INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2024-25

Course Name : B.Stat First Year
Subject : Numerical Analysis

Time: 2 hours **Date: 28th February, 2025** **Max Marks: 40**

Theorem/result used should be stated clearly. Non-programmable calculators are allowed. Books or notes are not allowed.

✪ Answer all the questions (there is a buffer of 5 marks). ✪

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1. Derive the general expression of error in polynomial interpolation formula, one using the $f^{(n+1)}$ and the other using divided difference. Hence show that Newton's forward interpolation formula *works*, i.e., in the general case error in the last significant figure is less than 1.

$[(3 \times 2) + 4 = 10]$

2. (a) Show that for four equispaced nodes $x_i = x_0 + ih, i = 0, 1, 2, 3, h > 0$,

$$y_2' = \frac{1}{6h}(y_0 - 6y_1 + 3y_2 + 2y_3) - \frac{h^3}{12} f^{iv}(\xi), \quad \xi \in (x_0, x_3).$$

(b) Given that there is a misprint in the following data, find the error and correct the data: 10, 8, 6, 10, 28, 60, 118, 206, 330, 496.

$[5 + 5 = 10]$

3. (a) What is the necessary and sufficient condition for a mechanical quadrature formula to achieve its highest degree of precision? Prove the necessary part.

(b) Compute the following integral

$$\frac{1}{\pi} \int_0^{\pi} \cos(8 \sin x - x)$$

correct to four places of decimals by the (composite) trapezoidal and Simson's rules by taking nine points.

$[(1 + 2) + 10 = 13]$

4. (a) Derive Lagrange's interpolation polynomial by the *method of undetermined coefficients* (i.e., using *partial fraction*). What is the most disadvantage of Lagrange's method?

   (b) Find a polynomial (in its simplest form) to fit the following data.

x	$y = f(x)$	$y' = f'(x)$
1	2	3
2	4	2
5	8	6

[3 + 1 + 8 = 12]