Indian Statistical Institute

Probability Theory I

B.Stat (hons) 1st year,
1st Midsemestral examination

Date: Sep 11 , 2025 Duration: 2hrs.

Attempt all questions. The maximum you can score is 50. Justify all your steps. This is a closed book, closed notes examination. You may use your own calculator.

1. A statistician needs to draw an SRSWOR of size 2 from a population of 6 people (labelled 1 to 6 and standing in a circle). She proceeds in two steps: First, she rolls a fair die and picks one person accordingly. Then she rolls the die once again (independently of course), and if the die shows a different number, she picks that person. But if the die shows the same number, she picks the next guy to the right. Let the labels of the guys thus picked in the two steps be X_1 and X_2, respectively. Then answer the following questions with justification.

   (a) What is $P(X_2 = 3)$?

   (b) Is this a valid scheme for drawing an SRSWOR of size 2?

   (c) Are the events $\{X_1 = 1\}$ and $\{X_2 = 5\}$ independent?

   [5+5+5]

2. I have 6 coins labelled 1 to 6. The i-th coin has $P(H) = p_i$. I pick one of the coins at random, and toss it twice. Given that both the tosses result in the same outcome, what is the chance that I had picked a coin with an even label? [10]

3. A cereal company is giving a free ball with every box. The ball is either red or green or blue or yellow, each being equally likely. A buyer has bought 10 boxes. What is the probability that (s)he has at least one ball of each colour?

   [10]

4. For a group of n people find the expected number of days of the year which are birthdays of exactly two people. (Assume 365 days in a year and that all arrangements are equally probable.)

   [10]

5. If $F(x)$ is the CDF of a random variable X, then show that $\forall a \in \mathbb{R}$ $\lim_{x \to a-} F(x) = P(X < a)$.

   [10]