE IN THE CLASS ARE ALSO ALLOWED.*** **SHOW ALL YOUR WORK AND SUBMIT YOUR FULL REPORT WITH CODES TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.**

1. (8+12+8=28) Consider the following Black-Scholes-Merton PDE

$$\frac{\partial G}{\partial t} + (r_d - r_f)S\frac{\partial G}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 G}{\partial S^2} = r_f G$$

where G is the price of a Put Option, r_d and r_f are the risk-free rate of interests of the domestic and a foreign country respectively, and the dynamics of the price process of the corresponding foreign currency (in terms of the domestic currency) $\{S_t\}$ is given by

$$dS_t = (r_d - r_f)S_t dt + \sigma S_t dB_t$$

with $\{B_t\}$ as the standard Brownian motion.

(a) Assume, $r_d = 6\%$, $r_f = 3\%$, $\sigma = 0.25$ per annum, $S_0 = 50$, $K = 50$, $S_{\max} = 100$ and $T = 3$ months. Using implicit difference method, dividing time unit into 30 parts and space unit into 40 parts compute the European and the American Put option prices at time $t = 0$ and $t = 1$ month.

(b) For the same data as in part (a), use the explicit difference method for the transformed B-S-M equation (using $Z = \log S$) to compute the European and the American Put option prices at $t = 0$ for the down and out Barrier at $L = 40$ (i.e., if $S_t < 40$ for some $t > 0$ the option would be invalid), and compare with that of (a).

(c) Use Monte Carlo method to simulate the path of asset for the data given in part (a) above, and find the European and the American Put option prices at $t = 0$, while reducing its variation using (i) antithetic method, and (ii) control variate method. Also provide the corresponding variances.

All the best.