INDIAN STATISTICAL INSTITUTE
Semester Examination 2024-25

M.Stat. - 2nd Year
Risk Management

28 April, 2025

Maximum Marks: 100 Time: 3 hours

Instructions

- This Question paper has two parts. Part A needs to be solved and submitted within the exam duration. Students may answer as much as they can. The **maximum one can score in Part A is 80**.
- Part B is compulsory which contains 20 marks. Its solution needs to be submitted by email on or before **11.59pm of 26th May, 2025**. Any late submission will get a **zero** in Part B.
- During the exam, students are allowed to use a calculator (that has no internet connection) and any **hand-written notes only**. Photocopies/print-outs of hand-written notes are allowed, but any copy of printed or published materials are not allowed.
- Notations in the questions are as used in the class. Students must clearly state all the results, defining the required notations, that they are using in their answers.

Part A

1. Let $\boldsymbol{X} = (X_1, \ldots, X_d)^T$ and $\boldsymbol{Y} = (Y_1, \ldots, Y_d)^T$ be random vectors having normal variance mixture distributions with the same dispersion matrix Σ and the same location vector $\mu_0 \mathbf{1}_d$ (represents the risk-free return). Vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ represent returns on $2d$ risky assets. Let $V_{\boldsymbol{X}}(\boldsymbol{w})$ and $V_{\boldsymbol{Y}}(\boldsymbol{w})$ denote the values at the end of the investment horizon for an investment of the capital V_0 in a portfolio consisting of the risk-free asset (one unit) and the assets with return vectors $\boldsymbol{X}$ or $\boldsymbol{Y}$, respectively, where $\boldsymbol{w}$ is a vector of monetary portfolio weights corresponding to the positions in the risky assets. Show that if ρ is a translation-invariant and positively homogeneous risk measure, then the ratio

$$\frac{\rho(V_{\boldsymbol{X}}(\boldsymbol{w}) - V_0\mu_0)}{\rho(V_{\boldsymbol{Y}}(\boldsymbol{w}) - V_0\mu_0)}$$

does not depend on the allocation of the initial capital V_0 or on the dispersion matrix Σ.

[10]

2. Let Z denote the daily log return of an asset. Empirical studies suggest that Z has zero mean, standard deviation 0.01, and a symmetric density function. Using some empirical data, an intern has computed $\mathrm{VaR}_{0.99}(Z) = 0.1$. Can you, as a manager of the intern, tell if his/her calculation is right without looking at the data? Justify your answer.

[10]

3. Suppose that (X, Y) follows a 2-dimensional elliptical distribution with location (μ_1, μ_2), scale Σ and characteristic generator $\psi \in \Psi_\infty$ (independent of the dimension), such that $\mathrm{Cov}(X, Y) = 0$. Prove that X and Y are independent if and only if (X, Y) is normally distributed.

[10]

4. Consider a latent variable model for a homogeneous portfolio of n risky loans. Let p be the default probability for each loan, let $X, X_1, \ldots, X_n$ be independent and standard normally distributed, and let $\rho \in (0, 1)$ be another parameter. The default indicators are modeled as

$$Y_k = I\left(\sqrt{\rho}X + \sqrt{1-\rho}X_k \leq \Phi^{-1}(p)\right), \quad k = 1, \ldots, n,$$

where Φ denotes the standard normal distribution function. Define the total number of defaults $M_n = \sum_{k=1}^{n} Y_k$.

(a) Determine the random variable $Q = g(X)$ such that the default indicators are conditionally independent and Bernoulli(q)-distributed given $Q = q$. Is this model exchangeable?

(b) Compute the VaR of Q, and hence the distribution function of Q.

(c) Derive the distribution of M_n.

(d) Device an algorithm for suitably estimating the parameters ρ and p from empirical data, specifying the type of data to be used.

[5+(5+5)+5+10]=30

5. Consider a put spread, i.e., a long positions and a short position in two put options on the same underlying security and the same maturity time (T) but with different strike prices (K_1 and K_2) and different current prices (p_1 and p_2). Assuming that the return on the underlying security is log-normally distributed with parameters μ and σ^2, compute the 95% VaR and 95% Expected Shortfall of the loss incurred by the put spread at the maturity time T.

[10+5] = 15

6. Historical observations of the daily claim amount (in million Rs.) for a type of insurance are collected. A quantile-quantile plot of the logarithm of the daily claim amount (y-axis) versus the quantiles of a standard Exponential distribution (x-axis) is given in Figure 1. Estimate the probability that the aggregated claim amount over 30 days exceeds 60. Assumptions must be clearly stated.

[10]

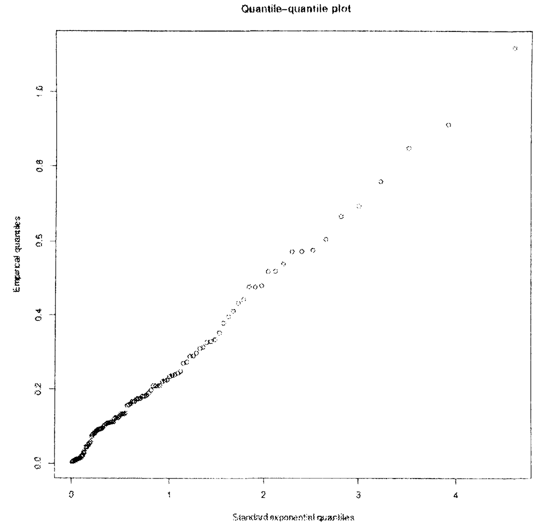

Figure 1: QQ-Plot for Question 6

Part B

Each student is required to select one of the following financial instruments, and write a report on its usage in risk management:

- Barrier Options
- Credit Default Swaps
- Exotic Default Options
- Total Return Swaps
- Credit Linked Notes
- Credit Spread Options
- Mortgage Backed Security (Bonds)
- Collateralized Swap Obligations
- Collateralized Bond Obligations
- Interest Rate Futures

You report should cover an introduction, layman's explanation of the chosen instrument, its mathematical formulation and derivation of its pricing formulas, and its practical applications in risk managements through a real data example. Final conclusion of the report should have student's own critical assessments of the chosen instrument. The computer code and the dataset used for illustrations must also be submitted along with the report. **No two students should choose the same financial instrument.**

[20]