

LIFE TABLE CONSTRUCTION FROM POPULATION AGE-DISTRIBUTIONS
SUFFERING FROM RESPONSE BIASES IN AGE-REPORTING : A NEW
TECHNIQUE (NOT REQUIRING AGE-SMOOTHING) WITH APPLICATION
TO INDIAN CENSUS AGE-RETURNS

by

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is organised into six chapters.

Chapter I deals with the preliminary concepts of life tables, its role in demographic analysis, a brief chronological review of various literatures on the construction of life table starting from John Graunt's investigation on mortality, along with an introduction to the basic problem and objectives of the present study.

Chapter II describes the concept and utility of cumulative census survival ratios, first introduced by Professors Ansley J. Coale and Paul Demeny (1967) as the ratio of the number of persons aged $x+10$ and above enumerated at time $t+10$ to the number of persons aged x and above enumerated at time t . These are compared to the usual census survival ratios with particular reference to the presence of heavy response biases in age-reporting in the censuses of developing countries.

Chapter III is devoted to the development of -
(i) various formulas for estimating 10-year cumulative life table survival ratios from 10-year cumulative census survival ratios under various assumptions regarding the age-structure of the population under study; (ii) an iterative procedure for life table construction from 10-year cumulative life table survival ratios above a certain childhood age; and (iii) a procedure for estimating infant and childhood mortality rates.

Chapter IV provides an empirical validation of the iterative procedure for life table construction and a discussion on the importance of the procedure. A mathematical treatment on convergence of the iteration process is shown in the appendix.

Chapter V illustrates an application of the technique developed in Chapter III with reference to Indian census age-returns and presents the all-India life tables for the decades 1951-61 and 1961-71 after describing the steps taken to control the errors due to age-misreporting.

Chapter VI contains the summary of the findings from this research investigation together with a brief discussions on the directions for future lines of research.

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CHAPTER I

INTRODUCTION AND OBJECTIVES

1.1. Some Preliminary Concepts and Definition of Life Table

A life table literally means summarisation of the life history of a group of persons born together in a systematic tabular form as it diminishes by deaths. Before going to more precise definition of a life table and its various methods of construction, one should have some preliminary ideas behind the creation of such a table in the study of mortality. These are discussed in the following paragraphs.

The urge for survival is a very common and natural instinct among all living beings. Since time immemorial, man is keenly interested to know the maximum possible length of human life span. This interest in longevity which has never been trimmed off in the history of mankind is shown by the frequent references in the daily news to the span of life, and to the tremendous gains which have been made in the average duration of life, particularly during the last century due to the spectacular improvement in

public health measures and medical sciences. It is needless to say that the sure knowledge that after all 'Man is mortal' in no way diminishes the world-wide search for methods of increasing the length of human life as much as possible. Thus, we find that the study of mortality has its own historical importance in the development of mankind. This was perhaps the first subject in the study of human populations brought under systematic and rigorous scientific research and the results have found commercial application in the field of insurance.

No doubt "human life is a very personal affair" (Dublin et al., 1949, p.3). A demographer is not interested in the life pattern of any particular person; rather his/her interest lies in the behaviour of a group or groups of persons who follow on an average a set pattern of mortality from the beginning to the end of life. This set pattern starts with infancy, passes through fairly definite stages, namely, childhood, adolescence, maturity and old age. Ideally, death should take place more or less at the end of this set pattern when 'the biological energy vested in each individual has been spent and the continuing function of the body's vital organs has reduced them to a state of general deterioration.' Under this natural pattern of mortality, 'deaths would tend to accumulate in the terminal period, probably at the ages between 70 and 90 years'(United Nations, 1955). But conditions are rarely ideal and due to accidents or acute illness many lives are nipped in the bud and some are cut short considerably before completing the ideal span of life. In this context, the

following observations made by Dorn are worth noting:

"Each individual begins his separate existence with the fertilisation of the ovum by the sperm cell. Immediately thereafter, lethal factors begin to operate. By the time embryo has developed to the stage when it can exist outside the body of its mother, perhaps as many as one out of every five embryos has died. These are foetal deaths. Of those who survive births, some died almost at once, so ill-prepared are they for a separate independent existence. The duration of life of the remainder varies greatly. Only a few will live to greet the first century after their births." (Dorn, 1959, p.440).

From the foregoing discussions, one may easily conclude that the incidence of death will vary from one age segment of a population to the other. This basic fact of mortality variation from one age segment to the other has to be incorporated in the construction of life table. Population of a community increases in number by births and immigration and decreases by deaths and emigration. In the absence of migration, the age structure of a population will be determined by the fertility as well as mortality schedules prevailing during the period under study (Coale, 1972, pp. 3-15). Any change in the mortality level will have an impact on the whole age span because death can occur at any stage of life; whereas any change in the fertility level during a particular period will have impact only on some very early age sector. Coale found that 'the decline in mortality, though it had nearly half as great an effect on growth as fertility, had a much smaller effect on the age distribution' (Coale, 1956, pp.79-114). The above discussions lead to the conclusion that mortality, as one of the major components of

population change, has an important role not only in determining the growth of population but also in influencing its composition by age, sex, race, nativity, etc. Thus, the main interest of a demographer in the field of mortality lies in its effect upon the size, composition and future growth of population.

Keeping in mind the general pattern of mortality experiences of human lives at various age sectors of the whole life span as discussed in the foregoing paragraphs, a life table may be defined more precisely as a mathematical model that portrays mortality experiences of a cohort or group of persons born at the same time, closed to migration, in terms of probabilities and provides a basis for estimation of longevity at various ages until they all die (Keyfitz, 1977, p.3; Bogue, 1966, p.554). To clear the idea of a life table as defined above, let us consider 100,000 babies born at the same time in a certain community. If a life table is constructed from the life history of these 100,000 babies, one can easily answer the following questions which normally arise to the mind of a common man: (i) How many out of these 100,000 babies will complete their first year of life? (ii) How many will reach age 5, 10, 15, 20 and so on? And when all these 100,000 babies pass through various ages until death, (iii) what would be the average length of life per person?

Life table can be of two types, namely, (i) cohort or generation life table and (ii) current or conventional life table. Strictly speaking, a cohort life table requires the information on

the actual mortality experience of a particular group of persons born at the same time (called the birth cohort) when the cohort is followed up from birth until the death of the last member of the group (Chiang, 1968, pp.189-190; W.H.O., 1977, pp.76-77; Spengler and Duncan, 1956, p.91). From the basic requirements, as stated above, for construction of a cohort life table, it appears that such a life table is quite difficult to generate in practice because the requisite statistics for a period of 100 years or more are rarely available and even if available, are likely to be defective due to various difficulties encountered in keeping track of them throughout their life. Thus such life tables, though useful in studies of animal populations and analysis of patient survival in studies of treatment effectiveness, are rather unpopular for human populations.

The current or conventional life tables which are most popular in demographic analysis give a cross-sectional view of the mortality experience at different ages in a given population, closed to migration, during a specified short period of time, usually not more than ten years. It scans out the life span of each individual of a hypothetical cohort (called radix) on the assumption that an individual at any particular age of his life will be subject to the same mortality as actually experienced by the given population for that age during the specified period and it also provides a basis for measuring what the longevity would be of a new-born infant if the same mortality schedule

could be assumed throughout his life (Chiang, 1968; Lahiri, 1972; W.H.O., 1977). However, the statements, quoted below, made by Campbell in the WHO Manual on Mortality Analysis is worth noting:

"The current life table is thus a fictitious pattern reflecting the mortality experience of a real population during a calendar year. However, it is an effective means of summarizing mortality and survival experience of a population and is a sound basis for statistical inference about the specific population under study for that calendar year"(W.H.O.,1977, p.59).

A lucid discussion on current life tables and the underlying assumptions in the construction of such life tables may be found elsewhere (Barclay, 1958).

There are two usual ways of presenting a life table, namely, complete and abridged. In a complete life table, the life table functions are computed for each year of life; whereas in an abridged life table, the functions are presented for age intervals more than one year in length, excepting possibly the first year or first five years of life. The standard notations of the various life table functions and their inter-relationships may be found in any standard text book on demography or vital statistics.

At the present era, there is a tendency among demographers to evaluate the worth of research in terms of the policy relevance of its results. Preston correctly pointed out that: "This inclination has been accompanied by and is largely responsible for the decline of mortality studies. Mortality is presumed to be

outside the policy sphere, since nations will(or atleast should) never spend less to promote human health and longevity" (Preston, 1976, pp. ix-xi). He further commented that the mortality situation of a community may be considered as a product of social and individual choice made under budget constraints and in the presence of competing alternatives like schools, roads, homes and recreations, etc., which apparently make our life more pleasant and attractive.

However, the importance of life table which is largely a product of actuarial science should not be undermined in any case since its application is not restricted merely to the computation of insurance premium. It has considerable value in terms of policy relevance too in terms of public health programmes. It is a useful analytical tool for demographers, epidemiologists, physicians and research workers in other areas of public health which in turn will help the policy makers to suggest the areas of concern of the governments in shaping their future plan in a fruitful manner as per need and resources of a country. It would not be over-emphasized if we say that the modern developments in demography would have been impossible if the concept of life table was not developed earlier. The need for development planning is particularly felt in less developed countries like India. Incidentally, in many of these countries there is considerable age misreporting which is to be given special attention in the construction of life table.

1.2. Various Measures of Mortality Versus Life Table and Its Role in Demographic Analysis

1.2.1. General mortality

Mortality in a community or a population can be measured in a number of ways. The measures developed so far may be categorised broadly into two groups according to their purposes, namely, (a) quantitative description for examining levels and trends and (b) analytical study of mortality.

A wide variety of measures, which are basically rates and ratios are used for the purpose of quantitative description of mortality situation of a community or a population (B. Benjamin, 1968, pp.92-101; W.H.O., 1977, pp.34-57). The most commonly used and widely available index of mortality is 'crude death rate' which is nothing but the ratio of deaths during a year to total mid-year population. It is the crudest possible but most simple type of rate based on registered deaths. It provides a measure of gross-deletion from the population through death, and is useful for giving a very general idea of short-term mortality trends for a specified area and for classification of countries according to mortality levels. However care must be taken in comparing mortality levels among various subdivisions of a country or different countries if their age-sex compositions are different. The crude death rate combines together various age-sex groups in the form of an average without taking into account the relative size of the groups among which mortality varies

widely (from one age-group to the other). In this context the following observations made by the United Nations are worth noting:

".....some developing countries have attained death rates lower than those now prevailing in industrialised countries. However, if age structure of the population is taken into account the mortality rates of the industrialized countries are, in general, lower. This paradox is explained by the higher proportion of older persons in the populations of the industrialized countries" (United Nations, 1973, p.109).

Further discussions of the effect of age-structure on crude death rate may be found elsewhere (United Nations, 1963). It is important to note that the crude death rate which is independent of the accuracy of age reporting, provides a valuable information of overall mortality situation of the countries where the age-returns are seriously distorted by age-misreporting. It is also useful for detecting yearly or seasonal fluctuations because normally the age-sex composition of the population under study is not expected to change abruptly during such short period of time.

To avoid the difficulty in comparing the mortality situation through crude death rates of two or more populations having different age structure as mentioned in the earlier paragraph, 'age-specific death rates' and 'standardised death rate' are often used for this purpose. Let us first consider the age-specific death rates which are nothing but crude death rates at various age-groups. Undoubtedly, this age-specific rates will provide a better picture of mortality situation of a country compared to that revealed by crude death rate because

(i) these specific rates contain more detailed information on mortality and (ii) risk of death is very much related to age as there is systematic metabolic change of the human body with advancement in age. It is true that the age-specific death rates are not vitiated by the age-structure of a population and they give more insight into the mortality situation of a population for further investigation, but when our aim is to compare the overall mortality situation of two or more populations, then the use of age-specific rates for purpose of comparison is nothing but to beat about the bush since it becomes very complex job for the human mind which has to comprehend the totality of age-specific rates as a single entity. Even if it is possible to compare, it may give some conflicting results. For example, it may happen that for a particular age-sex group the mortality rate of a region 'A' may be greater than that of another region 'B' while for another age group, the reverse may be the case. Thus we find that it is rather difficult to draw any precise conclusion regarding the general mortality situation of 'A' in comparison with 'B'. It is, therefore, necessary to combine the different age-specific mortality rates in such a fashion so that the true picture is revealed and comparison becomes valid. This can be done through a standardization technique in which the same standard weights are used for different countries so as to combine the age-specific death rates of each of the actual populations into a single index. Such indices are unaffected by the peculiarities in the age-compositions of the original populations under comparison (Wolfenden, 1923;

1957 & 1962; Stouffer, 1951; Yerushalmy, 1951; Silcock, 1959; Kitagawa, 1964; Spiegelman, and Marks, 1966). Death rate so obtained is known as 'standardized death rate'. The fundamental concepts of standardization are discussed briefly in the next paragraph.

There are two different approaches for obtaining standardized death rate which are commonly known as "direct" and "indirect" methods of standardization. The direct standardization (explained in the last paragraph) consists of applying the observed age-specific death rates of the given populations to a standard population so as to summarise two or more sets of age-specific rates independently of the age composition of the individual population under study for purpose of making valid comparisons. When the sets of age-specific rates of the populations whose mortalities are to be compared are not available but the total number of deaths occurring in each of the populations during a specified period is available, one may apply the indirect standardization. This approach consists of applying a standard set of age-specific rates to different populations classified by age so as to bring out the relative influence of the age-composition of different populations on their crude death rates. The computational procedures of standardized death rate through direct and indirect approaches may be found in any standard text book on Demography or Vital Statistics. It may be noted here that the selection of a standard population in the standardization technique is rather arbitrary which has considerable influence on

the values of the standardized death rates obtained by applying the technique particularly when the populations being compared have very marked compositional differences with respect to age. There are various other types of mortality indices depending upon the weights used for summarising the mortality experiences at various ages for the purpose of comparing mortality experiences of the two or more populations. A good deal of discussions on various measures of mortality may be found elsewhere (see, Cox, 1966, pp.120-143; Anderson and Daw, 1952, pp.194-206; and Benjamin and Haycock, 1970, pp.1-34).

We have already seen from the earlier paragraph that there are various ways of measuring and comparing mortality situations of various populations. Another way probably the best way of summarising a mortality experience or comparing several mortality experiences is by means of life tables. The life table is one of the most comprehensive expressions of the mortality pattern of a community prevailing at a particular point of time or period. The following statements made by Dublin and Lotka are worth noting, particularly, in the context of the foregoing discussions mentioned in the earlier paragraphs:

"The crude death rate, for well-known reasons, is not a good measure, because it is quite seriously affected by differences in age composition. Standardized death rates, on the other hand, have the disadvantage that they depend on an arbitrarily selected standard population. The life table is free from this arbitrary feature, and of course, with its several columns, exhibiting, for a "cohort" or "generation" traced from birth through life, the number of survivors, the number of deaths, the death rate and the expectation of life at each age; such a table gives much more detailed information than a general death rate, whether crude or standardized" (Dublin and Lotka, 1937, p.43).

The expectation of life at births which is the average number of years of life that a new-born infant is expected to live under the prevailing schedule of age-specific mortality at a particular point or period of time, is independent of the age-structure of the actual population. This is seen from the fact that the fundamental concept of a 'current' life table is to follow the life history of a hypothetical cohort of individuals born at the same time. The expectation of life at birth is considered widely as a superior measure of the overall impact of mortality of a population and it may be used in international and other comparison (Bogue, 1968, p.553; Jordan, 1967, p. 174). It can be easily shown that life table death rate (m_1), defined as the crude death rate in the hypothetical stationary population based upon the life table, is simply the reciprocal of the expectation of life at birth (e_0^0). If 5^Lx and 5^dx are the life table population and life table deaths in the age-interval $(x, x+5)$ respectively then by definition life table death rate (m_1) can be written as -

$$\begin{aligned} m_1 &= \frac{\text{(Total number of deaths in a hypothetical Stationary population)}}{\text{(Total number of persons in the stationary population)}} \\ &= \left(\sum_{x=0}^W 5^dx \right) / \left(\sum_{x=0}^W 5^Lx \right) \dots\dots\dots (1.1) \\ &= l_0/T_0, \text{ since } l_0 = \sum_{x=0}^W 5^dx \text{ and } T_0 = \sum_{x=0}^W 5^Lx. \end{aligned}$$

Now by definition, the expectation of life at birth(e_0^0) can be obtained by the ratio T_0/l_0 . Thus, the above relation

becomes $m_1 = 1/e_0^0$. Now, the numerator of the RHS of (1.1) can be written as -

$$\sum_{x=0}^W 5^d_x = \sum_{x=0}^W 5^L_x \cdot 5^m_x \dots\dots\dots (1.2)$$

where $5^m_x = \frac{5^d_x}{5^L_x}$ is the life table death rate in the age-interval $(x, x+5)$.

Now using (1.1) and (1.2) we get,

$$m_1 = \left(\sum_{x=0}^W 5^L_x \cdot 5^m_x \right) / \sum_{x=0}^W 5^L_x.$$

Thus m_1 is nothing but the weighted average of the schedule of age-specific death rates taking 5^L_x as weights. Whenever it is required to obtain weighted average of a schedule of age-specific mortality rates for the purpose of comparison, the use of 5^L_x values as weights are considered to be ideal for theoretical as well as practical considerations (Bogue, 1968, p.558). This is particularly useful in the case of analysing cause of death statistics. Thus, we see that the life table death rate may be considered as an age-adjusted death rate since the value of ' m_1 ' depends on e_0^0 which is independent of age-structure of the actual population. Thus the life table death rate may also be used in place of the age-standardized death rate for the purpose of international or other comparison. But it does not add any new information compared to that of e_0^0 since m_1 is nothing but reciprocal of e_0^0 . Furthermore, if age-specific death rates of a population are double (say) those of another

population, the relation may not be ^{exactly} reflected in the life table death rate. For these reasons, it is seldom used in practice.

Furthermore, the ratio l_{x+n} / l_x , where l_x is the number of survivors at exact age x out of the initial hypothetical birth cohort l_0 , serves as a useful measure in comparing the chance of survival between two populations belonging to the age interval $(x, x+n)$. For example, we would like to compare the mortality rate of Indian women in childbearing ages (15-45) to that of Ceylonese women in the same age-interval during certain period. For this purpose, we need to have life tables for India and Ceylon during the specified period; and this can be done by comparing the ratios $(1 - l_{45}/l_{15})$ obtained from the corresponding life tables.

1.2.2. Infant mortality

The children below one year of age require special attention from the demographic as well as social point of view, since the wastage of human life during the first year of life is glaringly high, due to various endogenous and exogenous causes (Bourgeois-Pichat, 1952; United Nations, 1954). It is needless to emphasise here the importance of infant mortality in mortality analyses and other allied fields since it has been universally accepted that the incidence of infant mortality can be treated as one of the most sensitive indicators of health and level of living which are heavily influenced by the environmental and socio-economic conditions (Titmuss, 1943; Logan 1953 & 1956;

United Nations, 1961, 1973 and 1980; W.H.O., 1977). However, some deviation with regard to the above fact has been observed from the recent trends of infant mortality in developing countries (United Nations, 1973, pp.123-125).

During the last fifty years, the infant mortality rates (q_0) per 1,000 live births in India have declined sharply from 249 for males and 232 for females in 1921-31 to 130 for males and 123 for females in 1961-71; the child mortality rates in the age interval 1 to 4 (${}_4q_1$) have also declined considerably from 199 to 68 and 182 to 75 for males and females respectively during the aforementioned time span of forty years. (For the estimates of q_0 and ${}_4q_1$ in 1921-31 - see Vaidyanathan, 1960, pp. 366 & 368; for 1961-71 - see Office of the Registrar General of India, 1977a, pp.14-15).

The studies made in other developing countries also show that the infant and child mortality had declined considerably during the last few decades (for discussions see United Nations, 1973, pp.123-128; 1980, pp.92-96). Such a sharp decline in infant and child mortality is mainly due to spectacular improvements in medical sciences with special reference to discoveries of some life saving drugs and various other antibiotics and to some extent, due to public health programmes, such as immunisation programme, etc. It is disheartening to note, in this context, that there was no significant decline in the level of infant mortality during the period 1970-78 in India, - specially in rural areas (Office of the Registrar General of India, 1980, p.33).

Probably, after certain stage of demographic transition, merely intensifying the immunisation programme and use of modern broad spectrum antibiotics and/or any other modern medical technology may not have significant effect in further decline in infant and child mortality without sufficient improvement in public health programmes other than immunisation and socio-economic development of the general mass (Coale and Hoover, 1958, pp.14-15; Dorn, 1959, p.457; United Nations, 1973, pp.152-155; Adlakha and Kirk, 1974, pp.381-400; Visaria and Jain, 1976; Palloni, 1981). It has been reported in one of the recent publications of the Office of the Registrar General of India that ".... mortality among female infants is more than that among male infants, especially in the rural areas. A question may be asked whether the preference for a male child results in female infants not receiving as much care and nourishment as bestowed on the male infants" (Office of the Registrar General of India, 1979, p.33). A good deal of qualitative and quantitative discussions on preference for children of particular sex in India and abroad may be found elsewhere (Lahiri, 1974; 1975; 1977; 1979 & 1980; Mukherjee, 1976; Williamson, 1976 & 1978; Srinivasan et al., 1978; Bhattacharya, 1978; Kabir, 1980; Pullum, 1980).

From the discussions in the foregoing paragraphs, we find that infant and child mortality have considerable correlation with the general health and social conditions of a community. Furthermore, the expectation of life at birth, which is often used for international comparison of mortality level, is a weighted average of

age-specific death rates as shown earlier and hence the life expectancy at birth is highly influenced by the death rates in infancy and childhood periods where the incidence of deaths are quite high compared to other ages. Furthermore, it may be noted that "a death at say age 65 represents the loss of few years of expected life, a death in infancy may entail a loss of life perhaps 60 years or more" (Logan, 1953, p.30). Thus, special attention is necessary in estimating child mortality, particularly that of infants while constructing life table of any population.

The term 'infant mortality rate', in true sense, refers to the probability that a child born will die before the completion of one year of age. The ideal way of estimating this probability, that is, infant mortality rate, is to follow up a large number of children born alive at a point of time for exactly one year and to record the number of death during this one year period; and then the infant mortality rate may be obtained by dividing these deaths by the initial number of children born alive. Since in actual practice, this is rarely possible, conventionally the infant mortality rate is defined as the number of infant deaths (D_o^Z) occurring in a community during a calendar year (Z) per 1000 live births (B^Z) during the same calendar year (Z). Symbolically,

$$IMR = \frac{D_o^Z}{B^Z} \times 1000 \dots\dots\dots (1.3)$$

It may be noted that the conventional infant mortality rate defined by the R.H.S. of (1.3) is not a probability rate ^{1/} as used in life table since some of the infant deaths D_0^Z during the calendar year Z may ^{relate to the infants} born in the preceding calendar year $(Z-1)$; similarly, some of the deaths below one year ^{of} age out of the total number of births (B^Z) during the calendar year Z may occur during the following calendar year $(Z+1)$ (Logan, 1953; Spiegelman, 1970). However, if the yearly fluctuation of births is not significant, then the conventional rate is expected to give sufficiently reliable measure of infant mortality rate. To distinguish between the 'conventional infant mortality rate' and 'life table infant mortality rate', the former may be called 'infant death rate' (Barclay, 1958). For obtaining an estimate of the true probability rate, that is, the life table infant mortality rate, the time-interval between births and deaths should be adjusted in such a fashion that they correspond to each other. The details of matching between the live births during a specified period and infant deaths may be found elsewhere (Logan, 1953; Spiegelman, 1953; Salinas, 1953).

From the foregoing discussions on various mortality indices (including that of infant mortality) and measures based on life table, it appears that the life table technique is the most useful method in summarising mortality data and to make valid comparisons between various populations as compared to other available measures of mortality provided, of course, fairly

^{1/} Henceforth this probability rate will be called as 'life table infant mortality rate'.

reliable life tables are available for the populations under study during the specified period.

There are several uses of life table in demographic analysis. Apart from detailed mortality investigation as discussed in the earlier paragraphs, life table is mostly used for estimating population of a community at some future date, its composition and distribution. Life table is essential for computing measures of population growth, such as net reproduction rate, intrinsic rate of natural increase, replacement index. Furthermore, the life table technique is extensively used in the studies of fertility, migration, evaluation of family planning programmes, analysis of cause of death statistics and solving various problems in public health. In other words, any analytical approach of the study of population requires the help of life table. A good deal of discussions on various application of life tables in demographic analysis may be found elsewhere (Dublin et al., 1949, pp.236-283; Shryock and Siegel, 1973, pp.449-459).

1.3. A Brief Chronological Review of Various Literatures on Construction of Life Table

1.3.1. A brief history of life table

The study of mortality which is the oldest subject as mentioned earlier in demography was first brought under systematic and rigorous analysis through the concept of life table. Truly speaking, most of the techniques for analytical study of human population are in some way or other related to life table.

It is not exactly known when and by whom the concept of life table was conceived for the first time. However, from various literatures on mortality investigation and other allied topics it appears that probably the first rudimentary life table was prepared by a Roman author, Ulpian sometime during the third century A.D. which apparently was used to determine the provisions of life annuity contracts (Trenerry, 1926; Dublin et al., 1949; Dorn, 1959). But several demographers are of the opinion that the harbinger of life table was John Graunt, haberdasher at London in the seventeenth century, who used to study in his spare moments, the bills of mortality during the plague periods in London (Dublin et al., 1949; Grebrik, 1959; Lorimer, 1959; Thomlinson, 1965; Benjamin and Haycocks, 1970; W.H.O., 1977). Graunt culled an enormous amount of interesting and enlightening information, while analysing, with great care for several years, the weekly returns of christenings and burials in the city of London which he presented in his remarkable book entitled "Natural and Political Observation mentioned in a Following Index and Made upon the Bills of Mortality" published in 1662 in London (Graunt, 1662). This book was highly appreciated by the members of Royal Society who brought Graunt's contribution to the scientific world (Lorimer, 1959, p.127.)

The basic idea which Graunt presented in his book of what later became popular as life table, was of a group of births followed through life and gradually reduced in number by deaths. However, it may be noted in this context that the reduction by death was not so gradual in those days; around

two-fifths were dead before attaining age six. The rough outline of a life table presented by Graunt as shown in Table 1.1 should be treated only as an exceedingly crude approximation ~~made~~ on the basis of the bills of mortality in London which were rather imperfect in nature.

According to Benjamin and Haycocks (1970, p.385) "....This was not a life table in the proper sense of the term and it was not correctly calculated, but it represented a tremendous leap forward from the simple death rate to a new and graphic method of representing the age pattern of mortality".

Table 1.1

Graunt's Table of Survivors Based on Bill of
Mortality in London

Age in years	Number Surviving	Age in years	Number Surviving
0	100	46	10
6	64	56	6
16	40	66	3
26	25	76	1
36	16	86	0

Source: Harold Westergaard, Contributions to the History of Statistics, P.S. King & Sons, London, 1932, p.22

It may be noted that while obtaining the number survivors out of the cohort of 100 birth at various ages as shown in Table 1.1, Graunt probably pre-supposed that the population under study was stationary. Though Graunt's data are too rough and too lacking in detail, yet in the absence of

better data an approximate computation shows that the average length of life was around 18.2 years (not supplied by Graunt) (Dublin et al., 1949, p.33). It is interesting to note that Graunt's data which were unquestionably faulty due to various types of error involved in the data attracted the attention of several researchers in other countries and his table was used for illustrative purpose of human mortality pattern even more than a century after the publication of his book (Westergaard, 1932; Dublin et al., 1949).

Later on, many academicians and Government officials were attracted by the John Graunt's remarkable work on the basis of bills of mortality in London and the life table gradually came to occupy the key position in population studies as a device of description and as an analytical tool (Coale and Demeny, 1966, p.6). The next important advancement in the history of life table was made by Graunt's successor, Edmund Halley (1656-1742), a member of the Royal Society in London, who laid the corner stone of life insurance business and constructed the first empirical life table, about thirty years after Graunt's famous publication, based on births and age-specific deaths collected from the church records of Breslau, Silesia. Halley's valuable work was first published by the Royal Society, London, in 1693 and later on reprinted by John Hopkins Press, Baltimore, in 1942 [Halley, 1693]. According to Dublin and others "..... Graunt was a layman, who, by exceptional talent was able to do productive works. Halley was the first more definitely

scientific worker in this field" (Dublin et al., 1949, pp.33-34). Halley's life table in which males and females were not distinguished was constructed on the basis of deaths alone under the assumption that the population of Breslau was stationary. However it was pointed out by Greenwood (1948) that Halley realised that the assumption made by him, mentioned just above, for the construction of life table, was not proper and to satisfy himself he made some correction for the excess of births over deaths. But the modification made by him was not sufficient to make the life table totally valid from theoretical as well as practical aspects.

It may be noted that Graunt could not obtain the expected number of persons by ages from his data. Nevertheless, Halley's life table has considerable historical importance not only as the earliest document of this kind, but also as a remarkable technical work on the construction of a life table from fairly current data, under the direction of a competent academic scholar. A summary of the Halley's life table is presented in the table 1.2 .

Among various other mortality investigations after Halley, the historically notable life tables are the French Tables (Walford, 1876, p.321), prepared by Deparcieux, Buffon, Mourgue and Duvillard, the Northampton Table (Sutton, 1874, p.107) (England), published by Dr. Richard Price in 1783, that of Wigglesworth Table (Wigglesworth, 1793, p.131) for Massachusetts

Table 1.2

Life Expectancies at Various Ages
According to Halley's Life Table*

Age in years	Expectation of Life in Years	Age in years	Expectation of Life in Years
0	33.50	45	19.22 .
5	41.55	50	16.81
10	39.99	55	14.51
15	36.86	60	12.09
20	33.61	65	9.73
25	30.38	70	7.53
30	27.35	75	5.99
35	24.51	80	5.74
40	21.78		

*Based on mortality experience in the city of Breslau during the period 1687 to 1691.

Source: Philosophical Transaction, Royal Society of London, Vol.17, 1693, p.596; R. Pearl, The Biology of Death, J.B.Lippincott Co., Philadelphia, 1927, p.83; See also Louis I. Dublin and others, Length of Life: A Study of the Life Table, (Revised Edition), p.34.

and New Hampshire for various periods of years before 1789 published in 1793 and that of a part of Philadelphia, U.S.A. by William Barton (1793, p.56), based on the mortality experience in the years 1782, 1788, 1789 and 1790.

