Consumer Welfare and Privatization in Mixed Markets: A Developing Country Perspective

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Chapter 1

Introduction

1.1 Motivation

This thesis centers on a broader and timely theme: the existence and relevance of mixed markets, and the formulation of optimal policies to guide the transition toward private provision, aimed at improving consumer welfare in developing countries. Mixed markets are defined by the coexistence of public and private firms. Historically, public firms have dominated the markets in developing and transition economies. Empirical evidence suggests that public firms tend to be more inefficient compared to their private counterparts (Megginson and Netter, 2001). As a result, many of these economies embarked on the privatization of public enterprises in the hope of improving market efficiency. However, private firms usually prioritize their own profits, while public firms maximize the social welfare. Mixed markets, therefore, offer a blend of both, combining efficiencies of private firms with welfare maximizing efforts of public firms. Does this lead to a better equilibrium outcome for consumers compared to fully private or a fully public market regime? We dedicate the first three chapters of this thesis to investigate this question.

The debate on the merits and demerits of mixed markets and privatization in developing, as well as in developed and transition economies—which underwent privatization in the 1980s and 1990s—is not new (see, for example, the Introduction in Anderson, De Palma, and Thisse, 1997). However, the debate has largely centered around production efficiency, profitability, and proceeds from privatization accruing to the government. On those measures, the balance of evidence indicates that most privatization programs, especially in developed economies, have been successful. Efficiency and profitability improved with privatization, and well-designed privatization programs generated significant revenues for the government (Megginson and Netter, 2001; Estrin, Hanousek, Kocenda, and Svejnar, 2009).

But what about consumers? Consumer interests play a prominent role in policy discussions as well as academic literature on mergers and competition policy (Farrell and Shapiro, 1990; Nocke and Whinston, 2010, 2013). Unless efficiencies are significant enough to outweigh anti-competitive effects and reduce consumer prices, mergers are unlikely to be approved by competition authorities in conventional markets.[1] However, consumer interests, or more generally, distributional consequences, do not feature prominently in privatization decisions (Duggan, Gupta, Jackson, and Templeton, 2023). As privatization travels from the developed to the developing world, distributional concerns are gaining more attention (Estrin and Pelletier, 2018). Even if prices increase with privatization, the adverse impact on consumers in developed economies can be partially offset because proceeds from privatization substitute for taxes. Furthermore, a part of increased profits accrues to consumers who hold shares in firms (either directly or indirectly, e.g., through pension funds). Neither of these channels is robust in developing countries.[2] Thus, privatization has little benefit for a large fraction of the population in developing economies unless it improves access or lowers prices for consumers (Stiglitz, 2002; Birdsall and Nellis, 2003; Piketty, 2014). Therefore, it is essential to put forth consumer interests at the

[1]Competition authorities have approved the majority of mergers in digital markets, which are typically multi-sided and characterized by strong network effects (OECD, 2023). Since merger assessment in digital markets relies on a distinct framework for evaluating anti-competitive effects, we restrict our analysis to conventional (non-digital) markets.

[2]According to a 2019 survey by the Reserve Bank of India (Rajan, 2023, June 18), only 3% of the Indian population and 13 % of the Chinese population invest in the stock market. The corresponding figure for the US is 55%.

core of discussion around transition toward private provisions in the context of developing economies.

The first three chapters of this thesis place consumer welfare at the core of the analysis on the desirability and design of mixed markets and privatization strategies. Using static models of oligopoly under varying assumptions—homogeneous versus differentiated products, cost heterogeneity, and vertical market structures—these chapters examine whether and when mixed markets offer superior outcomes. Together, they provide a comprehensive justification for the persistence of mixed markets in developing economies, and underscore the importance of timing, firm selection, and welfare considerations in the privatization process.

Building on this foundation, the fourth chapter introduces an additional layer of complexity by examining mixed markets in the presence of congestion. For several products and services, as a firm serves more customers, the utility from the product or service declines for each individual customer. We refer to this phenomenon as congestion disutility, which is evident in sectors such as healthcare, education, telecommunications, public transport, banking, and many others. Larger class sizes in a school contribute to reduced learning outcomes (Card and Krueger, 1992; Angrist and Lavy, 1999; Duflo, Dupas, and Kremer, 2015; Muralidharan and Sundararaman, 2015). More patients in a hospital lead to longer waiting times (Siciliani and Hurst, 2005; Dixon and Siciliani, 2009; Kreindler, 2010). Heavier traffic on roads results in longer and more costly delays (Schrank and Lomax, 2007). An increased number of internet users slows down connectivity for everyone (Kim, Chan, and Gupta, 2007). In these congestion-prone sectors, markets are frequently served by both public and private providers. While public firms may strive for maximizing social surplus, private firms, motivated by profit, may have higher incentives to address congestion efficiently. This creates a new tradeoff in mixed markets, where the objectives of public provision must be balanced against the efficiency advantages of private providers.

Does the presence of congestion tilt the balance in favor of private provision? The fourth chapter investigates this question by analyzing the role of congestion in shaping the optimal choice between public and private provisions in such markets.

Together, these chapters aim to broaden the lens through which privatization and market design are evaluated—shifting the focus from social welfare or efficiency to consumer welfare, and offering policy-relevant insights for developing economies navigating the transition toward greater private sector participation.

1.2 Summary

This thesis is presented in four chapters, each examining a distinct aspect of mixed market structures, with a consistent focus on consumer surplus as the primary metric for evaluating the desirability of privatization or private provision.

The first chapter introduces a symmetric Cournot oligopoly model in which both public and private firms produce a homogeneous good and compete in quantities. We find that consumer surplus is maximized at the two extremes: when the market consists solely of public firms or solely of private firms. In contrast, mixed regimes consistently yield intermediate outcomes, never achieving either the highest or the lowest level of consumer surplus. This result is robust to the level of competition, the specific objective functions assigned to public firms, and holds across all log-concave demand functions and convex cost functions. Importantly, when cost functions are strictly convex, we show that, contrary to conventional wisdom, an increase in the number of firms does not necessarily support privatization; in fact, it may weaken the case for introducing private firms.

The first chapter further extends the analysis to Bertrand competition with differentiated products, where firms compete in prices rather than quantities. Unlike in Cournot settings, firms' strategies are strategic complements under Bertrand competition. Despite this fundamental difference in the nature of competition, the key finding persists: mixed oligopolies never maximize consumer surplus. Moreover, in some cases, mixed markets can yield the lowest consumer surplus, even compared to fully public or private regimes. This counterintuitive outcome stems from the regime-contingent behavior of public firms. That is, an inefficient public firm with welfare concerns may respond to rival pricing by setting a higher price in a mixed regime than it would in a fully public one, thereby dampening consumer surplus.

While the first chapter shows that mixed markets never yield the highest or lowest consumer surplus, this finding appears at odds with their widespread existence and institutional support across the globe. Several explanations may account for this disconnect. First, privatization decisions are often driven by objectives such as profitability or broader welfare considerations, rather than consumer surplus alone. Second, governments concerned with consumer surplus may still prefer a mixed regime in settings with weak competition policy, where full privatization could increase the risk of collusion. However, in the next two chapters, we demonstrate that mixed regimes can, in fact, top the consumer surplus ranking without resorting to alternative welfare metrics or relying on collusion-based explanations. What is required is to move beyond the symmetric, single-stage oligopoly framework used in the first chapter.

The second chapter relaxes the assumption of symmetric firms by introducing cost heterogeneity, a key real-world feature, as firms often differ in cost structures for reasons unrelated to ownership. While privatization can improve efficiency, it may not fully eliminate these underlying cost differences. In this setting, we show that mixed regimes can deliver the highest consumer surplus. For instance, in a duopoly, privatizing the inefficient firm while retaining public ownership of the efficient one can outperform both fully public and fully private regimes. Conversely, if privatization targets the more efficient firm, leaving the less efficient one public, consumer surplus can be the lowest among all ownership structures. These results suggest that ownership design should account for firm-specific efficiency differences, and that optimal privatization policy may be highly context-dependent.

The third chapter addresses another limitation of earlier models by considering a vertically related market, with upstream and downstream monopolists interacting in a two-stage production process. Public firms in both sectors introduce two layers of inefficiency, while private firms in both sectors generate two rounds of markups, a classic double marginalization problem. We show that a mixed regime can yield the highest consumer surplus by eliminating one layer of inefficiency and one round of markup. This outcome is most likely when markups are moderate and inefficiencies are unevenly distributed across the two sectors. If the inefficiency of public firms is symmetric across the upstream and downstream sectors, mixed regimes tend to perform intermediately or even poorly in terms of consumer surplus. However, when public firm inefficiencies differ sufficiently across sectors, and markup levels are not too high, a mixed structure can outperform both extremes. Extending the model to a richer setting with both upstream and downstream oligopoly shows that the desirability of mixed regimes persists and, in fact, becomes stronger as competition intensifies. Thus, the interaction between vertical structure, firm efficiency, and market power plays a critical role in shaping welfare outcomes in privatization decisions.

While first three chapters evaluate privatization using consumer surplus as a welfare metric under standard oligopoly assumptions, the fourth chapter introduces a new dimension: congestion. In many markets such as healthcare, education, telecommunications, transportation etc., congestion disutility arises as firms serve more consumers, reducing individual utility. We model a Cournot oligopoly with congestion under both mixed and fully private regimes and characterize equilibrium outcomes in the presence of congestion. We conduct comparative static analysis with respect to market size and competition and determine a consumer surplus threshold: a fully private regime yields a higher consumer surplus if the relative cost inefficiency of the public firm exceeds this threshold. We show that congestion and increases in market size both lower this threshold, making privatization more favorable. However, the effect of competition is more nuanced. An increase in competition facilitates privatization only when the initial level of competition is low. Beyond a certain point, additional competition could in fact facilitate public provision. These results highlight how market structures and conditions influence optimal ownership structure in the presence of congestion.

Collectively, the four chapters of this thesis underscore the importance of evaluating privatization and ownership design through the lens of consumer welfare, particularly in the context of developing economies. The results challenge simplistic assumptions about public versus private provisions and offer a nuanced framework for understanding when mixed markets can be not just a compromise, but an optimal institutional structure.

Chapter 2

(When) Are Mixed Markets Good for Consumers?

2.1 Introduction

The literature on mixed oligopoly shows that privatization can improve welfare in various oligopoly environments. De Fraja and Delbono (1989) demonstrate that in the presence of decreasing returns to scale, privatization, by reducing the scale of production of public firms, improves production efficiency, which can enhance welfare. Using a spatial model of product differentiation, Cremer, Marchand, and Thisse (1991) show that the presence of a public firm could result in a product configuration that is too concentrated. Privatization can improve welfare by reducing this concentration. Introducing endogenous cost differentials between public and private enterprises, Matsumura and Matsushima (2004) show that private firms are indeed more efficient than public ones. However, increased efficiency comes at the expense of excessive investment in cost-reducing activities. Privatization improves welfare by mitigating the loss arising from excessive cost-reducing investment. Anderson et al. (1997) show how entry considerations can give rise to welfare-improving privatization in a differentiated Bertrand oligopoly. Low price set by the public firm deters entry of new varieties which lowers welfare. Privatizing the public firm enables more entry but that improves welfare if and only if the public firms make losses in free-entry equilibrium.

However, consumer concerns have largely been absent from this discussion. Given the growing relevance of distributional issues in developing countries, incorporating consumer welfare into the evaluation of privatization policies is becoming increasingly important. This chapter brings consumer surplus to the forefront of privatization discussion. We also shed light on the desirability of mixed markets—a market structure often overlooked in private versus public debate. Are consumers' interests best served in a purely private regime with efficient but profit-maximizing private firms, a purely public regime with inefficient but welfare-maximizing public firms, or a mixed regime that blends the two virtues—efficiency of private firms with the welfare orientation of public firms? That is a key question we address in this chapter.

We consider oligopoly environments and examine how consumer surplus (*CS* hereafter) in a mixed oligopoly, comprising both public and private firms, compares with *CS* in a purely public or a purely private oligopoly, where all firms are public and private respectively. Public firms are assumed to be less efficient but more welfare-oriented than their private counterparts. The efficiency assumption is motivated by empirical evidence suggesting that private firms are either more or at least as efficient and profitable as public firms (see, for example, Megginson and Netter, 2001). Welfare orientation of public firms aligns with the tradition of the mixed markets literature, which assumes that public firms maximize welfare or a weighted average of welfare and profits.[1]

We start with symmetric Cournot oligopoly (Section 2.2) where public and private

[1] See, for example, De Fraja and Delbono (1989), Basu (1993), Anderson et al. (1997), and Matsumura (1998). One might be skeptical about the public interest view that public enterprises maximize welfare or a weighted average of welfare and profits. A competing public choice view is that public enterprises are driven by politicians' objectives, resulting in inefficiency. See, for example, Boycko, Shleifer, and Vishny (2013), where such inefficiencies appear in the form of excess employment. We take such inefficiencies as given. Indeed, our starting point is that public firms are less efficient than their private counterparts. However, in representative democracies, even politicians cannot ignore the interests of the broader polity for which *CS* serves as a proxy.

firms produce homogeneous products and compete in quantities. We find that *CS* is maximized either when all firms are public (i.e., public oligopoly) or when all firms are private (i.e., private oligopoly). Irrespective of the number of public and private firms, all mixed oligopolies lie in the middle: they are neither the best nor the worst from consumers' perspective. The result holds for an arbitrary number of firms, various objective functions of public firms, and all logconcave demand and convex cost functions. When costs are strictly convex, we find that, contrary to common wisdom, competition does not necessarily facilitate privatization. Our finding has a similar flavor to Ritz (2024) who demonstrates that a positive association between competition and pass-through—which arises in an imperfect competition framework with constant marginal costs (Anderson and Renault, 2003; Weyl and Fabinger, 2013; Mrázová and Neary, 2017)—does not necessarily hold under convex costs.[2]

Section 2.3 revisits the comparison among public, private and mixed oligopolies in the presence of Bertrand competition with differentiated products. In contrast to Cournot competition, firms' choice variables (i.e., prices) are strategic complements (rather than strategic substitutes) under Bertrand competition. Despite this substantive difference, we find that, similar to Cournot, a mixed oligopoly never maximizes *CS*. In fact, *CS* in a mixed market can be lowest under Bertrand. This possibility arises because of a regime-contingent response from public firms. Responding to rival firms' prices, an inefficient public firm with some welfare orientation sets a higher price in a mixed regime than in a purely public regime.

For canonical environments considered in the industrial organization literature—e.g., homogeneous product Cournot (Section 2.2) and differentiated Bertrand (Section 2.3)—we find that consumer surplus (*CS*) is the highest either when *all* firms are public or when *all* firms are private. Alongside their virtues, mixed markets also inherit the vices of public and private firms: the inefficiency of public firms, and the profit-orientation of private firms.

[2] See the discussion after Proposition 2.4 for details.

That puts mixed regimes in the middle and, occasionally, at the bottom of the consumer surplus ranking. As a by-product of our analysis, we uncover (a) a nuanced relationship between competition and privatization under Cournot, and (b) a novel regime-contingent best response function of public firms under Bertrand.

The chapter is organized as follows. Section 2.2 outlines the model under the cournot competition, characterizes the equilibrium and compares the consumer surplus under different regimes—public, private, and mixed. In section 2.3, we characterize the equilibrium and compare the different regimes under Bertrand competition. Section 2.4 concludes. The detailed proofs are developed in the Appendix.

