

Indian Statistical Institute
Second Semestral Examination 2024-25

M. Stat. II yr
Statistical Inference III

Date: April 25, 2025

Maximum marks: 100

Duration: 3 hrs.

1. Find the set of values of p such that Shannon's entropy of the two point pmf $(p, 1 - p)$ exceeds entropy of the three point pmf $(1/16, 1/16, 7/8)$.
[10]
- 2(a) Define deficiency between two statistical experiments under the same measurable space. Show that for an experiment $\mathcal{E}$ if there is a sufficient statistic T then deficiency of $\mathcal{E}^T$ with respect to $\mathcal{E}$, where only T is observed, is zero.
(b) Show that the deficiency of $\text{Bin}(n, \theta)$ with respect to $\text{Bin}(n + 1, \theta)$ goes to zero as $n \rightarrow \infty$ by deriving a suitable upper bound.
[15+ 20=35]
- 3 (a) Describe the three basic principles of statistical inference namely, Sufficiency, Conditionality and Likelihood, and the Evidence with appropriate examples. (*Answer should be brief and to the point.*)
(b) Consider a data generated as follows. First generate three independent normal variables X_1, X_2, X_3 with a common mean $\theta \in \mathbb{R}$ and variances $\sigma^2, 4\sigma^2$ and $9\sigma^2$, $\sigma^2 > 0$, respectively. Next a discrete variable U is generated independently which assumes values in $\{1, 2, 3\}$ with equal probabilities. Observed data D are recorded as
$$\begin{aligned} D &= (X_1, \min(X_2, X_3), \max(X_2, X_3)) \quad \text{if } U = 1 \\ &= (\min(X_1, X_3), X_2, \max(X_1, X_3)) \quad \text{if } U = 2 \\ &= (\min(X_1, X_2), \max(X_1, X_2), X_3) \quad \text{if } U = 3. \end{aligned}$$
Derive appropriate t -statistics for testing $H_0 : \theta = 0$ in the two cases when (1) U is observed and (2) U is not observed, using conditionality and sufficiency principles.
(c) Let $x = (x_1, x_2, \dots, x_n)$ be a sample of n independent observations from $N(\theta, \sigma^2)$, Let $D = (x_2 - x_1, x_3 - x_1, \dots, x_n - x_1)$ be the difference statistic. Show that the class $\{h(D)\}$ of measurable functions of D does not exhaust the family of σ^2 -oriented statistics.

[5+20+10=35]

P.T.O..

- 4(a) Describe True positives, False positives, Bonferroni corrections and FDR in a formal multiple testing setting with m hypotheses.
- (b) Describe the procedure suggested by Benjamini and Hochberg and state the necessary result which establishes that BH procedure controls FDR in independent multiple hypothesis setting.

[5+ 15=20]