Dr. Price's Northampton Table published in 1873, which was based on the death returns for a parish in Northampton during 1735-80 was the first life table used for life insurance. While constructing the Northampton Table (1735-80), Price, initially, overestimated the mortality and thereby underestimated the expectation of life as he was overlooked the fact that there were large number of Baptists living in Northampton whose burial

took place in churchyard but their baptisms were not registered at birth. However, this error was corrected by him later on. According to the Northampton Table, as originally constructed, the life expectancy at birth was around 25 years which became 30 years after the correction. Unfortunately, the then Government had to bear heavy loss in its annuity scheme before the error was corrected (Dublin et al., 1949, p.35; Benjamin, 1968, p.105; W.H.O., 1977, p.79). Dr. Price also constructed a life table from the data on population and deaths by age in Sweden, which was the first national life table ever made and redounds much more to his fame than the Northampton Table (Benjamin, 1968, p.105).

The investigation on life table technique through appropriate scientific principles taking into consideration both deaths and the population exposed to risk of death classified by age, was carried out, for the first time in the history of life table, by Milne in 1815 on the basis of the mortality experience of two parishes in Carlisle (England) during the period 1779-1787 (Sutton, 1883, p.110; King, 1883; p.186). The expectation of life at births of the general population in Carlisle during 1779-1787 was 38.72 years (Sutton, 1883, p.118), which is close to that of India for the decade 1951-61 estimated by Coale and Hoover (1958), Immerwarh and Sinha (1971) and Lahiri (1972). However, later on, life table became intimately bound up with the analysis of stable and stationary (with fixed total size) populations as a by-product of the valuable work carried out in 1760 by Leonhard Euler, a Swiss mathematician on the basic

principles of the stable population - a population with fixed mortality, fertility, and age-sex distribution [Euler, 1970, (1760)]. Considerable sophistication in life table techniques were brought after the noteworthy studies made by Lotka (1907a, 1907b, 1922; Sharpe and Lotka, 1911; Dublin and Lotka, 1925; Lotka, 1931).

The various methods, developed so far, for the construction of life tables^{2/} can be divided into the following three broad categories:

- a) Life table based on death records only,
- b) Life table based on death records as well as census returns, and
- c) Life table based on census returns only.

Enormous amount of research has already been done in the techniques of construction of life table. It is not possible to make a very exhaustive review of all the literatures in this field. However, an attempt has been made here to summarise the historically important and recently devised techniques on life table construction.

1.3.2. Life table based on death records only

If the population under study can be considered as a stationary one, and the ages at death occurring in the population during a specified period are known, then the deaths at age x provides the fundamental information on the life table

^{2/} Here we are discussing, only the 'current or conventional life tables'.

deaths (d_x) at age x by using the following formula:

$$l_x = \sum_{t=x}^{\infty} d_t \dots\dots\dots (1.4)$$

and thus the l_x column of the life^{table} can be obtained by using the formula (1.4) and p_x , the probability of survival from the exact age x to the exact age $(x+1)$ can be estimated by the following formula:

$$p_x = \frac{l_x + 1}{l_x} \dots\dots\dots (1.5)$$

and hence, the other columns of the life table can be constructed by usual procedure (Spiegelman, 1970; W.H.O., 1977; Misra, 1980).

It may be noted, in this connection, that the life tables prior to the Carlisle Table (1779-87) constructed by Milne were based on the assumption of stationariness of the population. In modern times, this method has been abandoned for the purpose of constructing life tables of human population since the underlying assumptions of a stationary population can seldom be justified.^{3/} However, if the population under study can be considered as approximately stable,^{4/} Preston and others (1980) showed that it is possible to construct a reasonably good life table from the distribution of deaths even if the under-registration of deaths is

^{3/} The archaeologists or zoologists are using this technique to find the age at death of individuals from the skeletons of men or animals of which population cannot be known, but may be assumed as stationary (See, W.H.O., 1977, p.79; Spiegelman, 1970, pp.139-140).

^{4/} A population is said to be stable if its age-sex distribution alongwith the mortality and fertility schedules acting on it remain steady over time. This concept and the mathematical treatment of it are fundamentally due to Lotka (1931).

high; provided, of course, the completeness of registration remains almost the same at different ages beyond the earliest years of life and rate of growth (r) of the population under study is known. But the estimation of life table functions during infancy and childhood ages have to be treated separately. These are discussed in Section 1.3.5.

Now, we shall discuss some common but important techniques for construction of life tables under the second and third categories with more emphasis on the third since the present thesis is related to the area where death statistics are very poor in quality.

1.3.3. Life tables based on death records as well as census age returns

1.3.3(a) Death records and census age returns are both sufficiently reliable

The methods belonging to this category are applicable to the areas where the vital registration statistics and age-sex distribution of the population under study are fairly reliable. In this case, an abridged life table is based on the age-specific death rates (5^M_x) observed in the population under study. The age-specific death rates (5^M_x) may be obtained as follows:

$$5^M_x = \frac{5^D_x}{5^P_x} \dots\dots\dots (1.6)$$

where 5^D_x is the average annual number of registered deaths in the age group $(x, x+4)$ during the specified one-year

period under study and $5\bar{P}_x$ is the mid-year population in the age group $(x, x+4)$

Now, a 'current or conventional life table' may be constructed by following a closed cohort of persons^{5/} (male/female or both) born together, at various ages under the assumption that the people belonging to the cohort will be dying according to the mortality schedule, defined by l_x which is fixed in advance and does not change. Before going to discuss about various methods of construction, we should know the answer to two more questions without which one cannot proceed. These questions are stated below:

1. How can the life table age-specific death rates be related to actual recorded experience?

and

2. How can we follow up a particular cohort throughout its life time because the available mortality experience which we have is nothing but a snap shot of a particular moment or at the most for a very short period of time compared to the whole life span?

The first problem may be solved by assuming that the age-specific death rates in the life table population ($5^m x$) is the same as the age-specific death rates in the actual population ($5^M x$). That is symbolically -

$$5^m x \doteq 5^M x \dots\dots\dots (1.7)$$

^{5/} By a closed cohort of persons, we mean here a group of persons born together whose membership remains unchanged due to migration, ~~and~~ ^{and} ~~if any~~ ^{if any}, excepting the losses due to death. The cohort, called the radix of a life table, usually begins with some standard number of births, such as 1,000, 10,000 or 100,000. However, it may be noted that the cohort may start from any age.

However, to construct an abridged life table, we need to know the age-specific mortality rates ${}_5q_x$ which may be obtained from age-specific death rates (${}_5m_x$) under various assumptions leading to different formulas for obtaining ${}_5q_x$ from ${}_5m_x$.

To answer the second question, one may assume that mortality rates at each age even if they relate to the specific year or period, may be applied one after another to form the survival history of a hypothetical cohort. In General, life tables do not represent the life history of an actual cohort. Instead, a life history is generated synthetically from mortality rates by age during a specific year or short period of time and thus represents short segments of the experience of many cohorts.

There are various methods of constructing abridged life tables from death and population records by age which essentially makes use of some reasonable relationships between ${}_5m_x$ and ${}_5q_x$. In the simple case, if it is assumed that the deaths are evenly distributed in each quinquennial age group excepting the first age group (0, 4), it can be easily shown that the age-specific mortality rates (${}_5q_x$) can be estimated from the age-specific death rates (${}_5m_x$) by using the following relationship (Barclay, 1958; Spiegelman, 1970; W.H.O., 1977):

$${}_5q_x = \frac{10 \cdot {}_5m_x}{2 + 5 \cdot {}_5m_x} \dots\dots\dots (1.8)$$

$${}_5p_x = \frac{2 - 5 \cdot {}_5m_x}{2 + 5 \cdot {}_5m_x} \dots\dots\dots (1.9)$$

6/ The term 'age-specific mortality rate' means the probability of dying in certain quinquennial age-group.

Applying the relationship (1.9), one may easily construct an abridged life table (Barclay, 1958; W.H.O, 1977). The Registrar General of England & Wales has adopted the above relationship for the construction of abridged life tables (W.H.O., 1977; p.69; Benjamin, 1968, p.112). However, there are many other ways of constructing abridged life tables from the registered deaths and population statistics - among them the methods suggested by King (1914), Snow (1914), Reed & Morrell (1939), Greville (1943), are quite popular. These methods are well-discussed and brief descriptions of these methods may be found elsewhere (Benjamin, 1968; Spiegelman, 1970; W.H.O., 1977). In another attempt, Souto and Somoza proposed a technique in 1953 for construction of abridged life table from μ_x , the instantaneous death rate at age x , which was estimated by interpolating the value of ${}_nM_x$ for $n = 0$ from several values of ${}_nM_x$ considering the interval 'n' as a variable, through a third degree polynomial (Souto and Somoza, 1953; Arriaga, 1968; Myint, 1982). The recent developments in the life table techniques made by Chiang (1960, 1968), Sirken (1964), Keyfitz (1966, 1968, 1975 and 1977), Keyfitz and Frauenthal (1975), and Mitra (1974, 1977 and 1979) are noteworthy. In each of the above methods suggested by these authors, excepting that of Souto and Somoza, a direct relationship between ${}_5q_x$ and ${}_5M_x$ was established. However, Keyfitz (1977) disagreed with the assumption of equality between ${}_5M_x$ and ${}_5m_x$ and therefore he made necessary modification and obtained a relationship between them under the assumption of "local stability" of the age-structure in an increasing population. The introduction of the concept of the mean

proportion (${}_5a_x$) of the last year of life lived, by Chiang (1968) to find a functional relationship between ${}_5q_x$ and ${}_5M_x$ for the purpose of constructing abridged life table is also worthy of attention. Estimation of life table functions from life expectancies by Mitra (1974) is also interesting to note.

1.3.3(b) Death records are incomplete but census age returns are sufficiently reliable

The methods discussed above are applicable to those areas having quite good death records and the population statistics by age of the areas are also sufficiently reliable. But there are some countries where the death registration is rather incomplete but the census age returns are reasonably good. For such countries, one can also use the standard procedure for constructing life table based on death records and census age returns provided the death statistics by age could be adjusted for under-registration. Courbage and Fargues (1979) developed a technique for obtaining age-specific death rates from incomplete vital statistics under the assumption - (i) rate of under-registration of deaths is independent of age above certain initial age considered and (ii) the age-structure of deaths of a population under study is similar to a set of standard age-specific death rates which is very flexible according to the opinion of the authors. This method does not require adjustment of the basic age data even if it is affected, not seriously of course, by age-misreporting (Courbage and Fargues, 1979; Murthy, 1980). Various methods have also been developed in the recent past for correcting death registration statistics for under-reporting. The following methods are worth noting: (i) Brass's method (1975) of sectional growth balance equation; (ii) methods

devised by Preston and his colleagues (Preston et al., 1979; Preston and Hill, 1980; Preston et al., 1980); (iii) the technique of forward projections; (iv) an inter-censal cohort survival method developed by Preston and Hill (1980) and elaborated by Brass (1979b) and Trussell and Menken (1979); (v) a modified growth balance method by Martin (1980) and (vi) a modification of the method discussed in Preston et al., (1980) by Bennett and Horiuchi (1981).

It may be noted that the above-mentioned methods including that of Courbage and Fargues, provide estimates of the completeness of death registration relative to that of census enumeration and are based on the following assumptions (a) the population under study is closed; (b) the completeness of death registration is invariant over age above certain initial age considered; and (c) age reportings in census and death registration are sufficiently accurate.^{7/}

Furthermore, the assumption of stability is needed for the methods (i) and (ii) whereas the other methods including that proposed by Courbage and Fargues do not require this assumption. One of the major drawbacks of these methods is that they are unable to provide information regarding completeness of death registration below the initial age considered particularly for infants among whom the proportion of under-registration of death is considerably high, if not the highest. Moreover, the assumption

^{7/} Courbage and Fargues claim that their method is also to the population affected by the error of age-misprovided it is not of a serious order (see Courbage 1979, p.167).

constancy of rate of completeness of death registration beyond the initial age which is the crucial assumption, needs more careful and detailed investigation. It is worthwhile to mention here that the completeness could be associated with the socio-economic status of the families of the dead and that there exist socio-economic differentials in mortality. Furthermore, there could be strong rural-urban differential in completeness of death registration. There are examples that death registration in some countries are almost complete in urban areas whereas it is very incomplete in rural areas. Under the situations, mentioned above, the assumption of approximately constant rate of under-registration of death over the ages may be seriously violated (Preston et al., 1980, and Bennet and Horiuchi, 1981).

1.3.4. Life table based on census returns only:

The methods belonging to this category are usually used for the areas where the death registration statistics are either non-existent or very poor in quality, if existing; but the age-distribution of the population under study obtained through census or survey, is fairly reliable. Various methods based on census age returns are briefly described in the following lines:

1.3.4(a) Life table based on age returns of two consecutive censuses:

In this case, the method of construction is based, in principle, on tracing cohorts living at the time of any census date to their survivor at the time of the next census. The

underlying logic behind the method is as follows:

Let,

P_x^Z be the population at age x on the census date Z ,

P_{x+n}^{Z+n} be the population at age $x+n$ on the next census taken after n years of the first census date Z .

Then, under the assumption of closed population, that is, no migration (in or out) during the intercensal period, the population (P_{x+n}^{Z+n}) at age $x+n$ on the census date ($Z+n$) may be considered as the survivors of the population (P_x^Z) at age x on the first census date (Z), provided the two censuses are exact in terms of coverage of the population under study and age-recording or P_x^Z and P_{x+n}^{Z+n} are subject to same degree of error. Consequently, the n -year census survival ratio (${}_nS_x$) can be obtained by dividing the population at age $x+n$ in the latter census (P_{x+n}^{Z+n}) to the population at age x in the earlier census (P_x^Z). Algebraically,

$${}_nS_x = \frac{P_{x+n}^{Z+n}}{P_x^Z}$$

Now, the age-specific mortality rates (q_x) during the intercensal period ($Z, Z+n$) are derived from the n -year census survival ratios (${}_nS_x$) under the assumption that there is a common mortality scale applicable to all cohorts passing through the intercensal period. More specifically, persons, who arrive at age x on any time during the intercensal period, are subject to the mortality rate at age x that is, q_x according to their common mortality scale. Thus, the ~~enumerated~~ population on the

first census date moves on to the population at ages n and above on the next census date, under the operation of this common mortality scale, provided there is no migrations during the intercensal period. These common age-specific mortality rates which are to be estimated by certain procedure, can be used to obtain various functions of the desired life table, such as L_x which stands for the number of persons living between the exact ages x and $x+1$ at any time in a hypothetical stationary population, subject to those common rates of mortality acting on the actual population under study. Thus, under the assumption that the population under study will remain approximately stationary or stable during the intercensal period, then the following relation^{8/} holds true:

$$\frac{P_{x+n}^{Z+n}}{P_x^Z} \div \frac{L_{x+n}}{L_x} \dots\dots\dots (1.10)$$

A good deal of discussion on the rational of the methodology may be found elsewhere (Wolfenden, 1954, pp.115-117; Office of the Registrar General of India, 1960). The basic principle discussed above has been used for constructing life tables of various developing countries, such as Egypt (Kisor, 1944); Brazil (Mortara, 1949); Western Samoa (Raghavachari, 1974; Huagututia, 1979); other Latin American countries (Somoza, 1965) and India^{9/} (Davis, 1951; Office of the Registrar General of India,

^{8/} The proof of this relation under the assumption of stationarity and stability may be found in Chapter III.

^{9/} In India, this approach has been used by the Census Actuaries since 1881.

1960, n.d., 1977a; Lahiri, 1972; Singh, 1981). Usually, in most of the countries, the population census takes place at an interval of five or ten years, that is 'n' is either 5 or 10.

(i) A brief outline of the methodology adopted by the census actuaries of India:

In developing countries, like India where the population census takes place every ten years, the relationship (1.10) becomes

$$P_{x+10}^{Z+10} / P_x^Z = L_{x+10} / L_x$$

Using the usual approximation $L_x \doteq l_{x+\frac{1}{2}}$, where l_x

denotes the life table number of survivors at exact age x , then,

$$P_{x+10}^{Z+10} / P_x^Z \doteq \frac{l_{x+10+\frac{1}{2}}}{l_{x+\frac{1}{2}}} \doteq {}_{10}p_{x+\frac{1}{2}} \dots\dots\dots (1.11)$$

which is the probability that a person exactly aged $x+\frac{1}{2}$ will survive for 10 years. This 10-year survival probability may be used to estimate 5-year and one-year survival probabilities (Office of the Registrar General of India, 1954a; Kohli, 1977). Strictly speaking, for age below 10 the above procedure cannot provide the age-specific mortality rates (${}_5q_x$ or q_x) which reflects the average experience during the whole intercensal period. However, for all practical purposes, the above procedure may be used for estimating mortality rates at ages 5 and above; because the mortality rates q_5 , q_6 , q_7 , q_8 and q_9 obtained through the above procedure by comparing the two consecutive decennial census age returns represent the average mortality experience during the period of 5 years, 6 years, 7 years, 8 years and 9 years respectively before the date of the second census. Thus the mortality rates at ages 5 to 9 do not represent overall rates of

mortality prevailing during the entire intercensal period, but are based on the actual mortality experiences of only in certain portions of the decade (Z, Z+10) (Office of the Registrar General of India, 1954a).

The above procedure, however, does not yield infant and childhood mortality rates. The estimation of these rates should be treated separately since the nature of the mortality curve during infancy and early childhood is quite different from that in other stages of human life. In the absence of reliable information on death records by age, various indirect techniques^{10/} have been developed for estimating infant and childhood mortality rates from the data on children ever born and living. A brief review of these techniques may be found later on. However, while constructing life tables for India, the census actuaries first of all, estimate the l_x function of the relevant life table from age 5 onwards on the basis of the mortality rates estimated for the ages 5 and above. The values of l_x below age 5 were obtained by extrapolating the l_x - curve by fitting a curve of the following form:

$$l_x = A + Hx + BC^x + \frac{m}{nx+1}$$

where A, H, B, C, m and n are constants. The details of estimation of the constants may be found elsewhere (Office of the Registrar

^{10/} In this thesis, a new technique for estimating infant and childhood mortality has been developed, based on the population in childhood ages (excluding the data for infant and early childhood ages), overall birth rate, natural rate of growth and survival probabilities during childhood period. Details may be found in Chapter III.

General of India, 1954a; pp.2-3 & pp.18-21). Actual investigation shows that the value of B is a positive large number and C is a positive fraction and thus the term BC^x diminishes in geometric progression with increase in age, which shows that the function, of C is to regulate the decrease in the extra childhood mortality as age increases (Office of the Registrar General of India, 1954a). The term $\frac{m}{nx+1}$, which is introduced here so as to ensure that a heavy infant mortality, which is characteristic of Indian experience, should become negligibly small when x reaches age 5 and this objective is served, if 'n' is taken as 20 (Office of the Registrar General of India, 1954a). Various attempts have also been made by other authors to estimate by other means the values of l_x below age 5 (Lahiri, 1972; Sinha & Lahiri, 1976; Kohli, 1977; Singh, 1981).

(ii) Other methods of estimation of mortality from census survival ratios:

(a) Coale-Demeny Approach:

Let,

${}_5P_x^Z$ denote the population in the age group(x, x+4)

according to census date Z;

${}_5P_x^{Z+n}$ denote the population in the age group(x, x+4)

according to the next census taken after n-years of the earlier census, where n is either 5 or 10.

In the presence of age mis-reporting in the census age returns, Coale and Demeny have suggested to consider a different type of census survival ratios based on cumulated populations,

such as populations at ages 0 and above, (that is, total populations), 5 and above, 10 and above, etc., instead of the usual survival ratios $\frac{P_{x+n}^{Z+n}}{P_x^Z}$. They found that these new sets of survival ratios, 11/ that is, $\frac{P_{(x+n)+}^{Z+n}}{P_{x+}^Z}$, run more smoothly than the usual survival ratios. Realising the difficulty in translating these cumulated survival ratios into mortality rates since these values are highly dependent not only on mortality at various ages, but also on the age-distribution of the population in question, Coale and Demeny proposed to project the initial population on the Census date Z to the next Census date by applying the survival ratios of the model life tables prepared by them at various levels. From each set of projected population and the initial population at various ages, one can easily obtain n-year cumulative survival ratios provided n is either 5 or 10. By comparing the observed cumulative survival ratios with those obtained by projections with various model life tables one can easily obtain a series of estimates of the level of mortality (e_0^0) through interpolation between two levels, if necessary. They suggest that the medium level among the series of the estimates of the level of mortality (e_0^0) obtained by the above procedure may be considered as an estimate of the true mortality level (e_0^0) of the population under study (United Nations, 1967, pp.57-61). The life table corresponding to this level may be considered as an approximate life table of the relevant population, provided of course, that the pattern of mortality particularly during infancy and childhood period

11/ Henceforth these new sets of survival ratios will be called n-year cumulative survival ratios.

characterising the family of life table is very close to the average experience of the given population during the period under study.^{12/} It may be noted that in this procedure, an attempt has been made to locate a suitable life table age 5 onwards from the system of Coale-Demeny's model tables such that the cumulative survival ratios obtained from the estimated life table beginning with age 5 become consistent with the observed cumulative survival ratios but not necessarily consistent with the age-specific mortality rates (${}_5q_x$) which is actually prevailing in the population in question during the period under study.^{13/} It can be shown empirically that different combinations of the ${}_5q_x$ values may lead exactly to the same life table (5-year) cumulative survival ratios^{14/} (T_{x+5} / T_x).

(b) Brass technique:

Brass (1975) proposed a technique of estimating life table functions by using the census survival ratios (${}_5P_{x+n}^{Z+n} / {}_5P_x^Z$) from a priori knowledge of ${}_5L_0$. If the information on number of

^{12/} It may be noted here that the technique proposed by Coale and Demeny is not meant for construction of life table but for estimating overall death rate (strictly speaking death rate at ages 5 and above) and hence the birth rate by adding the intercensal rate of growth to the estimated death rate. However, the uncertainty regarding level of infant and child mortality may affect the estimated death and birth rates. For details see the comments made by Coale and Demeny (United Nations, 1967).

^{13/} Using the concept of cumulative survival ratios as suggested by Coale and Demeny, a new technique has been developed in this thesis for construction of life table which is not only consistent with the observed cumulative survival ratios, but also consistent with the age-specific mortality rates (${}_5q_x$) to which population is exposed. For details see Chapter III.

^{14/} For details, see the empirical validation of the proposed technique in Chapter IV.

children ever born and surviving by age of mother are available, one may easily obtain an estimate of ${}_5q_0$, the probability of dying from birth to age 5 through Brass technique of estimating mortality from the proportions surviving among children ever born (Brass and Coale, 1968, pp.88-150 ; United Nations, 1967, pp.34-36; Brass, 1975); and this ${}_5q_0$ can be used to estimate ${}_5L_0$ ^{15/}. The irregularities found in survival ratios due to age-misreporting will naturally be reflected in the values of ${}_5L_x$ which can be adjusted with the use of logit system. He also suggested a procedure for estimating ${}_5L_x$ function when the intercensal period is not a multiple of 5 (Brass, 1975).

(iii) Other methods:

There are several ways of constructing life tables. But it is not possible to mention them all here since it is not the main purpose of the present investigation. However, there are some important techniques which are briefly described in the following lines.

Arriaga (1966) proposed a technique for constructing life table based on the age distribution of a population which may be assumed to be almost stable. The basic equation based on the

^{15/} Alternately, an estimate of the survival ratio from birth to the age group (0,4) may be obtained from the births during the five-year period prior to the second census. These births may be roughly estimated from the knowledge of birth rate during the intercensal period. This survival ratio from birth to the age group (0, 4) may be used to obtain ${}_5L_0$.

theory of stable population, for the construction of life table proposed by Arriaga is -

$${}_5L_x = \frac{1}{b} c(x, x+5) \cdot e^{r(x+2.5)}$$

where,

$c(x, x+5)$ is the proportion of persons in the age interval $(x, x + 5)$,

'b' is the intrinsic births rate in the population,

'r' is the intrinsic growth rate of the population,

${}_5L_x$ is the total number of persons in the age interval $(x, x+5)$ in a life table with radix unity.

Obtaining the estimate of r at census date from intercensal geometric growth rate and other existing estimates of r, the smoothed values of $c(x, x+5)$ and b were estimated by fitting the following linear equation:

$$\log_e \left[\frac{c(x, x+5)}{{}_5L_x} \right] = \log_e b - r(x+2.5) \dots (1.12)$$

In the above equation (1.12), the values of $c(x, x+5)$ can be obtained from the census age return after necessary correction for irregularities in various age groups and ${}_5L_x$ values were borrowed from suitable model life table. The other details may be found elsewhere (Arriaga, 1966 & 1968).

In another attempt, Sinha and Lahiri (1976) constructed life tables for the Indian States during 1951-61 under the assumption that the natural population of each state is approximately stable. The technique adopted by them is described

briefly in the following lines:

From the fundamental equation of stable populations, it can be shown that the 5-year survival ratio (${}_5S_x$) between the age intervals $(x, x+5)$ and $(x+5, x+10)$ is given by -

$${}_5S_x = \frac{{}_5L_{x+5}}{{}_5L_x} = \frac{c(x+5, x+10)}{c(x, x+5)} e^{5r}$$

The survival ratio from birth to the age interval $(0, 4)$ is given by -

$${}_{0-4}S_B = \frac{{}_5L_0}{5.l_0} = \frac{c(0, 5)e^{r \cdot \xi}}{5.b}$$

where ξ is the mean age of persons belonging to the age interval $(0, 4)$ and an approximate value of ξ may be obtained from a suitable model life table through the following equation -

$$\xi \doteq \frac{0.5 * L_0 + 3 * {}_4L_1}{{}_5L_0}$$

Using the above equations one can easily obtain the values of ${}_5L_x$ for all x excepting for the open age interval 70 and above. The value of L_{70+} ($= T_{70}$) can be obtained by using the following formula -

$$T_{70} = \frac{{}_5S_{65+}}{1 - {}_5S_{65+}} * {}_5L_{65}$$

where ${}_5S_{65+}$, the survival ratio between the two age intervals 65 and above and 70 and above, can be obtained by the following

formula -

$${}_5S_{65+} \doteq \frac{c(70+)}{c(65+)} \text{Exp} \left[r \left\{ 5 - (e_{65}^0 - e_{70}^0) \right\} \right]$$

where e_{65}^0 and e_{70}^0 , the life expectancies at ages 65 and 70

respectively, can be obtained from the life table of earlier decade or from a suitable model life table selected on the basis of the very rough value of mortality level given by the ratio $\frac{16}{T_0 - T_{70}}$. The other details may be found elsewhere (Sinha and Lahiri, 1976).

Before applying the technique proposed by Sinha and Lahiri, it is necessary to smooth the census age returns thoroughly; whereas, in the case of Arriaga's method, a smoothed series of $c(x, x+5)$ is obtained through the linear equation (1.12) after removing the irregularities of the basic age data. It appears that the smoothed series of the age distribution is heavily influenced by the selection of the values of ${}_5L_x$ from model life tables or life table of some other population having more or less the same characteristics as in the population under study. However, it may be noted in this context that the smoothing procedures of age data, in general are rather arbitrary and subjective (United Nations, 1967).

1.3.4(b) Life tables based on single census :

The methods based on the theory of stable population which has already been described in the last section can also be used with the information given by a single census on the age-sex distribution provided (i) the population under study can be considered

 1/ For computing ${}_5S_{65+}$ we need to know the difference $e_{65}^0 - e_{70}^0$.

It can be shown empirically that the variation in the difference over the various mortality levels in Coale and Demeny model life tables is quite small.

as stable for all practical purposes and (ii) the rate of growth and birth rate of the population during the census year are available or could be estimated from the available information.

However, Stolnitz (1956) has developed a technique of construction of life table from a single census age returns which does not require the assumption of stability. The basic equation developed by him under assumption of closed population and exact census counts for estimating age-specific survival probabilities, is given by -

$$\frac{{}_5N_{i+5}}{{}_5N_i} = \frac{{}_5B_{i+5}}{{}_5B_i} * \frac{S_{i+2.5}^{T-5}}{S_{i+2.5}^T} * {}_5P_{i+2.5}, \quad i = 0, 5, \dots$$

where, ${}_5N_i$ denotes the population in the age-interval $(i, i+5)$, ${}_5B_i$ represents the number of births between i and $i+5$ years before the census, $S_{i+2.5}^T$ and $S_{i+2.5}^{T-5}$ stand for the survival ratios from birth to the age-interval $(i, i+5)$ on the census dates T and 5 years prior to the census date T respectively and lastly, ${}_5P_{i+2.5}$ is the probability of survival from the exact age $i+2.5$ to the exact age $i+7.5$.

To solve for ${}_5P_{i+2.5}$ from the above equation, we need to know three ratios : ${}_5N_{i+5}/{}_5N_i$, ${}_5B_{i+5}/{}_5B_i$ and $S_{i+2.5}^{T-5}/S_{i+2.5}^T$. The ratio $({}_5N_{i+5}/{}_5N_i)$ can easily be obtained from the census counts. But, for estimating other two ratios, that is, birth ratio $({}_5B_{i+5}/{}_5B_i)$ and survival ratio $(S_{i+2.5}^{T-5}/S_{i+2.5}^T)$, we need to have reliable information on birth statistics (for the

specific years prior to the census) which are usually very poor in quality, if available. However, Stolnitz (1956, pp.49-94) has suggested some approximate procedures for estimating these ratios.

1.3.5. Estimation of life table functions during infancy and childhood period.

When life table is constructed on the basis of census returns in the absence of reliable data on birth and death statistics for the population under study, it is rather difficult to estimate the infant and child mortality and, hence, the other function of life table. However, there are a number of indirect methods for estimating mortality rates during infancy and childhood period. These are discussed briefly in the following lines:

1.3.5(a) Indian official life tables :

While constructing the Indian life tables for various decades from the age distribution of two consecutive censuses, the values of l_x below 5 years are usually extended by fitting the following curve:

$$l_x = A + Hx + BC^x + \frac{m}{nx+1}$$

Further details have already been discussed earlier under section 1.3.4(a) (iii).

1.3.5(b) Arriaga's Method :

In order to determine the values of l_x for 10 years of age and below, Arriaga, first of all, interpolated certain convenient values for l_x by single year age groups through the application of Beers' multipliers (Beers, 1945) on ${}_5E_x$ values obtained earlier

by Arriaga's method. Then, the values of l_1 , l_5 and l_{10} can be estimated from the relevant l_x values through a third degree function (Arriaga, 1968, p.14).