2.2 Cournot competition

2.2.1 Model

Consider an industry with n firms producing a homogeneous product and competing in quantities. We assume that m (out of n) firms are private, and the remaining $n-m$ firms are public, where $0 \leq m \leq n$ and $n \geq 2$. Label the firms in a way such that the first m firms are private.

Let x_i and y_k respectively denote the amount of output produced by a private firm i and a public firm k, where $i \in \{1, 2, ..., m\}$ and $k \in \{m+1, ..., n\}$. A private firm i's cost of producing x_i units of output is $C_r(x_i)$. We assume that $C_r(.)$ is twice-differentiable, and furthermore, ${C_r}'(x_i) > 0$ and ${C_r}''(x_i) \geq 0$ for all $x_i \geq 0$. Similarly, a public firm k's cost function, given by $C_u(y_k)$, is assumed to be strictly increasing and convex in y_k for all $y_k \geq 0$.[3] There are no fixed costs: $C_u(0) = C_r(0) = 0$. To capture the relative inefficiency of public firms, we assume that marginal cost is higher for the public firms, that is, for any

[3]As both p*r*ivate and p*u*blic start with p, we use the second letter in these words—r and u—to distinguish between the two.

$q \geq 0$, $C_u'(q) \geq C_r'(q)$ where equality holds (if at all) only at $q = 0$.[4]

Firms face a twice-differentiable inverse demand function $P(Q)$ where $P'(Q) < 0$ and $Q = \sum_{i=1}^{m} x_i + \sum_{k=m+1}^{n} y_k$ denotes aggregate output. We work with strictly log-concave demand functions or equivalently inverse demand functions that satisfy[5]

Assumption 1: $P'(Q) + P''(Q)Q < 0$, for all $Q > 0$.

This assumption implies that quantities are strategic substitutes. To ensure that equilibrium output is strictly positive and finite, we assume $P_0 > \max\{C_r'(0), C_u'(0)\} > P_\infty = 0$, where $P_0 \equiv \lim_{Q\to 0} P(Q)$ and $P_\infty \equiv \lim_{Q\to\infty} P(Q)$.

Profits of a private firm i and a public firm k are given by

$$\pi_i = P(Q)x_i - C_r(x_i), \tag{2.1}$$

and

$$\pi_k = P(Q)y_k - C_u(y_k), \tag{2.2}$$

respectively. Consumer surplus (CS) and welfare (W) respectively are:

$$\begin{aligned} CS &= \int_0^Q P(z)dz - PQ, && (2.3) \\ W &= CS + \sum_{i=1}^{m} \pi_i + \sum_{k=m+1}^{n} \pi_k, \\ &= \int_0^Q P(z)dz - \sum_{i=1}^{m} C_r(x_i) - \sum_{k=m+1}^{n} C_u(y_k). && (2.4) \end{aligned}$$

[4]A remark is in order regarding cost function. Increasing marginal cost (that is, strictly convex costs) is necessary for privatization to be welfare-improving if public and private firms have identical costs. Welfare-improving privatization can arise with constant marginal costs if public firms are inefficient and/or they place some weight on profits. Our cost specification admits both possibilities. We assume that at least one of them—increasing marginal cost ($C''(.) > 0$) or strictly positive weight on profits—holds which is sufficient for our results (related to CS) to go through.

[5]Let $Q(P)$ be the direct demand function corresponding to inverse demand function $P(Q)$. Assumption 1 holds if and only if $Q(P)$ is strictly log-concave.

2.2.2 Cournot equilibrium

Firms compete in quantities. Each private firm i chooses x_i to maximize its own profits given by (2.1). A public firm however does not typically maximize its own profits. Early papers in the literature on mixed markets assume that public firms maximize welfare which is simply the sum CS and industry profits (see Anderson et al., 1997 for a discussion).[6] Later papers have assumed that a public or semi-public/partially privatized firm maximizes a weighted average of welfare and its own profits (see, for example, Matsumura, 1998). In other words, each public firm k chooses y_k to maximize

$$f_k = (1-\lambda)W + \lambda \pi_k,$$

where π_k and W are as defined in (2.2) and (5.2.7), and $\lambda \in [0,1)$ reflects the extent of profit orientation of public firms. $\lambda = 0$ implies public firms maximize welfare.

Suppose all firms produce strictly positive amounts of output. The relevant first-order conditions, $\frac{\partial \pi_i}{\partial x_i} = 0$ and $\frac{\partial f_k}{\partial y_k} = 0$, respectively, are:

$$P(Q) + P'(Q)x_i - C_r'(x_i) = 0, \quad (2.5)$$

$$P(Q) + \lambda P'(Q)y_k - C_u'(y_k) = 0. \quad (2.6)$$

Equation (2.5) reflects the quantity choice of a profit maximizing private firm which equates marginal revenue $P(Q) + P'(Q)x_i$ with its marginal cost. Equation (2.6) says that a public firm equates its marginal cost with a weighted average of price and marginal revenue, $(1-\lambda)P(Q) + \lambda(P(Q) + P'(Q)y_k)$ $(= P(Q) + \lambda P'(Q)y_k)$.

Let $x_i(Q)$ denote the value of x_i that satisfies (2.5) for a given Q. It denotes the optimal output of private firm i that is consistent with aggregate output Q. Similarly, let $y_k(Q)$—the

[6] See Harris and Wiens (1980) and De Fraja and Delbono (1989) who consider welfare-maximizing public firms. See also Bennett and Maw (2000) and Bennett and La Manna (2012) for objective function where CS plays a more prominent role. We discuss such objectives later in this section.

value of y_k that satisfies (2.6)—denote the optimal output of public firm k that is consistent with aggregate output Q.[7] Differentiating (2.5) and (2.6) yields

$$\frac{dx_i(Q)}{dQ} = -\frac{P'(Q)+P''(Q)x_i}{P'(Q)-C_r''(x_i)} < 0, \tag{2.7}$$

$$\frac{dy_k(Q)}{dQ} = -\frac{P'(Q)+\lambda P''(Q)y_k}{\lambda P'(Q)-C_u''(y_k)} < 0, \tag{2.8}$$

where the inequalities follow from noting that $P'(Q) < 0$, $C_r''(x) \geq 0$, $C_u''(y) \geq 0$, and $P'(Q)+\lambda P''(Q)z < 0$ where $z = x, y$ and $\lambda \in [0,1)$.[8] Equations (2.7) and (2.8) respectively reflect negative associations between (a) $x_i(.)$ and Q, and (b) $y_k(.)$ and Q. This implies that there is a unique $(x_i(Q), y_k(Q))$ which is consistent with a given Q.

Recall that m denotes the number of private firms in a n-firm oligopoly. We will abuse the notation slightly and use m subscript to denote both a mixed oligopoly as well as a mixed oligopoly with m private firms. The usage will be clear from the context. Private and public oligopolies are special cases corresponding to $m = n$ and $m = 0$ respectively. Unless the cost difference between public and private firms is too large, there exists a unique, interior Cournot equilibrium where each private firm i produces $x_i = x_i(Q_m^*)$, each public firm k produces $y_k = y_k(Q_m^*)$, and $Q = Q_m^*$ uniquely solves the following:

$$Q = \sum_{i=1}^{m} x_i(Q) + \sum_{k=m+1}^{n} y_k(Q). \tag{2.9}$$

All private firms produce the same amount of output. Similarly, all public firms produce the same amount of output. Hereafter, we use $x_m^*(\equiv x_i(Q_m^*))$ and $y_m^*(\equiv y_k(Q_m^*))$ to denote a private firm's output and a public firm's output in a mixed oligopoly Cournot

[7] According to aggregative games terminology in Anderson, Erkal, and Piccinin (2020), the functions $x_i(Q)$ and $y_k(Q)$ are inclusive best response functions for private firm i and public firm k respectively. Selten (1970) and Acemoglu and Jensen (2013) refer to these as cumulative best reply functions. Also see Vives (2001) for a discussion (p.97, chap.4).

[8] As $P'(Q) < 0$, $P''(Q) < 0 \Rightarrow P'(Q)+P''(Q)z < 0$. When $P''(Q) > 0$, the negative sign follows from noting that $P'(Q)+\lambda P''(Q)z < P'(Q)+P''(Q)z \leq P'(Q)+P''(Q)Q < 0$, where the penultimate inequality uses $z \leq Q$, and the last inequality holds by Assumption 1.

equilibrium. Aggregate output is $Q_m^* = mx_m^* + (n-m)y_m^*$. For convenience, we use $r(u)$ subscript instead of $m = n(0)$ to denote variables/outcomes associated with private (public) oligopoly. We use x_r^* and $Q_r^*(= nx_r^*)$ to denote individual output and aggregate output respectively in a private oligopoly equilibrium, while their counterparts in a public oligopoly equilibrium are denoted by y_u^* and $Q_u^*(= ny_u^*)$.

2.2.3 Consumer Surplus

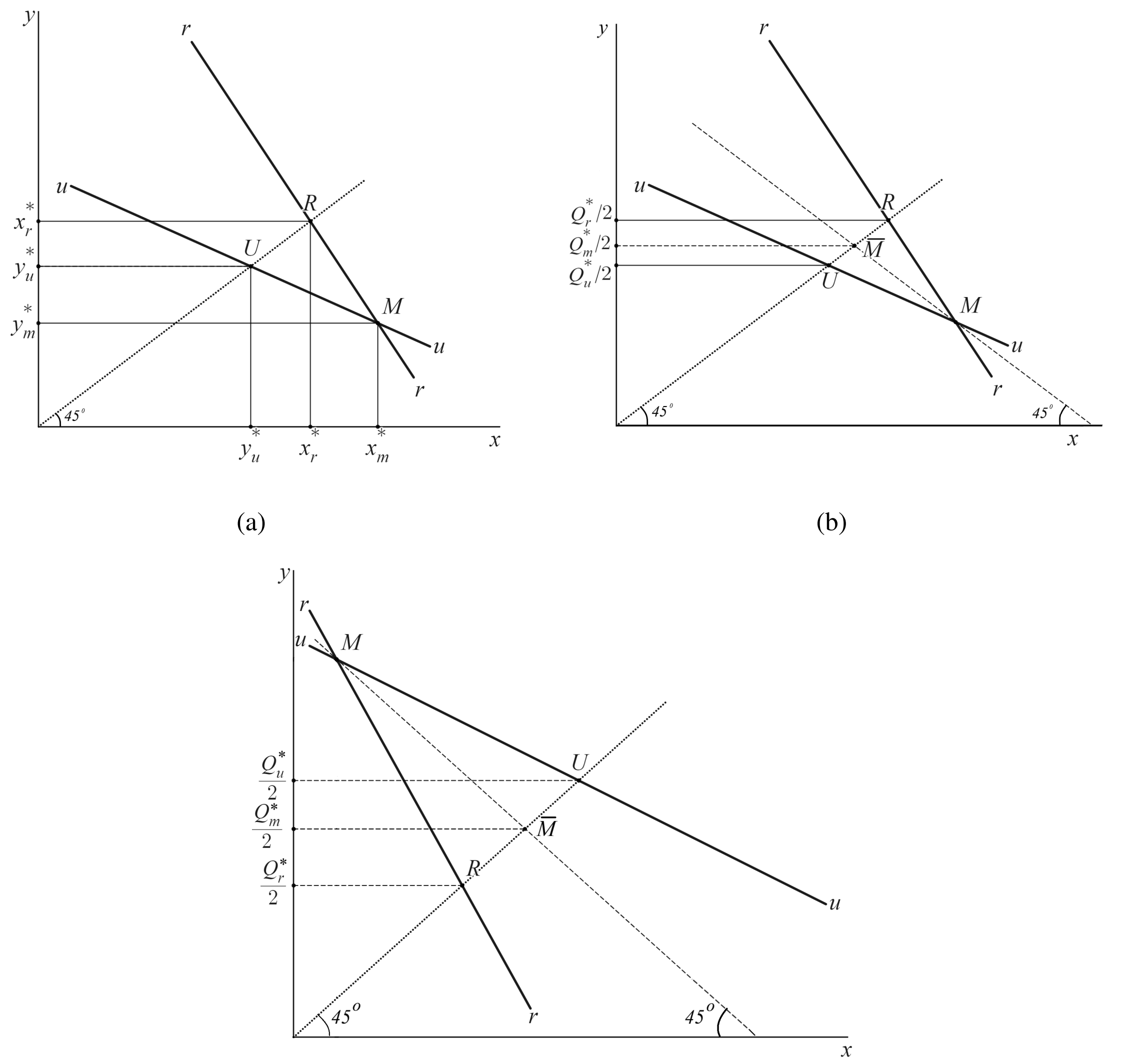
Which oligopoly regime delivers maximum *CS*—private, public, or the mixed one? As aggregate output Q is a sufficient statistic for *CS* in a homogeneous product oligopoly, an equivalent question is: which one is the largest among the three – Q_r^*, Q_u^*, or Q_m^*?

Public firms are more welfare-oriented but less efficient than their private counterparts. Welfare orientation encourages public firms to produce more while cost inefficiency discourages production. Thus, Q_u^* could be the same, higher, or lower than Q_r^*. Notwithstanding this ambiguity about relative ranking of Q_u^* and Q_r^*, we find that Q_m^* always lies in the middle, i.e.,

$$\min\{Q_u^*, Q_r^*\} \leq Q_m^* \leq \max\{Q_u^*, Q_r^*\}.$$

To better appreciate the middle-ness of the mixed regime, consider a Cournot duopoly where firm 1 is private and firm 2 is public. The reaction functions of the private and the public firms are given by rr and uu respectively in Figure 2.1. Assumption 1 implies that rr and uu are negatively sloped, and furthermore, the absolute value of the slope of $rr(uu)$ is greater (smaller) than unity.

Reaction functions rr and uu intersect at M which denotes the mixed duopoly equilibrium: private firm 1 produces x_m^* and public firm 2 produces y_m^*. If firm $2(1)$ were also private (public), reaction functions of firms 1 and 2 would have intersected on the upward sloping 45-degree line at $R(U)$. Points R and U denote the equilibrium in a private and public duopoly respectively.



(a) (b)

(c)

Figure 2.1: Cournot equilibrium

Figure 2.1(a) shows equilibrium outputs in different regimes when underlying parameter values are such that $Q_r^* > Q_u^*$. Observe that $x_m^* > y_m^*$ for all such parameter values. Figure 2.1(b) recasts Figure 2.1(a) in terms of average aggregate output ($\frac{Q}{2}$). The translation is immediate for private and public duopoly as individual outputs and the average output are the same, i.e., $x_r^* = \frac{Q_r^*}{2}$ and $y_u^* = \frac{Q_u^*}{2}$. To denote average output under mixed duopoly, $\frac{x_m^*+y_m^*}{2} = \frac{Q_m^*}{2}$, we draw a downward sloping 45-degree line $x_m + y_m = Q_m^*$ which passes through (x_m^*, y_m^*) and intersects the upward sloping 45-degree line at $x_m = y_m = \frac{Q_m^*}{2}$.

From the coordinates of U, R, and $\overline{M}$, it is immediate that $Q_u^* < Q_m^* < Q_r^*$ which implies $CS_u < CS_m < CS_r$. Similarly, when $Q_r^* < Q_u^*$, M is on the other side of 45-degree line, i.e., $x_m^* < y_m^*$, and we get $CS_r < CS_m < CS_u$. Figure 2.1(c) illustrates this case. Thus, a mixed duopoly lies in between a private and public duopoly when it comes to CS ranking.

This middle-ness of mixed regimes is not limited to duopoly. Proposition 2.1 says that the result holds for an arbitrary number of firms.

