

Indian Statistical Institute
First Semester Examination, 2025-26
Post Graduate Diploma in Business Analytics
Inference

Date: 12/12/2025

Maximum Marks: 50

Duration: 1.5 Hours

Part B

Q1. Suppose you are collecting data about the salaries of the workers in a multinational company. You know that the distribution of salaries for the workers at the company follows a Pareto distribution with the probability density function:

$$f(x; \alpha, \delta) = \frac{\alpha \delta^\alpha}{x^{\alpha+1}}, x \geq \delta$$

with both $\alpha > 0$ and $\delta > 0$ being parameters.

[2+5+8=15]

- Suppose the salaries of n individuals in the company are denoted as $X_1, \dots, X_n$ each having this distribution. Are they independent?
- Write down the joint log-likelihood for the observed data.
- Find suitable estimators for the parameters using the likelihood function, or otherwise.

Q2. On your trip to Goa with friends, you indulge in a betting game at a casino. You can bet either on red or on black. A ball will be dropped on a spinning wheel with equal number of black and red partitions and if the ball ends up on a black partition, then black wins or else red wins. According to the organizers both colours are equally likely to be winners for every spin. But your friend believes that black has a higher chance of winning. To test the claim, you partake in 11 such spins while betting on black. You note if you win or not.

[2+2+(3+3)+2+4+4=20]

- What distribution appropriately represents each of these outcomes? Mention the parameter(s) involved.
- What would be the parameter space Θ for testing the claim?
- What are the two competing hypotheses here in terms of the parameter(s) involved? Are they simple or compound?
- Suppose you take the claim of the organizers to be true and you decide to reject the null hypothesis if the number of times you have won is greater than 8. What does the rejection region look like symbolically? Explain all notations.
- What is the probability of committing a type I error with the above rejection region?
- What does the power function look like? Give the functional form in terms of the parameter(s). No need to draw a curve.

Q3. A cohort of 18 young college students with high blood pressure were part of a study about a blood pressure reducing drug. 9 people were given the actual drug and 9 people received a placebo. Their blood pressures were only recorded after six months of continuously taking the prescribed drugs. Consider the blood pressures for the people in the placebo group to be $Y_{11}, Y_{12}, \dots, Y_{19}$ and those in the treatment group to be $Y_{21}, Y_{22}, \dots, Y_{29}$. You want to test whether the drug works or not. You have $\bar{Y}_1 = 137, \bar{Y}_2 = 107, \hat{\sigma}^2 = 270$.

[5+2+8=15]

- a) Write this down as a one-way layout, explaining all the symbols and suitable assumptions.
- b) What are the competing hypotheses (using your notations)?
- c) Write down the corresponding ANOVA table.