1.3.5(c) Sinha and Lahiri's approach

After obtaining the values of ${}_5L_0$ and ${}_5L_5$ by Sinha and Lahiri's (1976) approach, the values of l_1 and l_5 can be obtained through the following linear equations:

$$\begin{aligned} l_1 &= a.l_0 + (1-a) \frac{{}_5L_0}{5} \\ l_5 &= b. \frac{{}_5L_0}{5} + (1-b) \frac{{}_5L_5}{5} \end{aligned}$$

where 'a' and 'b' are constants which can be estimated from a suitable life table or model life table.

The validity of the above relationships were examined empirically by Immerwarh and Sinha (1971).

1.3.5(d) Stolnitz's technique

For extending the life table below age 2, Stolnitz (1956) developed the following formula for estimating survival probabilities for single-year age groups:

$$\frac{{}_1N_{i+1}}{{}_1N_i} = \frac{{}_1B_{i+1}}{{}_1B_0} \times \frac{S_{i+0.5}^{T-1}}{S_{i+0.5}^T} * p_{i+0.5}, \text{ for } i = 0 \text{ and } 1.$$

The notations used in the equation all refer to single-year age groups. Knowing the values of $p_{0.5}$ and $p_{1.5}$ from the above equations, p_0 which is the probability of survival from birth to the exact age 1, can be estimated through the following equation :

$$p_{0.5} = \frac{L_1}{L_0} = p_0 \frac{c+dp_1}{a+bp_0} \quad \text{where} \quad p_0 = \frac{l_1}{l_0} \quad \text{and} \quad p_1 = \frac{l_2}{l_1},$$

and the constants a , b , c and d are such that $L_0 = al_0 + bl_1$ and $L_1 = cl_1 + dl_2$. The values of a , b , c and d can be obtained from a suitable life table. Values of p_1 , the probability of survival from the exact age 1 to exact age 2, can be obtained from $p_{1.5}$ by fitting a linear regression of p_1 on $p_{1.5}$ 17/

The probability of survival for the half-year age interval between 2 and 2.5 may be estimated from life tables selected on the basis of p_0 and p_1 . Therefore, the number of survivors l_1 , l_2 and $l_{2.5}$ can be found by successive multiplication of the survival probabilities by the corresponding values of l_x starting with l_0 , the radix of the life table. The number of survivors at later ages can be obtained as before from the 5 year survival rates (Stolnitz, 1956, pp.25-29).

1.3.5(c) Estimation of infant and child mortality from children ever born and surviving to mothers at various quinquennial age groups:

Owing to the lack of reliable information on birth and death registration statistics, demographers' attention have been focused, in the recent past, to develop indirect techniques of estimating infant and child mortality from the information on the number of children ever born and surviving to mothers at various ages

17/ Analysing the relationship between p_1 and $p_{1.5}$ with respect to a large number of life tables having a broad range of mortality condition, Stolnitz (1956, p.27) found that correlations between them for both the sexes between p_1 and $p_{1.5}$ are very close to unity.

which are of better quality than the civil registration statistics and mostly available from census and/or demographic survey. The pioneering work in this field was carried out in 1968 by William Brass (1968, pp. 104-120) and subsequently modified and refined by Sullivan (1972), Trussell (1975), Preston and Palloni (1977), Feeney (1980) and Palloni (1979, 1980 and 1981). These methods are described very briefly in the following paragraphs.

The Brass's technique for estimating infant and childhood mortality essentially consists of converting retrospective mortality statistics, the proportions dead among children born alive to women at various ages starting with age 15, into probabilities of dying from birth to various exact ages in infancy and childhood which in life table terminology are denoted by x^q_0 . Brass developed a table of multipliers, dependent on some assumed shape of fertility schedule of the population in question, for transforming the statistics on proportion dead (D_i) among children ever born reported by women in the quinquennial age-groups (i) 15-19, 20-24, 25-29 and 30-34 into approximate estimates of q_0 , $2q_0$, $3q_0$ and $5q_0$ respectively (Brass and Coale, 1968, p. 198). A set of multiplying factors may be selected on the basis of the index P_1 / P_2 , the ratio of the average number of children born to women in the age group (15,19) to that of the women in (20,24), or $\bar{m}$, the mean age of fertility schedule. Brass's approach rests on two basic assumptions, namely (i) mortality has remained unchanged in the recent

past and (ii) the women belonging to various age groups have been exposed to the same fertility regime. Later on, Sullivan (1972) made an attempt to improve the Brass's multipliers by examining the relationships between D_i and ${}_xq_0$ through regression analysis. For this purpose, he used observed fertility distributions instead of the fertility schedule suggested by Brass^{18/} (1968), in combination with the model life tables prepared by Coale and Demeny (rather than a fixed mortality schedule). After a good deal of calculations, he found that the fertility parameter P_2/P_3 , the ratio of the average parity of women aged 20-24 to women aged 25-29, produce better result than the Brass parameter P_1/P_2 . Sullivan finally concluded that though the Brass-multiplying factors produce satisfactory results, but the regression equations provided somewhat better results as well as a simpler procedure for obtaining the estimates. Trussell (1975) found that the Brass fertility model and Sullivan's observed fertility schedules, which were gathered by the Princeton Office of Population Research, were of restrictive nature. He, therefore, used a set of model fertility schedules, developed by Coale and Trussell (1974), which takes into account the wide range of variation in fertility pattern to test the performance of the Brass model compared to that studied by Sullivan (1972).

Trussell (1975) commented that the new set of the multiplying factors developed by him as a result of the test are significantly superior to those provided by either Brass or Sullivan.

^{18/} The Brass Fertility Schedule is defined by -

$$f(x) = \begin{cases} K \cdot (x-s) \cdot (s+33-x)^2, & \text{for } s \leq x \leq s+33 \\ 0, & \text{otherwise} \end{cases}$$

Where s is the earliest age of child-bearing and k is "scale" factor that determines the total number of children, ever born by the end of childbearing (at age $s + 33$).

The Brass technique and its subsequent modifications by Sullivan and Trussel rest on the assumption of constant mortality and fertility schedules in the recent past. But the above techniques fail to provide reasonably good estimates of mortality levels when the population under study is disturbed by any of the following elements - (i) fertility trends, (ii) unusual fertility patterns, (iii) flows of women among social categories, (iv) mis-statement of women's age, (v) small size of reporting women, (vi) uncertainty about the appropriate model mortality pattern and mortality trends. Preston and Palloni (1977) developed a technique of reducing the errors from these sources when the data are available on the ages of surviving children born to reporting women. The basic approach involves backward projection of surviving children by age until one has reproduced the reported number of children ever born. The age distribution of the living children born to mothers at various age groups may be obtained by matching children to mothers as per the census household records. If it is possible to match accurately all the surviving children to their respective mothers, the technique suggested by Preston and Palloni is expected to produce better result than the conventional one proposed by Brass which, of course, requires minimal information for converting the proportion dead to the corresponding probability of dying below certain age. Using regression technique for predicting mortality on the basis of age distributions

of surviving children born to mothers at various age groups, Preston and Palloni indentified the age groups of mothers that provide best estimates of mortality among children below certain age. Various sets pf regression model were generated taking into account the mortality pattern of Coale-Demeny model life tables and various models of fertility history produced by Coale and Trussel for the purpose of predicting mortality among children below certain age. After a good deal of empirical calculations, Preston and Palloni (1977) found that in the absence of knowledge of actual mortality pattern of the population in question, the use of the regression model under the assumption of West mortality pattern will produce reasonably good estimates of child mortality (with a small margin of error) compared to other models provided, of course, the actual mortality pattern belongs to any of the four mortality pattern considered by Coale and Demeny (1968).

Under the assumption of constant mortality and fertility in the recent past, the Brass estimates of infant and child mortality refer to the time of census or survey which provides the basic Brass-type information. But, when mortality is declining, these estimates no more correspond to the time of the census or survey, but they correspond to the period few years prior to the census or survey, as pointed by Brass (1968, p.115). Considering the conventional Brass-type information, Coale and Trussel (1978) developed a technique to determine the period of time (rather than the cohort) for which the Brass estimates of mortality correspond. The mortality patterns of Coale-Demeny model life

tables and Coale-Trussel fertility models were used to generate the Brass-type data. In another attempt Feeney (1980) has carried out similar experiment on the basis of Brass's general standard model life table family and the fertility polynomial suggested by Brass (1968). He presented various values of infant mortality rate and the years prior to census to which the infant mortality rate corresponds in a tabular form - for various values of the percentage of deceased children by age group of women (Q_i) and the age at onset of child-bearing(s) with the help of the following equation :

$$Q_i = \sum_{t=1}^{n(i)} q(t, r, w) \cdot C_i(t), \quad i = 1, 2, \dots, N$$

where Q_i denotes the proportion dead among children born alive to women belonging to the i -th age group, $C_i(t)$ represents the proportion of all children born to women in the i -th age group who were born during the t -th year prior to the census or survey, 'w' is the infant mortality rate at the time of the census and 'r' denotes the rate of linear decline in infant mortality rate per year. This table is quite useful for analysing the trends in infant mortality in some populations (Feeney, 1980).

Under the assumption of linear and non-linear decline in mortality, Palloni modified the Brass technique and presented alternative approach for estimating infant and child mortality. He has presented a procedure for obtaining the number of years prior to the census to which the level of mortality, estimated under

the assumption of constancy of fertility, corresponds (Palloni, 1979, 1980 and 1981).

In another study, Roy and Gotpagar (1976) proposed a micro simulation model to assess the effectiveness of the techniques suggested by Brass, Sullivan and Trussel (discussed earlier), under the various assumptions regarding changes in the mortality over time and in recall lapse errors. This study reveals that Sullivan's estimates are affected more by the incidence of recall lapse. This is so because the Sullivan's technique has a tendency to understate the xq_0 values even in the absence of recall lapse. Later on, it was brought out by them in another paper that the use of additional information on children ever born and surviving during the last 5 years to mothers in the reproductive age span, improves the estimate of infant mortality (Roy and Gotpagar, 1978).

In an another attempt, Mukerji (1977a) developed a linear programming technique for estimating infant mortality rates (IMR) by age of mother and an overall estimate of IMR on the basis of Brass-type information. There are number of studies regarding the problems of estimation of overall and infant mortality rates from census and survey data (Ramachandran and Nair, 1972; Ramakumar, 1972; Ruzicka and Kanitkar, 1973; Ruzicka and Hansluwka, 1983; Srinivasan, 1979; Vaidyanathan and Kanitkar, 1979).

1.4. Basic Problems of Construction of Life Tables for Developing Countries and Objectives of the Present Investigation

1.4.1. The basic problem

In many developing countries of the present world vital registration statistics are extremely poor, not only in terms of completeness, but also in terms of coverage of the area under study. According to United Nations and World Health Organization,

accurate registration of vital events is mainly restricted to the more developed countries which consist of less than one-third of the world's population. Furthermore, the situation regarding registration of infant deaths is much more grave, about 98 per cent of the world's infant deaths remain unregistered (United Nations & W.H.O., 1973). In India, for example, even though the system of registering births and deaths has been in operation for more than a century, yet the vital registration is far from being satisfactory (Office of the Registrar General of India, 1972). The under-registration of deaths during 1969-70 in urban areas of Maharashtra, an Indian State, where registration is relatively better compared to that of other States (urban areas), was around 30 per cent (Ranganathan, 1974). Thus, one may easily imagine the situation in the entire country (all-India urban and rural combined) with more than 80 per cent of the people living in rural areas where conditions are relatively worse, the magnitude of under-registration would be very large. Comparing the Civil Registration death rates with those of Sample Registration System (SRS), Dange (1977) found that the percentages of deaths under-reported in civil registration in 1969 were around 57 and 35 in rural and urban India respectively. However, there is a general feeling that SRS deaths are also under-registered to some extent; if it is so, then the under-registration of deaths by the civil registration would be more than the above-mentioned figures. An evaluation of birth and death registration in India during 1961-71 was carried out by the Office of the Registrar General of India (1980b) though Brass technique (1979b); and this

study showed that about 64 per cent of the deaths occurring in India (excluding the two States - Orissa and Rajasthan) were under-reported in civil registration. The state-wise variation in under-registration of deaths is quite revealing - it is as high as 94 per cent in Assam and about 23 per cent in Maharashtra (Office of the Registrar General of India, 1980, p.2). However, one may feel when there are a number of methods available, as mentioned before, to adjust the faulty death records why not use them to derive, if possible, fairly reliable age-specific death rates which are fundamental for the construction of life table. It may be noted that the methods developed so far for estimating completeness of death registration are based on the assumption of constant rate of under-registration over ages, and in addition to this, some of the methods assume stability of the populations under study. The assumption of constant under-registration rate over ages which is the most crucial assumption, needs more justification. Dange's (1977) study on under-registration of deaths in India by age in civil registration in comparison to that in SRS shows that the extent of under-registration decreases with age; this finding contradicts the assumption of constant rate of under-registration over ages. In this context, the comment made by Preston and others (1979) is pertinent: "If the same tendency for increasing registration completeness with age occurs in SRS, then all indirect completeness estimates for SRS are biased upwards". Moreover, it is a well-known fact that in many developing countries, the census age returns are

heavily affected by age-misreporting. The Indian census age returns show a marked evidence of such kind of error in age-reporting. Now, question may arise, if the census age returns are seriously affected by age-misreporting, how far are we justified to use various indirect methods of estimating the rate of completeness of death registration? While examining the extent of under-registration of deaths in India during 1969-72 through SRS data, Preston and others (1979) found that the general levels of death registration completeness become more than unity in many cases, particularly in the Brass's and Martin's techniques. The authors express their doubts regarding usefulness of these methods in a situation of such serious age-misreporting which characterises the Indian censuses.

From the foregoing discussions, it appears that in countries like India, where death registration statistics are extremely poor in quality, the direct method of constructing life tables by comparing the deaths in an age sector with the relevant exposed-to-risk population is ruled out. Therefore, the investigation into the mortality experience of the population under study having very poor quality of death records has to be made through some indirect method based on census age-returns only. But, in addition to very poor quality of death registration statistics, when the age distribution of the population under study is also severely affected by age-misstatements, the construction with high certainty of a fairly precise life table is, in a sense, extremely difficult. Even then the need for life tables is so great

in social and economic planning that one has to attempt at obtaining as good an approximation as possible. However, in general, it is found that in many of the developing countries, the census age-returns are relatively much better than civil registration statistics at least in term of completeness and coverage. Thus, for constructing life tables of these countries, one has to depend upon census age-returns only.

The observed age distributions of such countries obtained through population census are so much distorted by age-misstatement that the survival ratios, based on quinquennial age groups, computed from two consecutive census age distributions are usually also highly distorted. The order of distortions which will be discussed in greater details in the next chapter, is in general so high that it cannot be used for the purpose of construction of life tables without making any adjustment to these ratios. Thus, before applying any technique for construction of life tables, the age distributions or the raw survival ratios have to be adjusted so as to remove the effects of age-misreporting. It is worthwhile to mention here that in the presence of serious distortion in census age-returns of a country, the procedures usually adopted for adjusting the enumerated age-data to remove the effect of age-misreporting are rather subjective and arbitrary. Thus the age pattern of mortality embodied in the life-table estimated from the two consecutive censuses of the population under study is heavily influenced by the smoothing procedure adopted to remove the effect of age-misreporting. Coale and Demeny (1967)

observed that if the census survival ratios, not of the usual form, are obtained from consecutive decennial censuses as a ratio^{19/} of the population at age $x+10$ and above in the second census to that at age x and above in the first census, the effect of age-misreporting reduces considerably. This will be discussed in more details in the following chapter.

1.4.2. Objectives of the present study :

The discussions in the foregoing paragraphs reveal that in the presence of serious age-misreporting in the census age-returns, it is quite difficult to construct a precise life table since the smoothing procedures applied on the age data to remove the irregularities due to response biases in age-reporting are rather arbitrary, subjective and even sometimes heavily influenced by personal judgement. Then, there is every possibility that the actual mortality schedule, acting on the population during the period under study, may get vitiated by such an adjustment of the basic age data. Therefore, for developing nations, whose age data are heavily distorted by age-misreporting, it is necessary to develop a technique for constructing life table which does not require through smoothing of the age data.

^{19/} Henceforth, this ratio will be called 'ten-year cumulative census survival ratio' to distinguish from the usual form of census survival ratio. It may be noted here that the cumulative survival ratio does not mean cumulation of the usual form of survival ratios, but it may be obtained as a ratio of cumulated population (at ages 0+ , 5+ , 10+.....) in two consecutive censuses.

The concept of cumulative census survival ratios introduced by Coale and Demeny, has been used in a special way here to develop a new technique for the construction of life table ^{20/} above certain childhood age through an iterative procedure, from the enumerated census age distribution of two consecutive censuses. An attempt has also been made here to develop a technique for estimating the probability of survival from birth to the childhood age above which life table can be obtained through some iterative procedure. It has also been shown here how this probability of survival from birth to some childhood age can be utilized to estimate infant and childhood mortality rates. The methodological details of the various steps along with various assumptions involved in it are discussed in Chapter III.

In a nutshell, the technique developed here for the construction of life table beyond certain childhood age may be broadly divided into two steps. In the first step, 10-year cumulative census survival ratios (10-cum-CSR) are converted into 10-year cumulative life table survival ratios (10-cum-LSR) under certain assumptions and in the second step under certain condition these 10-cum-LSRs are used to obtain survival probabilities (that is, ${}_5P_x$ in life table terminology) which determine a life table uniquely. In the presence of heavy response biases

^{20/} Coale and Demeny (1967) gave some vague idea about construction of life tables from this 10-year cumulative survival ratios through the use of model life tables prepared by them. For detail discussion, see the section under the heading "other methods of Estimation of Mortality from Census Survival Ratio"

in age-reporting in the censuses of developing countries like India, the assumptions involved here to construct life table from the observed 10-cum-CSR, seem to be reasonable and more scientific than the usual procedure for constructing life tables from the two consecutive census age-returns after adjusting the enumerated age-data for age-misreporting through some massive smoothing procedures which are usually rather arbitrary and very much subjective in nature. The effects of different smoothing procedures so as to adjust the basic age-data, which are heavily distorted due to age-misreporting are discussed in details in the next chapter.

It should be emphasised here that by no means one may claim that the method, developed here, is the best and when applied to some faulty age-data of a population produces the exact life table of the population under study which is virtually impossible to have in the presence of heavy distortion in the reported age-data mainly due to serious age-misreporting. On the contrary an attempt has been made here to obtain as good an approximation of the true life table as possible through a procedure which does not require smoothing of the basic age-data before applying the method; and thereby it reduces to a large extent the effects of arbitrariness, subjectivity and personal biases as frequently come across while smoothing the basic age-data. It is worthwhile to mention here that the relative utility of matter of judgement depending not only upon the merits and demerits of the method as compared to others but the nature of results obtained by its application.

CHAPTER II

IMPORTANCE OF THE CUMULATIVE CENSUS SURVIVAL RATIOS COMPARED TO USUAL CENSUS SURVIVAL RATIOS

2.1. Effect of Smoothing of Age-Data on Survival Ratios:

It has already been mentioned in the first chapter that the census age-returns of developing countries like India are heavily affected by age misreporting and thereby the census survival ratios computed from the two consecutive censuses for the purpose of constructing life tables, will also be heavily distorted. Thus with a view to the minimization of this distortion, usually the census age-returns are adjusted so as to remove or, rather minimize, the effect of age-misstatements before computing the survival ratios. It is, therefore, worthwhile to examine how far the census survival ratios are influenced by the smoothing procedure which are rather subjective and arbitrary in nature, as mentioned in the previous chapter. To examine the situation, one may consider the case of India, as an example, which is the most populous country in the world after China. Furthermore, it may be noted here that though the system of conducting decennial censuses in India is century old, still, the Indian census age-returns suffer from serious errors of age-reporting, and under-enumeration particularly in the younger ages, mainly due to mass illiteracy. A good deal of discussions on plausible reasons for age-misstatements may be found elsewhere (Office of the Registrar General of India, 1963; Jain, 1980 and United Nations, 1955).

Table 2.1

Single Year Age-Distribution of All-India According to
1961 Census for Males and Females

Age	Population		Age	Population		Age	Population	
	Male	Female		Male	Female		Male	Female
0	6,802,773	6,761,939	31	1,130,822	866,121	62	558,795	465,713
1	5,848,438	5,838,989	32	3,375,178	2,838,035	63	218,497	175,732
2	6,784,946	6,659,394	33	1,186,257	934,171	64	251,443	222,324
3	7,030,979	7,115,542	34	1,308,539	1,128,620	65	1,756,212	1,743,453
4	6,720,020	6,539,618	35	7,693,190	6,849,306	66	199,234	160,710
5	7,879,090	7,365,057	36	2,017,189	1,614,835	67	169,028	137,903
6	7,180,513	6,803,988	37	1,090,785	834,163	68	232,091	229,340
7	6,192,107	6,092,289	38	1,972,323	1,856,461	69	116,167	107,766
8	7,041,697	6,634,313	39	830,145	705,517	70	1,708,420	1,866,400
9	4,781,047	4,703,858	40	8,005,243	7,513,991	71	100,327	90,417
10	7,557,054	6,829,801	41	772,694	581,253	72	209,825	191,326
11	3,659,710	3,467,972	42	1,844,390	1,495,954	73	68,873	54,962
12	7,036,989	5,711,399	43	720,238	570,648	74	68,640	55,188
13	3,673,183	3,261,758	44	743,946	611,837	75	629,326	670,152
14	4,347,618	3,760,701	45	6,213,924	5,382,192	76	76,453	64,053
15	4,578,042	3,839,196	46	925,219	721,804	77	40,227	30,054
16	4,228,373	3,968,656	47	690,594	519,384	78	74,400	71,413
17	2,382,661	2,267,347	48	1,357,180	1,243,869	79	33,092	31,203
18	5,139,432	5,100,020	49	549,531	455,246	80	615,292	733,240
19	2,270,190	2,108,619	50	6,419,013	5,952,578	81	33,168	30,630
20	6,202,145	7,414,995	51	558,628	404,048	82	48,134	43,380
21	2,390,293	2,155,133	52	1,125,127	849,396	83	16,778	12,720
22	4,645,385	4,683,125	53	466,300	336,564	84	20,584	16,422
23	2,287,589	2,149,023	54	561,941	437,186	85	140,687	151,048
24	2,673,445	2,731,422	55	3,288,668	2,938,778	86	16,583	13,994
25	8,396,675	8,701,959	56	662,310	521,464	87	11,786	10,373
26	2,899,622	2,581,770	57	384,450	281,840	88	12,322	11,891
27	2,163,882	1,330,062	58	636,878	564,388	89	8,439	8,536
28	3,799,600	3,827,068	59	311,001	242,602	90	126,462	156,115
29	1,272,594	1,108,692	60	4,323,967	4,414,833	91+	116,799	129,566
30	8,986,855	9,087,656	61	354,446	273,630	ANS	94,766	81,703
Total							226,146,101	212,790,217

Note: The above age-data excludes 297,053 persons (147,100 males and 150,753 Females) of N.E.F.A. for whom All-India Census Schedule was not canvassed.

Source: Office of the Registrar General of India, 1964, Social and Cultural Tables, Census of India, 1961, Vol. I (India), Part-II-C(i), Government of India, pp.412-413.

Table 2.2

Single Year Age-Distributions of All-India According
to 1971 Census for Males and Females

Population			Age			Population			Age			Population		
Male	: Female	:	:	:	:	Male	: Female	:	:	:	:	Male	: Female	:
8,306,375	8,213,147		31	1,158,107	928,392	62	638,727		566,377					
5,545,843	5,432,239		32	3,833,843	3,449,967	63	277,070		216,222					
5,842,528	5,563,675		33	1,351,945	1,196,094	64	280,384		225,275					
5,749,966	5,796,054		34	1,409,312	1,320,020	65	2,607,235		2,531,653					
5,759,204	5,350,485		35	10,144,101	9,243,220	66	248,921		205,506					
5,565,939	5,845,342		36	2,254,109	1,919,571	67	215,647		171,500					
5,145,418	5,606,678		37	1,316,105	1,104,078	68	335,919		317,565					
7,562,941	7,417,041		38	2,613,744	2,584,861	69	146,650		130,467					
5,635,576	5,952,585		39	908,289	810,224	70	2,445,330		2,392,922					
5,201,423	5,974,329		40	10,378,330	9,412,737	71	145,329		97,119					
5,415,209	9,435,240		41	788,348	591,505	72	264,727		219,643					
4,901,826	4,613,811		42	2,229,550	1,311,259	73	91,907		61,302					
5,610,728	7,953,961		43	861,051	725,292	74	87,722		69,563					
5,277,133	4,801,757		44	800,338	689,074	75	870,823		856,302					
5,208,408	5,469,761		45	8,277,900	6,926,231	76	97,062		77,975					
5,495,342	5,336,386		46	1,049,084	835,231	77	51,503		37,714					
5,719,323	5,105,382		47	808,085	619,302	78	95,953		91,200					
5,344,866	3,068,258		48	1,740,767	1,547,527	79	36,104		30,973					
6,732,463	6,148,471		49	591,674	488,832	80	824,611		896,637					
2,320,704	2,587,957		50	8,163,176	7,202,706	81	35,783		28,590					
7,253,788	8,117,681		51	553,141	388,732	82	58,842		49,855					
3,142,980	2,513,656		52	1,284,678	961,163	83	21,456		15,622					
5,434,853	5,338,805		53	521,333	396,749	84	23,414		20,874					
2,122,597	2,589,100		54	593,559	465,607	85	203,059		209,874					
2,360,201	2,968,693		55	4,535,007	4,034,379	86	20,973		17,801					
3,146,961	9,662,794		56	774,921	608,932	87	17,138		13,387					
3,120,205	2,861,054		57	435,161	322,051	88	17,754		16,591					
2,356,505	2,067,843		58	813,698	735,580	89	10,272		9,291					
4,376,758	4,685,358		59	317,637	251,023	90	168,177		194,248					
1,330,942	1,204,030		60	5,916,196	5,633,567	91+	157,293		168,576					
11,560,134	10,972,603		61	322,344	247,870	ANS	56,239		60,025					
Total: 234,049,276 264,110,376														

Source: Office of the Registrar General of India, 1976, Social and Cultural Tables (Table C-II, C-III, Part A & B and C-IV), Series-1, India, Part II-C(ii), Government of India.