Proposition 2.1. *Consumer surplus is maximum or minimum either when all firms are public or when all firms are private. Mixed oligopoly lies in the middle of consumer surplus ranking. More formally, let CS_u, CS_r, and CS_m refer to equilibrium consumer surplus in public, private, and mixed oligopoly regimes respectively. Then*

$$\min\{CS_u, CS_r\} \leq CS_m \leq \max\{CS_u, CS_r\},$$

where equality holds if and only if $CS_u = CS_r$.

Proof. As the ranking of CS mirrors the ranking of aggregate output, we prove an equivalent statement:

$$\min\{Q_u^*, Q_r^*\} \leq Q_m^* \leq \max\{Q_u^*, Q_r^*\}, \tag{1a}$$

where the equality holds if and only if $Q_u^* = Q_r^*$.

Suppose (1a) does not hold. In particular, suppose $Q_m^* > \max\{Q_u^*, Q_r^*\}$. From (2.7) we know that $x_i(Q)$ and Q have a negative association. Thus

$$Q_m^* > Q_r^* \Rightarrow x_m^* < x_r^*.$$

Similarly, from (2.8) we know that $y_k(Q)$ and Q are inversely related which yields

$$Q_m^* > Q_u^* \Rightarrow y_m^* < y_u^*.$$

Multiplying both sides of the inequality $x_m^* < x_r^*$ by m, and both sides of $y_m^* < y_u^*$ by $n-m$, and adding them together we get

$$mx_m^* + (n-m)y_m^* < mx_r^* + (n-m)y_u^*.$$

By definition, $mx_m^* + (n-m)y_m^* = Q_m^*$, $x_r^* = \frac{Q_r^*}{n}$, and $y_u^* = \frac{Q_u^*}{n}$. Substituting these expressions in the above inequality, and noting that the weighted average of any two distinct numbers is strictly less than the maximum of those two numbers, we get

$$Q_m^* < \frac{m}{n}Q_r^* + \left(1 - \frac{m}{n}\right)Q_u^* < \max\{Q_u^*, Q_r^*\}.$$

However, this contradicts our supposition $Q_m^* > \max\{Q_r^*, Q_u^*\}$. Similarly, we can show that $Q_m^* < \min\{Q_r^*, Q_u^*\}$ cannot hold either.

□

That mixed regimes cannot deliver the highest *CS* might seem surprising as it blends two positives—efficiency of private firms and welfare orientation of public firms. However, alongside the positives, mixed regimes also carry public and private firms' negative features (at least from consumers' perspective): inefficiency of public firms and profit-orientation of private firms. That puts mixed regimes in the middle of *CS* ranking.

2.2.4 Discussion

The key message of Proposition 2.1 is that *CS* is the highest (lowest) either when *all* firms are public (private) or when *all* firms are private (public). Mixed market lie in the middle—it is neither the best nor the worst from consumers' perspective. This key finding is robust to several alternative specifications including (a) quality-augmented model of Cournot competition where private and public firms' products differ in quality,[9] (b) an alternative cost function of the public firm, where a public firm's cost decreases as it puts more weight on profit, and finally (c) alternative objective functions for public firms where *CS* plays a more prominent role. We present (a) and (b) in Section 2.6.1, under Supplementary materials. Below we explore (c) in detail.

We focus on consumer surplus (*CS*), but, following the mixed oligopoly literature, assume that public firms are welfare-oriented. The key distinction is that in a homogeneous product oligopoly, *CS* depends solely on total output, whereas welfare takes into account both output and production costs. Regardless of whether *CS* or welfare is used as the metric, the policy implications are the same when efficiency gains from privatization are either sufficiently large or sufficiently small. If the gains are large, both welfare and *CS* are maximized under a purely private regime; if small, under a purely public regime. A divergence between *CS* and welfare arises for intermediate efficiency gains. While a mixed regime never maximizes *CS*, it can—under an endogenous choice of λ—maximize welfare. This distinction is policy-relevant: if efficiency gains from privatization increase over time, a welfare-oriented policymaker will begin privatization early and proceed gradually, whereas a *CS*-oriented policymaker will start late but implement it all at once.

What is the appropriate metric—consumer surplus (*CS*) or welfare? While the answer depends on the context, our dual focus on both aligns with institutional reality: competition authorities, which often evaluate the effects of privatization, typically emphasize consumer outcomes, whereas national industrial policy departments, which set objectives for public firms, tend to prioritize social welfare. The welfare standard underlying many industrial policies, including privatization, is often guided by the objective of "maximizing the size of the pie," regardless of its distribution. This logic follows the Kaldor-Hicks criterion,

[9] See Laine and Ma (2017) for a comprehensive discussion of endogenous quality provision in a mixed duopoly model with vertical differentiation where the valuation for quality differs across consumers and in equilibrium either the private or public firm can offer higher quality depending on parameter values. We consider a simpler model in section 2.6.1 under Supplementary materials where the qualities are exogenously given and private firms offer higher quality products.

which deems a policy desirable if the gainers could hypothetically compensate the losers without eroding the total surplus (Directorate for Financial and Enterprise Affairs Competition Committee, OECD, 2023). In advanced economies, where firms are primarily owned by investment and pension funds, the line between producers and consumers is blurred (Albæk, 2013), leading to limited divergence between total welfare and consumer welfare objectives. In developing economies, however—where ownership is concentrated and redistribution through taxes or subsidies is costly—the distribution of gains becomes central (Hovenkamp, 2019).

Tensions between industrial policy and competition policy have intensified as governments pursue targeted interventions to maximize welfare while competition authorities emphasize consumer surplus (Aiginger, 2014; Millot and Rawdanowicz, 2024). Industrial policy often seeks to promote specific sectors or firms, whereas competition enforcement focuses on maintaining a level playing field and preventing consumer harm. In case of privatization policies, Australian Competition & Consumer Commission Chair Rod Sims cautioned that privatization without regulation "will see higher prices for users... and can effectively be a tax on consumers" (Australian Competition & Consumer Commission, 2016). Once a strong proponent of privatization, Sims later acknowledged that poorly designed asset sales have "severely damaged our economy" (The Guardian, 2016), urging at the Australian Competition & Consumer Commission/Australian Energy Regulator (ACCC/AER) Regulatory Conference, 2021 that "we should either privatize to improve the efficiency of our economy... or not privatize at all" (Australian Competition & Consumer Commission, 2021).

Does it make a difference if, indeed, the interest of competition authorities and the public firm are more aligned? The answer is no, at least not necessarily. Specifically, we consider two examples where public firms' objective function assigns more weight to consumer surplus. First, suppose public firm maximizes CS subject to making non-negative profits. Proposition 2.1 still holds as long as average cost, $C_u(y)/y$, is increasing in y (see section 2.6.1).

In the main model as well as in all the alternative scenarios considered so far, aggregate output (Q) and a public firm's output (y) have a negative relationship. This translates to a downward-sloping reaction function for the public firm in a duopoly. Next, we consider a duopoly setting where the public firm's reaction function is either entirely upward-sloping or has upward-sloping segments reflecting possible strategic complementarity for some output vectors. Proposition 2.1 continues to hold, provided the degree of strategic complementarity is not too high.

Consider a mixed duopoly where private firm 1 chooses x to maximize its own profits π_1 and public firm 2 chooses y to maximize $f_2 = (1-\lambda)CS + \pi_2$ where $\lambda \in [0,1)$.[10] Firm 2's reaction function is upward (downward) sloping at (x,y) if and only if

$$\frac{\partial^2 f_2(x,y)}{\partial x \partial y} = \lambda (P^{'}(Q) + QP^{''}(Q)) - xP^{''}(Q) > (<) 0.$$

As $P^{'}(Q) + QP^{''}(Q) < 0$ (by Assumption 1), $\frac{\partial^2 f_2(x,y)}{\partial x \partial y} > 0$ holds only if $P^{''}(Q) < 0$.

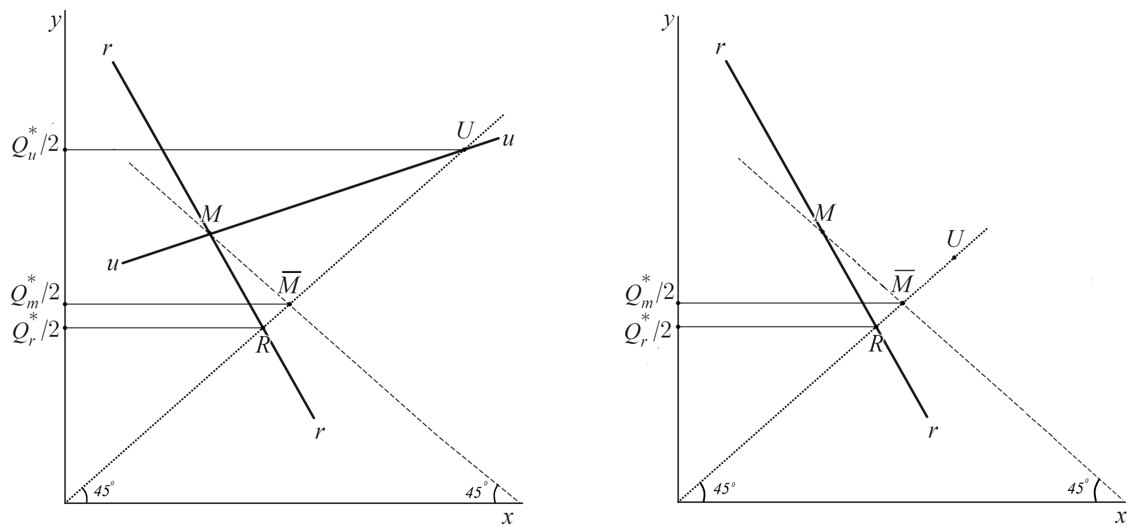
Suppose $\lambda = 0$ and $P''(Q) < 0$. Then $\frac{\partial^2 f_2}{\partial x \partial y} > 0$ holds for all (x,y) which in turn implies an upward-sloping reaction function for a public firm. As depicted in Figure 2.2(a), Q_m^* remains positioned between Q_u^* and Q_r^* when the public firm's reaction function is monotone and upward-sloping.

Continue with $P^{''}(Q) < 0$, but now suppose $\lambda > 0$. For $x \approx 0$, $\frac{\partial^2 f_2}{\partial x \partial y} < 0$, while for $y \approx 0$ (i.e., $x \approx Q$), $\frac{\partial^2 f_2}{\partial x \partial y} > 0$ provided λ is not too close to unity. For such non-monotone reaction functions, Proposition 2.1 continues to hold as long as

$$\frac{\partial^2 f_2(z,z)}{\partial y^2} + \frac{\partial^2 f_2(z,z)}{\partial x \partial y} < 0 \tag{2.10}$$

[10]Earlier, we worked with $f_2 = (1-\lambda)W + \lambda \pi_2 = (1-\lambda)(CS + \pi_1) + \pi_2$. Here we drop π_1 from f_2.

holds for all symmetric output vectors $(x,y) = (z,z)$.



(a) (b)

Figure 2.2: Cournot equilibrium under alternative objective functions

To see why, consider a mixed duopoly equilibrium where $y_m^* > x_m^*$. In Figure 2.2(b), point M denotes such an equilibrium. By definition, M must lie on the private firm's reaction function rr. The downward sloping 45-degree line passing through M intersects the upward-sloping 45-degree line at $\overline{M} \equiv (\frac{Q_m^*}{2}, \frac{Q_m^*}{2})$. Since rr is steeper than the negatively sloped 45-degree line, it is immediate from Figure 2.2(b) that $Q_m^* > Q_r^*$. To prove $Q_m^* < Q_u^*$, consider the following expression:

$$\frac{\partial f_2}{\partial y} = -(1-\lambda)P'(Q)Q + P(Q) + P^{'}(Q)y - C_u^{'}(y). \tag{2.11}$$

By definition, $\frac{\partial f_2}{\partial y} = 0$ at M. Starting from $M \equiv (x_m^*, y_m^*)$, as we slide downward on the negatively sloped 45-degree line, Q remains unchanged at $Q_m^* = x_m^* + y_m^*$ but x increases and y declines. From (2.11), it follows that $\frac{\partial f_2}{\partial y} > 0$ at $\overline{M}$. At any symmetric public duopoly equilibrium $(\frac{Q_u^*}{2}, \frac{Q_u^*}{2})$ we must have $\frac{\partial f_2}{\partial y} = 0$. From (2.10), it follows that $\frac{\partial f_2}{\partial y}$ strictly decreases as we move up diagonally along the upward sloping 45-degree line. Thus, as shown in Figure

2.2(b), point U depicting the public duopoly equilibrium must lie to the northeast of $\overline{M}$ on the 45-degree line. This implies $Q_m^* < Q_u^*$.[11]

Finally, we consider a Stackelberg duopoly. Many public firms operate under predetermined output or pricing targets, or are subject to quantity floors or price ceilings, limiting their strategic discretion. Since these targets are often chosen by regulators with welfare objectives, a sequential-move game provides a natural approximation. Without loss of generality, firm 1 is the leader and firm 2 the follower. The public regime consists of two welfare-maximizing public firms, the private regime of two profit-maximizing private firms, and the mixed regime of a welfare-maximizing public leader and a profit-maximizing private follower. Public and private firms have constant marginal costs c_u and c_r, respectively, and market demand is linear, i.e., $P(Q) = a - Q$.

In the private regime, the Stackelberg equilibrium satisfies $x_1^* = 2x_2^*$, implying aggregate output $Q_r^* = \frac{3(a-c_r)}{4}$. In the public regime, both firms maximize welfare, yielding aggregate output $Q_u^* = a - c_u$. In the mixed regime, equilibrium outputs are: $x_m^* = 2(c_u - c_r)$ and $y_m^* = (a - c_r) - 4(c_u - c_r)$, so that aggregate output is $Q_m^* = (a - c_r) - 2(c_u - c_r)$.

Unlike Cournot model, the public leader's reaction function is regime-contingent—the response of leader public firm depends on its follower's ownership type. This leads to

$$\left.\frac{\partial W}{\partial y}\right|_{\text{public}} - \left.\frac{\partial W}{\partial y}\right|_{\text{mixed}} = (a - Q - c_u) - (a - Q - c_u - \frac{x_m}{2}) = \frac{x_m}{2} > 0,$$

where the additional term reflects the fact that the public leader internalizes the effect of its output on the private follower's profit. Therefore, when $\left.\frac{\partial W}{\partial y}\right|_{\text{public}} = 0$, $\left.\frac{\partial W}{\partial y}\right|_{\text{mixed}} < 0$,

[11] Using similar arguments, we can show that $Q_u^* < Q_m^* < Q_r^*$ when M is on the other side of 45-degree line (i.e., $x_m^* > y_m^*$).

implying that $y_m^* < y_1^*$. Additionally, at $y_2 = y_2^*$,

$$\left.\frac{\partial \pi}{\partial x}\right|_{\text{mixed}} = a - Q - c_r - x_m = (c_u - c_r) - x_m < 0,$$

yielding $y_2^* > x_m^*$. It follows immediately that $Q_m^* = x_m^* + y_m^* < y_1^* + y_2^* = Q_u^*$.

