

INDIAN STATISTICAL INSTITUTE
Back Paper Examination : 2025-26
M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY

[Symbols have their usual meaning]

Date: 18.12.2025

Full marks:100

Time: 3 hours

Note : Answer all the questions

1. a) Find a measure of skewness of a binomial distribution with parameter n and p . Hence prove that the distribution is symmetric when $p = q$, where $q = 1 - p$.
b) At an art exhibition there are 15 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 7 out of 10 cases on an average. Given that the expert decides that the painting is authentic what is the probability that it is really original?

[12+8 = 20]

2. a) State and prove the Weak Law of Large numbers due to Khintchine.
b) State and prove the Central Limit Theorem due to Lindberg and Levy. Explain its application in the field of SQC.

[8+(10+2) = 20]

3. a) Derive the p.d.f of χ^2 distribution with n degrees of freedom.
b) Let $X \sim \text{Gamma}(1, \mu)$ and $Y \sim \text{Gamma}(1, \lambda)$ and X and Y are independently distributed. Find the pdf of $Z = \frac{X}{X+Y}$. Identify the distribution of Z .

[10+10 = 20]

4. a) State and prove Bonferroni's inequality.
b) Define monotonic sequence of events. Let $\{A_n\}$ be monotonic sequence of events, then prove that,

$$\lim_{n \rightarrow \infty} P(A_n) = P(\lim_{n \rightarrow \infty} A_n).$$

[10+(2+8) = 20]

5. a) Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

- b) Let $X_i \sim P(\lambda_i)$, $i=1,2$. X_1 and X_2 are independent. Let $Y = X_1 + X_2$. Find the distribution of Y . Find the conditional distribution of X_1 given $Y = y$.

[10+(5+5) = 20]