FIGURE 2.1

Population by Single Year of Age, All-India -
Males, 1961 Census

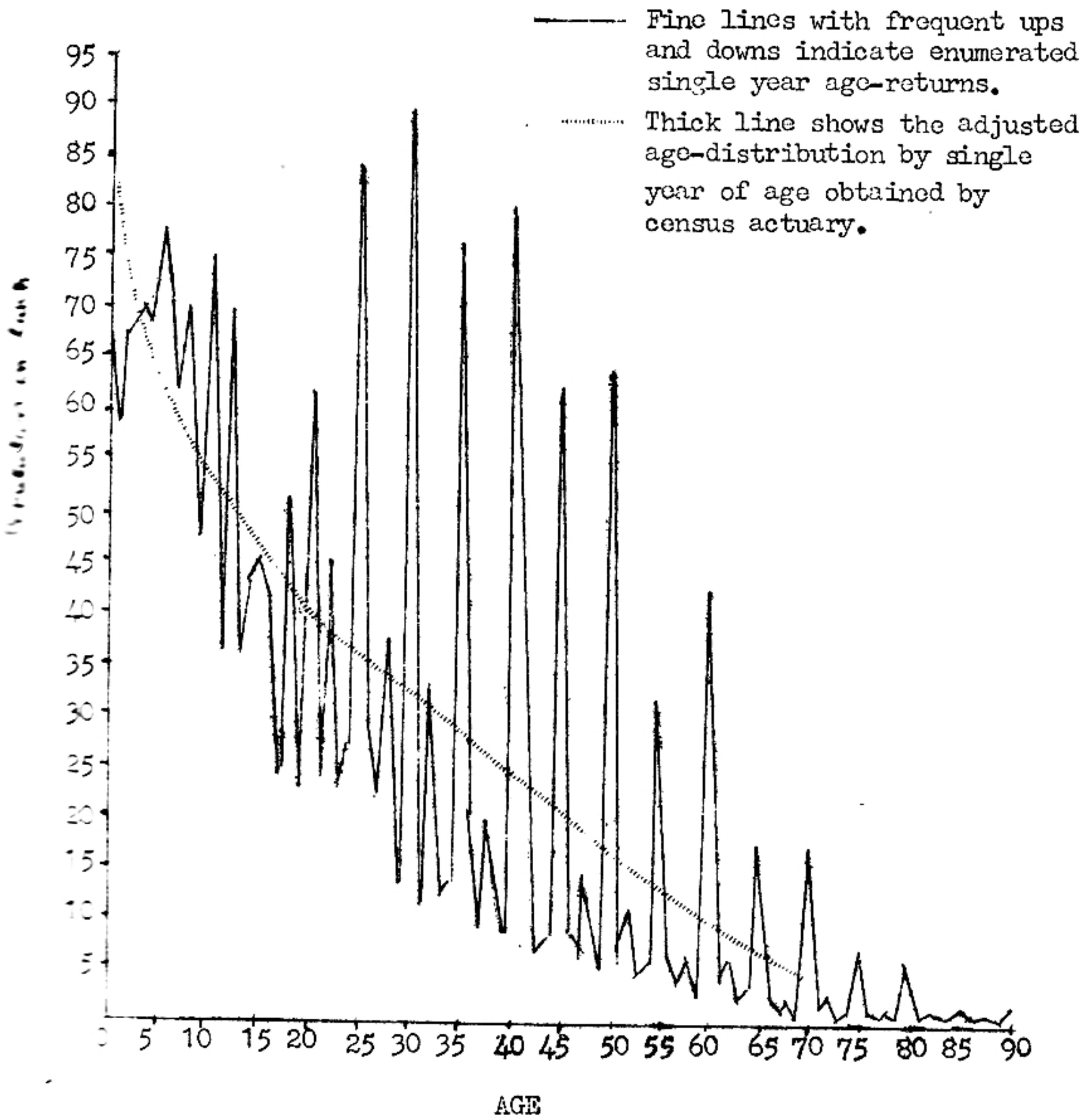
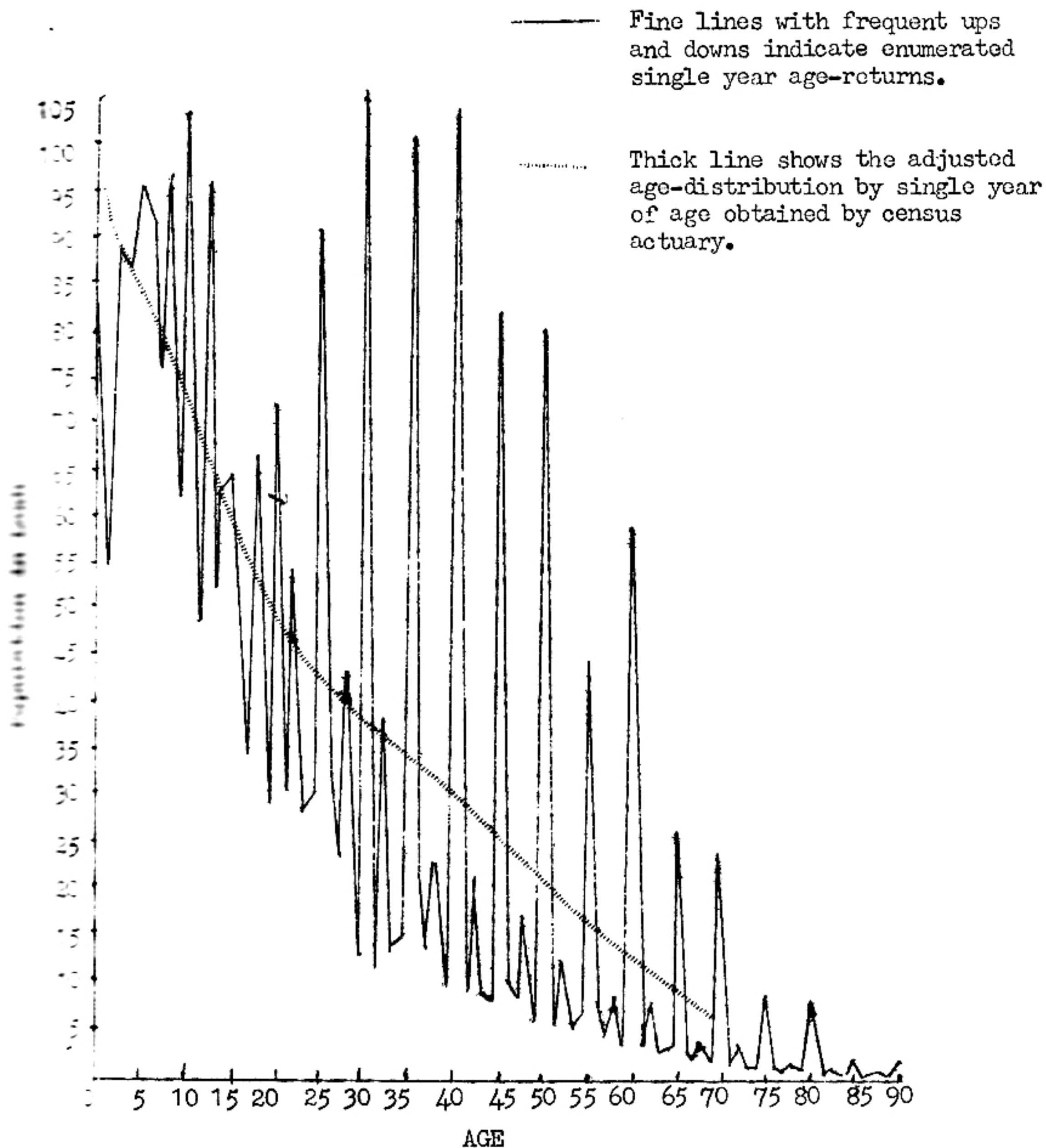


FIGURE - 2.2

Population by Single Year of Age, All-India,
Males, 1971 Census



2.1.1. An illustration showing the effect of smoothing on age structure and census survival ratio by using Indian census age returns:

The single-year enumerated age returns for Indian males and females as well as the corresponding smoothed age distributions obtained by the Indian census actuaries according to 1961 and 1971 Censuses are presented in Tables 2.1 and 2.2 respectively. These tables clearly show that the age returns in Indian censuses are heavily distorted by age-misstatements. In order to have an overall idea at a glance regarding the extent of age-misstatements in Indian census age-returns, the single-year enumerated age data for Indian males according to 1961 and 1971 Censuses are presented graphically in Figures 2.1 and 2.2. For the purpose of visual comparison the smoothed single-year age distributions for Indian males in 1961 and 1971 Censuses are superimposed on the corresponding graphs shown in Figures 2.1 and 2.2. Similar type of age distortion is also found among Indian females (not shown graphically). The graphical representations of the enumerated as well as smoothed single-year age data for males, shown in Figures 2.1 and 2.2, clearly indicate that very drastic and rigorous smoothing procedures are required to obtain the smoothed series of single-year age data after removing the effects of such serious errors in age-reporting. Thus, when the census age-returns are notoriously distorted by age-misreporting, one may naturally suspect that the adjusted age distribution of a population may depend heavily on the smoothing procedures applied to the raw age data so as to remove or minimize the effect of age-misreporting. Such possibilities are discussed in the following paragraphs.

Table 2.3

All India Adjusted Population in Five Year Age Groups for Males and Females obtained by Two Different Procedures of Adjusting Enumerated Age Returns according to 1951 Census

Age-Group*	Adjusted Population of 1951 Census for			
	Males obtained by		Females obtained by	
	Census Actuary: Kohli		Census Actuary: Kohli	
		:		:
0-7	38,098,400	39,113,382	33,017,700	33,834,840
8-12	21,826,100	21,552,330	20,608,900	20,652,928
13-17	19,456,500	19,126,960	18,191,900	17,747,440
18-22	17,042,500	16,960,160	16,401,900	15,441,677
23-27	15,319,100	14,972,934	14,822,700	13,815,138
28-32	13,985,800	13,177,819	13,137,700	12,275,137
33-37	12,460,500	11,583,964	11,308,200	10,819,040
38-42	10,787,400	10,031,611	9,558,700	9,428,735
43-47	9,128,700	8,585,000	8,048,900	7,918,524
48-52	7,460,900	7,231,077	6,664,400	6,652,681
53-57	5,789,700	5,903,380	5,327,400	5,492,681
58-62	4,312,800	4,633,061	4,149,800	4,390,941
63-67	2,927,500	3,425,099	2,933,600	3,483,149
68+	4,690,800	6,989,923	9,340,200	11,559,089
Total	183,286,700	183,286,700	173,512,000	173,512,000

Note: * This age-grouping is adopted by Kohli for presenting the smoothed and graduated population of 1951 and 1961 censuses and for the purpose of comparison the smoothed and graduated populations obtained by the census actuary are also grouped into the same age-groups as adopted by Kohli

Source: (i) Office of the Registrar General of India, 1954, Age-Table - 1951 Census, Paper No.3 of 1954, Government of India, New Delhi, 1954, Table C, pp. 144-146, and

(ii) K.L. Kohli, Mortality in India: A State-wise Study, Sterling Publishers Pvt. Ltd., New Delhi, 1977, 'Appendix', Table C-1, p.212.

Table 2.3

All India Adjusted Population in Five Year Age Groups for Males and Females obtained by Two Different Procedures of Adjusting Enumerated Age Returns according to 1951 Census

Age-Group*	Adjusted Population of 1951 Census for			
	Males obtained by		Females obtained by	
	Census Actuary: Kohli	:	Census Actuary: Kohli	:
0-7	38,098,400	39,113,382	33,017,700	33,834,840
8-12	21,826,100	21,552,330	20,608,900	20,652,928
13-17	19,456,500	19,126,960	18,191,900	17,747,440
18-22	17,042,500	16,960,160	16,401,900	15,441,677
23-27	15,319,100	14,972,934	14,822,700	13,815,138
28-32	13,985,800	13,177,819	13,137,700	12,275,137
33-37	12,460,500	11,583,964	11,308,200	10,819,040
38-42	10,787,400	10,031,611	9,558,700	9,428,735
43-47	9,128,700	8,585,000	8,048,900	7,918,524
48-52	7,460,900	7,231,077	6,664,400	6,652,681
53-57	5,789,700	5,903,380	5,327,400	5,492,681
58-62	4,312,800	4,633,061	4,149,800	4,390,941
63-67	2,927,500	3,425,099	2,933,600	3,483,149
68+	4,690,800	6,989,923	9,340,200	11,559,089
Total	183,286,700	183,286,700	173,512,000	173,512,000

Note: * This age-grouping is adopted by Kohli for presenting the smoothed and graduated population of 1951 and 1961 censuses and for the purpose of comparison the smoothed and graduated populations obtained by the census actuary are also grouped into the same age-groups as adopted by Kohli

Source: (i) Office of the Registrar General of India, 1954, Age-Table - 1951 Census, Paper No.3 of 1954, Government of India, New Delhi, 1954, Table C, pp. 144-146, and

(ii) K.L. Kohli, Mortality in India: A State-wise Study, Sterling Publishers Pvt. Ltd., New Delhi, 1977, 'Appendix', Table C-1, p.212.

Table 2.4

All-India Adjusted Population in Five Year Age Groups for Males and Females Obtained by Two Different Procedures of Adjusting Enumerated Age Returns According to 1961 Census

Age-group*	Adjusted Population of 1961 Census for			
	Male obtained by		Female obtained by	
	Census Actuary	: Kohli	Census Actuary	: Kohli
0-7	54,883,100	54,420,930	53,446,600	52,774,706
8-12	26,972,900	27,034,512	25,787,600	26,574,640
13-17	23,412,800	23,955,808	22,062,500	22,983,440
18-22	20,424,100	21,130,432	19,308,100	19,860,176
23-27	18,295,000	18,514,848	17,576,700	17,069,696
28-32	16,518,000	16,083,882	15,537,100	14,680,922
33-37	14,461,700	13,896,176	13,063,900	12,562,758
38-42	12,343,000	11,881,039	10,826,700	10,742,224
43-47	10,412,300	10,063,171	9,036,200	9,056,454
48-52	8,547,600	8,381,492	7,412,700	7,564,610
53-57	6,743,900	6,819,401	5,927,400	6,147,174
58-62	5,023,300	5,364,488	4,506,900	4,827,887
63-67	3,470,400	3,946,663	3,323,500	3,603,032
68+	4,304,900	4,320,145	4,752,600	4,010,832
Total	225,813,000	225,812,987	212,458,500	212,458,559

Note: * See footnote of Table 2.3

Source: (i) Office of the Registrar General of India, 1963, Age-Tables - 1961 Census, Paper No.2 of 1963, Government of India, New Delhi, 1963.

(ii) K.L. Kohli, Mortality in India: A State-wise Study, Sterling Publishers Pvt. Ltd., New Delhi, 1977, 'Appendix', Table C-1, p.212.

The Tables 2.3 and 2.4 give two sets of smoothed age distributions for all-India males and females obtained by two different procedures for adjusting the 1951 and 1961 census age-returns. In one of the procedures, Kohli (1977) obtained smoothed age distributions for males and females according to 1951 and 1961 Censuses by applying the technique developed by Chanana and others (1969), and the other sets of adjusted age-sex distributions were obtained by the smoothing technique proposed by the then census actuary of India (Office of the Registrar General of India, 1954c; 1963). The essence of the method developed by Chanana and others is that it takes care of large deficit or excess in quinquennial age groups due to shifting of persons in the ages 10 and above by making age ratios nearly 100; the remaining errors were sought to be removed by obtaining population in single-year ages and applying to it certain summation formulas so as to obtain smoothed series of single-year age distribution. ^{1/} The technique mentioned above provides smoothed population by single-year of ages starting with age 8. Further details of the procedure may be found elsewhere (Chanana et al., 1969; Kohli, 1977). This type of adjustment in age data has also been made for many Asian countries where serious distortions in the enumerated age-returns were found (Rao, 1971).

The smoothing procedures adopted by the census actuaries of India to adjust the Indian census age-returns are rather

^{1/} This method was developed when the authors were under training at the International Institute for Population Studies (IIPS) (earlier known as the Demographic Training and Research Centre), Bombay as a part of the training programme at IIPS, Bombay.

complicated. The details of the methods of smoothing for the 1951 and 1961 census age-returns may be found in the respective Age Tables published by the Office of the Registrar General of India (Office of the Registrar General of India, 1954c and 1963). However, the techniques adopted by the census actuary for 1951 and 1961 Censuses are described very briefly in the following lines. In the case of 1951 Census, the single-year age data were first clubbed into quinquennial age-groups starting with the group 3-7 which was considered relatively satisfactory compared to the other possible quinary groupings for the preparation of the Age Tables. The group totals, based on 3-7 grouping, when plotted on a graph paper showed a systematic stair like appearance with more concentration of population for groups having a multiple of 10 at the mid-age than the groups having mid-age ending with the digit 5. The group totals were corrected for such systematic biases through a system of transferring persons from groups in excess to those in deficit. The details of the system of transfers may be found elsewhere (Office of the Registrar General of India, 1954c, pp. 12-15). In 1961, 3-7 age-grouping was also adopted but the group totals were not adjusted for systematic biases through the system of transfers as used in 1951 Census; however, the single-year age-returns in 1961 Census were first smoothed by 11-term moving average prior to forming the aforementioned quinary groups. The other steps leading to the final smoothed single-year age distribution were almost similar in both 1951 and 1961 censuses. The group totals so obtained were

further smoothed by taking a weighted average of three successive groups as equal to the middle group, the weight being 1 : 2 : 1. The finally-adjusted group totals were then distributed into single-year of age by using the osculatory interpolation formulas developed by Kozakeiwicz in connection with Canadian Census. In 1951, Kozakeiwicz's interpolation formulas were used to estimate the population in single-year of ages between 5 to 69. The population for age 70 was extrapolated. The population at ages 0 to 4 were kept as enumerated without any adjustment. In 1961, aforementioned osculatory interpolation formulas were used to estimate population in single-year of ages between 8 to 67. The populations at ages 68 and 69 were estimated by fitting a second degree curve in the age-range 63 to 67. The population aged 70 and above was obtained by subtracting the estimated populations at ages 68 and 69 from the enumerated population in the age group 68 and above. The population at ages 0 to 7 were estimated by a special procedure (Office of the Registrar General of India, 1963). Various other techniques, have also been developed recently for adjusting census age-returns suffering from age-misstatements; the details may be found elsewhere (Brass, et al, 1968; Dyson, 1976; Mukerji, 1977; Peeney, 1978; Chandra, 1980 and Saxena and Gogte, 1983).

Comparing the two sets of smoothed age distributions for 1951 and 1961 Censuses, shown in Tables 2.3 and 2.4, which have been obtained by using two different procedures mentioned earlier, one may easily find that the two different smoothing

techniques give rise to two different age distributions of the same population. This gives an indication that probably either or both of the smoothing techniques, even though with some theoretical basis, are not fully satisfactory to depict the true picture of the age-structure of the population under study in the presence of serious errors of age-reporting, otherwise, they would have given practically the same age distribution.

Now, let us examine how far the smoothing procedures affect the ten-year census survival ratios which are the fundamental inputs for estimating the various life table functions. The ten-year census survival ratios during the decade 1951-61, obtained from the two sets of adjusted age data (given in Tables 2.3 and 2.4) are shown in Table 2.5.

Table 2.5

All-India Ten-year Census Survival Ratios During the Decade 1951-61 for Males and Females Calculated from the Adjusted Age-sex Data for 1951 and 1961 Censuses Obtained by Two Different Techniques of Smoothing the Enumerated Census Age>Returns.

Age-Group*	All-India Ten-year Census Survival Ratios During 1951-61 Calculated from the Corresponding Census Age Returns Smoothed by						
	Census Actuary			Kohli			
	Male	:	Female	:	Male	:	Female
8-12	0.935765		0.936882		0.980424		0.961616
13-17	0.940303		0.955189		0.967997		0.961812
18-22	0.969224		0.947274		0.948333		0.950734
23-27	0.944031		0.881344		0.928086		0.909347
28-32	0.882538		0.824094		0.901594		0.875120
33-37	0.835624		0.799084		0.868716		0.837085
38-42	0.792369		0.775492		0.835508		0.802294
43-47	0.738758		0.736423		0.794339		0.776303
48-52	0.673283		0.689770		0.741866		0.725705
53-57	0.599409		0.623850		0.668543		0.655970

* See footnote of Table 2.3

Source: Computed from Tables 2.3 and 2.4

The data presented in the Table 2.5 clearly show that the all-India age-specific ten-year survival ratios during the period 1951-61 calculated from the adjusted age data for 1951 and 1961 Censuses, obtained by the census actuary, differ widely from those during the same period estimated by Kohli on the basis of his adjusted age data for the aforementioned census years. Incidentally it may be noted that the census age distributions of India and its zonal sub-divisions according to 1961 and 1971 Censuses were smoothed by different procedures.^{2/} While constructing the life tables for India and its constituent zones for the decade 1961-71 using the above mentioned adjusted age data for 1961 and 1971 Censuses, the actuary of the 1971 Census found that in some cases the ten-year survival probabilities (${}_{10}P_{x+\frac{1}{2}}$) were close to unity, even sometime more than one. Thus, he found it desirable to smooth afresh the enumerated age-returns of 1961 Census for some of the zones where the ten-year survival probabilities were found to be inconsistent, by using the procedure adopted to adjust the 1971 Census age-returns (Office of the Registrar General of India, 1977a, p.2).

^{2/} In 1971 Census, the 'best' quinary age-grouping, where the errors due to age-misstatements were expected to be considerably low, was selected on the basis of Myer's blended sums and U.N. age-ratio score. The quinquennial group totals, so obtained, were adjusted by the three-point formula as used in the 1951 and 1961 Censuses. The adjusted five-year group totals were then distributed into single-year of ages through Grebill's weighted average of Spreque's coefficients (Office of the Registrar General of India, 1977b, pp. 23-63.)

The foregoing discussions lead to the conclusion that in the presence of serious age-misstatements, the survival ratios computed from the adjusted age data of two consecutive censuses are likely to be highly influenced by the arbitrary and subjective smoothing procedures which may be applied on the age-data of two consecutive censuses so as to remove the effect of age misreporting. Now, question may arise, can we tackle the situation without smoothing the primary age data at least for the purpose of construction of life tables from the age-distributions of two consecutive censuses? When the census age-returns are seriously distorted due to age-misstatements, obviously, the survival ratios computed from the single-year enumerated age-returns of two consecutive censuses will also be highly erratic in nature. This erratic nature of the survival ratios could be reduced to some extent, if the enumerated age data of the two consecutive censuses were grouped into quinquennial or decennial age groups before computing survival ratios. The selection procedure for appropriate quinquennial age-grouping is described in the following section.

1.2. Selection of Appropriate Quinquennial Age-Grouping:

There are five possible systems of 5-year age-groupings in which the lower limits of the first age intervals of the systems are below 5-years of age, namely, 0-4, 1-5, 2-6, 3-7 and 4-8. An arbitrary selection of any one of the aforementioned 5-year age-groupings may not give satisfactory results with regard to reduction in erratic behaviour of the survival ratios

based on the selected quinquennial age-grouping. For example, the Indian Census age-returns are characterized by very heavy concentration of populations at ages ending with the digits '0' and '5' (c.f. figures 2.1&2.2). While adjusting the Indian age data of 1951 census, the census actuary plotted the 5-year group totals on a graph paper against the age groups with respect to each of the five sets of quinquennial age-groupings as mentioned before, and he found that the group total with respect to 3-7 grouping run more smoothly in comparison to other groupings (Office of the Registrar General of India, 1954c). Thus, one way of selecting suitable age-grouping is to plot the group totals against the age-groups for all possible age-groupings and to choose the grouping in which the group totals run more smoothly in comparison to other age-groupings. This type of graphical selection of suitable age-grouping is rather subjective; since it involves certain amount of personal judgement in recognising the smoothness of the curves of age-structures with respect to various age-groupings. To avoid the personal bias in selecting the suitable quinquennial age-grouping among the five sets of age-groupings, a more scientific approach for selecting an appropriate one would be to choose the age-grouping in which the sum total (algebraic sum) of the digital biases at individual ages included within the group becomes negligibly small; and at the same time the 'extent of inaccuracy' of the grouped age data as per the selected age-grouping should be

considerably low, compared to those in the other age-groupings. This type of selection, adopted in 1971 Census of India before applying certain smoothing procedure to adjust the age data, can be made with the help of Myers' (1940) "blended" measures for digital preferences and age-ratio score, a measure of accuracy of census age-returns grouped into 5-year age-interval proposed by the United Nations (1952). Before going into the details of the selection procedure, one should have some idea regarding Myer's "blended" measures and 'U.N. age-ratio score' which are discussed briefly in the following paragraphs.

2.2.1. Myers' "blended" measures for digital preference:

To assess the digital preferences in the reported ages in a census or a survey, one may think of comparing the 10 totals P_i , i ranging from 0 to 9, where P_i denotes number of individuals recorded at ages ending in the digit i . But such a simple method, however, does not serve the purpose; since these sums will tend to decrease even in the absence of digital preference with the advancing terminal digits. This is so, because any two consecutive terminal digits, i and $i+1$, the population included in P_{i+1} is 'one-year older' than the preceding one (that is, the population included in P_i). To avoid this difficulty, Myers introduces the concept of "blended" sums of populations. To obtain the "blended" sum for each terminal digit, first of all, the 10-term series of sums of populations at ages terminating in the ten digits 0 to 9

are determined separately for the two half - open intervals - 10 and above and 20 and above^{3/} and then these two series are fused into a single series as a weighted sum of the two by assigning the successive weights 1, 2, 3, 9 & 10 to the former and 9, 8, 7,1 & 0 to the latter. In the absence of any digital bias, the "blended" sums should ~~be~~ almost same for the terminal digits. In other words, if the ages are correctly reported, the "blended" sum for each of the ten digits should be very close to 10 per cent of their grand total. Absolute deviations for each "blended" sum from 10 per cent of their grand total, when added together gives an overall measure of digital preference known as Myers' Index which can vary theoretically between 0 and 180 (Myers, 1940). There are two other types of measures indicating digital bias in the reported ages, namely, Whipple's index (Office of the Registrar General of India, 1924), which measures the extent of bias towards ages ending with the digits '0' and '5' and the other is known as Bachi's (1953) index which is more or less similar to Myer's index, but theoretically more sound; but its computational part is more laborious and time-consuming and it provides almost the same results in practice, as those obtained by the application of Myer's index (United Nations, 1955b, p.42).

^{3/} In India, the age-returns at very old ages are very unreliable. Therefore, the calculation for Myers' blended sums for each of the ten digits are restricted to the age groups 10 to 69 and 20 to 79.

2.2.2. Age-ratio score as a measure of accuracy of census age-returns grouped into five-year age-intervals

In case, the single-year census age-returns are seriously distorted due to age-misstatements, one may feel that if age data are grouped into five-year age intervals then the intensity of age-distortion (rather, age-group distortion) may not be very serious; because the errors (excess or deficit) at individual ages may get cancelled out substantially within the groups. Therefore, it should be examined how far in practice these quinquennial group totals of the raw census age-returns can be regarded as reasonably correct. For this purpose two tests based on age-ratios and sex-ratios at various quinquennial age groups, have been recommended by the United Nations Secretariat (1952). Age ratio may be defined as the number of persons reported in a particular age-group per 100 of the arithmetic mean of numbers in the two age groups adjacent to that age-group. It is calculated for males and females separately. When the ages are reported correctly, the age-ratios should be very close to 100 unless the population happens to be rather abnormal. Thus, any large deviation in the age-ratio from 100 may be taken as an indication of inaccurate age-reporting and/or omission to the enumeration.^{4/} The effect of the two factors cannot be separated out from the age-ratio. For the purpose of comparing the extent of accuracy

^{4/} Caution is needed in interpreting the results based on age-ratios because irregularities in the normal pattern of ratios may occur due to factors like war casualties, temporary birth deficits or age-selective migration; random fluctuations due to smallness of the total population may also affect the age-ratio.

in age-reporting among two or more populations whose primary age-returns are tabulated in the same system of quinquennial age-grouping, it is necessary to have an overall measure of age accuracy known as 'age-ratio score' which is defined as the average of the absolute deviations of the age-ratios from 100 at various age-groups. The other measures -(i) 'sex-ratio score', based on sex-ratios at successive age-groups, gives an idea regarding overall situation of relative age distortions between the two sexes and (ii) 'joint score' which is a combined index of sex-wise age-ratio scores and sex-ratio score. These scores are not considered here.^{5/}

It follows from the preceding paragraphs that the first step for selecting the appropriate quinquennial age-grouping is to calculate the Myers' "blended" percentages for each of the ten digits 0 to 9 from the enumerated age-data of the population under study. This will help in identifying the appropriate grouping, out of the five possible sets of quinquennial groupings, viz., 0-4, 1-5, 2-6, 3-7 and 4-8; for which the sub-total of the "blended" percentages corresponding to the five-terminal digits in the grouping is closest to 50 since, in the absence of any preference for or dislike to a particular digit, over the others the "blended" ~~sum~~ for the digit should be around 10 per cent of the grand total

^{5/} For the purpose of evaluation of census age-returns, United Nations have suggested certain standards on the basis of the age-ratio and sex-ratio scores calculated from Swedish Population having highly accurate age data. These standards are -- the age ratio scores of 2.6 for males, 2.4 for females, the sex ratio score of 1.5, and the joint score of 9.5 (United Nations, 1952, pp.69-70).

of the "blended" sums for all the ten digits taken together. It may happen in practice that the sub-totals of the "blended" percentages for more than one quinquennial age-groupings are very close to 50. In such a situation we make use of the age-ratio score which gives an indication of the extent of accuracy of the enumerated age data grouped into a specified 5-year age interval. Now, the procedure is to select the age-grouping out of those age-groupings chosen primarily through Myers' "blended" measures for digital preferences, that age-grouping which records the lowest age-ratio score; sometimes, however, additional grounds may favour another grouping with a score little higher than the lowest one.

In developing countries like India, where the extent of age-misstatements in census age-returns is of very high order, such a choice of quinquennial age-grouping as mentioned above, will undoubtedly reduce the effect of age-distortion, at the group level, considerably; but may not necessarily be sufficient enough to cancel out completely the positive and negative effects of single-year age-distortions within each of the quinquennial group totals. For example, while adjusting the Indian age-returns of 1971 Census, the census actuary found that the adoption of the best age-grouping 6/ reduces the age-distortion

6/ While analysing the accuracy of 1971 census age-returns for India and its constituent States, the census actuary observed that the groupings 2-6 and 4-8, in general, were found to be suitable in most of the States. The groupings 3-7 and 1-5 were most unsuitable in all the States. For special reasons few States were in favour of the grouping 0-4 which is most commonly used in tabulation. For further discussion (Office of the Registrar General of India, 1977 b, p.27).

considerably, but still the group totals when plotted on a graph paper do not appear to run sufficiently smoothly as shown by some ups and downs in the successive totals. The census actuary of 1951 Census adopted the 3-7 age-grouping for the purpose of adjusting the census age-returns and he found that the population concentration is relatively more in the age groups whose mid-ages have '0' as the terminal digit.

2.3. Utility of Cumulative Population at Age x in Estimating Cumulative Census Survival Ratio

Let us assume that the quinquennial age-grouping starting with the age group $(a, a+4)$, where $0 \leq a \leq 4$, is the most suitable one, selected on the basis of the procedure mentioned in the Section 2.3. It has already been mentioned in the previous paragraphs that in the presence of serious errors of age-reporting such a choice of age-grouping is definitely helpful in reducing the age-distortions at the group level; but mere classification of the census age-returns into some suitable quinquennial age-groups may not be sufficient enough to remove all sorts of errors due to age-misstatements. However, the errors remaining after presenting the age-data in suitable groups could be reduced further to a very large extent by considering the 'cumulated population at age x ' ^{7/} (where $x = a, a+5, a+10, \dots$). It is worthwhile to note in this connection that the 'cumulated population at age x ' is likely to be distorted by age-misstatements that transfer

^{7/} 'Cumulated population at age x ' means population at age x and above.

persons across age x , but it is not affected by any other forms of age-misstatements which causes transfer but not across age x (United Nations, 1967). It may be noted that the transfer across age x may be in either direction, and the direction and magnitude of distortion of the 'cumulated population' depends upon the net effect of both types of transfers. It should be emphasized here that the net effect of transfers of persons across the age x in the cumulated total is likely to be controlled to a large extent if x is at an end point of the 'best' age-intervals selected by the procedure described elsewhere in this chapter.

Now obtaining the cumulated populations at ages a , $a+5$, $a+10$ etc.^{8/} for any two successive census dates ten-years apart, for example, one can easily calculate the 'cumulative census survival ratios' ^{9/} during the intercensal period as a ratio of the 'cumulated population at age $x+10$ ' in the later census to the 'cumulated population at age x ' according to the earlier census. It may be noted that these ratios which may be called '10-year cumulative census survival ratios', are likely to be distorted to some extent by the joint effect of net transfers of persons across age $x+10$ in the later census and across age

^{8/} Where 'a' is the lower end-point of the age-interval specifying the 'best' age-grouping.