Let $c_u - c_r = \Delta$. Comparing the aggregate outputs under different regimes, we get:

$$Q_u^* \gtreqless Q_r^* \Leftrightarrow (a - c_u) \gtreqless \frac{3(a-c_r)}{4} \Leftrightarrow \Delta \lesseqgtr \frac{(a-c_r)}{4},$$
$$Q_r^* \gtreqless Q_m^* \Leftrightarrow \frac{3(a-c_r)}{4} \gtreqless (a-c_r) - 2\Delta \Leftrightarrow \Delta \gtreqless \frac{(a-c_r)}{8},$$

and

$$Q_u^* - Q_m^* = (a-c_r) - \Delta - (a-c_r) + 2\Delta = \Delta > 0.$$

Furthermore, assuming, $\Delta < \frac{(a-c_r)}{4}$ ensures that all the outputs are positive in the equilibrium. Define,

$$\overline{\Delta} = \frac{(a-c_r)}{4}, \text{ and } \Delta_0 = \frac{(a-c_r)}{8},$$

where $\Delta_0 = \frac{\overline{\Delta}}{2} < \overline{\Delta}$.

Since aggregate output is a sufficient statistic for consumer surplus, the consumer-surplus ranking follows directly from the output ranking. That is,

$$CS_u^* > CS_m^* > CS_r^* \Leftrightarrow Q_u^* > Q_m^* > Q_r^*, \text{ if and only if } \Delta \in (0, \Delta_0),$$
$$CS_u^* > CS_r^* > CS_m^* \Leftrightarrow Q_u^* > Q_r^* > Q_m^*, \text{ if and only if } \Delta \in (\Delta_0, \overline{\Delta}).$$

The intuition differs from that of the Cournot model. The regime-contingent response of the public leader always implies that the public regime generates more aggregate output than the mixed regime for $\Delta \in (0, \overline{\Delta})$. When the public-private efficiency gap is sufficiently small, welfare considerations outweigh the efficiency disadvantage of the public firm, so the mixed regime continues to outperform the private regime and lies in the middle of the consumer surplus ranking. However, as the efficiency gap increases, the public leader's weaker output response dominates, causing the mixed regime to produce less output than the private regime. Thus, Proposition 2.1 continues to hold only for sufficiently small efficiency differences. More importantly, the sequential-move structure introduces a possibility absent under Cournot competition: although the mixed regime never maximizes consumer surplus, it can minimize consumer surplus—which arises due to the regime-contingent response of the leader public firm. As shown in Section 2.3, the same mechanism reappears under differentiated product Bertrand competition.

2.2.5 Scale of privatization

Proposition 2.1 says that all mixed oligopolies, irrespective of their composition of public and private firms, lie between the two extremes—public and private oligopoly. However, it is silent about how two mixed oligopolies with different numbers of firms compare with each other. Proposition 2.2 addresses that question.

Proposition 2.2. *Consumer surplus in mixed oligopoly is monotone in the number of private firms. Let CS_m denote the consumer surplus in a mixed regime with m private firms. If $CS_u < (>) CS_r$, CS_m is strictly increasing (decreasing) in the number of private firms m. Else, if $CS_u = CS_r$, $CS_m = CS_u = CS_r$.*

Proof. As aggregate output is a sufficient statistic for CS, it is equivalent to show that Q_m^* is monotone in m. Suppose not. That is, suppose $Q_m^* - Q_{m-1}^* > 0$ as well as $Q_m^* - Q_{m+1}^* > 0$ hold for some m.

Express $Q_m^* - Q_{m-1}^*$ as

$$Q_m^* - Q_{m-1}^* = (m-1)(x_m^* - x_{m-1}^*) + (x_m^* - y_{m-1}^*) + (n-m)(y_m^* - y_{m-1}^*),$$

where x_z^*, y_z^* and Q_z^* respectively denote a private firm's output, a public firm's output and aggregate output in Cournot equilibrium where the first z firms are private and $z \in \{m-1, m, m+1\}$. From (2.7) and (2.8) we know that $Q_m^* - Q_{m-1}^* > 0$ implies the following:

$$x_m^* - x_{m-1}^* < 0 \quad \text{and} \quad y_m^* - y_{m-1}^* < 0.$$

But then

$$x_m^* - y_{m-1}^* > 0$$

must hold as, otherwise, $Q_m^* - Q_{m-1}^* < 0$. Combining $x_m^* - y_{m-1}^* > 0$ and $y_m^* - y_{m-1}^* < 0$ we get

$$x_m^* > y_m^*. \tag{2a}$$

Now express $Q_m^* - Q_{m+1}^*$ as

$$Q_m^* - Q_{m+1}^* = m(x_m^* - x_{m+1}^*) + (y_m^* - x_{m+1}^*) + (n-m-1)(y_m^* - y_{m+1}^*).$$

Proceeding similarly (as above) we can show that $Q_m^* - Q_{m+1}^* > 0$ implies

$$x_m^* - x_{m+1}^* < 0, \quad y_m^* - y_{m+1}^* < 0, \quad \text{and} \quad y_m^* - x_{m+1}^* > 0.$$

The first two inequalities follow from (2.7) and (2.8) respectively. Given those, the third inequality must hold as, otherwise, $Q_m^* - Q_{m+1}^* < 0$. Combining $y_m^* - x_{m+1}^* > 0$ and $x_m^* - x_{m+1}^* < 0$, we get

$$x_m^* < y_m^*$$

which contradicts (2a). □

Proposition 2.2 implies that if full-scale privatization increases CS, then any scale of privatization does that too. Similarly, if full-scale privatization reduces CS, so does any scale of privatization. A cautious/constrained government might be willing/able to privatize at most only one firm at a time. Proposition 2.2 suggests that such limited-scale exercise might still be valuable, as the qualitative impact of privatization on consumers—whether they are worse off or better off—are the same irrespective of the number of firms that are privatized.

Proposition 2.1 places CS_m between CS_u and CS_r. Proposition 2.2 unpacks CS_m and establishes a monotone relationship between CS and the number of private firms. Together, Propositions 2.1 and 2.2 imply that unless $CS_u = CS_r$, one of the following two orderings hold:

$$CS_u < CS_1 < CS_2 < \ldots < CS_r, \quad \text{or} \quad CS_u > CS_1 > CS_2 > \ldots > CS_r.$$

Proposition 2.3 completes our ranking by identifying which one of the two holds and when. For sharp characterization, we consider

$$C_u(q) = c_u q + C(q), \text{ and } C_r(q) = c_r q + C(q) \tag{2.12}$$

as the cost functions for public and private firms respectively, where $c_u > c_r > 0$, $C'(q) > 0$ and $C''(q) \geq 0$ (the inequality is strict when $\lambda = 0$).

Proposition 2.3. *Let $\Delta = c_u - c_r$ be the degree of relative inefficiency of public firms where c_u and c_r are as defined in (2.12). For any $n \geq 2$ and $c_r > 0$, there exists $\Delta = \hat{\Delta}(c_r, n) \geq 0$ such that*

$$CS_u < CS_1 < CS_2 < \ldots < CS_r \text{ if and only if } \Delta > \hat{\Delta}(c_r, n).$$

That is, CS *strictly increases with the scale of privatization when public firms are suffi-*

ciently inefficient.

Proof. See Appendix 2.5.1. □

From Proposition 2.2 we know that CS is increasing in the scale of privatization if and only if $CS_r > CS_u$. Proposition 2.3 says that this inequality holds when public firms are sufficiently inefficient. Welfare orientation of public firms encourages them to increase production, whereas inefficiency—as reflected in $\Delta = c_u - c_r > 0$—prompts them to curtail production. These two concerns balance each other when the difference in marginal benefit of public firms and private firms from an additional unit of output, i.e.,

$$\left(P(Q_u^*) + \frac{\lambda Q_u^* P'(Q_u^*)}{n}\right) - \left(P(Q_r^*) + \frac{Q_r^* P'(Q_r^*)}{n}\right),$$

equals the difference in marginal cost Δ. This occurs at

$$\Delta = -\frac{(1-\lambda) Q_r^* P'(Q_r^*)}{n} \equiv \hat{\Delta}(c_r, n).$$

CS_u is decreasing in Δ but CS_r is invariant. Thus, $\Delta > (<) \hat{\Delta}(c_r, n) \Leftrightarrow CS_u < (>) CS_r$.

2.2.6 Competition and privatization

We conclude this section by discussing how competition impacts privatization when the latter is guided by consumer surplus.

Proposition 2.4. *An increase in competition—measured by an increase in the number of firms—facilitates privatization, i.e., the threshold $\hat{\Delta}(c_r, n)$ is strictly decreasing in n if and only if*

$$\frac{\mu}{e_d} > \eta e_{mc}, \tag{2.13}$$

where $\mu = \frac{P(Q)}{C'(q)}$ denotes markup, $e_d = -\frac{P(Q)}{QP'(Q)}$ denotes elasticity of demand, $\eta = \frac{QP''(Q)}{P'(Q)}$ denotes elasticity of slope (of demand), $e_{mc} = \frac{qC''(q)}{C'(q)}$ is the elasticity of marginal cost, and *all are evaluated at $Q = Q_r^*$ and $q = \frac{Q_r^*}{n}$. The inequality in (2.13) holds for all n when marginal cost is constant ($e_{mc} = 0$) or demand is convex ($\eta \leq 0$).*

Proof. See Appendix 2.5.1. □

Proposition 2.4 tells us that an increase in competition may or may not facilitate privatization.[12] That is, $\hat{\Delta}$ might decrease or increase with n. To understand this ambiguity, we substitute $Q = Q_r^*$ and $x_i = \frac{Q_r^*}{n}$ in the private firm's first-order condition in equation (2.5) and rewrite it as follows (assuming that the cost function is given by equation (2.12)):

$$-\frac{Q_r^* P'(Q^*)}{n} = P(Q_r^*) - c_r - C'(\frac{Q_r^*}{n}).$$

Using this substitution we can express the threshold value as

$$\hat{\Delta}(c_r, n) = (1-\lambda)(P(Q_r^*) - c_r - C'(\frac{Q_r^*}{n}))$$

implying whether competition facilitates or hinders privatization depends on whether the difference between price and marginal cost decreases or increases with competition.

As competition increases, price unambiguously declines. So does the difference between price and marginal cost when $C''(.) = 0$. However, when $C''(.) > 0$, marginal cost declines too. As n increases, $\frac{Q_r^*}{n}$ declines due to familiar business stealing effect (Mankiw and Whinston, 1986) which in turn lowers $C'(.)$. Thus, whether the difference between price and marginal cost increases or decreases with increased competition depends on demand and cost curvatures. Equation (2.13) expresses this dependence in terms of elasticities and markup. Observe that the condition always holds when marginal cost is constant ($e_{mc} = 0$) or demand is convex ($\eta \leq 0$). However, the condition might fail to hold for strictly concave

[12] Note that an increase in total number of firms may arise from an increase in private firms, increase in public firms or both. Proposition 2.4, therefore, encompasses all possible changes in market composition of public-private firms, without assuming any specific change in the mix of two types of firms.

demand ($\eta > 0$) and strictly convex costs ($e_{mc} > 0$) as the price might decline at a slower rate than the decline in marginal cost.

That competition does not necessarily facilitate privatization resembles Ritz (2024) who demonstrates that an increase in competition does not necessarily imply an increase in pass-through. The presence of strictly convex costs can break the positive association between competition and pass-through. In our framework too, a strictly convex cost function is necessary for competition not to facilitate privatization. However, that alone is not enough; we also need strictly concave demand.

2.3 Bertrand competition

From Cournot competition with homogeneous products, we now turn our attention to another important setting: Bertrand competition with differentiated products, which deserves separate attention for several reasons.[13] Differentiated oligopoly fits well with the description of several industries involving public firms (e.g., banking, telecommunications) and it is a natural setting to explore price competition. Public and private firms can charge different prices and still sell strictly positive amounts of output—a possibility that typically does not arise in quantity competition with homogeneous products.

Importantly, as we move from Cournot to Bertrand competition, we also shift from an environment where firms' actions (i.e., quantities) are typically strategic substitutes to one where they (i.e., prices) are strategic complements. Despite this substantive difference with Cournot competition, we find that, as in Cournot, consumer surplus is maximum either when all firms are public or when all firms are private. However, unlike Cournot competition, a mixed regime does not necessarily lie in the middle of the consumer surplus ranking. Consumer surplus in a mixed regime can be lower than that in both private and

[13] Following the tradition of industrial organization literature, we pair quantity competition with homogeneous products, and price competition with differentiated products. An exception is Dastidar (1997) which compares Bertrand and Cournot in a homogeneous product setting.

public regimes.

Consider an economy with two sectors: a perfectly competitive sector producing the numeraire good z and an oligopolistic sector with $n(\geq 2)$ firms, each producing a different variety. For simplicity, we focus on $n = 2$.[14] Let p_i and q_i denote firm i's price and quantity respectively, where $i \in \{1,2\}$. A representative consumer maximizes $V(q,z) \equiv U(q) + z$ subject to $p_1q_1 + p_2q_2 + z \leq I$ where $q \equiv (q_1, q_2)$ and I denotes income. We assume that U is smooth and strictly concave, and income I is large enough so that $q_i > 0$ holds for both $i(=1,2)$. Inverse demands follow from first-order conditions of the utility maximization problem: $p_i = \frac{\partial U}{\partial q_i}$. Inverting the inverse demand system yields direct demands: $q_i = D_i(p)$ where $p \equiv (p_1, p_2)$. Demand functions are downward sloping (i.e., $\frac{\partial D_i(p)}{\partial p_i} < 0$) and the goods are gross substitutes (i.e., $\frac{\partial D_i(p)}{\partial p_j} > 0, j \neq i$).

For a given price vector p, the profits of firm i, consumer surplus, and welfare respectively are given by

$$
\begin{aligned}
\pi_i(p) &= p_iD_i(p) - C_i(D_i(p)), \\
CS(p) &= U(D(p)) - \sum_{i=1}^{2} p_iD_i(p), \text{ and} \\
W(p) &= CS(p) + \sum_{i=1}^{2} \pi_i(p) = U(D(p)) - \sum_{i=1}^{2} C_i(D_i(p)),
\end{aligned}
$$

where $D(p) = (D_1(p), D_2(p))$, and firm i's cost of production, $C_i(.)$, is a twice differentiable function satisfying $C_i(0) = 0$, $C_i'(.) > 0$, and $C_i''(.) \geq 0$. To capture the relative inefficiency of public firms, as in Section 2.2, we assume that marginal cost is higher for the public firms; that is, for any $q \geq 0$, $C_u'(q) \geq C_r'(q)$ where equality holds (if at all) only at $q = 0$.[15]

[14] Duopoly ($n = 2$) simplifies our analysis. In section 2.6.2, under Supplementary materials, we show that the main results hold in a differentiated oligopoly with $n(> 2)$ varieties.

[15] Here we abuse notation slightly as we use the subscript (associated with C) to denote both identity of firms (i.e., 1 or 2) as well as type of firms (i.e., public or private). Which one we refer to—identity or type—will be clear from the context.

2.3.1 Bertrand equilibrium

Firm i chooses price p_i to maximize

$$f_i(p) = (1-\lambda_i)W(p) + \lambda_i \pi_i(p),$$

where $\lambda_i = 1$ if firm i is private, and $\lambda_i = \lambda \in [0,1)$ if firm i is public. A private firm maximizes its own profits whereas a public firm maximizes a weighted average of welfare and its own profits. We assume that $f_i(p)$ satisfies the following properties for all $p \in \mathbb{R}^2_{++}$.

Assumption 2: (i) $\frac{\partial^2 f_i(p)}{\partial p_i \partial p_j} > 0$, and (ii) $\frac{\partial^2 f_i(p)}{\partial {p_i}^2} + \frac{\partial^2 f_i(p)}{\partial p_i \partial p_j} < 0$ where $i, j \in \{1,2\}$, $j \neq i$.

