

INDIAN STATISTICAL INSTITUTE

First Semester Examination : 2025-26

Course Name : B.Stat. 3rd Year

Subject Name : Sample Surveys

Date : Nov 27, 2025 Total Marks : 50 Duration : 3 hrs.

Answer Q.4 and any 2 from Q.1 to Q.3.

Symbols and notations are as usual. Only hand calculator is allowed.

1. (a) State the procedures of the cumulative total method and D.B. Lahiri's method of drawing a sample with probability proportional to a known size variable x having $x_i > 0$ for all units $i = 1, \dots, N$ in population U .
- (b) Show with proof how Des Raj's ordered estimator can be employed to estimate unbiasedly the population total Y of a variable of interest y from a PPSWOR sample of size n . Also obtain an unbiased variance estimator of this estimator.
- (c) Define symmetrized Des Raj's estimator. What is its advantage over Des Raj's ordered estimator? Obtain symmetrized Des Raj's estimator for a sample s of size $n = 2$ and show that this estimator is independent of the order of occurrences of the units.

[4+8+3 = 15]

2. (a) State the Horvitz and Thompson (1952)'s (HT) estimator for population total Y of a variable of interest y . Show that it is unbiased for Y . Derive its variance and variance estimator. Mention clearly the necessary conditions that need to be fulfilled.
- (b) Consider a general sampling design that has been planned to provide estimates of population parameters of a big region U having N units. The sampling design has the positive first and second order inclusion probabilities for all units and pair of units in the population. Suppose the overall population can be split into D disjoint domains. It is required later to provide estimates of the population mean $\bar{Y}_d$'s all these domains $d = 1, 2, \dots, D$. Show how to solve this problem and also show how the standard errors can be obtained for these domain estimates.

[10+5 = 15]

3. (a) For estimating the population ratio $R = \frac{Y}{X}$ through SRSWOR, derive Hartley and Ross's unbiased estimator. In addition, obtain an unbiased estimator for Y utilizing Hartley and Ross's ratio estimator.
- (b) Explain with proof the Hansen and Hurwitz's double sampling technique of handling the non-response errors to estimate the population mean $\bar{Y}$ in SRSWOR of size n out of N units. Find the variance of this estimator.

[8 + 7 = 15]

4. For estimating the total yield (Y) of a particular crop in a big area, the entire area was divided into two disjoint strata and within every stratum a **two-stage cluster** sampling scheme was adopted. For two-stage cluster sampling within each stratum, at the 1st stage **4 villages** were selected with **PPSWR**, size measure being the geographical area of that village; then within each selected village, at the 2nd stage **4 plots** were selected by **circular systematic** sampling. For every selected plot, the yield of the crop in kg (y) was collected. The following data gives the collected sample data. Based on this sample data, give an unbiased estimate of the total yield Y in the entire area and obtain its standard error ($\widehat{SE}$). Also obtain the relative standard error ($RSE = \frac{\widehat{SE}}{|\bar{Y}|}$) as a percentage. [20]

No proof is needed. Only write the formulae and the computation results.

Yield of the crop obtained from the sample plots

Stratum	Sample village	Inverse of probability ($1/p_i$)	Total no. of plots	Yield in kg			
				Plot 1	Plot 2	Plot 3	Plot 4
1	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119
2	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	138
	3	24.76	222	264	78	144	56
	4	49.99	69	300	114	68	111