^{9/} The term 'cumulative census survival ratio' does not mean the cumulation of the usual census survival ratios, but it can be obtained as a ratio of the cumulated population at age $x+10$ in the second decennial census to the cumulated population at age x according to the immediately previous census. The term has been introduced here so as to distinguish it from the usual survival ratios during the intercensal period.

x in the earlier census, but these ratios remain unaffected by all other transfers due to age-misstatement. But, if the pattern of age-misreporting differs significantly in the two consecutive censuses under consideration, then it will be rather difficult to compute the 10-year cumulative census survival ratios $\frac{10}{10\text{-cum-CSR}}$. If for example, it is found on the basis of Myers' "blended" measures for digital preference and U.N. age-ratio score that the age-grouping starting with 2-6 is most suitable for the first census; whereas 4-8 grouping is found to be the best for the second census, then it is not possible to obtain 10-cum-CSR directly from the cumulated populations of the two consecutive censuses. Because the ages at the end-points of the systems of groupings, at which cumulation are to be carried out, are different in the two censuses. However, this type of difficulties can be overcome by making one of the age-groupings compatible with the other and this can be done by adopting the procedure described in the following lines. The group totals of, say, the second census as per the selected 4-8 grouping are kept intact. But the quinquennial group totals of the first census according to the selected grouping starting with the group 2-6 are first converted into single-year age distribution by using Sprague multipliers or Beers multipliers for "ordinary" interpolation (Shryock and Siegel, 1960, pp. 876-877). These estimated populations at single-years of

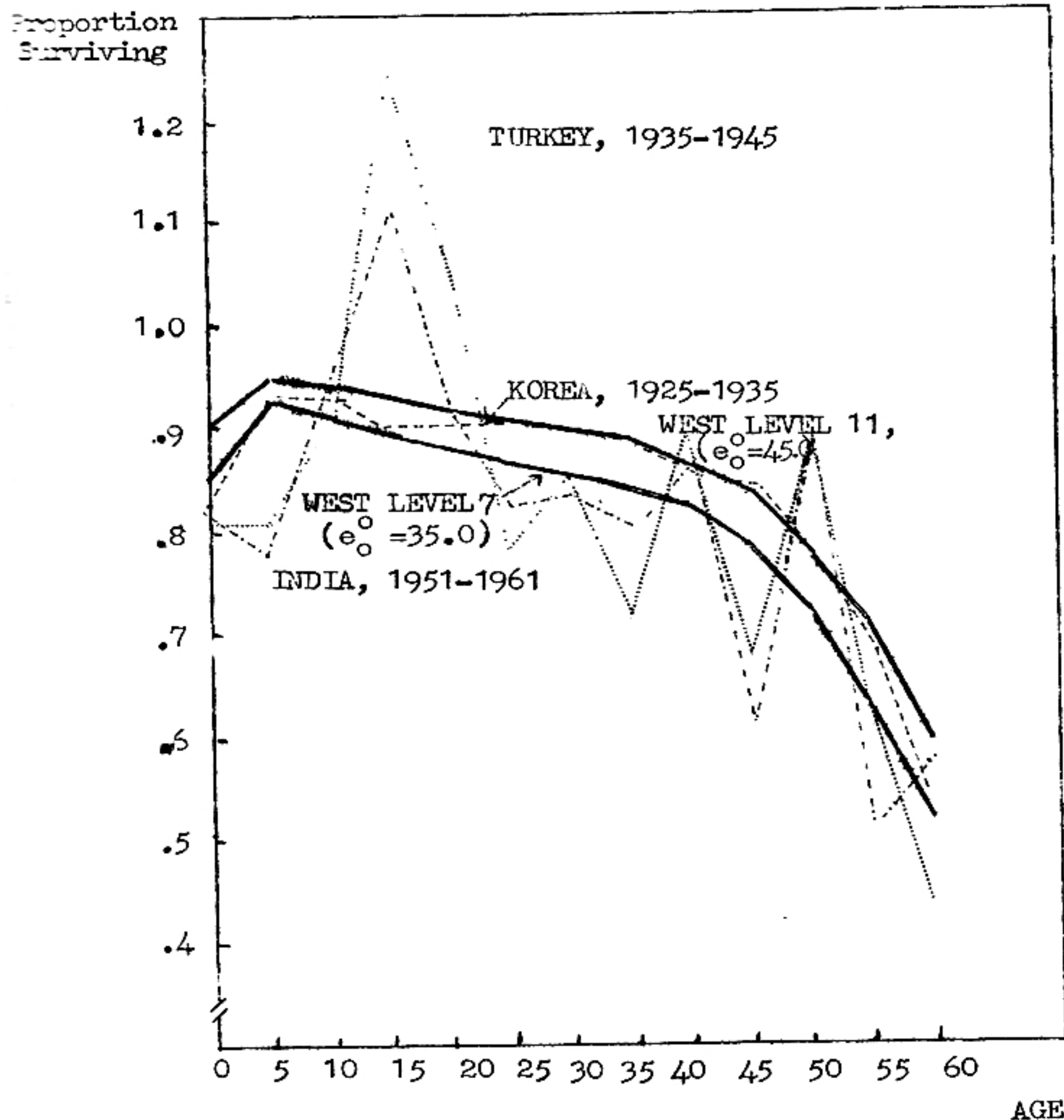
Henceforth, the term '10-year cumulative census survival ratio' will be denoted by '10-cum-CSR' and the usual 10-year census survival ratios will be indicated simply by the symbol '10-CSR'.

age are grouped according to the 4-8 grouping so as to make it comparable with the grouped populations according to the second census. Now, these new sets of quinquennial group totals pertaining to the first census together with those of the second census may be used in the standard way to obtain the desired '10-cum-CSRs'.

From the foregoing discussions on cumulative census survival ratios, it appears that the effect of age-misreporting is considerably low in '10-cum-CSR' due to dampening effect of cumulated populations compared to that in the usual census survival ratios (10-CSR). However, it would be more enlightening if one could justify the above finding empirically on the basis of some actual age-returns which suffer from very heavy age-misreporting. Coale and Demeny presented graphically 10-CSRs and 10-cum-CSRs in the standard form for three developing countries, namely, India (1951-61), Korea (1925-35) and Turkey (1935-45) calculated from their respective census age-returns (United Nations, 1967, pp. 9-11). For the benefit of the readers the graphs are reproduced here (c.f. Figures 2.3 & 2.4). It is interesting to note from the graphical representation of the 10-CSRs of the above three countries shown in Figure 2.3, borrowed from UN Manual IV, that the extents of age-misreporting in India and Turkey are of very high order, whereas that of Korea is considerably low. Furthermore, the graphical representation of '10-cum-CSR' of the above three countries depicted in Figure 2.4,

FIGURE - 2.3

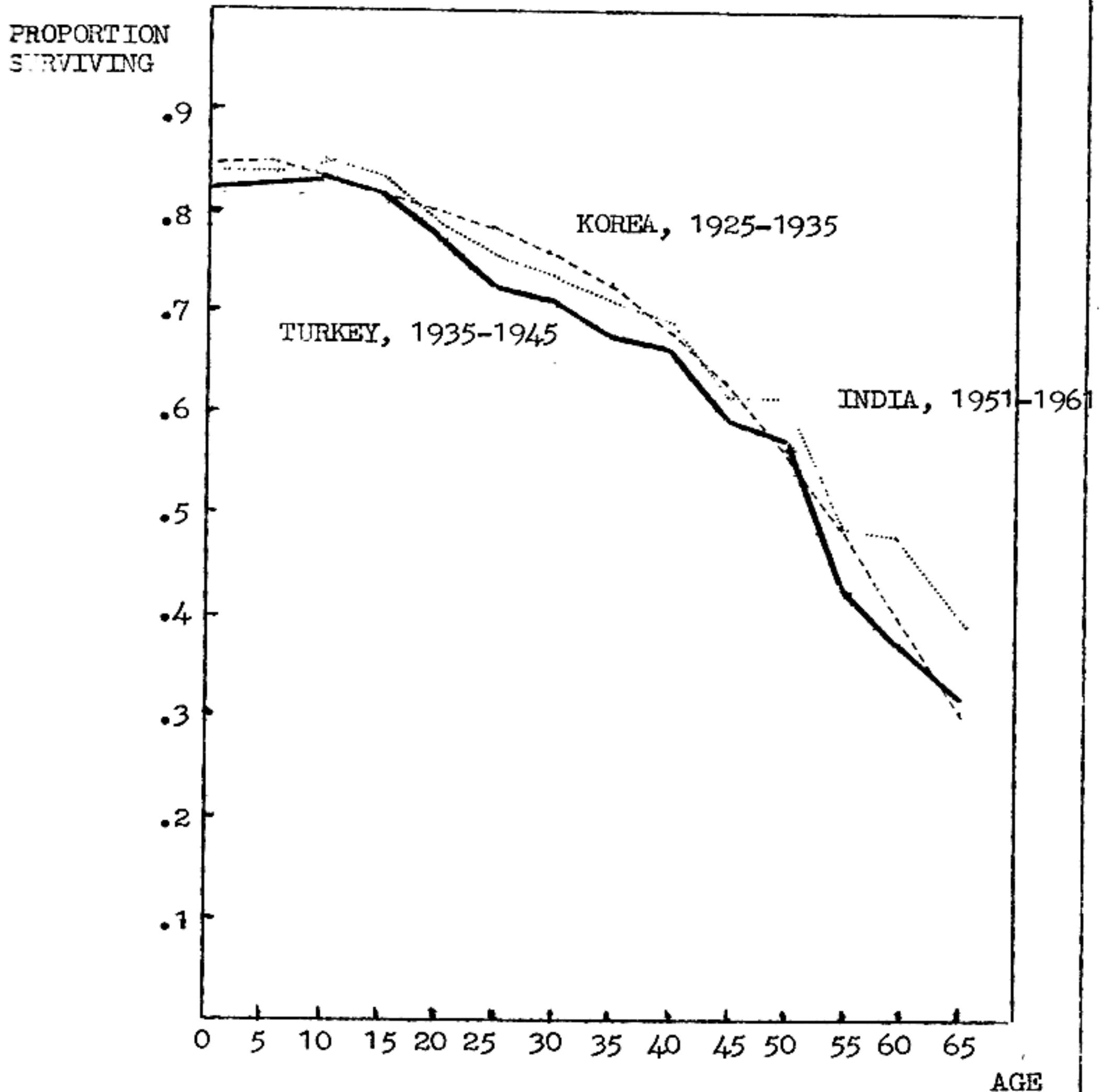
Usual Census Survival Ratios of Females from Age x to $x+5$ at time t to age $x+10$ to $x+15$ at time $t+10$, according Censuses of India, Korea and Turkey, and according to Selected "West" model life tables



Source: United Nations, 1967. Methods of Estimating Basic Demographic Measures from Incomplete Data, Manual IV, (Population Studies No.42: ST/SGA/SER.A/42), New York, p.9.

FIGURE 2.4

Cumulative Census Survival Ratios of Females
from Age x and above at time t to Age $x+10$ and
above at time $t+10$ according to Censuses of India,
Korea and Turkey



Source: United Nations, 1967, Methods of Estimating Basic Demographic Measures from Incomplete Data, Manual IV, (Population Studies No.42: ST/SGA/SER.A/42), New York, p.10.

clearly shows that dampening effect of cumulation on '10-cum-CSR' is remarkable. It may be noted here that the 10-CSRs and the '10-cum-CSRs' are based on the census age-returns grouped in standard quinquennial age-grouping that is, (0-4 grouping) and cumulated populations at the corresponding quinquennial ages respectively. Some zigzags found in the graphs of '10-cum-CSR' as shown in Figure 2.4, could probably be controlled considerably if the quinquennial age-groupings were chosen suitably through the procedure, mentioned in the Section 2.3, prior to obtain the '10-CSR' and '10-cum-CSR',^{11/}.

The above discussions reveal that the presence of serious age-misreporting makes the '10-CSRs' highly erratic, whereas, the '10-cum-CSRs' are much less affected by such age-misreporting. Furthermore, the adjusted series of 10-CSRs, commonly used to construct life table, are highly influenced by the smoothing procedures applied on raw census age-returns so as to remove the effect of age-misreporting. Therefore, in such a situation, if it is possible to construct the life table from '10-cum-CSRs' instead of '10-CSRs', the resultant life table is expected to be more close to the actual one. It should be noted here that unlike the usual census survival ratio, the '10-cum-CSRs' cannot be easily translated into survival probabilities (${}_5p_x$) because the latter survival ratios are not only affected by the mortality at various age-sectors but also influenced by the age distribution of the population under study. In the next chapter an attempt has been made to develop a technique for constructing life table from '10-cum-CSR'.

^{11/} For further discussion see chapter V.

CHAPTER III

DEVELOPMENT OF THE NEW TECHNIQUE

The technique developed here may be applied to any population, closed to migration, whose age-sex distributions, collected through population censuses are available at two different points of time - five or ten-years apart. The present chapter is divided into three parts. The first part deals with the development of various formulas for estimating usual and cumulative life table survival ratios from the corresponding usual and cumulative population survival ratios under various assumptions (described later on). In the second part, an attempt has been made to search for an appropriate method, after discussing various possibilities, for construction of life table from cumulative life table survival ratios and finally an iterative procedure has been proposed for life table construction (beyond certain childhood age) from the cumulative life table survival ratios.^{1/} A new technique for estimating infant and childhood mortality rates is shown in the third part.

^{1/} The procedure developed here for life table construction by using cumulative life table survival ratios is different from the well-known ones.

3.1. Relationship Between Population Survival Ratios (Usual and Cumulative) with those of Life Table Survival Ratios (Usual and Cumulative)

To establish the relationship between population survival ratios and life table survival ratios, it is necessary to take into account the intercensal period in advance. Let us assume that the intercensal period is ten-years. Procedure similar to that described below, can be followed when the interval between the two consecutive censuses is five or some integral multiple of five.^{2/}

Notations used:

In addition to the standard life table notations, the following notations will be used for the purpose of establishing the requisite relationships between population survival ratios and life table survival ratios. Some subsidiary notations may be used whenever needed; and those notations will be described at the stage where they are required. However, the major notations, used here, are described below. The notations which are not usually found in the literature of life table should be noted carefully. The term 'age $\underline{x}$ ', used in connection with a census, stands for age $\underline{x}$ last birthday (l.b.d.). But when $\underline{x}$ has reference to a theoretical or mathematical model

^{2/} In most of the countries of the present world where there is a system of population census, the interval between any two consecutive censuses is five or ten-years. In the present study, this interval is taken as ten-years because the technique, proposed here, is applied on the age-data of India where population census takes place at an interval of ten-years.

it ($\underline{x}$) stands for the exact age $\underline{x}$. Thus $\underline{x}$ is discrete (integral) variable for a census, but a continuous one for a model.

Census dates: A superscript of $\underline{t}$ has reference to the earlier or first census, and $t+10$ indicates the next census taken 10 years later. Sometimes $\underline{t}$ may be used for the variable -- time $\underline{t}$; this will be clear from the context.

w : Maximum age attainable by a person in the population under study.

P_x^t : Number of persons at age $\underline{x}$ according to the census date t .

P_x^{t+10} : Number of persons at age $\underline{x}$ according to the census date $t+10$ taken 10-years after the first census date t .

${}_5P_x^t$ and ${}_5P_x^{t+10}$ } Number of persons in the age group $(x, x+4)$ according to the census dates t and $t+10$ respectively

P_{x+}^t and P_{x+}^{t+10} } Number of persons at age x and above according to the census dates t and $t+10$ respectively.

P^t and P^{t+10} } Total number of persons according to the census dates t and $t+10$ respectively. Thus $P^t = P_{0+}^t$ and $P^{t+10} = P_{0+}^{t+10}$.

- r : If $p^{t+10} = e^{10r} \cdot p^t$, the r is called the annual (exponential) growth rate of the population during the intercensal decade.
- ${}_5r_x$: Annual (exponential) growth rate of the population in the age group $(x, x+4)$ during the intercensal decade.
- r_{x+} : Annual (exponential) growth rate of the population of age $x+$, i.e., population belonging to the age-interval (x, w) during the intercensal decade.
- $10^S_{x+2.5}$: 10-year usual census survival ratio (10-CSR) between the population in the age group $(x+10, x+14)$ in the second census to that in the age group $(x, x+4)$ in the first census.
- 10^S_{x+} : 10-year cumulative census survival ratio (10-cum-CSR) between the population in the age group $(x+10, w)$ in the second census to that in the age group (x, w) in the first census.
- ${}_5L_x$: Number of person - years lived in the age-interval $(x, x+5)$ according to life table (stationary) population model.
- $10^S_{x+2.5}$: 10-year usual life table survival ratio (10-LSR) between life table population in the age-interval $(x+10, x+15)$ to that in the age-interval $(x, x+5)$.

T_x : Number of person - years lived at age $\underline{x}$ and above according to life table (stationary) population model.

10^S_{x+} : 10-year cumulative life table survival ratio (10-cum-LSR) between life table population in the age-interval $(x+10, w)$ to that in the age-interval (x, w) .

e_x^0 : Complete expectation of life at age $\underline{x}$ (exact) according to the life table under consideration.

$5E_x$: Ratio of the expectation of life at exact age $x+5(e_{x+5}^0)$ to that at exact age $\underline{x}(e_x^0)$.

N_x^t } The age-structure of the model (which the
 $5N_x^t$ } study population is assumed to follow), may
 be described by a function N_x^t of $\underline{x}$ such
 that the number of persons between the exact
 ages $\underline{x}$ and $x+dx$ at time $\underline{t}$ is given by
 $N_x^t dx$. It follows that the number of persons
 $5N_x^t$, for example, between the exact ages $\underline{x}$
 and $x+5$ at time $\underline{t}$ is

$$5N_x^t = \int_x^{x+5} N_x^t dx.$$

It may be noted here that the superscript $\underline{t}$ of N_x , used here, is a variable (time) and not the first census indicator.

N_{x+}^t : Number of persons at age x and above at time t in the population according to the assumed mathematical model of the age-structure.

r_x^t : If $\frac{1}{N_x^t} \frac{d}{dt} N_x^t = r_x^t$, then r_x^t is called the (instantaneous) relative growth rate of persons aged x at time t according to the model.

$\left. \begin{array}{l} r \\ 5r_x \\ r_{x+} \end{array} \right\}$ These symbols have been used already with reference to the census. However, the symbols will be used in connection with mathematical models also - but no confusion is likely to arise because the correct meaning will be clear from the context in which they are used. The defining equations for the model are the same as those for the census excepting for the fact that the P's appearing in the census relations are to be replaced by the corresponding N's.

$10S'(x+2.5)$: 10-year usual survival ratio in the population (10-PSR) according to the assumed mathematical model of the age-structure.

$10S'(x+)$: Ten-year cumulative survival ratio in the population (10-cum-PSR) according to the assumed mathematical model of the age-structure.

Using the notations, described above, the 'ten-year usual

population survival ratio' (10-PSR) and 'ten-year usual census survival ratio' (10-CSR) from the age-interval $(x, x+5)$ to $(x+10, x+15)$ during the period $(t, t+10)$ are given by

$$\begin{aligned} & \text{10-PSR between } (x, x+5) \text{ to } (x+10, x+15) \\ & = {}_{10}S'(x+2.5) = \frac{{}_5N_{x+10}^{t+10}}{{}_5N_x^t} \dots \dots (3a) \end{aligned}$$

$$\begin{aligned} & \text{10-CSR between } (x, x+4) \text{ to } (x+10, x+14) \\ & = {}_{10}S_{x+2.5} = \frac{{}_5P_{x+2.5}^{t+10}}{{}_5P_x^t} \dots \dots (3b) \end{aligned}$$

The 'ten-year cumulative population survival ratio' (10-cum-PSR) at age $\underline{x}$ during time period $(t, t+10)$ and 'ten-year cumulative census survival ratio' (10-cum-CSR) at age $\underline{x}$ during the intercensal decade are given by

$$\text{10-cum-PSR at age } \underline{x} = {}_{10}S'(x+) = \frac{N_{(x+10)+}^{t+10}}{N_{x+}^t} \dots (3c)$$

$$\text{10-cum-CSR at age } \underline{x} = {}_{10}S_{x+} = \frac{P_{(x+10)+}^{t+10}}{P_{x+}^t} \dots (3d)$$

Similarly, the '10-LSR' between the age intervals $(x, x+5)$ and $(x+10, x+15)$ and '10-cum-LSR' at age $\underline{x}$, denoted by ${}_{10}S_{x+2.5}$ and ${}_{10}S_{x+}$ respectively, are defined by

$${}_{10}S_{x+2.5} = \frac{{}_5L_{x+10}}{{}_5L_x} \text{ and } {}_{10}S_{x+} = \frac{T_{x+10}}{T_x}$$

In this section an attempt will be made to establish the relationships between 10-PSR $\left[{}_{10}S'(x+2.5) \right]$ and 10-LSR $\left[{}_{10}S_{x+2.5} \right]$ and also between 10-cum-PSR at age x $\left[{}_{10}S'(x+) \right]$ and 10-cum-LSR at age x $\left[{}_{10}S_{x+} \right]$ under the assumptions that the age-structure of the population under study can be adequately described by the following theoretical model of age-structure corresponding to (i) stationary population, (ii) stable population developed by Lotka (1939), and (iii) more generalised model developed, most recently, by Preston and Coale (1982).

3.1.1. The population under study is stationary

A population is said to be stationary if the number of births occurring every year is same as that of deaths within the population under study with an unchanging schedule of mortality rates. According to the definition of a stationary population, stated above, the number of persons at each age (N_x) remains unaltered over time. Thus, one may easily find the following properties of a stationary population.

$$N_x^t = N_x \text{ for all } t.$$

$$\frac{1}{N_x} \frac{dN_x}{dx} = - \mu(x) \quad \dots \quad (3.1)$$

where $\mu(x)$ is the age specific mortality rate (or instantaneous force of mortality) at exact age x . Since

N_x is independent of t , $\mu(x)$ is also so. The age-structure of such a population is given by the L_x column of the appropriate life table which is a mathematical model, as described in Chapter I, based on the age specific mortality rates $[\mu(x)]$ defined by the equation (3.1).

According to the definition of life table, the person - years which will be lived, by a cohort of l_x persons now at exact age x , in the age-interval $(x, x+5)$ and in the age-range x and above are respectively

$${}_5L_x = \int_x^{x+5} l_y dy \quad \dots \dots (3.2a)$$

$$T_x = \int_x^w l_y dy \quad \dots \dots (3.2b)$$

where l_y is the number of persons surviving to exact age y out of l_0 births and the meaning of the symbol w has already been stated earlier.

The ten-year usual survival ratio (${}_{10}S_{x+2.5}$) and ten-year cumulative survival ratio (${}_{10}S_{x+}$), as defined earlier, in the life table population (stationary), generated through the age-specific mortality rates at various ages defined by the equation (3.1), are given by

$$10^S_{x+2.5} = \frac{5L_{x+10}}{5L_x} = \frac{\int_{x+10}^{x+15} l_y dy}{\int_x^{x+5} l_y dy} \dots\dots (3.3)$$

$$\text{and } 10^S_{x+} = \frac{T_{x+10}}{T_x} = \frac{\int_{x+10}^w l_y dy}{\int_x^w l_y dy} \dots\dots (3.4)$$

Since the population under study is considered as stationary, the number of persons at exact age x (N_x) may be treated as an observed value of l_x , the number of survivors at exact age x out of l_0 infants born at the same point of time and exposed to the risk of dying defined by (3.1) throughout their life time. Thus, by replacing l_y by N_y in (3.3) and (3.4), we get

$$10^S_{x+2.5} = \frac{5L_{x+10}}{5L_x} = \frac{\int_{x+10}^{x+15} N_y dy}{\int_x^{x+5} N_y dy} = \frac{5N_{x+10}}{5N_x} = 10^{S'}(x+2.5) \dots\dots (3.5)$$

Since in the present case $5N_{x+10}^t = 5N_{x+10}$ and $5N_x^t = N_x$ by the virtue of the stationary nature of the population,

$$\text{and } 10^S_{x+} = \frac{T_{x+10}}{T_x} = \frac{\int_{x+10}^w N_y dy}{\int_x^w N_y dy} = \frac{N_{(x+10)+}}{N_{x+}} = 10^{S'}(x+) \dots\dots (3.5a)$$

It may be noted here that in the case of a stationary population, ${}_5N_x^t$ and N_x^t remain unaltered over time, so that ${}_5N_{x+10} = {}_5N_{x+10}^t$, ${}_5N_x = {}_5N_x^t$ and $N_{(x+10)+} = N_{(x+10)+}^{t+10}$, $N_{x+} = N_{x+}^t$. Now, the P's given by the censuses can be regarded as the 'observed' values of the corresponding N's. Here, ${}_{10}S'_{(x+2.5)}$ and ${}_{10}S'_{(x+)}$ may be estimated by ${}_{10}S_{x+2.5}$ and ${}_{10}S_{x+}$ respectively. Thus, we find that in the case of a stationary population '10-CSR-' (${}_{10}S_{x+2.5}$) and 10-cum-CSR- (${}_{10}S_{x+}$) are identical to (in the sense of 'estimate of'), '10-LSR-' (${}_{10}S_{x+2.5}$) and 10-cum-LSR- (${}_{10}S_{x+}$). Furthermore, in the case of a stationary population ${}_{10}S'_{(x+2.5)}$ and ${}_{10}S'_{(x+)}$ can be estimated from any single census, (say, at time t) since ${}_5N_{x+10}^{t+10}$, ${}_5N_x^t$, $N_{(x+10)+}^{t+10}$, N_{x+}^t may be estimated by the 'observed' values of ${}_5P_{x+10}^t$, ${}_5P_x^t$, $P_{(x+10)+}^t$, P_{x+}^t as the N's remain unchanged over time.

To obtain the relationships between 10-PSR- $[{}_{10}S'_{(x+2.5)}]$ and 10-LSR- (${}_{10}S_{x+2.5}$) and between 10-cum-PSR- $[{}_{10}S'_{(x+)}]$ and 10-cum-LSR- (${}_{10}S_{x+}$) for a non-stationary population, an allowance must be made for the distribution of population within each age group of the whole life span, because the numerical distribution (not percentage distribution) of persons by age of the population

under study (non-stationary) changes over time which is in sharp contrast to that of a stationary population. The requisite relationships in the case of a non-stationary population are dealt with in the next paragraphs.

3.1.2. The population under study is stable

A population is said to be stable if the fertility and mortality schedules acting on the population remain unaltered over time and the number of births occurring in the population changes with time at a constant rate $\underline{r}$. In such population, it can be shown that the number of persons at each individual age also changes over time at the constant rate $\underline{r}$. The following relations hold good for a stable population.

$$B(t_0+t) = B(t_0) e^{rt} \quad \dots \quad (3.6)$$

$$N_x^t = B(t) e^{-rx} p(x) \quad \dots \quad (3.7)$$

$$\text{and } N_x^{t_0+t} = N_x^{t_0} \cdot e^{rt} \quad \dots \quad (3.8)$$

where,

$N_x^t dx$: Number of persons between the exact ages $\underline{x}$ and $\underline{x}+dx$ at time $\underline{t}$

$B(t)dt$ Number of births occurring in the population in the time interval $(t, t+dt)$,

r : Intrinsic rate of growth per person,

$p(x)$: Probability of surviving from birth to exact age $\underline{x}$ independent of time $\underline{t}$.

According to the life table notation $p(x) = l_x/l_0$. The symbols l_x and l_0 have already been mentioned earlier.

It may be noted here that $p(x)$ remains unaltered over time t ; whereas N_x^t and $B(t)$ change over time through the equations (3.8) and (3.6) respectively. Now, putting $p(x) = l_x/l_0$ in (3.7), we get

$$N_x^t = B(t) e^{-rx} l_x, \text{ where } l_0 = 1 \quad \dots \dots (3.9)$$

(without loss of generality l_0 is assumed to be unity).

3.1.2(a) Relationship between '10-PSR' and '10-LSR'

Integrating both sides of (3.9) in the age-range $(x, x+5)$, we get

$${}_5N_x^t = \int_x^{x+5} N_y^t dy = B(t) \int_x^{x+5} e^{-ry} l_y dy$$

Now, by using the mean value theorem^{3/} of integral calculus, the equation becomes

$${}_5N_x^t = B(t) e^{-r \cdot C_x} \int_x^{x+5} l_y dy$$

^{3/} According to the first mean value theorem of integral calculus, if the function $f(x)$ is continuous and bounded

$$\text{or } {}_5N_x^t = B(t) e^{-r \cdot C_x} {}_5L_x \quad \dots \quad (3.10)$$

where, C_x is an intermediate value between x and $x+5$.

It may be noted here that the mean value theorem of integral calculus only states that there exists a value C_x lying between x and $x+5$ which satisfies the equation (3.10). But it does not give any explicit expression for the exact value of C_x . Thus, in practical situation, it is quite difficult to obtain the exact value of C_x ; however, a reasonably good approximation to C_x

(Ref. 3/ contd.)

in the interval (a, b) and the function $\phi(x)$ is integrable and keeps the same sign positive or negative, throughout the interval of integration (a, b) , there exists a value of x (say, C_x) in the interval such that $\int_a^b f(x) \cdot \phi(x) dx = f(C) \int_a^b \phi(x) dx$, where $a < C < b$. In the present case, let us take $f(x) = e^{-rx}$ and $\phi(x) = l_x$. It may be noted that the function $f(x)$, being an exponential function of x , is a continuous function. Furthermore, one can easily show that the minimum and maximum values of $f(x)$ as x varies between a to b are e^{-b} and 1 respectively, since r , being a growth rate, lies between 0 and 1. Thus, $f(x)$ is also bounded in (a, b) . The function $\phi(x)$ is continuous in (a, b) , since the life table function l_x is known to be continuous. Thus, $\phi(x)$ is integrable in (a, b) , since all continuous functions are integrable. (For details, see Ferrar, 1958; Chatterjee, 1970).

may be taken as $x+2.5$, the mid-point of the five-year age-interval $(x, x+5)$ - provided it does not contain the first few years of life -- because the population within such a five-year interval may be assumed to be fairly uniformly distributed over the interval. (The problem of earlier ages will be considered separately).

Thus, the equation (3.10) becomes,

$${}_5N_x^t \doteq B(t) \left[e^{-(x+2.5)r} \right] {}_5L_x, \quad \text{for all } x > 0$$

... (3.11)

Such an approximation of C_x which minimizes the maximum possible error in the estimate, since $x < C_x < x+5$ has been used by several authors (Lopez, 1961; Office of the Registrar General of India, 1963; and Coale and Demeny, 1966). The expression for ${}_5N_x^{t+10}$ can be obtained from (3.11) by replacing t by $t+10$. That is,

$${}_5N_x^{t+10} \doteq B(t+10) \left[e^{-(x+2.5)r} \right] {}_5L_x, \quad \text{for all } x > 0$$

By using the relation (3.6), we get

$${}_5N_x^{t+10} \doteq B(t) e^{10r} \cdot e^{-(x+2.5)r} {}_5L_x, \quad \text{for all } x > 0$$

... (3.12)

Now,

$${}_{10}S'(x+2.5) = \frac{{}_5N_{x+10}^t}{{}_5N_x^t} \doteq \frac{e^{10r} e^{-(x+10+2.5)r} \cdot {}_5L_{x+10}}{e^{-r(x+2.5)} \cdot {}_5L_x}$$

or ${}_{10}S'(x+2.5) \doteq {}_{10}S_{x+2.5}$, for all $x > 0$ (3.13)

3.1.2(b) Relationship between '10-cum-PSR' at age x and '10-cum-LSR' at age x

Integrating (3.9) in the interval (x, w) , we get

$$N_{x+}^t = B(t) \int_x^w e^{-ry} l_y dy$$

Making use of mean value theorem of integral calculus, we get

$$N_{x+}^t = B(t) e^{-r \cdot C_{x+}} \int_x^w l_y dy$$

or $N_{x+}^t = B(t) (e^{-r \cdot C_{x+}}) T_x$ (3.14)

where, $x < C_{x+} < w$ and $T_x = \int_x^w l_y dy$

It is quite difficult to obtain the true value of C_{x+} . However, a reasonably good approximation to C_{x+} may be obtained as a weighted average of the approximations $x+2.5$, $x+7.5$, etc., to the C_x 's for the quinquennial age-intervals covering the interval (x, w) , the weights being

the populations in the corresponding age intervals.