The first part of Assumption 2 says that prices are strategic complements. This is standard in Bertrand competition with profit-maximizing firms. Requiring part (i) to hold for arbitrary $f_i(p)$ is restrictive but it simplifies the analysis significantly.[16] The second part of Assumption 2 is akin to the dominant diagonal condition. It ensures the uniqueness of equilibrium. Both parts (i) and (ii) hold for a differentiated duopoly where cost functions are linear and demand functions are derived from a widely used quadratic utility specification (see, for example, Singh and Vives, 1984):

$$U(q) = a(q_1+q_2) - \frac{q_1^2+q_2^2}{2} - bq_1q_2, \tag{2.14}$$

where $a > 0$ and $b \in (0,1)$.

A price vector $p^* \equiv (p_1^*, p_2^*)$ is a Bertrand equilibrium if and only if $f_1(p^*) \geq f_1(p_1, p_2^*)$ for all $p_1 \neq p_1^*$ and $f_2(p^*) \geq f_2(p_1^*, p_2)$ for all $p_2 \neq p_2^*$. We assume that a Bertrand equilibrium exists where both p_1^* and p_2^* are strictly positive, and furthermore, $p_1^* = p_2^*$ if $f_1(p) = f_2(p)$ implying that there is a symmetric equilibrium in both public and private

[16] Towards the end of this section we discuss what happens when we relax strategic complementarity of prices for arbitrary $f_i(p)$.

duopolies. Equilibrium prices satisfy the following first-order conditions (i.e., $\frac{\partial f_i(p)}{\partial p_i} = 0$):

$$(p_i - C_i'(D_i(p)))\frac{\partial D_i(p)}{\partial p_i} + \lambda_i D_i(p) + (1-\lambda_i)(p_j - C_j'(D_j(p)))\frac{\partial D_j(p)}{\partial p_i} = 0, \qquad (2.15)$$

where $i, j \in \{1,2\},\ j \neq i$, and $\lambda_i = 1(\lambda), C_i(.) = C_r(.)(C_u(.))$ if firm i is private (public). Assumption 2(ii) guarantees that Bertrand equilibrium in any duopoly regime is unique.

Figure 2.3 illustrates the reaction functions of different firms and equilibrium prices in different regimes. Point M in Figure 2.3 denotes the equilibrium price vector (p_1^*, p_2^*) in a mixed duopoly. It lies at the intersection of rr and mm which respectively denote the reaction functions of private firm 1 and public firm 2 in a mixed duopoly. That the reaction functions are upward sloping, and rr is steeper than mm follows from Assumption 2.

Point R in Figure 2.3 denotes the equilibrium price vector (p_r^*, p_r^*) in a private duopoly. It lies where rr intersects the 45-degree line. If firm 2 were also private, then the mirror image of rr (with respect to 45-degree line) would have constituted private firm 2's reaction function, and those reaction functions of private firms would have intersected at point R.

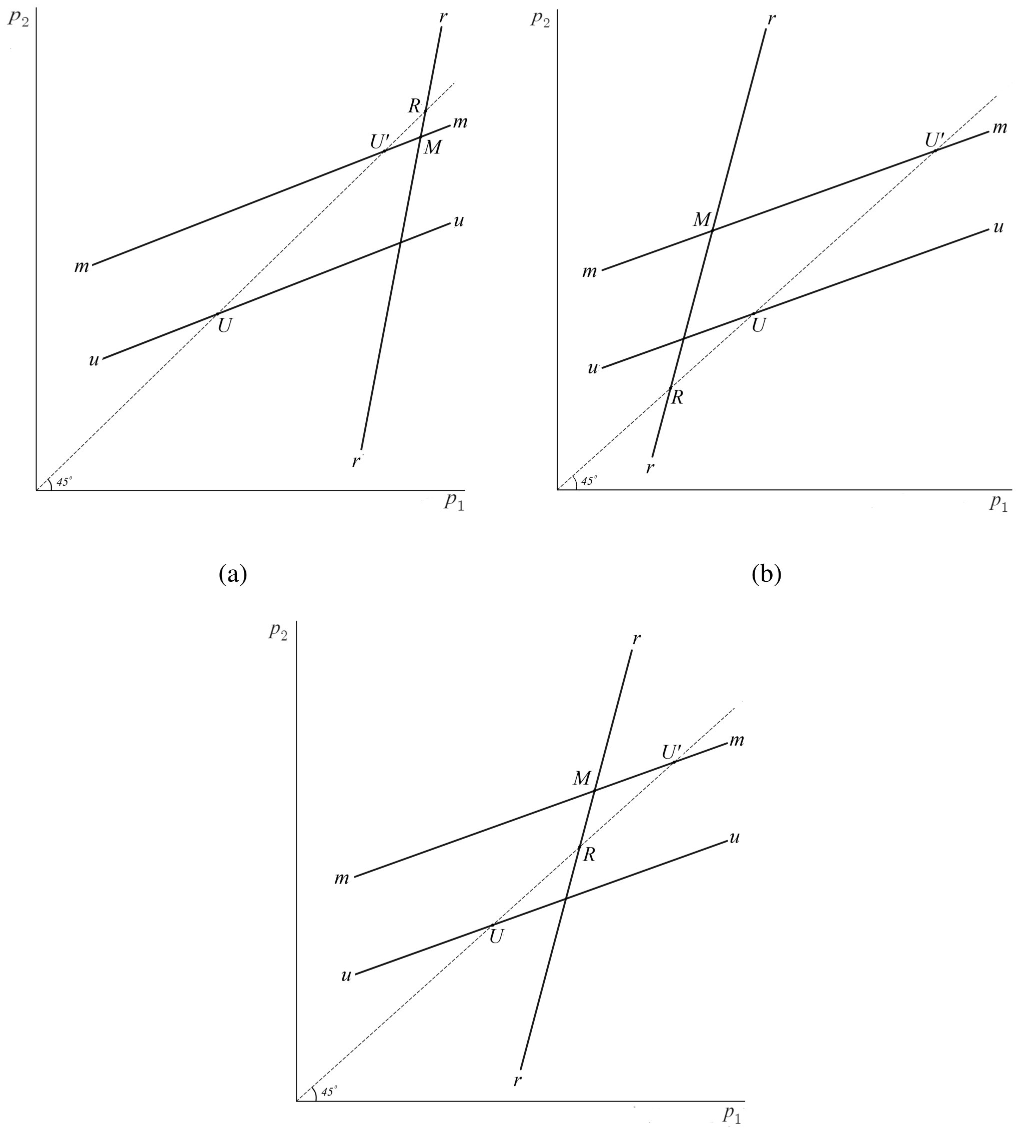
Reasoning as in private duopoly, one might conclude that point U'—where mm intersects the 45-degree line—denotes the equilibrium in a public duopoly. That conclusion is incorrect. Firm 2's reaction function in a public duopoly is given by uu, not mm. Point U, where uu intersects the 45-degree line, denotes the equilibrium price vector in a public duopoly.

To see why uu differs from mm, or more specifically why uu lies below mm, suppose $\lambda_2 = 0$ in the left hand side of equation (2.15). As public firm 2 raises p_2, demand for its own product decreases by $\frac{\partial D_2}{\partial p_2}$. Irrespective of the type of duopoly—public or mixed—welfare changes by the same amount: $(p_2 - C_2'(D_2(p))\frac{\partial D_2}{\partial p_2}$. Demand for firm 1's product increases by $\frac{\partial D_1}{\partial p_2}$ which impacts welfare differently depending on the regime. Suppose $p_1 - C_u'(D_1(p)) \geq 0$. It follows that welfare in a public duopoly goes up by $(p_1 - C_u'(D_1(p))\frac{\partial D_1}{\partial p_2}$. Welfare in a mixed duopoly increases by a larger amount, $(p_1 - C_r'(D_1(p))\frac{\partial D_1}{\partial p_2}$, as $C_r'(.) <$

$C_u'(.)$, and consequently price cost margin is higher in a mixed duopoly. Evaluating at the same p, we get

$$\frac{\partial f_2}{\partial p_2}|_{mixed} - \frac{\partial f_2}{\partial p_2}|_{public} = \left(C_u'(D_1(p)) - C_r'(D_1(p))\right)\frac{\partial D_1}{\partial p_2} > 0. \tag{2.16}$$

Thus, when $\frac{\partial f_2}{\partial p_2}|_{public} = 0$, $\frac{\partial f_2}{\partial p_2}|_{mixed} > 0$. A welfare-maximizing public firm 2 sets a relatively higher p_2 in a mixed duopoly which explains why uu lies below mm.



(a) (b)

(c)

Figure 2.3: Bertrand equilibrium

This regime-contingent response from a public firm arises because of three key features that are embedded in the model: (a) Bertrand competition, (b) a strictly positive weight on welfare ($\lambda_i \neq 1$), and (c) cost asymmetry, or more specifically, marginal cost asymmetry. As in Bertrand competition, rival firm j's cost function, $C_j(q_j)$, appears in public firm i's objective function in Cournot competition also. However, the marginal cost of rival firm, $C_j'(q_j)$, does not influence firm i's best response because a public firm i takes q_j, and consequently $C_j'(q_j)$, as given. If firm i is private, weight on welfare $1-\lambda_i = 0$. In that case, as (2.15) shows, $C_j'(q_j)$ has no impact on firm i's pricing decision. Finally, if there is no asymmetry in marginal cost, i.e., $C_u'(.) = C_r'(.)$, public firm i's choice of p_i is the same in both mixed duopoly and public duopoly.

The regime-contingent response of a public firm holds not only for $\lambda_2 = 0$ but also for any $\lambda_2 < 1$. This plays an important role in the comparison of equilibrium prices, which in turn influences the consumer surplus ranking under Bertrand competition.

2.3.2 Consumer Surplus

Unlike Cournot analysis in Section 2, there is no aggregate variable like aggregate output (Q) or market price (P)—at least not for general demand functions—that can act as a sufficient statistic for consumer surplus. This limitation notwithstanding, symmetry in public and private duopoly simplifies the comparison as there is only one relevant price in each of those regimes—p_u^* in public duopoly, and p_r^* in private duopoly. Lemma 1 compares equilibrium prices across regimes which sets the stage for consumer surplus ranking in Proposition 2.4.

Lemma 2.1. *Equilibrium prices are the lowest in either a public or private duopoly. In fact, prices in both duopoly regimes—public and private—can be lower than both prices in a mixed duopoly. More formally, let p_u^*, p_r^*, and p_i^* denote firm i's equilibrium price in a public, private, and mixed duopoly respectively where $i \in \{1,2\}$, and firm 1(2) is private (public) in a mixed duopoly. Either p_u^*, or p_r^*, and in some cases both p_u^* and p_r^*, are lower than both p_1^* and p_2^*.*

Proof. See Appendix 2.5.2. □

The essence of Lemma 2.1 is captured in Figure 2.3. Observe that the reaction function of private firm 1, *rr*, and the two reaction functions of public firm 2, *uu* and *mm*—which are relevant for public and mixed duopoly regimes respectively—intersect the 45-degree line at R, U, and U' respectively. Recall that U and R respectively denote equilibria in a public and private duopoly. If R coincides with U' or it lies to the northeast of U' on a 45-degree line—as in Figure 2.3(a)—then U must lie to the southwest of M implying $p_u^* < \min\{p_1^*, p_2^*\}$. Else, if R lies to the southwest of U' on 45-degree line—as in Figures 2.3(b) or 2.3(c)—then R lies to the southwest of M as well implying $p_r^* < \min\{p_1^*, p_2^*\}$. Thus, either p_u^* or p_r^* is lower than both prices in the mixed duopoly—p_1^* and p_2^*. Indeed, if R lies between U and U' on the 45-degree line—as in Figure 2.3(c)—then, both R and U lie to the southwest of M, implying both p_u^* and p_r^* are lower than the mixed duopoly prices. In Appendix 2.5.2, we demonstrate this possibility in a linear differentiated duopoly.

Proposition 2.5 spells out the implication of Lemma 2.1 for ranking different regimes according to consumer surplus.

Proposition 2.5. *Consider a differentiated duopoly where firms compete in prices and Assumption 2 holds. Consumer surplus is maximum either in a public or a private duopoly. While consumer surplus in a mixed duopoly can never be the maximum, it can be the minimum for some parameterizations. More formally, let CS_u, CS_r, and CS_m denote equilibrium consumer surplus corresponding to public, private, and mixed duopolies respectively. Then*

$$CS_m < \max\{CS_u, CS_r\}.$$

There exist parameterizations for which $CS_m < \min\{CS_u, CS_r\}$ holds.

Proof. The proof follows immediately from Lemma 2.1 once we recognize that consumer surplus is decreasing in each firm's price. □

2.3.3 Discussion

A common theme across Bertrand and Cournot competition is that consumer surplus is maximum either in a public or in a private duopoly. However, unlike Cournot competition where a mixed duopoly lies in the middle, a mixed duopoly can be at the bottom of the consumer surplus ranking under Bertrand competition.

Duopoly, as assumed above, simplifies the analysis, but it is not necessary for our results. In section 2.6.2, under Supplementary materials, we show that Proposition 2.5 holds in a differentiated oligopoly with $n(>2)$ varieties, where a representative consumer's preference over n varieties is given by (see Example 1, pp. 145-147, in Vives, 2001):

$$U(q) = a\sum_{i=1}^{n} q_i - \frac{1}{2}\sum_{i=1}^{n} q_i^2 - b\sum_{j\neq i} q_i q_j,$$

where $a > 0$, $b \in (0,1)$. Consumer surplus continues to be the maximum under either a fully public or a fully private regime. The possibility of mixed oligopoly delivering the minimum consumer surplus also remains.

This possibility however does not necessarily survive under all alternative objective functions. As explained above, the underlying reason for CS_m to be the minimum is that the public firm's best response to a rival firm's price depends on the rival firm's marginal cost. This dependence no longer holds when we drop welfare from a public firm's objective function and focus on consumer surplus instead. Suppose a public firm maximizes consumer surplus subject to non-negative profits constraint and its marginal cost is constant, c_u say.[17] Irrespective of whether its rival is public or private, a public firm will set its price equal to c_u. As long as the private firm's reaction function slopes upward, it is easy to show that CS_m

[17] In the absence of a non-negative profit constraint, the public firm would set $p_2 = 0 < c_u$, thereby incurring losses. Allowing persistent losses introduces broader public finance considerations, as any deficit must ultimately be financed through taxation, making the choice of tax instrument and the shadow cost of public funds relevant. Moreover, public firms, like private firms, do not have unlimited financial resources. Imposing a non-negative profit constraint is therefore a natural and widely adopted assumption in this setting (Ishida and Matsushima, 2009).

lies between CS_u and CS_r. The same holds—at least in a linear differentiated duopoly with constant marginal cost—where a public firm i maximizes $(1-\lambda)CS+\pi_i$ where $\lambda \in [0,1)$.

One assumption that might seem restrictive—even with welfare in the objective function—is Assumption 2(i). It implies that prices are strategic complements which gives rise to upward sloping reaction functions for both public and private firms. Strategic complementarity of prices is a standard assumption for profit-maximizing private firms. Assuming the same for a public firm though might be restrictive, as it might not hold for all demand functions or for all $\lambda \in [0,1)$, at least not for all price vectors. Below, we relax Assumption 2(i) for a public firm i's objective function f_i and show that CS_m continues to be lower than $\max\{CS_u, CS_r\}$.

Consider a duopoly environment with constant marginal costs and welfare-maximizing public firms. Marginal costs of public and private firms are given by c_u and c_r respectively, where $c_u > c_r > 0$. Suppose firm 2 is public. The first-order condition for firm 2's optimization problem, $\frac{\partial W}{\partial p_2} = 0$, is

$$(p_2 - c_u)\frac{\partial D_2}{\partial p_2} + (p_1 - c_1)\frac{\partial D_1}{\partial p_2} = 0, \tag{2.17}$$

which must hold in public as well as mixed duopoly equilibria.