Since we are mainly interested to estimate 10-cum-PSR- defined by

$$10\text{-cum-PSR} = \frac{N_{(x+10)+}^{t+10}}{N_{x+}^t}$$

we need to know a reasonable good estimate of C_{x+} for the decade $(t, t+10)$. Such an approximation of C_{x+} which gives an average picture during the decade $(t, t+10)$ is given below:

$$\hat{C}_{x+} = \frac{\sum_{y=x}^{x+5} (y+2.5) {}_5N_y^* + \sum_{y=1}^w (y+0.5) {}_1N_y^*}{\sum_{y=x}^{x+5} {}_5N_y^* + \sum_{y=1}^w {}_1N_y^*} \dots (3.15)$$

where, $(1, w)$ is the last interval whose width is less than 5 years and ${}_5N_y^* = \left[{}_5N_y^t + {}_5N_y^{t+10} \right] / 2$, and the meaning of ${}_1N_y^*$ is obviously similar. This approximation of C_{x+} , as mentioned before, seems to be quite reasonable and simple. It may be noted, while obtaining the approximation of C_{x+} , through the formula (3.15), that the average of the population at time t and $t+10$ has been used for the purpose of weighting so as to obtain an estimate $\hat{C}_{x+}$ during the period $(t, t+10)$. Some points in favour of

this sort of approximation of C_{x+} ^{4/} for the decade (t, t+10), as mentioned below, may be noted.

Since we are interested to know an approximation of C_{x+} which gives an average picture during the decade (t, t+10), it may be assumed that C_{x+} relates to the time t+5, mid-point of the decade (t, t+10). Let us consider the following integrals:

$$I = \int_x^w e^{-ry} \cdot N_y^{t+5} dy \quad \dots \quad \dots \quad (3.15a)$$

where $N_y^{t+5} dy$, the number of persons in the interval (y, y+dy) at time t+5, may be defined by (i) stationary (life table) model, (ii) stable population model, or (iii) Preston-Coale generalised model. In the case of stationary model $N_y^{t+5} = l_y$, in the cases of stable and Preston-Coale generalised models N_y^{t+5} 's are defined by the equations (3.7) and (3.21) respectively.

4/ Preston and Bennett (1983) proposed a different procedure for estimating C_{x+} based on the principle of successive approximation under the assumption of stability of the population under study. This approach for estimating C_{x+} is not utilised here since the present investigation on life table construction is based on cumulative survival ratios under the assumption of Preston-Coale generalised model. This will be clear from the context of the discussions, described later on [see the Section 3.1.4(b)].

Now applying mean value theorem of integral calculus, the integral (3.15a) becomes

$$I = e^{-r \cdot C_{x+}} \int_x^w N_y^{t+5} dy, \quad \text{where } x < C_{x+} < w$$

$$\text{or } I = e^{-r \cdot C_{x+}} \cdot N_{x+}^{t+5}, \quad \text{where } x < C_{x+} < w \quad \dots (3.15b)$$

Once again the integral (3.15a) may be sub-divided as follows:

$$\begin{aligned} I = \int_x^w e^{-ry} \cdot N_y^{t+5} dy &= \int_x^{x+5} e^{-ry} \cdot N_y^{t+5} dy + \int_{x+5}^{x+10} e^{-ry} \cdot N_y^{t+5} dy \\ &+ \int_{x+10}^{x+15} e^{-ry} \cdot N_y^{t+5} dy + \dots \end{aligned}$$

Now, applying mean value theorem within each of the sub-intervals we obtain,

$$\begin{aligned} I \doteq e^{-r \cdot C_x} \cdot 5N_x^{t+5} &+ e^{-r \cdot C_{x+5}} \cdot 5N_{x+5}^{t+5} \\ &+ e^{-r \cdot C_{x+10}} \cdot 5N_{x+10}^{t+5} + \dots \end{aligned}$$

where, $C_x (\doteq x+2.5)$ lies in the interval $(x, x+5)$ [see section 3.1.2(a)]

Now writing $e^{-rz} \doteq 1 - rz$ under the assumption that

rz is small (it may be noted $\underline{r}$, the rate of growth,

is a very small quantity), the above expression for the integral I may be written as,

$$I \doteq (5^{N_x^{t+5}} + 5^{N_{x+5}^{t+5}} + 5^{N_{x+10}^{t+5}} + \dots) + r \left[C_x \cdot 5^{N_x^{t+5}} + C_{x+5} \cdot 5^{N_{x+5}^{t+5}} + C_{x+10} \cdot 5^{N_{x+10}^{t+5}} + \dots \right]$$

$$\text{or } I \doteq N_{x+}^{t+5} + r \left[C_x \cdot 5^{N_x^{t+5}} + C_{x+5} \cdot 5^{N_{x+5}^{t+5}} + C_{x+10} \cdot 5^{N_{x+10}^{t+5}} + \dots \right] \dots (3.15c)$$

$$\text{where } N_{x+}^{t+5} = 5^{N_x^{t+5}} + 5^{N_{x+5}^{t+5}} + 5^{N_{x+10}^{t+5}} + \dots$$

Similarly, (3.15b) can be written as

$$I \doteq (1 - r \cdot C_{x+}) N_{x+}^{t+5} \dots \dots (3.15d)$$

Now, using (3.15c) and (3.15d), we get

$$C_{x+} \doteq \frac{C_x \cdot 5^{N_x^{t+5}} + C_{x+5} \cdot 5^{N_{x+5}^{t+5}} + C_{x+10} \cdot 5^{N_{x+10}^{t+5}} + \dots}{5^{N_x^{t+5}} + 5^{N_{x+5}^{t+5}} + 5^{N_{x+10}^{t+5}} + \dots} \dots \dots (3.15e)$$

Now, since the population at time $t+5$ is not known, an estimate of $5^{N_x^{t+5}}$ may be obtained as follows:

$$5^{N_x^*} = 5^{N_x^{t+5}} = \frac{1}{2} (5^{N_x^t} + 5^{N_x^{t+10}})$$

Now, the 10-cum-PSR at age $\underline{x}$ during $(t, t+10)$, obtained by using the relation (3.14) is given by

$${}_{10}S'(x+) = \frac{N_{(x+10)+}^{t+10}}{N_{x+}^t} = \frac{B(t+10)e^{-r \cdot C_{(x+10)+}}}{B(t)e^{-r \cdot C_{x+}}} \cdot \frac{T_{x+10}}{T_x},$$

for all $x > 0$ (3.16)

where C_{x+} and $C_{(x+10)+}$ approximated through the formula (3.15), are some intermediate values belonging to the intervals (x, w) and $(x+10, w)$ respectively. Now, replacing T_{x+10}/T_x by ${}_{10}S_{x+}$ and using the relation $B(t+10) = B(t)e^{10r}$, the relation (3.16) becomes:

$${}_{10}S'(x+) = \text{Exp} [(10 - C_{(x+10)+} + C_{x+})r] \cdot {}_{10}S_{x+},$$

for all $x > 0$ (3.17)

Knowing the estimate or approximate values of

${}_{10}S'(x+)$, C_{x+} , $C_{(x+10)+}$ and $\underline{r}$, one can easily obtain the estimated value of ${}_{10}S_{x+}$, the 10-cum-LSR- at age $\underline{x}$, from the equation (3.17). The value of ${}_{10}S'(x+)$ can be estimated by ${}_{10}S_{x+}$ which can be obtained from the two consecutive census age data through the formula (3d). The estimates of C_{x+} and $C_{(x+10)+}$ can be obtained

through (3.15) by replacing ${}_5N_y^t$ by ${}_5P_y^t$ and $\underline{x}$, from the equation $p^{t+10} = p^t \cdot e^{10r}$.

Comparing the relations (3.13) and (3.17) in the case of stable population to those in (3.5) and (3.5a) respectively under the assumptions of stationary population, one may easily find that the equality between 10-PSR- and 10-LSR- holds good in both the populations- stable and stationary; whereas the equality between the 10-cum-PSR- and 10-cum-LSR- holds good only in the case of stationary population, but not in the case of stable population.

3.1.3. The population under study is approximately stable

It may be noted that the formula (3.17) is applicable strictly to a stable population in which the population at any age group changes over time exponentially with a constant rate $\underline{r}$. But, in reality, a stable population in the strict sense, with the same rate of growth at all ages, along with the other conditions of stability (3.6) and (3.7), is very rare. In some countries, particularly developing ones, all the conditions of stability are not satisfied closely; yet, due to non-availability of reliable vital registration statistics, the theory of stable population, introduced by Lotka, had been used very extensively to estimate various demographic

parameters. Fortunately, it has been found that the estimates of various demographic parameters, obtained through the application of stable population theory are rather robust against some small deviations from the assumptions of stability as long as our interest lies in estimating certain characteristics of the entire population. But, when we are interested in certain characteristics of only a part of a population which is not sufficiently stable then, necessary care should be taken about the variation in age-specific rate of growth, $r(x)$, which is likely to affect significantly the estimates of the characteristics under study of the sub-population. An example of this kind may be found from the relation (3.17), where we are interested to estimate $10\text{-cum-LSR-}(10s_{x+})$ from $10\text{-cum-PSR-}[10s'(x+)]$ for the sub-population at age x and above for the purpose of life table construction. The formula (3.17) is expected to provide reasonably good estimate of $10s_{x+}$, when the age-specific rate of growth remains almost constant at all ages; but, when it is not so, as usually found in many cases, proper care should be taken in formulating the relationship between $10s_{x+}$ and $10s'(x+)$ which takes into account the possible variation in the age-specific growth rates. This type of situation has been discussed in the following paragraph.

A population in which the age-specific growth rate $[r(x)]$ varies with age, may be termed as 'destabilised population' as proposed by Martin (1980). The need for age-specific growth rates in demographic analysis has been discussed by several authors (Martin, 1980; Preston et al., 1980; Bennett and Horiuchi, 1981; Preston and Coale, 1982). The difficulty as mentioned in the earlier paragraph for estimating $10S_{x+}$ values using the relations (3.17) in the case of a 'destabilised population' may be avoided by introducing the concept of cumulative rate of growth at age x . This concept needs precise definition, which may be met in two ways. The cumulative rate of growth ($\bar{r}_{x+}$) at age x may be defined as the weighted average of the exponential growth rates (${}_5r_x$) during the period $(t, t+10)$ for the quinquennial age-intervals covering the age-interval (x, w) , the weights being the population in the corresponding age-intervals. Alternatively, the cumulative rate (r_{x+}) of growth at age x may be defined as the exponential growth rate of the cumulated population at age x during the period $(t, t+10)$. Mathematically, the first definition of r_{x+} may be written as

$$\bar{r}_{x+} = \frac{\sum_{y=x}^{1-5} {}_5r_y \cdot {}_5N_y^* + \bar{r}_{1+} \cdot N_{1+}^*}{\sum_{y=x}^{1-5} {}_5N_y^* + N_{1+}^*} \dots (3.18)$$

where, (l, w) is the last open interval,

$${}_5N_y^* = \left[{}_5N_y^t + {}_5N_y^{t+10} \right] / 2 \quad , \quad \dots \quad \dots \quad (3.18a)$$

$$N_{l+}^* = \left[N_{l+}^t + N_{l+}^{t+10} \right] / 2 \quad \dots \quad \dots \quad (3.18b)$$

and $\bar{r}_{l+}$, the exponential growth rate of the population at age l and above, is defined by

$$\bar{r}_{l+} \doteq 0.1 \log_e \left[N_{l+}^{t+10} / N_{l+}^t \right] \quad \dots \quad \dots \quad (3.18c)$$

It appears from (3.18) to (3.18c) that the formula for obtaining $\bar{r}_{x+}$, according to the first definition is rather complicated. Let us now examine the formula for cumulative growth rate according to its second definition. The algebraic expression of the formula for estimating $\bar{r}_{x+}$ as per the second definition is:

$$r_{x+} = 0.1 \log_e \left[N_{x+}^{t+10} / N_{x+}^t \right] \quad \dots \quad \dots \quad (3.19)$$

Comparing the two formulas, shown in (3.18) and (3.19) for measuring the cumulative growth rate, it is easily found that the formula according to the second definition shown in (3.19) is much simpler than the formula according to the first definition. It has already been stated earlier that proper care should be taken in the case of a 'destabilised

population' regarding the variation in the age-specific growth rates while formulating the relationship between ${}_{10}S'(x_+)$ and ${}_{10}S_{x_+}$. Compared to r_{x_+} it seems that $\bar{r}_{x_+}$ gives a better summary of the different age-specific growth rates because $\bar{r}_{x_+}$ takes into separate account the different quinquennial rates whereas r_{x_+} merely gives the overall net picture. Thus, logically, one would feel that the use of $\bar{r}_{x_+}$ provides a better picture regarding the relationship between ${}_{10}S'(x_+)$ and ${}_{10}S_{x_+}$ than that of r_{x_+} . However, it can be shown empirically that the values of $\bar{r}_{x_+}$ and r_{x_+} obtained through the formulas (3.18) and (3.19) are quite close to each other. Therefore, from a practical point of view, it is immaterial which one we accept for formulating the relationship between ${}_{10}S_{x_+}$ and ${}_{10}S'(x_+)$; however, we found it expedient to use r_{x_+} shown in (3.19) because of its simplicity.

From the relation (3.16), obtained under the assumption of stability, one would expect that the development of the relationship between ${}_{10}S_{x_+}$ and ${}_{10}S'(x_+)$ in the case of a destabilised population would require the cumulative growth rates r_{x_+} and $r_{(x+10)_+}$ for the sub-population aged x and above and $(x+10)$ and above respectively, provided the degree of destabilisation is not too

high. In such an approximately stable population where the age-specific growth rates may deviate from constancy as age advances (but not too erratically or highly, the trend more or less regular). A better relationship between $10^S(x_+)$ and $10^S x_+$, similar to that obtained under assumption of stability shown in (3.17), could be proposed for such approximately stable population by making use of the average of the cumulative growth rates at ages x and $x+10$, that is, $\frac{1}{2} [r_{x+} + r_{(x+10)+}]$ instead of the constant rate of growth $\underline{r}$. Thus, the modified relationship between $10^S x_+$ and $10^S(x_+)$ in the case of a destabilised population is given by:

$$10^S(x_+) = \text{Exp} \left[(10 - C_{(x+10)+} + C_{x+}) \bar{r}^*_{(x+5)+} \right] 10^S x_+,$$

for $x > 0$ (3.20)

where $\bar{r}^*_{(x+5)+} = \frac{1}{2} [r_{x+} + r_{(x+10)+}] \dots \dots (3.20a)$

It may be noted that according to the definitions of cumulative rate of growth by age, the two statistics -- $\bar{r}^*_{(x+5)+}$ and $\bar{r}_{(x+5)+}$ are not identical; but, if the curve for r_{x+} may be considered as almost linear in the interval $(x, x+10)$, the quantities $\bar{r}^*_{(x+5)+}$ and $\bar{r}_{(x+5)+}$ will be very close to each other. The other symbols have already been explained elsewhere. For estimating $10^S x_+$

through the formula (3.20), it is necessary to know the estimated values of ${}_{10}S'(x_+)$, C_{x_+} , $C_{(x+10)_+}$ and $\bar{r}^*(x+5)_+$. The estimated values of ${}_{10}S'(x_+)$, C_{x_+} can be obtained by using respectively the formula (3d) -- ${}_{10}S_{x_+}$ being taken as an estimate of ${}_{10}S'(x_+)$ -- and formula (3.15). The estimated value of $\bar{r}^*(x+5)_+$ can be obtained through (3.19) and (3.20a) after replacing $N_{x_+}^t$ by the corresponding census data $(P_{x_+}^t)$ of the population under study.

So far, we have put forward some general arguments in favour of the relationship between ${}_{10}S_{x_+}$ and ${}_{10}S'(x_+)$, shown in the formula (3.20), for any approximately stable population. However, the mathematical justification of the relationship (3.20) would require a generalised model of age-structure of any population. Very recently Preston and Coale (1982) have developed a generalised mathematical model of age-structure of any population in an interesting research monograph. The mathematical treatment for justifying the validity of the formula (3.20), on the basis of Preston-Coale generalised model for age-structure of any population, has been discussed in the following paragraphs.^{5/}

^{5/} The formula (3.20) was originally derived here without any prior knowledge of the generalised model for age-structure developed by Preston and Coale (1982).

3.1.4. The population under study is neither stationary nor stable (or approximately stable)

According to Preston-Coale generalised model which is applicable to any population, N_x^t , the function describing the age-structure of the population at time t is given by

$$N_x^t = B(t) \text{Exp} \left[- \int_0^x r_y(t) dy \right] \cdot p^t(x) \quad \dots\dots (3.21)$$

where,

$B(t)dt$: Number of births in the time interval $(t, t+dt)$;

$r_y(t)$: Relative growth rate of persons aged y (exact) at time t ;

$p^t(x)$: Probability of surviving from birth to exact age x according to the life table prevailing at time t .

It may be noted that in the case of a stable population the growth rate $r_y(t)$ remains constant, over age and time, and is therefore denoted simply by r and also $p^t(x)$ is independent of time t ; but $B(t)$, the birth function changes over time with the constant rate r .

In the case of Preston-Coale generalised model, the relative growth rate $r_x(t)$ may be defined, analogous to instantaneous force of mortality at age x (μ_x),

as follows:

$$r_x(t) = \lim_{\Delta t \rightarrow 0} \frac{N_x^{t+\Delta t} - N_x^t}{N_x^t \cdot \Delta t} = \frac{1}{N_x^t} \frac{d N_x^t}{dt} \quad \dots (3.22)$$

so that $N_x^t = \text{const.}$ Exp $[\int r_x(t) dt]$, where the constant is independent of t .

While constructing abridged life table of a country from the age-returns of its two consecutive censuses, 10-years apart, at time t and $t+10$ say, tabulated in some suitable system of quinquennial age groups, 10-year usual census survival ratios ($_{10}S_{x+2.5} = \frac{5P_{x+10}^{t+10}}{5P_x^t}$) are normally used under the assumption that the 10-year usual life table survival ratios (10-LSR) ($_{10}S_{x+2.5} = \frac{5L_{x+10}}{5L_x}$) are equal to the corresponding 10-year usual census survival ratios (10-CSR) of the population under study. It has already been established earlier that the assumption of equality between 10-CSR- and 10-LSR- is fully justified for stationary population and stable population. In the next section, the validity of the assumption has been examined in the context of the generalised model proposed by Preston and Coale (1982).

When the assumption is not tenable, what should be the appropriate relationship between 10-CSR- and 10-LSR- in the case of the generalised model applicable to any population? This has also been developed in the following section.

3.1.4(a) Relationship between '10-CSR' and '10-LSR' in the case of any population described by Preston-Coale generalised model for age-structure

It has already been mentioned earlier that, given the distribution of persons by age of a population at two points of time, 10-years apart, say t and $t+10$, a life table during the decade $(t, t+10)$ may be constructed on the basis of life table survival ratios obtained under certain assumption from the population survival ratios during the period $(t, t+10)$. The life table, so obtained, depicts an average experience of mortality during the 10-years period $(t, t+10)$ and thus may be treated as the life table at time $t+5$, the mid-point of the decade $(t, t+10)$.

According to the definition, the 10-LSR- at age x during the decade $(t, t+10)$ in the population undergoing the Preston-Coale generalised model and the

10-LSR at age x at time $t+5$ are given by

$$10S'(x+2.5) = \frac{5N_{x+10}^{t+10}}{5N_x^t}$$

and

$$10s_{x+2.5}^{t+5} = \frac{5L_{x+10}^{t+5}}{5L_x^{t+5}}$$

The formula (3.22) can be written as

$$\frac{d}{dt} \log_e N_x^{t'} = r_x(t'), \quad t \leq t' \leq t+10$$

Integrating both sides with respect to t' from t to $t+10$, we get

$$\int_t^{t+10} d[\log_e N_x^{t'}] = \int_t^{t+10} r_x(t') dt' = 10 \varepsilon_x \quad (\text{say})$$

$$\text{or } \log_e \frac{N_x^{t+10}}{N_x^t} = \int_t^{t+10} r_x(t') dt'$$

$$\text{or } \log_e \frac{N_x^{t+10}}{N_x^t} = \sum_{i=t}^{t+9} \int_i^{i+1} r_x(t') dt', \quad \text{using the}$$

additive property of integral calculus. It follows

that

$$N_x^{t+10} = N_x^t \text{Exp} \left[\sum_{i=t}^{t+9} \int_1^{i+1} r_x(t') dt' \right] = N_x^t e^{10g_x} \dots (3.23)$$

It may be noted that $r_x(t')$ is the (instantaneous) relative growth rate at (exact) age x at time t' . Now, if x is not small $r_x(t')$ may be treated as almost constant within each unit interval of time $(i, i+1)$. Therefore, the above equation becomes

$$N_x^t \cdot e^{10g_x} = N_x^{t+10} \div N_x^t \text{Exp} \left[\sum_{i=t}^{t+9} r_x(i + \frac{1}{2}) \right] \dots (3.23a)$$

We write

$$\bar{r}_x = \frac{1}{10} \sum_{i=t}^{t+9} r_x(i + \frac{1}{2}) \dots \dots (3.23b)$$

The equation (3.23a) can be written as

$$N_x^{t+10} \div e^{10 \cdot \bar{r}_x} \cdot N_x^t \dots \dots \dots (3.23c)$$

Hence comparing (3.23) and (3.23c), we get

$$g_x \div \bar{r}_x \dots \dots \dots (3.23d)$$

From (3.23b) one can easily find that $\bar{r}_x$ is the arithmetic mean of the (instantaneous) relative growth rates $r_x(i+\frac{1}{2})$'s at the mid-points of all the ten one-year time intervals comprising the intercensal decade. Thus, $\bar{r}_x$ can be interpreted as an average experience of the annual population growth during the intercensal decade. Now, we are to present the intercensal mortality experience in a life table form. But strictly speaking, a life table depicts the mortality conditions prevailing at a point of time. As a consequence, we have to suppose that the intercensal experience summarised by $\bar{g}_x$ or the average, $\bar{r}_x$, operates at particular point of time, and the obvious choice is the mid-point, $t+5$, of the intercensal decade. In other words, our objective would be met if we consider a hypothetical population with (instantaneous) relative growth rate of $\bar{g}_x \doteq \bar{r}_x$ at (exact) age x at time $t+5$. In consonance with this we write $\bar{r}_x(t+5)$ for $\bar{r}_x$ to emphasise that in subsequent developments our reference time is $t+5$.

We rewrite (3.23c) as

$$N_x^t \cdot e^{10\bar{g}_x(t+5)} = N_x^{t+10} \doteq e^{10 \cdot \bar{r}_x(t+5)} \cdot N_x^t \quad \dots \quad (3.23e)$$

It should be clear in mind that $\bar{r}_x(t+5)$ may not necessarily coincide with $r_x(t+5)$ the relative growth rate of the

population, prevailing at time $t+5$. However, they are approximately equal as shown below:

We have seen previously that

$$10 \cdot g_x = \int_t^{t+10} r_x(t') dt'$$

By using the mean value theorem, we get

$$10 g_x = r_x(\underline{C}) \int_t^{t+10} dt' = 10 \cdot r_x(\underline{C}), \text{ where } t < \underline{C} < t+10$$

The exact value of $\underline{C}$ is not known, but we may take

$\underline{C} \doteq t+5$, the mid-point of the interval $(t, t+10)$. Thus using this approximation to $\underline{C}$ and the approximate relation $g_x \doteq \bar{r}_x$ shown in (3.23d), we get

$$r_x(t+5) \doteq g_x \doteq \bar{r}_x(t+5) \quad \dots \quad (3.23f)$$

In the subsequent developments $\bar{r}_x(t+5)$ is used instead of r_x .

Now writing $t+5$ for t and $\bar{r}_y(t+5)$ for $r_y(t+5)$ in (3.21) and noting that

$$p^{t+5}(x) = l_x(t+5)/l_o(t+5), \text{ we get}$$

$$N_x^{t+5} = \frac{B(t+5)}{l_o(t+5)} \text{Exp} \left[- \int_0^x \bar{r}_y(t+5) dy \right] l_x(t+5) \dots (3.24)$$

For the sake of simplicity in presenting various formulas, the 'argument' $t+5$ within the bracket will be omitted henceforth. However, all the functions subsequently used here pertain to the time $t+5$, unless otherwise mentioned. The superscript $t+5$, (as in L.H.S. of (3.24)) may also be omitted.

Integrating both sides of (3.21) in the age-interval $(x, x+5)$, we get

$${}_5N_x = \int_x^{x+5} N_y dy = (B/l_o) \int_x^{x+5} \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] l_y dy \dots (3.25)$$

According to the first mean value theorem of integral calculus there exists an age C_x in the interval $(x, x+5)$, such that

$$\int_x^{x+5} \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] l_y dy = \text{Exp} \left[- \int_0^{C_x} \bar{r}_z dz \right] \cdot \int_x^{x+5} l_y dy \dots (3.25a)$$

Using (3.25a) in (3.25), we get

$${}_5N_x = (B/l_0) \text{Exp} \left[- \int_0^{c_x} \bar{r}_z dz \right] {}_5L_x \quad \dots (3.25b)$$

where $x < c_x < x+5$ and ${}_5L_x = \int_x^{x+5} l_y dy$

The true value of c_x is not given by the mean value theorem which only asserts the existence of such a point in the interval $(x, x+5)$, but it does not give its exact magnitude. However, as before, a reasonably good approximation to c_x may be taken as $x+2.5$ provided x does not belong to the first few years of life. Thus, the equation (3.25b) becomes

$${}_5N_x \doteq (B/l_0) \text{Exp} \left[- \int_0^{x+2.5} \bar{r}_z dz \right] {}_5L_x, \quad x > 0 \quad \dots (3.25c)$$

Now, using (3.25c), we find

$$\frac{{}_5N_{x+10}}{{}_5N_x} \doteq \text{Exp} \left[- \int_0^{x+12.5} \bar{r}_z dz + \int_0^{x+2.5} \bar{r}_z dz \right] \frac{{}_5L_{x+10}}{{}_5L_x},$$

for $x > 0$

$$\text{or } \frac{5N_{x+10}}{5N_x} \div \text{Exp} \left[- \int_{x+2.5}^{x+12.5} \bar{r}_z dz \right] 10^S_{x+2.5}, \quad x > 0 \quad \dots (3.26)$$

$$\text{since } \int_0^{x+12.5} \bar{r}_z dz = \int_0^{x+2.5} \bar{r}_z dz + \int_{x+2.5}^{x+12.5} \bar{r}_z dz$$

according to the additive property of definite integral calculus, and $10^S_{x+2.5}$ ($= 5^L_{x+10}/5^L_x$) is the 10-LSR- at time $t+5$.

It may be noted here that all the functions in (3.26) relate to the time $t+5$. Thus, the L.H.S. of (3.26) represents the ratio of number of persons in the age-interval $(x+10, x+15)$ at the time $t+5$ to that in the age-interval $(x, x+5)$, also at time $t+5$. The ratio $5N_{x+10}^{t+5}/5N_x^{t+5}$, the L.H.S. of (3.26), is not the same as the 10-PSR- at age x $[10^{S'}(x+2.5)]$ during the period $(t, t+10)$, since the latter is $5N_{x+10}^{t+10}/5N_x^t$. Thus, to obtain a relationship between 10-PSR- at age x $[10^{S'}(x+2.5)]$ and 10-LSR- at age x ($10^S_{x+2.5}$), we need to know the relationship between $10^{S'}(x+2.5)$ $= 5N_{x+10}^{t+10}/5N_x^t$ and the ratio $5N_{x+10}^{t+5}/5N_x^{t+5}$ which is related to $10^S_{x+2.5} = 5^L_{x+10}^{t+5}/5^L_x^{t+5}$ through the equation (3.26).

as the average annual (exponential) growth rate of persons in the age-interval $(x, x+5)$, previously denoted by ${}_5r_x$, during the decade $(t, t+5)$. Thus, symbolically,

$${}_5r_x \doteq \bar{r}_{x+2.5} \quad \dots\dots (3.28)$$

It will be recalled that the relation defining ${}_5r_x$ (in relation to a model of age structure) is

$${}_5N_x^{t+10} = e^{10 \cdot {}_5r_x} \cdot {}_5N_x^t \quad \dots\dots (3.28a)$$

Now, our problem is how to evaluate the ratio ${}_5N_{x+10}^{t+10}/{}_5N_x^t$.

The L.H.S. of (3.28), ${}_5r_x$, being an average annual (exponential) growth rate, the R.H.S., $\bar{r}_{x+2.5}$ is also approximately an average annual rate. Hence for the purposes of approximation, a rate of ${}_5r_x$ may be considered to be operative separately within each one-year interval during the decade $(t, t+10)$. Thus, we deduce that -

$${}_5N_x^{t+5} \doteq e^{5 \cdot {}_5r_x} \cdot {}_5N_x^t \quad \dots\dots (3.29)$$

similarly,

$${}_5N_{x+10}^{t+10} \doteq e^{5 \cdot {}_5r_{x+10}} \cdot {}_5N_{x+10}^{t+5} \quad \dots\dots (3.29a)$$

where, 5^r_{x+10} is the annual (exponential) growth rate for the age-interval $(x+10, x+15)$ during the period $(t, t+10)$.