If firm 1 is public, $c_1 = c_u$. From (2.17) it follows that $p_1 = p_2 = c_u (\equiv p_u^*)$ in a public duopoly equilibrium, which is denoted by U in Figure 2.4. If firm 1 is private, equation (2.17) together with the fact that $p_1 - c_1 > 0$, $\frac{\partial D_1}{\partial p_2} > 0$, and $\frac{\partial D_2}{\partial p_2} < 0$ implies that $p_2^* > c_u$ must hold.

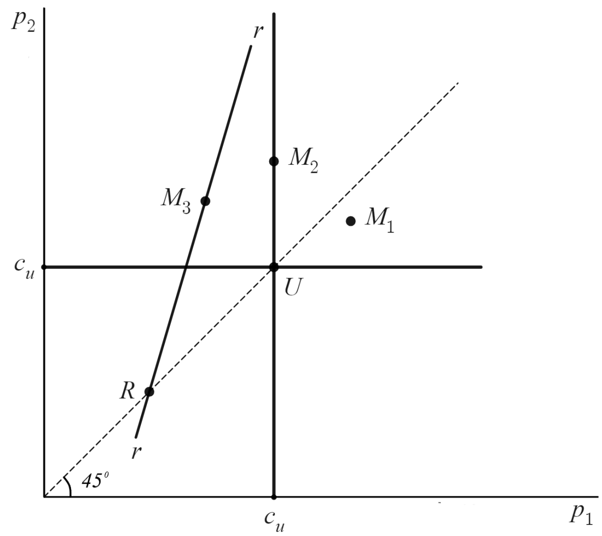


Figure 2.4: Bertrand equilibrium with welfare maximizing public firms

In addition, if $p_1^* \geq c_u$, then, as illustrated in Figure 2.4, the mixed duopoly equilibrium point is in the northeast (e.g., M_1) or north (e.g., M_2) of public duopoly equilibrium $U(\equiv (c_u, c_u))$. Else, if $p_1^* < c_u$ and M_3 denotes the mixed duopoly equilibrium, then, it must lie to the northeast of R, the private duopoly equilibrium. This follows from noting that the upward-sloping reaction function of private firm 1 must pass through M_3, and must intersect the 45-degree line at some point which by definition is R. As all possible candidate points for mixed duopoly equilibrium such as M_1, M_2, and M_3 lie either to the north or northeast of U and/or R, the top spot in consumer surplus ranking continues to be elusive for a mixed duopoly.

2.4 Conclusion

In this chapter, we present canonical models under homogeneous Cournot and differentiated Bertrand competition. In the standard oligopoly setting, we characterize the equilibrium across regimes—public, private and mixed, and demonstrate the comparison of consumer surplus among these three regimes. We find that when efficiency gains from privatization are significant, a purely private regime with efficient profit-maximizing firms deliver the highest consumer surplus while a purely public regime with inefficient, welfare oriented firms does that when such efficiency gains are negligible. The prospect of a mixed regime delivering the highest consumer surplus seems promising at the outset, as it combines the efficiency of private firms and the welfare orientation of public firms both of which work towards lowering prices and improving consumer surplus. The promise however is not met in standard symmetric oligopoly environments including homogeneous products Cournot and differentiated Bertrand. Consumer surplus is highest either when *all* firms are public or when *all* firms are private. A mixed regime always lies in the middle of the consumer surplus ranking in Cournot, whereas, in Bertrand, it is either in the middle or at the bottom.

Despite the lack of theoretical support from conventional oligopoly models in Sections 2.2 and 2.3, a government might opt for a mixed regime for various reasons. First, mixed markets can perform better when alternative metrics like profitability or overall welfare are used to compare different regimes. Second, economic considerations do not always dictate privatization decisions; political motives also play a significant role. For instance, as highlighted by Dinc and Gupta (2011), politicians might resist privatizing firms rooted in their own constituencies. Finally, weak enforcement mechanisms may prompt private firms to collude, thereby undermining the benefits of privatization for consumers (Glaeser and Scheinkman, 1996).[18] In such contexts, the presence of a public firm can serve as a disciplining device, and a mixed regime can deliver the highest consumer surplus.

In next two chapters, we show that we do not need to invoke any of the above reasons to find support for a mixed regime. We need to relax some of the assumptions used in previ-

[18]Private firms can engage in collusion or other anti-competitive practices, especially in developing economies, where the enforcement of competition laws is often lacking (Aydin and Büthe, 2016; Molestina, 2019). Several commentators have argued that a lack of well-functioning private markets and the absence of strong regulatory authorities often lead to less competition and increased income inequality post-privatization in developing countries. See Chapter 3 in Stiglitz (2002) and Chapter 5 in Piketty (2014) for discussions along these lines.

ous sections. So far we have assumed that (a) firms differ primarily due to their ownership structure, and (b) all firms operate within the same sector. In the subsequent chapters, we relax (a) and (b), one at a time. Chapter 3 considers heterogeneous firms that have inherent cost disparities unrelated to ownership, while Chapter 4 delves into vertically related markets comprising upstream and downstream sectors of production. We demonstrate that, in both environments, a mixed regime can beat a fully public or a fully private regime and secure the top spot in the consumer surplus ranking!

2.5 Appendix

2.5.1 Cournot competition

Proof of Proposition 2.3

Proof. From (2.5) we know that, in the symmetric equilibrium of a private oligopoly, each private firm produces $\frac{Q_r^*}{n}$ units of output where Q_r^* uniquely solves

$$P(Q_r^*) + P^{'}(Q_r^*)\frac{Q_r^*}{n} - \left(c_r + C^{'}\left(\frac{Q_r^*}{n}\right)\right) = 0. \tag{2.5.1}$$

From (2.6) we know that, in the symmetric equilibrium of a public oligopoly, each public firm produces $\frac{Q_u^*}{n}$ units of output where Q_u^* uniquely solves

$$P(Q_u^*) + \lambda P^{'}(Q_u^*)\frac{Q_u^*}{n} - \left(c_r + \Delta + C^{'}\left(\frac{Q_u^*}{n}\right)\right) = 0. \tag{2.5.2}$$

Using (2.5.1) and (2.5.2) we get

$$Q_u^* = Q_r^* \Leftrightarrow \Delta = (1-\lambda)\left(-\frac{Q_r^* P'(Q_r^*)}{n}\right) \equiv \hat{\Delta}(c_r, n). \tag{2.5.3}$$

Differentiating (2.5.2) we get

$$\frac{dQ_u^*}{d\Delta} = \frac{n}{nP'(Q_u^*) + \lambda[P'(Q_u^*) + Q_u^* P''(Q_u^*)] - C''\left(\frac{Q_u^*}{n}\right)} < 0,$$

where the inequality follows from noting that $P^{'}(Q_u^*) < 0$, $C^{''}\left(\frac{Q_u^*}{n}\right) \geq 0$, and $P^{'}(Q_u^*) + Q_u^* P^{''}(Q_u^*) < 0$ (due to Assumption 1). As Q_r^* is independent of Δ (from (2.5.1)) and Q_u^* is decreasing in Δ, we find that

$$Q_u^* < Q_r^* \Leftrightarrow \Delta > \hat{\Delta}(c_r, n). \tag{2.5.4}$$

Then $CS_u < CS_1 < CS_2 < \ldots < CS_r \Leftrightarrow \Delta > \hat{\Delta}(c_r, n)$ follows from the following chain of implications:

$$CS_u < CS_1 < CS_2 < \ldots < CS_r \Leftrightarrow CS_u < CS_r \Leftrightarrow Q_u^* < Q_r^* \Leftrightarrow \Delta > \hat{\Delta}(c_r, n),$$

where the first $\Leftrightarrow$ follows from Proposition 2.2, the second $\Leftrightarrow$ follows from the fact that aggregate output is a sufficient statistic for CS, and the last $\Leftrightarrow$ follows from (2.5.4). □

Proof of Proposition 2.4

Proof. Differentiating $\hat{\Delta}(c_r, n)$ from (2.5.3), we get

$$\begin{aligned} \frac{d}{dn}(\hat{\Delta}(c_r, n)) &= (1-\lambda)\frac{d}{dn}\left(-\frac{Q_r^* P^{'}(Q_r^*)}{n}\right) \\ &= \frac{(1-\lambda)}{n}\left[\left(-\frac{dQ_r^*}{dn}\right)\left(P^{'}(Q_r^*) + Q_r^* P^{''}(Q_r^*)\right) + \frac{Q_r^* P^{'}(Q_r^*)}{n}\right]. \end{aligned} \tag{2.5.5}$$

Totally differentiating equation (2.5.1) and rearranging we find

$$\frac{dQ_r^*}{dn} = \frac{\frac{Q_r^*}{n}(P^{'}(Q_r^*) - C^{''}(\frac{Q_r^*}{n}))}{(n+1)P^{'}(Q_r^*) + Q_r^* P^{''}(Q_r^*) - C^{''}(\frac{Q_r^*}{n})},$$

which, upon substitution in (2.5.5), gives

$$\frac{d}{dn}(\hat{\Delta}(c_r,n)) = \left(\frac{Q_r^* P^{'}(Q_r^*)(1-\lambda)}{n^2}\right)\left(\frac{nP^{'}(Q_r^*) + \eta C^{''}(\frac{Q_r^*}{n})}{(n+1)P^{'}(Q_r^*) + Q_r^* P^{''}(Q_r^*) - C^{''}(\frac{Q_r^*}{n})}\right),$$

where $\eta = \frac{Q_r^* P^{''}(Q_r^*)}{P^{'}(Q_r^*)}$ denotes elasticity of slope (of demand).

As $P'(Q_r^*) < 0$, $C^{''}(\frac{Q_r^*}{n}) \geq 0$ and $P^{'}(Q_r^*) + Q_r^* P^{''}(Q_r^*) < 0$ (due to Assumption 1), it follows that

$$\frac{d}{dn}(\hat{\Delta}(c_r,n)) < 0 \Leftrightarrow nP^{'}(Q_r^*) + \eta C^{''}\left(\frac{Q_r^*}{n}\right) < 0. \tag{2.5.6}$$

Multiplying both sides by $\left(-\frac{q_r^*}{C^{'}(q_r^*)}\right)$, we rewrite (2.5.6) in terms of elasticity and markup as

$$\frac{nq_r^*}{Q_r^*}\left(-\frac{Q_r^* P^{'}(Q_r^*)}{P(Q_r^*)}\right)\left(\frac{P(Q_r^*)}{C^{'}(q_r^*)}\right) - \eta\left(\frac{q_r^* C^{''}(q_r^*)}{C^{'}(q_r^*)}\right) > 0 \Leftrightarrow \frac{\mu}{e_d} - \eta e_{mc} > 0 \Leftrightarrow \frac{\mu}{e_d} > \eta e_{mc},$$

where $\mu = \frac{P(Q)}{C^{'}(q)}$ denotes markup, $e_d = -\frac{P(Q)}{QP^{'}(Q)}$ denotes elasticity of demand, $e_{mc} = \frac{qC^{''}(q)}{C^{'}(q)}$ is the elasticity of marginal cost, and all expressions are evaluated at $Q = Q_r^*$ and $q = \frac{Q_r^*}{n}$.

Observe that this condition holds when marginal cost is constant (i.e., $C''(.) = 0$ which implies $e_{mc} = 0$), or demand function is convex (i.e., $P^{''}(.) \geq 0$ which implies $\eta \leq 0$). This condition does not necessarily hold for all n when the demand function is strictly concave (i.e, $P''(.) < 0$, or equivalently $\eta > 0$) and marginal cost is increasing in output ($C''(.) > 0$, or equivalently $e_{mc} > 0$). To see why, suppose the demand and cost functions are given by $P(Q) = a - bQ^2$ and $C(q) = zq^3$ respectively. Using $P(Q) = a - bQ^2, P^{'}(Q) = -2bQ$, and

$P''(Q) = -2b$ we get

$$e_d = -\frac{P(Q)}{QP'(Q)} = \frac{a-bQ^2}{2bQ^2}, \quad \eta = -\frac{QP''(Q)}{P'(Q)} = 2.$$

Similarly, using $C'(q) = 3zq^2$, $C''(q) = 6zq$, and $P(Q) = a - 2bQ^2$ we get

$$e_{mc} = \frac{qC''(q)}{C'(q)} = \frac{6zq^2}{3zq^2} = 2, \quad \mu = \frac{P(Q)}{C'(q)} = \frac{a-bQ^2}{3zq^2}.$$

Using Proposition 2.4 and substituting the expressions for μ, e_d, η and e_{mc} (derived above) in equation (2.13), we find that

$$\frac{d}{dn}(\hat{\Delta}(c_r,n)) > 0 \Leftrightarrow \frac{\mu}{e_d} < \eta e_{mc} \Leftrightarrow \frac{\left(\frac{a-bQ^2}{3zq^2}\right)}{\left(\frac{a-bQ^2}{2bQ^2}\right)} < 2 \Leftrightarrow n < \left(\frac{3z}{b}\right)^{\frac{1}{2}}$$

where $Q = Q_r^*$ and $q = \frac{Q_r^*}{n}$ denote the equilibrium aggregate output and individual output of private firms in private oligopoly. Thus, for low values of n, an increase in competition does not necessarily facilitate privatization when the demand function is strictly concave and marginal cost is increasing in output, i.e., $\eta > 0$ and $e_{mc} > 0$. □

2.5.2 Bertrand competition: Proof of Lemma 2.1

Lemma 2.1 states that either (i) $p_r^* < \min\{p_1^*, p_2^*\}$, or (ii) $p_u^* < \min\{p_1^*, p_2^*\}$, or (iii) possibly both, i.e., $\max\{p_u^*, p_r^*\} < \min\{p_1^*, p_2^*\}$. We prove (i) and (ii) using Claims 2.1 and 2.2 below. We prove (iii) using an example in Claim 2.3.

Claim 2.1. $p_2^* > p_1^* \Rightarrow p_r^* < \min\{p_1^*, p_2^*\}$.

Proof. We have

$$\begin{aligned}
\frac{\partial\pi_1(p_1^*,p_2^*)}{\partial p_1}-\frac{\partial\pi_1(p_r^*,p_r^*)}{\partial p_1} &= \frac{\partial\pi_1(p_1^*,p_2^*)}{\partial p_1}-\frac{\partial\pi_1(p_1^*,p_1^*)}{\partial p_1}+\frac{\partial\pi_1(p_1^*,p_1^*)}{\partial p_1}-\frac{\partial\pi_1(p_r^*,p_r^*)}{\partial p_1} \\
&= \int_{p_1^*}^{p_2^*}\frac{\partial^2\pi_1(p_1^*,p_2)}{\partial p_1\partial p_2}dp_2+\int_{p_r^*}^{p_1^*}\left(\frac{\partial^2\pi_1(\tilde{p},\tilde{p})}{\partial p_1^2}+\frac{\partial^2\pi_1(\tilde{p},\tilde{p})}{\partial p_1\partial p_2}\right)d\tilde{p}.
\end{aligned}$$

As (p_1^*,p_2^*) and (p_r^*,p_r^*) are equilibrium prices in mixed duopoly and private duopoly respectively, we have $\frac{\partial\pi_1(p_1^*,p_2^*)}{\partial p_1}=0$ and $\frac{\partial\pi_1(p_r^*,p_r^*)}{\partial p_1}=0$. Using these values we get

$$\int_{p_r^*}^{p_1^*}\left(\frac{\partial^2\pi_1(\tilde{p},\tilde{p})}{\partial p_1^2}+\frac{\partial^2\pi_1(\tilde{p},\tilde{p})}{\partial p_1\partial p_2}\right)d\tilde{p}+\int_{p_1^*}^{p_2^*}\frac{\partial^2\pi_1(p_1^*,p_2)}{\partial p_1\partial p_2}dp_2=0.$$

Assumption 2 implies that the two integrands have opposite signs, which in turn implies that

$$\mathrm{sgn}(p_1^*-p_r^*)=\mathrm{sgn}(p_2^*-p_1^*)$$

By assumption, $p_2^*>p_1^*$. Then, the above equality implies $p_1^*>p_r^*$ which in turn implies that $p_r^*<\min\{p_1^*,p_2^*\}$. □

Claim 2.2. $p_1^*\geq p_2^*\Rightarrow p_u^*<\min\{p_1^*,p_2^*\}$.