Now, using the equations (3.29) and (3.29a), we obtain

$$\frac{5^N_{x+10}^{t+5}}{5^N_x^{t+5}} = \frac{5^N_{x+10}^{t+10} \cdot e^{-5 \cdot 5^r_{x+10}}}{5^N_x^t \cdot e^{5 \cdot 5^r_x}}$$

$$\text{or } \frac{5^N_{x+10}^{t+5}}{5^N_x^{t+5}} \doteq 10^{S'(x+2.5)} \cdot e^{-5 \cdot (5^r_x + 5^r_{x+10})} \dots (3.30)$$

Using (3.30) in (3.26), we get

$$10^{S'(x+2.5)} \cdot e^{-5(5^r_x + 5^r_{x+10})} \doteq \text{Exp} \left[\int_{x+2.5}^{x+12.5} \bar{r}_z dz \right] 10^{S_{x+2.5}}$$

$$\text{or } 10^{S_{x+2.5}} \doteq 10^{S'(x+2.5)} \cdot e^R, \quad x > 0 \dots (3.31)$$

$$\text{where, } R = -5(5^r_x + 5^r_{x+10}) + \int_{x+2.5}^{x+12.5} \bar{r}_z dz \dots (3.31a)$$

The above equation provides the requisite relationship between $10^{S'(x+2.5)}$ and $10^{S_{x+2.5}}$. The relationship (3.31) shows that in general,

$$10^{S_{x+2.5}} \neq 10^{S'(x+2.5)} \dots (3.32)$$

However, it can be verified easily from the relationship (3.31) that the equality between $10^S_{x+2.5}$ and $10^{S'}(x+2.5)$ which is normally assumed to construct life table from the distribution of persons by age of the population under study at two successive points of time ten-years apart, holds good when $\underline{R}$ vanishes. The vanishing of $\underline{R}$ is ensured in the following cases:

i) $\bar{r}_x = 0$ for all x :-- it follows from (3.22)^{6/}

that N_x^t and, therefore, $5N_x^t$ are both constants, independent of $\underline{t}$, namely, N_x^0 and $\int_x^{x+5} N_y^0 dy$ respectively. Hence

from the relation, $5N_x^{t+10} = e^{10 \cdot 5r_x} \cdot 5N_x^t$, ... (3.32a)

defining $5r_x$ one deduces that $5r_x = 0 = 5r_{x+10}$.

Also $\int_{x+2.5}^{x+12.5} \bar{r}_z dz = 0$, obviously. Hence $R = 0$.

ii) $\bar{r}_x = r$ (constant) for all x :-- again from (3.22), we get

$$N_x^t = N_x^0 e^{rt}$$

^{6/} Since we are dealing with the hypothetical population, $r_x(t)$ of (3.22) means $\bar{r}_x(t)$.

and

$${}_5N_x^t = \int_x^{x+5} N_z^0 e^{rt} dz = e^{rt} \int_x^{x+5} N_z^0 dz \quad \dots (3.32b)$$

Now, using (3.32b) and (3.32a) we obtain

$$e^{r(t+10)} \int_x^{x+5} N_z^0 dz = e^{10 \cdot 5^r x} \cdot e^{rt} \int_x^{x+5} N_z^0 dz,$$

that is, ${}_5^r x = r = {}_5^r x+10$. Also,

$$\int_{x+2.5}^{x+12.5} \bar{r}_z dz = \int_{x+2.5}^{x+12.5} \bar{r}_z dz = 10 r. \quad \text{Hence } R = 0.$$

iii) The function $\bar{r}_x$ can be expressed as a first degree polynomial. In other words, $\bar{r}_x$ is linear in $\underline{x}$. In such a situation trapezoidal rule of integration can be applied thus,

$$\int_{x+2.5}^{x+12.5} \bar{r}_z dz = 10 \left[\frac{\bar{r}_{x+2.5} + \bar{r}_{x+12.5}}{2} \right] = 5({}_5^r x + {}_5^r x+10) \quad \dots (3.33)$$

In deriving the R.H.S. of (3.33), (3.28) has been used.

Hence, $R = 0$.

Using (3.33) in (3.31) we get

$$10^{S_{x+2.5}} \neq 10^{S'(x+2.5)}$$

The cases (i) and (ii) clearly show that the equality between $10^{S_{x+2.5}}$ and $10^{S'(x+2.5)}$ holds good when the population under study is stationary or stable. According to case (iii), $\bar{r}_x$ is of the linear form -

$$\bar{r}_x = A(t) + B(t).x \quad \dots\dots (3.34)$$

where 'A' and 'B' are constants independent of x . It may be noted that the cases (i) and (ii) are particular situations of the case (iii). Case (i) corresponds to $A = 0 = B$, and (ii) to $A = r$ and $B = 0$. If (3.34) were true then by solving the differential equation (3.22) it can be shown that the age-structure would be of rather special form, namely,

$$N_x^t = \text{const.} \cdot e^{\alpha(t) + x \cdot \beta(t)} \quad \dots\dots (3.35)$$

Close adherence to this special form in real situations is not ordinarily to be expected.

Thus, from the foregoing discussions we find that generally, $\bar{r}_x$ as used in (3.31) and before, cannot be considered as linear function of x . Let us now go back to

the formula (3.31). To make the formula (3.31) usable in practice for estimating $10^{S_{x+2.5}}$ from $10^{S'(x+2.5)}$, it is necessary to express in suitable approximate form of the integral on the R.H.S. of (3.31). Now, using Simpson's rule^{7/} of numerical integration, we get

$$\int_{x+2.5}^{x+12.5} \bar{r}_z dz \doteq \frac{5}{3} (\bar{r}_{x+2.5} + 4 \cdot \bar{r}_{x+7.5} + \bar{r}_{x+12.5}), \quad x > 0$$

.... (3.36)

Now, using (3.28), we find

$$\int_{x+2.5}^{x+12.5} \bar{r}_z dz \doteq \frac{5}{3} (5^{r_x} + 4 \cdot 5^{r_{x+5}} + 5^{r_{x+10}}), \quad x > 0$$

.... (3.37)

Thus, using (3.37) in (3.31), we obtain

$$10^{S_{x+2.5}} \doteq 10^{S'(x+2.5)} \cdot e^{-5(5^{r_x} + 5^{r_{x+10}})} \\ * \text{Exp} \left[\frac{5}{3} (5^{r_x} + 4 \cdot 5^{r_{x+5}} + 5^{r_{x+10}}) \right]$$

^{7/} While using the Simpson's rule of integration it should be kept in mind that the interval of integration must be divided into an even number of sub-intervals of equal width. In the present case, the interval of integration, viz., $(x+2.5, x+12.5)$, has been divided into two sub-intervals - $(x+2.5, x+7.5)$ and $(x+7.5, x+12.5)$ of width 5.

the formula (3.38); stated explicitly,

$$10^{\hat{s}}_{x+2.5} = \left[\frac{\left(\frac{5^{P^{t+10}}_{x+10} * 5^{P^{t+10}}_{x+5}}{5^{P^t}_{x+10}} \right)^2 / 5^{P^{t+10}}_x}{\left(\frac{5^{P^t}_x * 5^{P^t}_{x+5}}{5^{P^t}_{x+10}} \right)^2 / 5^{P^t}_x} \right]^{1/3} \dots (3.39c)$$

Let the age-returns of a community according to its two consecutive censuses, 10-years apart, be summarised into the standard quinquennial age groups (0,4), (5,9), (10,14), (15,19) (60,64), (65,69) and (70 and above).^{8/} Now, using the summary information, one can easily obtain an estimate of $10^{\hat{s}}_{x+2.5}$ through the formula (3.38) or (3.39c) after obtaining the values of $10^{\hat{s}'}_{(x+2.5)}$ and $_{+i}$, ($i = 0, 5, 10$) through the application of the formulas (3.39a) and (3.39b).^{9/}

Different form of age-grouping may be considered, if necessary. For example, if considerable digital preferences for ages ending with 0 or 5 are found as observed in many developing countries like India, the standard quinquennial age-grouping may give some distorted picture regarding the age-distribution. Appropriate age-grouping in which age-reporting errors due to digital biases are almost nil or sufficiently low, should be selected. A procedure of selecting appropriate age-grouping has already been discussed in Chapter II.

However, before applying the formula (3.38), one should make sure that the age-data in the quinquennial ages according to the two consecutive censuses of the population under study are sufficiently reliable in terms of its coverage, completeness and age-reporting. Furthermore, the formula (3.38) assumes the population under study is closed to migration during the period under consideration. Thus, in the presence of migration during the intercensal period, the census age-data should be suitably adjusted.

Now, for constructing an abridged life table during the period $(t, t+10)$, we need to know the 5-LSR (${}_5s_{x+2.5} = {}_5L_{x+5}/{}_5L_x$). The values of ${}_5s_{x+2.5}$ can be obtained from ${}_{10}s_{x+2.5}$ through the following formula (see Brass, 1975, p.112; Office of the Registrar General of India, 1954a, Paper No.2, p.17);

$${}_5s_{x+7.5} \doteq ({}_{10}s_{x+2.5} * {}_{10}s_{x+7.5})^{1/4} \dots (3.40)$$

The logic behind such an approximation is as follows:

$${}_{10}s_{x+2.5} = \frac{{}_5L_{x+10}}{{}_5L_x} = \frac{{}_5L_{x+10}}{{}_5L_{x+5}} * \frac{{}_5L_{x+5}}{{}_5L_x} = {}_5s_{x+7.5} * {}_5s_{x+2.5}$$

Similarly,

$${}_{10}s_{x+7.5} = {}_5s_{x+12.5} * {}_5s_{x+7.5}$$

Now,

$$\begin{aligned} ({}_{10}s_{x+2.5} * {}_{10}s_{x+7.5})^{1/4} &= \left({}_5s_{x+2.5} * {}_5s_{x+7.5} * {}_5s_{x+7.5} \right. \\ &\quad \left. * {}_5s_{x+12.5} \right)^{1/4} \end{aligned}$$

Taking logarithm both sides of the above formula, we get

$$\begin{aligned} \log ({}_{10}s_{x+2.5} * {}_{10}s_{x+7.5})^{1/4} &= \frac{1}{4} \left(\log {}_5s_{x+2.5} + 2 \log {}_5s_{x+7.5} \right. \\ &\quad \left. + \log {}_5s_{x+12.5} \right) \end{aligned}$$

or

$$\log \left(10^{s_{x+2.5}} * 10^{s_{x+7.5}} \right)^{1/4} = \frac{1}{4} \left[\sum_{t=x}^{x+4} \log s_{t+0.5} \right.$$

$$+ 2 \cdot \sum_{t=x}^{x+4} \log s_{t+5.5} \\ \left. + \sum_{t=x}^{x+4} \log s_{t+10.5} \right]$$

or

$$\log \left(10^{s_{x+2.5}} * 10^{s_{x+7.5}} \right)^{1/4} \doteq \sum_{t=x}^{x+4} \log s_{t+5.5},$$

$$\text{where } s_t = \frac{L_{t+1}}{L_t}$$

or

$$\log \left(10^{s_{x+2.5}} * 10^{s_{x+7.5}} \right)^{1/4} \doteq \log 5^{s_{x+7.5}}$$

Hence, we obtain the formula (3.40).

It may be noted, for the given set of age data presented in standard quinquennial age groups, the formula (3.38) provides the values of $10^{s_{x+2.5}}$, for $x = 5, 10, \dots, 45$; and hence, the formula (3.40) gives the values of $5^{s_{x+7.5}}$, for $x = 5, 10, \dots, 50$. The values of $5^{s_{7.5}}$

relationship between the two survival ratios, applicable to any population which is closed to migration during the period under study and whose age-returns according to the two consecutive censuses, 10-years apart^{10/}, are sufficiently reliable in terms of its coverage, completeness and age-reporting. However, there are number of countries, particularly developing ones, where the census age-returns are very much affected due to age misstatements and also due to underreporting, particularly, among infants and young children. Thus, for such countries, mentioned just now, the distribution of persons by age should be smoothed and graduated before applying the formula (3.38) or (3.39c) to estimate $10^S_{x+2.5}$. It has already been mentioned earlier that the smoothing procedure, used for the purpose of adjusting the distorted age data due to age-misstatements, are rather arbitrary and subjective and there is some uncertainty as to whether the adjusted age data, so obtained, depicts the real picture of the age-distribution sufficiently closely. Further discussion in this respect may be found in the second chapter.

^{10/} Similar type of formula can also be derived when the intercensal period is five years.

The difficulties, mentioned above, for estimating 10-LSR ($_{10}S_{x+2.5}$) through the equation (3.38) or (3.39c) for countries where the census age-returns are heavily distorted by age-misstatements, can be avoided to a great extent through the introduction of 10-cum-CSR at age $\underline{x}$ ($_{10}S_{x+}$) which is less likely to be affected by age-misreporting as discussed in the earlier chapter. Thus, in the following section an attempt has been made to develop a formula for converting 10-cum-PSR at age $\underline{x}$ $\left[_{10}S'_{(x+)}\right]$ in any population into the corresponding 10-cum-LSR at age $\underline{x}$ ($_{10}S_{x+}$) which in turn can be used to construct a life table during the intercensal period under study through an iterative procedure described elsewhere.

5.1.4.(b) Relationship between '10-cum-PSR' at age $\underline{x}$ and the corresponding '10-cum-LSR' at age $\underline{x}$

Integrating both sides of the equation (3.24) in the age-range (x, w) we find:

$$N_{x+} = \int_x^w N_y dy = (B/l_0) \int_x^w \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] l_y dy$$

.... (3.41)

It may be noted that each function in the above equation refers to the time $t+5$ and henceforth all the functions pertain to the time $t+5$ unless otherwise stated. Now, using the first mean value theorem of integral calculus, as before, we find that there exists a point c'_{x+}

lying between $\underline{x}$ and $\underline{w}$ such that

$$\int_x^w \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] l_y dy = \text{Exp} \left[- \int_0^{c'_{x+}} \bar{r}_z dz \right] \int_x^w l_y dy,$$

$$\text{where } x < c'_{x+} < w$$

$$\int_x^w \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] l_y dy = \text{Exp} \left[- \int_0^{c'_{x+}} \bar{r}_z dz \right] T_x, \quad \dots (3.41a)$$

$$\text{where } x < c'_{x+} < w \text{ and } T_x = \int_x^w l_y dy$$

To find out the approximate value of c'_{x+} during the period $(t, t+10)$, let us consider the following expression

$$F(x) = \int_x^w \text{Exp} \left[- \int_0^y \bar{r}_z dz \right] N_y^{t+5} dy$$

for convenience, we shall write, henceforth, simply N_y instead of N_y^{t+5} .

On

$$F(x) = \int_x^w \text{Exp} \left[- \int_0^x \bar{r}_z dz - \int_x^y \bar{r}_z dz \right] N_y dy$$

or

$$F(x) \cdot \text{Exp} \left[\int_0^x \bar{r}_z dz \right] = \int_x^w \text{Exp} \left[- \int_x^y \bar{r}_z dz \right] N_y dy \quad \dots (3.41b)$$

Applying the mean value theorem (MVT) to the R.H.S. of (3.41b), we get

$$\text{R.H.S.} = \text{Exp} \left[- \int_x^{c'_{x+}} \bar{r}_z dz \right] \int_x^w N_y dy$$

or

$$\text{R.H.S.} = \text{Exp} \left[- \int_x^{c'_{x+}} \bar{r}_z dz \right] N_{x+} \quad \dots \dots (3.41c)$$

where $x < c'_{x+} < w$ and, as usual, $N_{x+} = \int_x^w N_y dy$

Applying MVT once again, but now to (3.41c) we get

$$\text{R.H.S.} = \text{Exp} \left[- (c'_{x+} - x) \bar{r}_{a_{x+}} \right] N_{x+} \quad \dots \dots (3.41d)$$

where $x < a_{x+} < c'_{x+} < w$

The R.H.S. of (3.41b) can also be written as,

$$\left[\int_x^{x+5} + \int_{x+5}^{x+10} + \int_{x+10}^{x+15} + \dots \right] \text{Exp} \left[- \int_x^y \bar{r}_z dz \right] N_y dy$$

[for L.H.S. see overleaf]

$$= \text{Exp} \left[- \int_x^{c_x} \bar{r}_z dz \right] \int_x^{x+5} N_y dy + \text{Exp} \left[- \int_x^{c_{x+5}} \bar{r}_z dz \right] \int_{x+5}^{x+10} N_y dy \\ + \text{Exp} \left[- \int_x^{c_{x+10}} \bar{r}_z dz \right] \int_{x+10}^{x+15} N_y dy + \dots$$

where $x < c_x < x+5 < c_{x+5} < x+10 < c_{x+10} < x+15 < \dots$

Writing ${}_5N_x = \int_x^{x+5} N_y dy$, we get

$$\text{R.H.S.} = {}_5N_x \cdot \text{Exp} \left[- \int_x^{c_x} \bar{r}_z dz \right] + {}_5N_{x+5} \cdot \text{Exp} \left[- \int_x^{c_{x+5}} \bar{r}_z dz \right] \\ + {}_5N_{x+10} \cdot \text{Exp} \left[- \int_x^{c_{x+10}} \bar{r}_z dz \right] + \dots$$

Applying the MVT again, we obtain

$$\text{R.H.S.} = {}_5N_x \cdot \text{Exp} \left[- (c_x - x) \bar{r}_{a_x} \right] + {}_5N_{x+5} \cdot \text{Exp} \left[- (c_{x+5} - x) \bar{r}_{a_{x+5}} \right] \\ + {}_5N_{x+10} \cdot \text{Exp} \left[- (c_{x+10} - x) \bar{r}_{a_{x+10}} \right] + \dots \quad (3.41c)$$

where $x < a_x < c_x < x+5$, $x < a_{x+5} < c_{x+5} < x+10$,

$x < a_{x+10} < c_{x+10} < x+15$,

noting that

$$\text{Exp}[-(C - x)\bar{r}_a] \doteq 1 - (C - x)\bar{r}_a ,$$

since the $\bar{r}_a$'s are very small, we obtain by equating the two different expressions (3.41d) and (3.41e) for the R.H.S., the following relation:

$$\begin{aligned} N_{x+} [1 - (C'_{x+} - x)\bar{r}_{a_{x+}}] &\doteq 5^{N_x} [1 - (C_x - x)\bar{r}_{a_x}] \\ &+ 5^{N_{x+5}} [1 - (C_{x+5} - x)\bar{r}_{a_{x+5}}] \\ &+ 5^{N_{x+10}} [1 - (C_{x+10} - x)\bar{r}_{a_{x+10}}] + \dots \end{aligned}$$

Remembering that $N_{x+} = 5^{N_x} + 5^{N_{x+5}} + 5^{N_{x+10}} + \dots$

we deduce that

$$\begin{aligned} (C'_{x+} - x) N_{x+} &\doteq (C_x - x) U_{a_x} \cdot 5^{N_x} + (C_{x+5} - x) U_{a_{x+5}} \cdot 5^{N_{x+5}} \\ &+ (C_{x+10} - x) U_{a_{x+10}} \cdot 5^{N_{x+10}} + \dots \end{aligned}$$

where $U_{a_{x+5i}} = \bar{r}_{a_{x+5i}} / \bar{r}_{a_{x+}}$, $i = 0, 1, 2, \dots$

$$\begin{aligned} \text{Hence, } C_{x+} \cdot N_{x+} &\doteq [U_{a_x} \cdot C'_x \cdot 5^{N_x} + U_{a_{x+5}} \cdot C_{x+5} \cdot 5^{N_{x+5}} \\ &+ U_{a_{x+10}} \cdot C_{x+10} \cdot 5^{N_{x+10}} + \dots] \\ &+ x [N_{x+} - U_{a_x} \cdot 5^{N_x} - U_{a_{x+5}} \cdot 5^{N_{x+5}} \\ &- U_{a_{x+10}} \cdot 5^{N_{x+10}} + \dots] \end{aligned}$$

Now, using the approximate expression (3.15e) for c_{x+} derived before, viz.,

$$c_{x+} = \frac{c_x \cdot 5^{N_x} + c_{x+5} \cdot 5^{N_{x+5}} + c_{x+10} \cdot 5^{N_{x+10}} + \dots}{5^{N_x} + 5^{N_{x+5}} + 5^{N_{x+10}} + \dots}$$

we get,

$$c'_{x+} - c_{x+} = \frac{\epsilon_{ax} \cdot c_x \cdot 5^{N_x} + \epsilon_{ax+5} \cdot c_{x+5} \cdot 5^{N_{x+5}} + \epsilon_{ax+10} \cdot c_{x+10} \cdot 5^{N_{x+10}} + \dots}{5^{N_x} + 5^{N_{x+5}} + 5^{N_{x+10}} + \dots}$$

$$= x \frac{\epsilon_{ax} \cdot 5^{N_x} + \epsilon_{ax+5} \cdot 5^{N_{x+5}} + \epsilon_{ax+10} \cdot 5^{N_{x+10}} + \dots}{5^{N_x} + 5^{N_{x+5}} + 5^{N_{x+10}} + \dots}$$

where $\epsilon_{ax+5i} = U_{ax+5i} - 1$, $i = 0, 1, 2, \dots$

Now, in real situations, the $\bar{r}_{ax+5i}$ (for $i = 0, 1, 2, \dots$) are likely to be nearly $\bar{r}_{ax+}$, in the age-range (x, w) if x is not small (in the case of stable population $\bar{r}_{ax+5i}$ and $\bar{r}_{ax}$ are exactly identical). Hence the U_{ax+5i} , for $i = 0, 1, 2, \dots$ are nearly unity, so that the deviations ϵ_{ax+5i} , for $i = 0, 1, 2, \dots$ are quite small. Moreover these deviations are sometimes positive and sometimes negative. The net effect is a small R.H.S. - very small

in comparison to either of the C 's. Thus

$$C'_{x+} \doteq C_{x+} \quad \dots (3.41f)$$

It will be noted that C'_{x+} (or C_{x+}) is a weighted average of the C 's. Replacing ${}_5N_x$ by ${}_5N_x^* = \frac{1}{2} ({}_5N_x^t + {}_5N_x^{t+10})$ and approximating C_x by $x+2.5$, we obtain an estimate of C'_{x+} (or C_{x+}).

Now, using (3.41) and (3.41a) and (3.41f) we find that -

$$N_{x+} = (B/l_0) \exp \left[- \int_0^{C_{x+}} \bar{r}_z dz \right] T_x, \text{ where } x < C_{x+} < w$$

..... (3.42)

Now, using the above equation, we get

$$\frac{N_{(x+10)+}}{N_{x+}} = \exp \left[- \int_0^{C_{(x+10)+}} \bar{r}_z dz + \int_0^{C_{x+}} \bar{r}_z dz \right] \frac{T_{x+10}}{T_x},$$

$$\text{where } x+10 < C_{(x+10)+} < w$$

or

$$\frac{N_{(x+10)+}}{N_{x+}} = \exp \left[- \int_{C_{x+}}^{C_{(x+10)+}} \bar{r}_z dz \right] 10^{S_{x+}} \dots (3.43)$$

$$\text{since } {}_{10}S_{x+} = T_{x+10}/T_x \quad \text{and} \quad \int_0^{C(x+10)+} \bar{r}_z dz = \int_0^{C_{x+}} \bar{r}_z dz + \int_{C_{x+}}^{C(x+10)+} \bar{r}_z dz .$$

Since all the above functions in the above relations pertain to the time $t+5$, the expression in the L.H.S. of (3.43) represents the ratio of persons aged $x+10$ and above at time $t+5$ to those aged x and above at the same time $t+5$. Hence, the ratio $N_{(x+10)+} / N_{x+}$, which refers to the time $t+5$ is different from the 10-cum-PSR- at age x $[{}_{10}S'(x+)]$ during the period $(t, t+10)$ which is the ratio of persons aged $(x+10)$ and above at time $t+10$ to those aged x and above at time t . Since the ratio $N_{(x+10)+}^{t+5} / N_{x+}^{t+5}$ is related to ${}_{10}S_{x+}$ through the equation (3.43), the problem of obtaining a relationship between ${}_{10}S'(x+)$ and ${}_{10}S_{x+}$ amounts to find a relationship between the ratios $N_{(x+10)+}^{t+5} / N_{x+}^{t+5}$ and ${}_{10}S'(x+) = N_{(x+10)+}^{t+10} / N_{x+}^t$. The search for such a relationship is attempted in the following paragraphs.

Integrating both sides of (3.23c) in the age-range (x, w) we get,

$$N_{x+}^{t+10} \doteq \int_x^w e^{-10 \cdot \bar{r}_y} N_y^t dy \quad \dots\dots (3.44)$$

Now, applying the first mean value theorem of integral calculus in R.H.S. of (3.44), we get

$$N_{x+}^{t+10} \doteq \text{Exp} \left[10 \cdot \bar{r}_{k_{x+}} \right] N_{x+}^t \quad \dots \quad \dots \quad (3.44a)$$

where $x < k_{x+} < w$ and $N_{x+}^t = \int_x^w N_y^t dy$.

According to the notations used here, the annual (exponential) growth rate of persons aged $\underline{x}$ and above during the intercensal decade is denoted by r_{x+} . The relation defining r_{x+} (with respect to the model), is

$$N_{x+}^{t+10} = e^{10 \cdot r_{x+}} \cdot N_{x+}^t \quad \dots \quad \dots \quad (3.44b)$$

comparing (3.44a) with the above relation gives

$$\bar{r}_{k_{x+}} \doteq r_{x+} \quad \dots \quad \dots \quad (3.44c)$$

Thus, $\bar{r}_{k_{x+}}$ which is the average annual (exponential) growth rate of persons aged $\underline{x}$ and above during the decade $(t, t+10)$. The annual growth rate, r_{x+} , being an average experience during the period $(t, t+10)$, may be considered applicable for each one-year interval during the decade $(t, t+10)$ and therefore, we find that,

$$N_{x+}^{t+5} \doteq e^{5 \cdot r_{x+}} \cdot N_{x+}^t \quad \dots \quad \dots \quad (3.45a)$$

and

$$N_{(x+10)+}^{t+10} \doteq e^{5 \cdot r(x+10)+} \cdot N_{(x+10)+}^{t+5} \dots (3.45b)$$

Now, using the above two equations, we find that -

$$10^S(x+) = \frac{N_{(x+10)+}^{t+10}}{N_{x+}^t} \doteq \frac{N_{(x+10)+}^{t+5}}{N_{x+}^{t+5}} \cdot \frac{e^{5 \cdot r(x+10)+}}{e^{-5 \cdot r_{x+}}}$$

$$\text{or } 10^S(x+) \doteq \frac{N_{(x+10)+}^{t+5}}{N_{x+}^{t+5}} \cdot e^{5[r_{x+} + r(x+10)+]} \dots (3.46)$$

Now, using (3.46) and (3.43) we have -

$$10^S(x+) \doteq e^{5[r_{x+} + r(x+10)+]} \cdot \text{Exp} \left[- \int_{C_{x+}}^{C_{(x+10)+}} \bar{r}_z dz \right] 10^S x+ \dots (3.47)$$

To use the above relationship in practice, it is necessary to evaluate the integral in R.H.S. of (3.47). The integral

$$\int_{C_{x+}}^{C_{(x+10)+}} \bar{r}_z dz, \text{ denoted by } I(C_{x+}, C_{(x+10)+}), \text{ can be}$$

evaluated numerically under various assumptions regarding

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APPENDIX - I

A NOTE ON THE FIXATION ON $T_{b+5}^{(i)}$

The formula (3.62) really states

$$T_{b+5}^{(i)} = \frac{{}_5s_{b+}^{(i)}}{1 - {}_5s_{b+}^{(i)}} \cdot {}_5L_b^{(i)} \quad \dots (I. 1)$$

But in the formula (4.3), ${}_5s_{b+}^{(i)}$ is replaced by the constant $\hat{{}_5s}_{b+}$, and some clarification is needed.

It may be recalled that in the i -th, ($i = 1, 2, 3, \dots$), stage of iteration, for the i -th life table, $LT^{(i)}$, -where age x ranges over $a, a+5, a+10, \dots, b-5$ and b , - we first calculate ${}_5p_x^{(i)}$ through the formula ${}_5p_x^{(i)} = \hat{{}_5s}_{x+} / {}_5E_x^{(i-1)}$, then ${}_1l_x^{(i)}$ from ${}_1l_x^{(i)} = {}_5p_{x-5}^{(i)} \cdot {}_1l_{x-5}^{(i)}$, (${}_1l_a$ being taken to be 100,000, for all values of i). It may be noted that the above formula for ${}_1l_x^{(i)}$ permits calculation of ${}_1l_{b+5}^{(i)}$ even though the age $b+5$ is outside the contemplated age-range $a, a+5, \dots, b-5$ and b .

Then ${}_5L_x^{(i)}$ may be obtained by the formula ${}_5L_x^{(i)} = \frac{5}{2} ({}_1l_x^{(i)} + {}_1l_{x+5}^{(i)})$ in the case of conventional approximation for all x including ${}_5L_b^{(i)}$ since ${}_1l_{b+5}^{(i)}$ (which is outside the contemplated age-range) has already been calculated. Greville's approximation for ${}_5L_x$ can also be used to ${}_5L_x^{(i)}$ for all x including at $x = b$ provided ${}_1l_{b+10}^{(i)}$ can be obtained by extrapolating ${}_1l_x$ values

beyond age $b+5$. For the T_x column, however, there is a special difficulty, whereas ${}_5p_x^{(i)}$, $l_x^{(i)}$ and ${}_5L_x^{(i)}$ can be calculated from lower to higher ages, $T_x^{(i)}$ is to be built from higher to lower ages. To get the $T_x^{(i)}$ column for the contemplated age range $(a, a+5, \dots, b)$ we must, at every stage of iteration, somehow fix $T_{b+5}^{(i)}$ (which is outside the age-range), independently, before one can apply the formula $T_x^{(i)} = T_{x+5}^{(i)} + {}_5L_x^{(i)}$, (for the case $x = b$ first, and then for other x 's). Next $(i)e_x^o$ column can be obtained by using the formula $(i)e_x^o = T_x^{(i)} / l_x^{(i)}$. We are also to lay the basis for $LT^{(i+1)}$ and therefore we require ${}_5E_x^{(i)}$ from ${}_5E_a^{(i)}$ to ${}_5E_b^{(i)}$. Now, ${}_5E_x^{(i)}$ is given by the formula ${}_5E_x^{(i)} = (i)e_{x+5}^o / (i)e_x^o$. For ${}_5E_b^{(i)}$, therefore, we require $(i)e_{b+5}^o$, which in turn requires $T_{b+5}^{(i)}$ which is to be fixed independently. Finally the ${}_5s_{x+}^{(i)}$ column is obtained by using the formula ${}_5s_{x+}^{(i)} = T_{x+5}^{(i)} / T_x^{(i)}$, and for ${}_5s_{b+}^{(i)}$, therefore, we require again the independently fixed $T_{b+5}^{(i)}$.