Proof. Let f_2^m and f_2^u respectively denote the public firm 2's objective function in a mixed duopoly and a public duopoly. We have

$$\begin{aligned}
&\frac{\partial f_2^m(p_1^*,p_2^*)}{\partial p_2}-\frac{\partial f_2^u(p_u^*,p_u^*)}{\partial p_2} \\
&=\left(\frac{\partial f_2^m(p_1^*,p_2^*)}{\partial p_2}-\frac{\partial f_2^u(p_1^*,p_2^*)}{\partial p_2}\right)+\left(\frac{\partial f_2^u(p_1^*,p_2^*)}{\partial p_2}-\frac{\partial f_2^u(p_2^*,p_2^*)}{\partial p_2}\right)+\left(\frac{\partial f_2^u(p_2^*,p_2^*)}{\partial p_2}-\frac{\partial f_2^u(p_u^*,p_u^*)}{\partial p_2}\right) \\
&=\left(\frac{\partial f_2^m(p_1^*,p_2^*)}{\partial p_2}-\frac{\partial f_2^u(p_1^*,p_2^*)}{\partial p_2}\right)+\int_{p_2^*}^{p_1^*}\frac{\partial^2 f_2^u(p_1,p_2^*)}{\partial p_1\partial p_2}dp_1+\int_{p_u^*}^{p_2^*}\left(\frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_2^2}+\frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_1\partial p_2}\right)d\tilde{p}.
\end{aligned}$$

As (p_1^*,p_2^*) and (p_u^*,p_u^*) are equilibrium prices in mixed duopoly and private duopoly respectively, we have $\frac{\partial f_2^m(p_1^*,p_2^*)}{\partial p_2} = 0$ and $\frac{\partial f_2^u(p_u^*,p_u^*)}{\partial p_2} = 0$. Using these values we get

$$\left(\frac{\partial f_2^m(p_1^*,p_2^*)}{\partial p_2} - \frac{\partial f_2^u(p_1^*,p_2^*)}{\partial p_2}\right) + \int_{p_2^*}^{p_1^*} \frac{\partial^2 f_2^u(p_1,p_2^*)}{\partial p_1 \partial p_2} dp_1 + \int_{p_u^*}^{p_2^*} \left(\frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_2^2} + \frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_1 \partial p_2}\right) d\tilde{p} = 0.$$

The first term on the left-hand side of the above equation is strictly positive (see equation (2.16)). If $p_1^* \geq p_2^*$, then the second term is weakly positive as $\frac{\partial^2 f_2^u(p_1,p_2)}{\partial p_1 \partial p_2} > 0$ (by Assumption 2(i)). This implies that the third term must be strictly negative. By Assumption 2(ii), $\frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_2^2} + \frac{\partial^2 f_2^u(\tilde{p},\tilde{p})}{\partial p_1 \partial p_2} < 0$ implying $p_2^* > p_u^*$. Thus, $p_1^* \geq p_2^* \implies p_2^* > p_u^*$, which gives us

$$p_u^* < \min\{p_1^*, p_2^*\} = p_2^*.$$

□

Claim 2.3. *Suppose $U(q)$ is given by (2.14) and constant marginal costs of public and private firms are given by c_u and c_r respectively. Define $\Delta = c_u - c_r$. There exists $\underline{\Delta}$ and $\overline{\Delta}$ such that* $\max\{p_u^*, p_r^*\} < \min\{p_1^*, p_2^*\}$ *for all $\Delta \in (\underline{\Delta}, \overline{\Delta})$.*

Proof. Direct demand functions corresponding to (2.14) are

$$D_i(p) = \frac{a(1-b) - p_i + bp_j}{1-b^2}$$

where $i, j \in \{1,2\}$, $j \neq i$ and $b \in (0,1)$. Substituting $\frac{\partial D_i(p)}{\partial p_i} = -\frac{1}{1-b^2}$ and $\frac{\partial D_i(p)}{\partial p_j} = \frac{b}{1-b^2}$ in the first order conditions of optimization problems for firm i and firm j, rearranging, and solving the resultant equations gives us the equilibrium prices:

$$\begin{aligned}
p_u^* &= \frac{a\lambda(1-b)+(1-b(1-\lambda))c_r+(1-b(1-\lambda))\Delta}{(1+\lambda)-b}, \\
p_r^* &= \frac{a(1-b)+c_r}{2-b}, \\
p_1^* &= \frac{a(1-b)(1+\lambda(1+b))+(b+(1+\lambda)-b^2(1-\lambda))c_r+b\Delta}{2(1+\lambda)-b^2}, \\
p_2^* &= \frac{a(1-b)(b+2\lambda)+(2-b(1-2\lambda))c_r+2\Delta}{2(1+\lambda)-b^2}.
\end{aligned}$$

Comparing p_1^* and p_2^* we get

$$p_2^* > p_1^* \Leftrightarrow \Delta > \frac{(1-\lambda)(1-b)^2(a-c_r)}{(2-b)} \equiv \underline{\Delta}.$$

From Claim 1 it follows that for all such Δ,

$$p_r^* < \min\{p_1^*, p_2^*\} = p_1^*.$$

Comparing p_u^* and p_r^* we get

$$p_u^* < p_r^* \Leftrightarrow \Delta < \frac{(1-\lambda)(1-b)^2(a-c_r)}{(2-b)[1-b(1-\lambda)]} \equiv \overline{\Delta}.$$

As

$$\overline{\Delta} - \underline{\Delta} = \frac{b(1-\lambda)^2(1-b)^2(a-c_r)}{(2-b)[1-b(1-\lambda)]} > 0,$$

we have

$$p_u^* < p_r^* < \min\{p_1^*, p_2^*\} \Rightarrow \max\{p_u^*, p_r^*\} < \min\{p_1^*, p_2^*\}$$

for all $\Delta \in (\underline{\Delta}, \overline{\Delta})$.

$\square$

2.6 Supplementary materials

2.6.1 Cournot competition: Discussion

In Section 2.2 we have shown that consumer surplus is maximum or minimum when all firms are public or all firms are private (Proposition 2.1). Mixed oligopoly lies in the middle of the CS ranking. Below we show that the result is robust to several variations of the model, including (i) a quality-augmented model of Cournot competition, (ii) an alternative cost function, and (iii) an alternative objective function of the public firms.

Quality

In a homogeneous products model like ours, higher prices imply lower consumer surplus. However, a high price is not necessarily bad for consumers if accompanied by suitably high quality. In fact, a common perception is that private firms charge high prices but also offer high-quality products. Below, we extend our model of Cournot competition to incorporate quality differential between private and public firms. Once again we find that a mixed market—which offers a combination of high-quality, high-priced private firms' products and low-quality, low-priced public firms' products—cannot be optimal from consumers' point of view.

Assume that there is a unit mass of consumers who buys either one unit of the product or nothing. Consumer valuation θ is distributed according to $F(\theta)$ in $[\underline{\theta}, \overline{\theta}]$ where $F(\theta)$ is strictly increasing in θ, $F(\underline{\theta}) = 0$, and $F(\overline{\theta}) = 1$.

We assume that the market structure is a duopoly. Consumers are willing to pay a premium Δ for a private firm's product which is of higher quality. Let p_r and p_u denote the prices charged by the private and public firms respectively. If both firms are active in the market, it must be true that

$$\theta + \Delta - p_r = \theta - p_u,$$

as anything else would imply that one firm is not active.

Let ρ denote the common quality-adjusted price, i.e.,

$$p_r - \Delta = p_u = \rho.$$

This formulation implies that the public and private firms will charge different prices in a mixed duopoly but quality-adjusted prices will be the same.

A consumer with valuation θ buys the product if and only if $\theta \geq \rho$., This implies that aggregate demand is

$$Q = 1 - F(\rho)$$

which in turn gives

$$\rho = F^{-1}(1-Q) \equiv \rho(Q).$$

The private firm chooses x to maximize

$$\pi_r = p_r x - C_r(x) = (\rho(Q) + \Delta)x - C_r(x)$$

while the public firm, as before, chooses y to maximize

$$f_u = (1-\lambda)W + \lambda \pi_u$$

where π_u is as in (2.2) and W is as in (5.2.7), except that $P(Q)$ is replaced by $\rho(Q)$.

The first-order conditions corresponding to the maximization problems are

$$\rho(Q) + \Delta + \rho'(Q)x - C_r'(x) = 0, \tag{2.6.1}$$

$$\rho(Q) + \lambda \rho'(Q)y - C_u'(y) = 0. \tag{2.6.2}$$

These two equations are analogous to equations (2.5) and (2.6) in Section 2.2. As in the proof of Proposition 2.1, exploiting negative associations between (a) x and Q in (2.6.1), and (b) y and Q in (2.6.2), we can show that

$$\min\{Q_u^*, Q_r^*\} \leq Q_m^* \leq \max\{Q_u^*, Q_r^*\}.$$

This in turn implies

$$\min\{CS_u, CS_r\} \leq CS_m \leq \max\{CS_u, CS_r\}.$$

Alternative cost function

We have assumed that λ, i.e. profit-orientation of a public firm, has no bearing on its cost function $C_u(.)$. To allow for the public firm's costs to depend on λ, we can model the public firm's cost of producing y as a weighted average of $C_u(.)$ and $C_r(.)$, i.e.,

$$\begin{aligned}\tilde{C}_u(y) &= (1-\lambda)C_u(y) + \lambda C_r(y) \\ &= C_u(y) + \lambda(C_r(y) - C_u(y)).\end{aligned}$$

Since $C_r(y) - C_u(y) < 0$, $\tilde{C}_u(y)$ decreases as λ increases. In other words, a public firm's efficiency increases as it becomes more profit-oriented.

It follows from the specification that $\tilde{C}_u(.)$ has the same properties as $C_u(.)$ and $C_r(.)$. That is, $\tilde{C}_u(0) = 0$, $\tilde{C}_u'(y) > 0$, and $\tilde{C}_u''(y) \geq 0$ for all $y \geq 0$. All that happens with this alternative specification is that $\tilde{C}_u'(y)$ replaces $C_u'(y)$ in equation (2.6). The negative association between y and Q, which is key to the proof of Proposition 2.1, continues to hold. Thus mixed duopoly continues to be dominated by private or public duopoly in terms of CS.

Alternative objective function

Instead of maximizing the weighted average of welfare and profit, suppose a public firm maximizes CS subject to making non-negative profits. To make things concrete, consider a mixed duopoly where x and y denote outputs produced by the private and the public firm respectively. In this case, the first-order condition in equation (2.6) is replaced by the break-even condition:

$$P(Q)y - C_u(y) = 0.$$

Differentiating the zero-profit condition and rearranging we get

$$\frac{dy}{dQ} = \frac{P'(Q)y}{C_u'(y) - P(Q)} = \frac{P'(Q)y}{C_u'(y) - \frac{C_u(y)}{y}}$$

which is strictly negative as long as average cost, $C_u(y)/y$, is increasing in y. Increasing average cost implies that marginal cost is greater than average cost: $C_u'(y) > \frac{C_u(y)}{y}$. Hence, the numerator is positive. Then $\frac{dy}{dQ} < 0$ follows from noting that $P'(Q) < 0$. From equation (2.7) we already know that $\frac{dx}{dQ} < 0$. Exploiting the negative association between (a) y and Q, and (b) x and Q, and proceeding as in the proof of Proposition 2.1, it is straightforward to show that a mixed regime lies in the middle of the CS ranking.

2.6.2 Bertrand competition: Discussion

The main result in Section 2.3 is given by Proposition 2.5. It says that CS is maximum either in a public or a private duopoly. Furthermore, while CS in a mixed duopoly can never be maximum, it can be minimum for some parameterizations. Throughout Section 2.3 we assumed that the market structure is a duopoly, i.e., $n = 2$. Below we show that Proposition 2.5 holds even when $n > 2$.

We assume that a representative consumer maximizes $U(q) + z$ subject to a budget constraint where $q \equiv (q_1, q_2, ..., q_n)$ and z is the numeraire good produced under perfect competition. The representative consumer's preference over n varieties is given by:

$$U(q) = a\sum_{i=1}^{n} q_i - \frac{1}{2}\sum_{i=1}^{n} q_i^2 - b\sum_{j\neq i} q_i q_j.$$

We assume that income is sufficiently high so that the consumer optimization problem has an interior solution. Rearranging the first-order conditions of the optimization problem yields the indirect demand function of good i:

$$p_i = a - q_i - b\sum_{i\neq j} q_j$$

where $i,j \in \{1,2,\ldots,n\},\ j \neq i$. Inverting the indirect demand system yields the direct demand functions. Direct demand for variety i is given by

$$D_i = a_n - b_n p_i + e_n \sum_{i\neq j} p_j$$

where

$$a_n = \frac{a}{1+(n-1)b},\ b_n = \frac{1+(n-2)b}{(1+(n-1)b)(1-b)},\ e_n = \frac{b}{(1+(n-1)b)(1-b)},$$

$n \geq 2$, and $b \in (0,1)$. Private and public firms have constant marginal costs, given by c_r and c_u respectively. We assume that $a > c_u$. Firm i chooses price p_i to maximize

$$f_i(p) = (1-\lambda_i)W(p) + \lambda_i \pi_i(p)$$

where $\lambda_i = 1(\lambda)$ if firm i is private (public), $\lambda \in (0,1)$ and $i = 1,2,\ldots,n$.