From what has been stated above it is clear that independent fixation of $T_{b+5}^{(i)}$ is of crucial importance. But such fixation cannot be arbitrary. An essential requirement of the iteration process is that it converges in such a manner that ${}_5s_{x+}^{(i)}$ tends to ${}_5\hat{s}_{x+}$ as i increases. The fixation

of $T_{b+5}^{(i)}$ therefore should be in conformity with this essential feature that ${}_5s_{b+}^{(i)}$ tends to $\hat{{}_5s}_{b+}$ as $\underline{i}$ increases. In view of the relation (I.1), an obvious suggestion would be to fix $T_{b+5}^{(i)}$ by

$$T_{b+5}^{(i)} = \frac{\hat{{}_5s}_{b+}}{1 - \hat{{}_5s}_{b+}} \cdot {}_5L_b^{(i)} \quad \dots (I.2)$$

for all values of i . It will be noted that the relations (I.1) and (I.2) are (practically) the same when $\underline{i}$ attains the value M (say) at which the life table attains a (practically) steady state. A question remains open as to whether the fixation (I.2) is in conformity with the basic feature ${}_5s_{b+}^{(i)}$ tends to $\hat{{}_5s}_{b+}$. That is so, is shown by the relation (4.4) of Chapter IV in the main text.

APPENDIX - II

A NOTE ON GENERAL REVIEW OF THE STRATEGY FOR FIXATION OF ${}_5E_b$, THE LAST ENTITY OF THE SET $\{{}_5E_x\}$

Let us assume for the purpose of simplicity that the observed values of ${}_5S_{x+}$ are available at ages 5, 10, 15,, 60 and 65. It may be re-emphasized that the life table is completely determined once ${}_5E_{65}$ is fixed. Also, the same (or nearly the same) values of ${}_5E_{65}$ determines the same (or nearly the same) life table whatever be the process, elaborate or simple, of arriving at this value. Analysis of C-D (West) and S-G model life tables shows that, from a practical point of view, it is reasonable to suppose for the combined series of ${}_5E_{65}$'s corresponding to C-D (West) and S-G model that ${}_5E_{65}$ lies between 0.7557 and 0.7967. Choice of ${}_5E_{65}$ as their mid-value, viz, 0.7762 may entail a maximum error of 2.64%. This uncertainly appears rather wide, one has to seek methods by which narrower limits may be set for ${}_5E_{65}$. An examination of Table 3.1 shows that this can be achieved if, on the basis of ~~some~~ suitable criteria (discussed in Chapter V), it can be found as to which of the two C-D (West) or S-G model sets is likely to be more in accord, broadly speaking with the mortality behaviour of the population under study. The narrowing is to the extent of 0.23% of the mid-value of 0.7708 for the C-D series, and for the S-G models 1.31% of the mid-value of 0.7762.

We have not so far made use of the fact that the values of ${}_5E_5$, even when we consider the heterogeneous mixture of C-D and S-G models, lie within a very narrow range, -- so narrow that ${}_5E_5$ may be considered to be almost a constant. The choice of ${}_5E_5$ as the 'combined' mid-value, viz., 0.9367 may entail a maximum error of only 0.79%. For the C-D model alone it is only 0.30% of the mid-value 0.9321, and for S-G model alone only 0.68% of the mid-value 0.9377. One may use this fact of near-constancy of ${}_5E_5$ as a means to judge whether the ${}_5E_{65}$ fixed earlier is, from a practical point of view, satisfactory or not. In this connection it may be kept in mind that once ${}_5E_{65}$ is fixed, the life table is also fixed, and consequently ${}_5E_5$ is also fixed. With reference to the illustration given earlier use was made of ${}_5E_{65}$ values of West Model for levels 14 and 15. These ${}_5E_{65}$ values are, of course, within the 'observed' limits and consequently the selected or fixed ${}_5E_{65}$, obtained by linear interpolation also is within limits. But, even though West Model ${}_5E_5$'s must also be within 'observed' limits 0.9293 and 0.9349 there is no guarantee that the ${}_5E_5$ value of the life table, finally arrived at by the iteration process would also be within those limits. It must be pointed out that those 'observed' limits are not so sacrosanct that these cannot under any circumstances be crossed. From a practical point of view, values slightly outside these limits are acceptable.

Now, if after fixing a ${}_5E_{65}$ it is found that the 'final' life-table gives a ${}_5E_5$ very much outside the observed limits, then one may wonder whether it would not be proper to regard the fixation of ${}_5E_{65}$ as a tentative one, to be adjusted so that the value of ${}_5E_5$ becomes acceptable, without at the same time making ${}_5E_{65}$ value unacceptable. This simultaneous consideration of the two quantities can be made fully objective through the following measure δ , which is a function of both ${}_5E_{65}$ and ${}_5E_5$ of tentative or final table. The following notations are used here,

$M({}_5E_x)$ = 'observed' median of ${}_5E_x$ values of the (approximate) models.

$\mu({}_5E_x)$ = 'observed' mean deviation about the median of the ${}_5E_x$ values of the models

$$\delta = \left| \frac{{}_5E_5 - M({}_5E_5)}{\mu({}_5E_5)} \right| + \left| \frac{{}_5E_{65} - M({}_5E_{65})}{\mu({}_5E_{65})} \right|$$

In principle, that ${}_5E_{65}$ is to be chosen which leads to the smallest δ . In practice this involves (a) selection of a suitable set of ${}_5E_{65}$ values, (b) obtaining 'final' life tables for each ${}_5E_{65}$ value, (c) calculation of ${}_5E_5$'s for these life tables, and (d) finally calculation of the corresponding δ 's. The ${}_5E_{65}$ value which gives the smallest δ may be taken as the 'final' fixation. (Strictly, for further refinement, one should treat this as a fixation of a 'focal' or

'key' value around and near which further trial $5E_{65}$ values are to be taken for further examination).

The above idea can be pursued further if similar attention is to be placed not only on $5E_5$ and $5E_{65}$, but on the entire set of $5E_x$ values (c.f. footnote 14 in main text). For this purpose the definition of δ should be modified to cover not only $5E_5$ and $5E_{65}$ but also other $5E_x$'s. Incidentally, δ may also be made use of the selection of proper model set. Finally, it may be observed that the general quality of data may not justify the adoption of the above laborious procedure to its full length, and short-cuts may be desirable. An alternative procedure for the fixation of $5E_b$ can be found in Chapter V.

APPENDIX-III

ON CONVERGENCE OF THE ITERATION PROCESS FOR SOLVING SUCCESSIVELY A SET OF EQUATIONS EACH OF WHICH CORRES- PONDS TO DIFFERENT AGE GROUP CONTAINING $5P_x$ AS UNKNOWN: MATHEMATICAL TREATMENT

When a numerical equation $f(x) = 0$ can be expressed in the form -

$$x = \phi(x) \dots\dots\dots (1)$$

the real roots of the equation $f(x) = 0$ can be obtained by the process of iteration which has been discussed in greater detail elsewhere (Scarborough, 1958; and Dixon, 1974). A brief mathematical account of the process is given in the following lines. To begin with the process, it is necessary to know, somehow, an approximate value, say $x^{(0)}$, of the desired root. Substituting this in R.H.S. of (1), one may obtain an improved value say $x^{(1)}$, of the desired root given by -

$$x^{(1)} = \phi(x^{(0)})$$

Then the succeeding approximations are -

$$x^{(2)} = \phi(x^{(1)})$$

$$x^{(3)} = \phi(x^{(2)})$$

$$\dots\dots\dots$$

$$x^{(n)} = \phi(x^{(n-1)})$$

If $\phi'(x)$ denotes the first order derivative of $\phi(x)$, then the sequence $\{x^{(n)}\}$ converges to the desired root (real) under the condition that $|\phi'(x)|$ should be less than unity in the neighbourhood of the desired root. In practice the process is continued until two consecutive approximations become very close to each other. In mathematical terms, the process will be discontinued at the r -th iteration if the following condition gets satisfied,

$$|x^{(r-1)} - x^{(r)}| \leq \epsilon \dots\dots\dots (2)$$

where ϵ is a pre-assigned positive number however small. 1/

1/ To be on safe side, it may be desirable to ensure that the absolute values of the differences $x^{(r)} - x^{(r+1)}$ and $x^{(r+1)} - x^{(r+2)}$ should also be less than or equal to ϵ .

In the present case, however, we are not solving a single equation but a set of equations 2/, namely -

$$f({}_5p_x) = {}_5p_x - {}_5s_x + / {}_5E_x = 0; \text{ for } x = a, a+5, \dots b-5, b \dots (3)$$

successively for two unknowns ${}_5p_x$ and ${}_5E_x$ corresponding to each of the above equations mentioned in (3), on the basis of a known set of ${}_5s_x$ values, such that the solution set $\{ {}_5p_x; x = a, a+5, \dots b-5, b \}$ satisfies the properties of the ${}_5p_x$ column of a life table 3/. The foregoing discussion, mentioned in the earlier paragraphs, regarding any iteration process for solving an equation of the form $f(x) = 0$ indicates that it is necessary to find whether the equation $f({}_5p_x) = 0$, shown in (3), can be expressed in the form ${}_5p_x = \psi({}_5p_x)$, similar to that of the equation (1), so as to ensure the applicability of the process and to examine whether the iteration process converges or not. It may be noted here that the mathematical proof for convergence of the iteration process for life table construction, described below, is based only on the case where the process takes into account the conventional approximation for obtaining ${}_5L_x$ from l_x values.

2/ It should be clear in that the equations, mentioned in (3), are not simultaneous equations, because these equations correspond to different age groups and each of them contains two unknown ${}_5E_x$ and ${}_5p_x$ which are themselves variable over the equations.

3/ Since any equation belonging to the set of equations presented in (3), contains two unknowns, viz. ${}_5E_x$ and ${}_5p_x$, therefore, theoretically, each such equation may have large number solutions for ${}_5E_x$ and ${}_5p_x$. But we are not interested to all such solutions, rather, we like to have a particular solution set of ${}_5p_x$ values which satisfies certain fundamental properties of the ' ${}_5p_x$ column' of a life table as mentioned in the footnote 28 of the chapter III.

It has already been shown in earlier section that the equation $f(5^p_x) = 0$, a member of the set of equations presented in (3), is approximately equivalent to the equation (4.10). In mathematical terms, the equation $f(5^p_x) = 0$ can approximately be written as -

$$5^p_x \doteq 5^{s_{x+5}} * (A_{x+5} + B_{x+5} * 5^p_x), \text{ where } a \leq x \leq b-5 \dots (4)$$

where the set of s_{x+5} values is known, A_{x+5} is defined by the relation (4.10a) and B_{x+5} equals to $1 + A_{x+5}$. One may easily find from (4.10a) that A_{x+5} and B_{x+5} depends only on 5^p_{x+5} since the set of s_{x+5} values is already known to us. Thus, the expression in the R.H.S of (4) can be considered as a function of 5^p_x and 5^p_{x+5} . That is, the equation (4) can be expressed in the following form -

$$5^p_x \doteq \psi(5^p_x, 5^p_{x+5}), \text{ where } a \leq x \leq b-5 \dots (4a)$$

One may easily find that the above equation is still not in the form of (1) and hence, the equation $f(5^p_x) = 0$ cannot be solved by iterative method using the relation (4a). But, if we can find an age, say k , lying between a and b where 5^p_k (is known or can be estimated correctly by certain procedure) remains unaffected by the iteration process, the equation $f(5^p_{k-5}) = 0$ can be transformed in the following form :

$$5^{p(i)}_{k-5} \doteq \psi(5^{p(i-1)}_{k-5}) \dots \dots \dots (4b)$$

where,
$$\psi(5^{p_{k-5}}) = 5^{s(k-5)+} * (A_k + B_k * 5^{p_{k-5}}) \dots (4.c)$$

and
$$A_k = \frac{1 - 5^{s_k+}}{1 + 5^{p_k}} \text{ and } B_k = 1 + A_k \dots \dots \dots (4.d)$$

Since the equation $f(5^{p_{k-5}}) = 5^{p_{k-5}} - 5^{s(k-5)+} / 5^{E_{k-5}} = 0$ can be expressed in the form of (1) the equation $f(5^{p_{k-5}}) = 0$ can be solved through an iterative procedure by using the relation (4b). Let $5^{p_{k-5}^{(1)}}$ denote an initial value of $5^{p_{k-5}}$.

Now, substituting this initial value of $5^{p_{k-5}}$, that is, $5^{p_{k-5}^{(1)}}$ in R.H.S. of (4.b) one may obtain an improved value, say $5^{p_{k-5}^{(2)}}$ of $5^{p_{k-5}}$ given by -

$$5^{p_{k-5}^{(2)}} = \psi(5^{p_{k-5}^{(1)}})$$

The subsequent approximations can also be obtained by substituting the previous approximate value of $5^{p_{k-5}}$ to the R.H.S. of (4.b). Now, the question is whether the sequence $\{5^{p_{k-5}^{(n)}}\}$ converges to the true value of $5^{p_{k-5}}$ or not. Now, if we can show that $|\psi'(5^{p_{k-5}})|$ is less than one, as mentioned earlier, our purpose will be served.

Differentiating both sides of (4.c) with respect to $5^{p_{k-5}}$, we find that -

$$\psi'(5^{p_{k-5}}) = 5^{s(k-5)+} * B_k \dots \dots \dots (5)$$

Since A_k and B_k are independent of $5^{p_{k-5}}$ and remain unaltered

over iterations; and $5^s(k-5)_+$ is a known quantity. One may easily verify that $\psi'(5^p_{k-5})$ cannot be negative since both $5^s(k-5)_+$ and $B_k (= 1 + A_k)$ are always positive. Thus, $\psi'(5^p_{k-5})$ will be less than unity if and only if the product $[5^s(k-5)_+ * B_k]$ is less than unity. Using (4.d) we find,

$$\left(5^s(k-5)_+\right) * B_k = \frac{5^s(k-5)_+ * (2 + 5^p_k - 5^s_{k+})}{1 + 5^p_k}$$

now, the product $5^s(k-5)_+ * B_k$ will be less than unity if

$$(5^s(k-5)_+) * (2 + 5^p_k - 5^s_{k+}) < 1 + 5^p_k \dots\dots\dots (6)$$

since, 5^s_{k+} and $5^s(k-5)_+$ are all positive fractions,

after transposing the similar quantities in one side we find

that the inequality (6) will hold true if --

$$5^p_k > \frac{2 * 5^s(k-5)_+ - 5^s_{k+} * 5^s(k-5)_+ - 1}{1 - 5^s(k-5)_+} \dots\dots\dots (7)$$

or if,

$$5^p_k > \frac{5^s(k-5)_+ * (1 - 5^s_{k+})}{1 - 5^s(k-5)_+} - 1$$

or if,

$$1 + 5^p_k > 5^s(k-5)_+ + 2.5 = 5^s_{k-2.5} \dots\dots\dots (8)$$

since from (3.57d) of chapter III we find that -

$$5^s_{x+2.5} = \frac{5^s_{x+} [1 - 5^s(x+5)_+]}{1 - 5^s_{x+}}, \text{ for all } x$$

The above inequality is true for all positive values of ${}_5p_k$

since ${}_5s_{(k-5)+2.5} (= {}_5L_k / {}_5L_{k-5})$, being the 5-year usual life table survival ratio, ^{4/} lies between 0 and 1. Hence,

$\psi'({}_5p_{k-5})$ is less than unity for any positive value of ${}_5p_k$

which is known in advance and remains steady over the various iterations. The above finding indicates that the sequence

$\left\{ {}_5p_{k-5}^{(n)} \right\}$ of the successive approximations for ${}_5p_{k-5}$ obtained through the repeated application of the equation (4.b) will converge to the desired root, say ${}_5p_{k-5}'$, of the equation $f({}_5p_{k-5}) = 0$ subject to the condition that the value of ${}_5p_k$ is known and remain steady over iterations. Now, after obtaining the value of ${}_5p_{k-5}$ through the method of successive approximation ^{5/} by using the formula (4.b) one may easily find that the equation $f({}_5p_{k-10}) = {}_5p_{k-10} - {}_5s_{(k-10)+2.5} / {}_5E_{k-10} = 0 \dots (9)$

can be expressed in the following form as before -

$${}_5p_{k-10}^{(i)} = \psi({}_5p_{k-10}^{(i-1)}) \dots \dots \dots (10)$$

since the value of ${}_5p_{k-5}'$ obtained by the method of successive approximation becomes steady over the iterations. ^{6/} Thus it may

^{4/} It may be noted that the value of ${}_5s_{(k-5)+2.5}$ has been obtained from the given set of ${}_5s_{x+}$ values.

^{5/} The 'method of successive approximation' and the 'method of iteration' are synonymous.

^{6/} Where ${}_5p_{k-10}^{(i-1)}$ and ${}_5p_{k-10}^{(i)}$ are the values of ${}_5p_{k-10}$ at the (i-1)th and i-th iterations respectively.

be found easily, as shown earlier in the case of $5^{p_{k-5}}$, that the sequence of $5^{p_{k-10}^{(i)}}$, obtained by the method iteration through the use of the formula (10), will converge to the desired root, $5^{p_{k-10}}$, of the equation (9). Proceeding in this manner, one may obtain successively the values of $5^{p_{k-15}}$, $5^{p_{k-20}}$, ..., $5^{p_{a+5}}$, 5^{p_a} by the method of iteration.

The above mathematical treatments reveal if the value of $5^{p_{x+5}}$, where $a \leq x \leq b-5$, is known, remains unchanged over the iteration, it is possible to show that the following equation -

$$f(5^{p_x}) = 5^{p_x} - 5^{s_{x+5}} / 5^{E_x} = 0 \dots\dots (11 a)$$

can be approximately expressed as a linear function of 5^{p_x} , as shown below:

$$5^{p_x^{(i)}} \doteq \psi(5^{p_x^{(i-1)}}) = 5^{s_{x+5}} * \left[A_{x+5} + B_{x+5} * 5^{p_x^{(i-1)}} \right] \dots (11 b)$$

where $5^{p_x^{(i-1)}}$ and $5^{p_x^{(i)}}$ are the values of 5^{p_x} at the (i-1)th

and i-th iterations respectively and the co-efficients $\frac{1}{5^{s_{x+5}}} A_{x+5}$

and B_{x+5} , independent of 5^{p_x} , are given by,

7/ It may be noted that the values of the co-efficients A_{x+5} and B_{x+5} , which are dependent on $5^{p_{x+5}}$, also remain unaltered over the various iterations for a given value of $5^{s(x+5)+}$ because, the value of $5^{p_{x+5}}$ is known.

$$A_{x+5} = \frac{1 - 5^s(x+5)}{1 + 5^p_{x+5}} \text{ and } B_{x+5} = 1 + A_{x+5} \dots (11c)$$

Furthermore, it can be shown that the sequence of $5^p_x^{(i)}$ values, obtained by the method of iteration through the use of the formulas (11b) and (11c) will converge to the desired root, 5^p_x of the equation (11a). Thus, for example, if the value of 5^p_k , where $a + 5 \leq k \leq b$ is known and remains steady over the iteration, one can find all the values of 5^p_x , where, $a \leq x \leq k-5$, successively in a decreasing order of x starting with 5^p_{k-5} by the method of iteration through the formulas (11b) and (11c).

Now, let us go back to the iteration process which has been proposed for construction of life table at ages a onwards on the basis of the observed set of 5^s_{x+} values, that is,

$$\{5^s_{x+}; x = a, a+5, \dots, b-5, b\}. \text{ The process may}$$

start with any arbitrary set of 5^E_x values (positive) -

$$\{5^{E(0)}_x; x = a, a+5, \dots, b-5, b\}. \text{ Applying this initial}$$

set of 5^E_x values on the observed set of 5^s_{x+} values, one can obtain the first approximated set of 5^p_x values - $\{5^{p(1)}_x; x = a, a+5, \dots, b-5, b\}$ through the formula $5^p_x = 5^s_{x+} / 5^E_x$.

This first approximated set of 5^p_x values at ages a and above may be used to construct an abridged life table at ages a onwards which will, in turn, produce a new set of 5^E_x values at ages a and above. This new set of 5^E_x values may once

again be applied on the observed set of ${}_5s_{x+}$ values to obtain the second approximated set of ${}_5p_x$ values at ages a and above which will, in turn, produce another new set of ${}_5E_x$ values at ages a and above after constructing a life table at ages a and above with this new set of ${}_5p_x$ values. Similar operations may be repeated in a cyclical order until the two consecutive sets of ${}_5E_x$ are almost identical. It is interesting to note, as mentioned earlier, that the iteration process for construction of life table is such that it makes the last values of the initial set of ${}_5E_x$, namely, ${}_5E_b^{(0)}$, invariant over the various iterations and therefore, the last value of the first approximated set of ${}_5p_x$ values, namely, ${}_5p_b^{(1)}$ also remains invariant over the iterations.^{8/} Since, ${}_5p_b^{(1)}$ remains steady over the iterations, the formula (11b) starts acting implicitly,^{9/} from the second iteration onwards to obtain the values of ${}_5p_{b-5}$ at various iterations and finally the sequence of ${}_5p_{b-5}^{(i)}$ values will converge at certain stage of iteration to some value, a root of the equation $f({}_5p_{b-5}) = 0$; and until then the other ${}_5p_x$'s where $a \leq x \leq b-10$, go on changing their values at various iterations through the

^{8/} This invariant characteristics of ${}_5E_b^{(0)}$ and ${}_5p_b^{(1)}$ have already been shown empirically and mathematically. For details, see Section [4.1.1.(I)] of chapter IV.

^{9/} The word 'implicitly' has been used here to signify that though the formula is not apparently visible during the iteration process for life table construction, but actually it is acting latently during the iteration process. It carries the same meaning wherever it is used.

formula (4 a) ^{10/} implicitly. As soon as the value of ${}_5P_{b-5}$ converges at certain iteration, the formula (11 b) once again starts acting implicitly from the very next iteration to obtain the values of ${}_5P_{b-10}$ at various iterations and finally the sequence of ${}_5P_{b-10}^{(i)}$ values converges to certain value, a root of the equation - $f({}_5P_{b-10}) = 0$; whereas, the other ${}_5P_x$'s, where $a \leq x \leq b-15$ go on changing their values at various iterations through the formula (4 a) implicitly. This process will continue until all values of ${}_5P_x$ become steady over the various iterations. It should be mentioned here that the whole life table at ages a onwards is determined uniquely by the last value of the initial set of ${}_5E_x$ values, that is, ${}_5E_b^{(o)}$. Because the iteration process of life table construction converges for any arbitrary initial set of ${}_5E_x$ values (positive) of which only the last value of the initial set of ${}_5E_x$, namely, ${}_5E_b^{(o)}$ remains unaltered during the iteration process; whereas, the other values of ${}_5E_x^{(o)}$ for $x < b$ get modified successively during the process of iteration. Hence, one has to take utmost care in determining the value ^{11/} of ${}_5E_b^{(o)}$; the remaining values of ${}_5E_x^{(o)}$ for $a \leq x \leq b-5$ could be any arbitrary positive values need not necessarily be less than one even.

^{10/} It is worthwhile to mention here that though the structural forms of the two formulas (11 b) and (4 a) are apparently identical, but actually in the case of the former formula the co-efficients A_{x+5} and B_{x+5} , dependent on ${}_5P_{x+5}$, remain steady over the various iterations due to invariant characteristic of ${}_5P_{x+5}$, whereas in the case of latter one the behavior of A_{x+5} and B_{x+5} are just the reverse.

^{11/} A method of estimating appropriate value for ${}_5E_b^{(o)}$, the last value of the initial set of ${}_5E_x$'s, are discussed elsewhere - (See chapter V and see also Appendix-I)

While describing the iteration process, it has been mentioned that the process will be discontinued as soon as the following condition holds true, that is,

$$\left| \frac{5^{E_x^{(M-1)}} - 5^{E_x^{(M)}}}{5^{E_x^{(M-1)}}} \right| \leq \epsilon \quad \text{for all } x \dots (12a)$$

where, $5^{E_x^{(M)}}$ is the value of 5^{E_x} obtained from the M-th iterated life table, M is a positive integer, however large and ϵ is a pre-assigned positive number, however small. Now, our problem is how to obtain the condition (12a)

It has already been mentioned earlier that the iteration process, operated through the formula - $5^{p_x^{(n)}} = 5^{s_{x+}} / 5^{E_x^{(n-1)}}$... (12b) where the upper suffix within bracket denotes the iteration number, will converge for a given set of $5^{s_{x+}}$ values subject to the condition that the last value of initial set of 5^{E_x} values will remain unaltered during the process of iteration.

According to Cauchy's general principle of Convergence

(Narayan, Shanti, 1962), the necessary and sufficient condition for convergence of the sequence $\{5^{p_x^{(n)}}\}$ is that to every positive number, δ , however small, there corresponds a positive integer m , however large, such that -

$$|5^{p_x^{(n+p)}} - 5^{p_x^{(n)}}| < \delta, \quad \text{for all } x \dots (12c)$$

where $n \geq m$ and p has any positive integral values.

In particular, let us take an integer M which is greater than m . Now, taking $n = M$ and $p = 1$ the inequality (12c) becomes -

$$\left| 5^{p_x^{(M+1)}} - 5^{p_x^{(M)}} \right| < \delta \quad \text{for all } x$$

Now, dividing both sides of the above inequality by $5^{p_x^{(M+1)}} > 0$ we find -

$$\left| \frac{5^{p_x^{(M)}}}{5^{p_x^{(M+1)}}} - 1 \right| < \frac{\delta}{5^{p_x^{(M+1)}}} = \epsilon \quad (\text{say}), \quad \text{for all } x \quad \dots (12d)$$

Since $5^{p_x^{(M)}}$ and $5^{p_x^{(M+1)}}$ are obtained from the M -th and $(M+1)$ -th iterated life tables respectively, then as per formula (12b) we find that -

$$5^{p_x^{(M)}} = 5^{s_x} / 5^{E_x^{(M-1)}} \dots \dots \dots (12e)$$

$$\text{and } 5^{p_x^{(M+1)}} = 5^{s_x} / 5^{E_x^{(M)}} \dots \dots \dots (12f)$$

subtracting (12f) from (12e) and then dividing by (12f) we obtain,

$$\left| \frac{\begin{matrix} (M+1) & (M) \\ 5^p_x & - & 5^p_x \end{matrix}}{(M+1)} \right| = \left| \frac{\begin{matrix} (M) & (M-1) \\ 5^E_x & - & 5^E_x \end{matrix}}{(M-1)} \right| \text{ for all } x \dots (12g)$$

5^p_x 5^E_x

using (12d) and (12g) we obtain the desired condition (12a).

APPENDIX - IV

Table A 4.1

Single Year (Unsmoothed) Age-Sex-Distribution for
All-India, 1951 Census Adjusted for Boundary and
Age not Stated

Age	Population		Age	Population	
	Male	Female		Male	Female
0	6020960	5849767	19	1968505	1824799
1	4073407	4040665	20	5232126	5935223
2	4836317	4795289	21	1871814	1748329
3	4876693	4961432	22	3498431	3421924
4	4775040	4692588	23	1848150	1746397
5	5380386	5144416	24	2415316	2352423
6	4804915	4680246	25	6610760	6526009
7	4331654	4197854	26	2414027	2191931
8	5147567	4923940	27	1912191	1692003
9	3642410	3583081	28	3040675	2930306
10	6319986	5867563	29	1310466	1196500
11	3193071	3066791	30	6727535	6952360
12	6141171	5177607	31	1218005	1069771
13	3247828	3039210	32	2841362	2495806
14	4054786	3680640	33	1258784	1077905
15	4428376	3897532	34	1512265	1371193
16	3895396	3695952	35	5892806	5218902
17	2274505	2165305	36	1899090	1580295
18	4303822	4246275	37	1232447	1033255

Contd.... /

Table A 4.1 (Contd.)

Age	Population		Age	Population	
	Male	Female		Male	Female
38	1895025	1710016	56	661113	572191
39	999649	853221	57	493697	407397
40	6191770	5881805	58	710794	604842
41	951907	776969	59	421878	343279
42	1785718	1474477	60	2963801	3252414
43	899740	738449	61	380160	359277
44	1053095	899211	62	572737	558945
45	4599464	3947085	63	318461	283399
46	1006880	834543	64	373857	352275
47	852792	689197	65	1267573	1284980
48	1329458	1167516	66	257350	232963
49	682488	605559	67	228569	208051
50	4676328	4504461	68	273257	273998
51	696587	556577	69	170583	163900
52	1092585	906515	70+	3603250	3869659
53	593544	500042			
54	737935	641648			
55	2485093	2172171			
Total Population				185680185	175702522

Source: Borrowed from - Chanana, H.B., N.Y. Gore and G. Somasekharan Nair, 1969, A Demographic Study of India - Based on the 1951 and 1961 Census Data; submitted to the Demographic Training Research Centre, Bombay as a part of the training programme.

Table A 4.2

Single-Year (Unsmoothed) Age-Sex-Distribution for
All-India, 1961 Census Adjusted for Boundary and
Age not Stated

Age	Population		Age	Population	
	Male	Female		Male	Female
0	6807683	6772188	18	5146131	5107005
1	5852307	5846407	19	2275433	2113376
2	6789315	6667723	20	6215477	7426816
3	7035679	7123823	21	2397902	2159647
4	6724124	6546505	22	4657799	4691309
5	7883641	7372317	23	2294850	2153069
6	7184625	6810045	24	2680804	2736362
7	6195514	6098092	25	8411376	8713474
8	7045717	6639784	26	2906925	2586373
9	4783988	4708103	27	2169557	1833166
10	7561523	6835142	28	3806431	3831889
11	3662138	3470942	29	1275924	1110551
12	7041206	5716255	30	8998880	9096918
13	3675611	3265208	31	1133486	867664
14	4350300	3764119	32	3380059	2840801
15	4581088	3843482	33	1188531	935554
16	4231369	3973233	34	1311010	1130003
17	2384908	2271308	35	7701760	6854660

Contd.... /

Table A 4.2 (Contd.)

Age	Population		Age	Population	
	Male	Female		Male	Female
36	2020457	1616144	55	3290488	2940540
37	1092873	835228	56	662680	521981
38	1975296	1858403	57	384699	282153
39	831760	706551	58	637232	564958
40	8013308	7519204	59	311194	242881
41	773834	581886	60	4326192	4417255
42	1846387	1497143	61	354658	274070
43	721283	571351	62	559141	466206
44	744792	612575	63	218627	175727
45	6218456	5385432	64	251587	203114
46	926136	722602	65	1757007	1744357
47	691296	520068	66	199332	160939
48	1358254	1244868	67	169152	137993
49	550038	455923	68	232232	229484
50	6423029	5956354	69	116230	107826
51	559013	404408	70+	4178755	4446045
52	1125773	850113			
53	466646	336954			
54	562293	437521			
Total population				226293201	212941570

Note: The procedure for boundary adjustment has been described in Chapter V of the text and the relevant were borrowed from the census publication :
Office of the Registrar General of India, 1964, Social and Cultural Tables, Census of India, 1961, Vol. I (India) Part II-C(i), Government of India.

APPENDIX

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