Mixed Oligopoly

Suppose there are m private firms and $n-m$ public firms in a mixed oligopoly. The first-order conditions associated with private firm i and public firm k respectively are given by

$$(p_i - c_r)\frac{\partial D_i(p)}{\partial p_i} + D_i(p) = 0, \tag{2.6.3}$$

$$(p_k - c_u)\frac{\partial D_k(p)}{\partial p_k} + \lambda D_k(p) + (1-\lambda)\sum_{l=1}^{m}(p_l - c_r)\frac{\partial D_l(p)}{\partial p_k} + (1-\lambda)\sum_{j>m,\, j\neq k}(p_j - c_u)\frac{\partial D_j(p)}{\partial p_k} = 0. \tag{2.6.4}$$

We consider an equilibrium where all private firms charge P_1 and all public firms charge P_2. Substituting these in (2.6.3) and (2.6.4) and rearranging the first-order conditions we get

$$P_1 = \frac{a_n + b_n c_r + e_n(n-m)P_2}{2b_n - (m-1)e_n}, \tag{2.6.5}$$

$$P_2 = \frac{\lambda a_n + [b_n - (1-\lambda)(n-1)e_n]c_u + m(1-\lambda)e_n(c_u - c_r) + m e_n P_1}{(1+\lambda)b_n - (n-m-1)e_n}. \tag{2.6.6}$$

In Figure 2.5, rr and mm depict (2.6.5) and (2.6.6) respectively in (P_1, P_2) space. When $n=2$ and $m=1$, Figure 2.5 is the same as Figure 2.3 where rr and mm are the reaction functions of the private firm and the public firm respectively. While the reaction function interpretation does not quite hold for $n>2$, we use rr and mm as it enables diagrammatic representation of the equilibrium of different regimes. The two curves, rr and mm, intersect at point M whose coordinates give the equilibrium prices in a mixed oligopoly:

$$P_1^* = \frac{a_n[(1+\lambda)b_n-(n-m-1)e_n+\lambda e_n(n-m)]+e_n(n-m)[b_n-(1-\lambda)(n-m-1)e_n]c_u+[b_n((1+\lambda)b_n-(n-m-1)e_n)-(1-\lambda)(n-m)me_n^2]c_r}{[(1+\lambda)b_n-(n-m-1)e_n][2b_n-(m-1)e_n]-me_n^2(n-m)},$$

$$P_2^* = \frac{a_n[\lambda(2b_n-(m-1)e_n)+me_n]+[b_n-(1-\lambda)(n-m-1)e_n][2b_n-(m-1)e_n]c_u+[b_n-(1-\lambda)(2b_n-(m-1)e_n)]me_nc_r}{[(1+\lambda)b_n-(n-m-1)e_n][2b_n-(m-1)e_n]-me_n^2(n-m)}.$$ [19]

[19] It is easy to show that the numerators in the expressions for P_1^* and P_2^* are strictly positive. Denote the

Private Oligopoly

The first-order condition for a profit-maximizing private firm i is given by

$$(p_i - c_r)\frac{\partial D_i(p)}{\partial p_i} + D_i(p) = 0 \Rightarrow a_n - 2b_n p_i + e_n \sum_{j \neq i} p_j + b_n c_r = 0. \tag{2.6.7}$$

Even though all n firms are private with identical costs, we divide n private firms into two groups: Group 1 with first m private firms, each charging P_1 and Group 2 comprising remaining $(n-m)$ firms, each charging P_2. Grouping firms in this fashion facilitates the comparison between private and mixed oligopolies. Substituting these prices in (2.6.7) and rearranging the first-order conditions we get

$$P_1 = \frac{a_n + b_n c_r + e_n(n-m)P_2}{2b_n - e_n(m-1)}. \tag{2.6.8}$$

Note that (2.6.8) is the same as (2.6.5) which implies that if all firms in Group 2 charge P_2, the price response of a representative private firm in Group 1 remains the same irrespective of whether the regime is a private oligopoly (i.e., Group 2 firms are private) or mixed oligopoly (i.e., Group 2 firms are public). Hence, in a private oligopoly regime, rr continues to capture the best response of a representative private firm in Group 1. rr intersects the 45-degree line at $(P_1, P_2) = (P_r^*, P_r^*)$ where

$$P_r^* = \frac{a_n + b_n c_r}{2b_n - (n-1)e_n}$$

denote the equilibrium price in a symmetric, n-firm private oligopoly.[20]

denominator—which is same for P_1^* and P_2^*—as $[(1+\lambda)b_n - (n-m-1)e_n][2b_n - (m-1)e_n] - me_n^2(n-m) = D(\lambda)$. Since $D'(\lambda) = b_n(2b_n - (m-1)e_n) > 0$, $D(0) > 0 \Rightarrow D(\lambda) > 0 \Rightarrow P_i^* > 0$ for both $i(=1,2)$. Using the expressions of b_n and e_n derived above, we find $D(0) = \frac{2+(2n+m-5)b+(3-2(m+n)+mn)b^2}{(1+(n-1)b)^2(1-b)^2}$. That $D(0) > 0$ follows from noting $2 + (2n+m-5)b + (3-2(m+n)+mn)b^2 > 2 + [(2n+m-5) + (3-2(m+n)+mn)]b^2 = 2(1-b^2) + m(n-1)b^2 > 0$ as $b \in (0,1)$.

[20] Using expressions of b_n and e_n, we have $2b_n - (n-1)e_n = \frac{2(1-b)+(n-1)b}{(1+(n-1)b)(1-b)} > 0$ where the inequality follows from noting that $b \in (0,1)$ and $n > 2$.

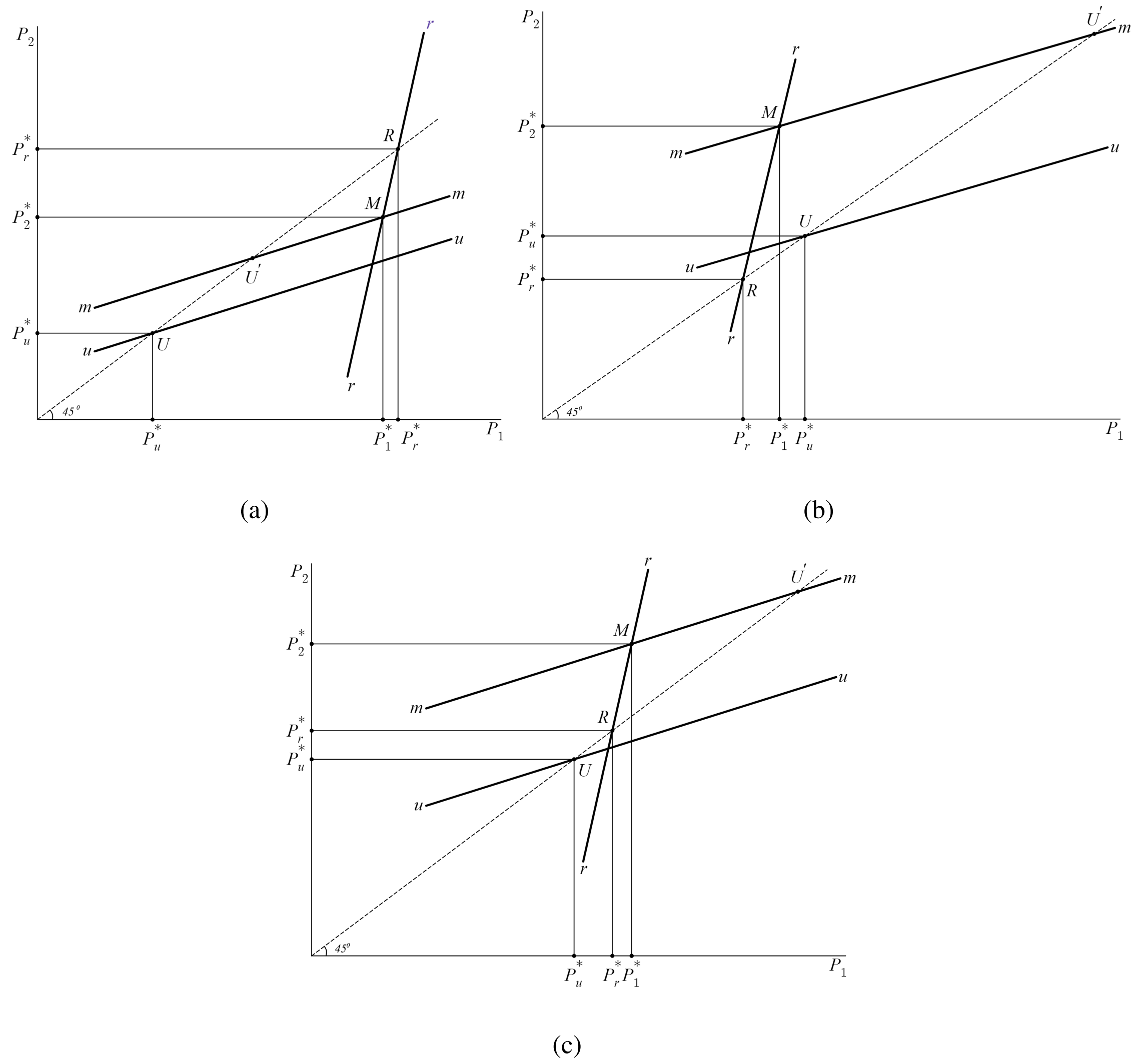


Figure 2.5: Bertrand equilibrium in n-firm oligopoly

Public Oligopoly

The first order condition for public firm k is given by

$$(p_k - c_u)\frac{\partial D_k(p)}{\partial p_k} + \lambda D_k(p) + (1-\lambda)\sum_{j\neq k}(p_j - c_u)\frac{\partial D_j(p)}{\partial p_k} = 0. \tag{2.6.9}$$

Once again, we divide n public firms into two groups: Group 1 consisting of m public firms, each charging P_1, and Group 2 comprising $n-m$ public firms, each charging P_2.

Substituting these prices in (2.6.9) and rearranging the first-order conditions we get

$$P_2 = \frac{\lambda a_n + [b_n - (1-\lambda)(n-1)e_n]c_u + m e_n P_1}{(1+\lambda)b_n - (n-m-1)e_n}. \tag{2.6.10}$$

Writing P_2 in (2.6.6) and (2.6.10) as P_2^{mixed} and P_2^{public} respectively and subtracting (2.6.6) from (2.6.10) we get

$$P_2^{public} - P_2^{mixed} = -\frac{m(1-\lambda)e_n(c_u - c_r)}{(1+\lambda)b_n - (n-m-1)e_n} < 0$$

This is reflected in the positioning of uu in Figure 2.5 where mm always lies above uu.

Observe that uu intersects the 45-degree line at $(P_1, P_2) = (P_u^*, P_u^*)$ where

$$P_u^* = \frac{a_n\lambda + (b_n - (1-\lambda)(n-1)e_n)c_u}{(1+\lambda)b_n - (n-1)e_n}$$

denotes the equilibrium price in a symmetric, n-firm public oligopoly.[21]

Comparing prices across different regimes, we can show algebraically that prices are lowest or equivalently consumer surplus is highest either in a public oligopoly or in a private oligopoly. Below we present our arguments with a diagram (similar to Figure 2.3) which is more intuitive.

Suppose mm intersects the 45-degree line at U'. Unless R coincides with U or U', rr can intersect the 45-degree line to the northeast of U', southwest of U, or in between U and U'. These three possibilities are considered in panels (a), (b), and (c) respectively in Figure 2.5. U lies to the southwest of M in Figure 2.5(a) which implies $P_u^* < \min\{P_1^*, P_2^*\}$, while in Figure 2.5(b), R lies to the southwest of M implying $P_r^* < \min\{P_1^*, P_2^*\}$. Figure 2.5(c) illustrates the possibility that both R and U lie to the southwest of M implying

[21] From expressions of b_n and e_n, we find that $b_n - (n-1)e_n = \frac{1-b}{(1+(n-1)b)(1-b)} > 0$ as $b \in (0,1)$. It follows that the denominator of P_u^* given by $(1+\lambda)b_n - (n-1)e_n$ is strictly positive since $(1+\lambda)b_n - (n-1)e_n \geq b_n - (n-1)e_n > 0$ for $\lambda \in [0,1)$.

$\max\{P_u^*, P_r^*\} < \min\{P_1^*, P_2^*\}$. Two possibilities are not covered by (a)-(c): R coincides with U, and R coincides with U'. When R coincides with U', U lies to the southwest of M implying $P_u^* < \min\{P_1^*, P_2^*\}$. When R coincides with U, both R and U lie to the southwest of M implying $P_r^* = P_u^* < \min\{P_1^*, P_2^*\}$. Since either $P_u^* < \min\{P_1^*, P_2^*\}$ or $P_r^* < \min\{P_1^*, P_2^*\}$, consumer surplus can never be the highest in a mixed oligopoly. In fact, as in section 2.3, consumer surplus in a mixed oligopoly can be the lowest among all regimes since there are parameterizations where Figure 2.5(c) holds, i.e., $P_u^* < P_r^* < \min\{P_1^*, P_2^*\}$.

Chapter 3

Privatization and Consumer Welfare in the Presence of Heterogeneous Firms

3.1 Introduction

In Chapter 2, we have demonstrated our main finding that mixed markets typically do not yield the highest consumer surplus (*CS* hereafter). This might seem at odds with the coexistence of public and private firms in developing countries. Even in advanced economies such as Europe, Canada, and Japan, public firms operate in various sectors, including electricity, telecommunications, banking, and even manufacturing (Christiansen, 2011). This chapter shows that resorting to alternative metrics such as welfare, or introducing collusion as a sign of weak competition policy post-privatization, are not necessary for a mixed regime to appear at the top of the *CS* ranking. What we need is to move beyond the symmetric, single-stage oligopoly setting considered in Sections 2.2 and 2.3.

Firms may exhibit innate cost differences that are unrelated to ownership. While privatization enhances firms' efficiency, some of the innate cost differences do not go away. In this chapter, we introduce heterogeneity by allowing for innate cost differences among firms that persist even when these firms have the same ownership structure. Two mixed regimes with the same number of private firms can yield different consumer surplus as not all private firms necessarily have the same costs. Neither do all public firms.

This possibility points toward the importance of firm selection in case of privatization. The design of privatization should not only focus on privatizing the public firms but also on the choice of the public firm(s) to be privatized. Adverse selection of public firms to be privatized can indeed end up harming the consumers more rather than benefiting them. For instance, in a duopoly, privatizing the inefficient firm but retaining public ownership of the efficient firm *can* result in a higher *CS* than purely public or private regimes, where ownership structure is uniform across all firms. Conversely, in a mixed regime, if privatization is of the opposite kind—where an efficient firm is privatized while the inefficient one remains public—the *CS* can be lowest.

Despite an increase in the number of possible mixed regimes, there is a sense in which middle-ness of the mixed regimes—as shown in Proposition 2.1—continues to hold. All mixed regimes cannot be strictly better (or worse) than both public and private regimes (Proposition 3.1). However, we find that a mixed regime can deliver the highest consumer surplus. In a duopoly, privatizing the relatively inefficient firm while retaining public ownership of the relatively efficient firm might yield the highest consumer surplus among all possible regimes. This occurs when efficiency gains from privatization are moderate and the cost difference between the two firms exceeds a threshold (Proposition 3.3). Interestingly, we observe that the threshold can be arbitrarily close to zero.

The chapter is organized as follows. Section 3.2 revisits Proposition 2.1 under the assumption of firm heterogeneity. We provide a brief description of the model in Section 3.3 and characterize the Cournot equilibrium in Section 3.4. The ranking of consumer surplus across different regimes are derived in Section 3.5. Section 3.6 concludes. Detailed proofs are developed in the Appendix.

3.2 Proposition 2.1 under firm heterogeneity

Without delving into the details of the model, we first revisit Proposition 2.1 under the assumption of firm heterogeneity. We assume that the underlying model features are the same as in Section 2.2 with one important exception: firms with similar ownership structures can have different costs. For simplicity, we consider a duopoly with two firms, firm 1 and firm 2, where for all $q \geq 0$ and $s \in \{u, r\}$, $C_{is}^{''}(q) \geq 0$, $i = 1, 2$, and $C_{1s}^{'}(q) > C_{2s}^{'}(q)$, i.e., firm 2 is more efficient than firm 1 under any ownership structure. Privatization reduces costs for both firms but the efficient firm 2 continues to be relatively more efficient when both firms are privatized. That is, $C_{iu}^{'}(q) > C_{ir}^{'}(q)$ for $i = 1, 2$, and $C_{1r}^{'}(q) > C_{2r}^{'}(q)$.

Here, the comparison is among four regimes: public, private, and two mixed duopolies—mixed duopoly of type 1 (m_1) where only the inefficient firm 1 is private, and mixed duopoly of type 2 (m_2) where only the efficient firm 2 is private. Let q_{is}^{z} denote equilibrium output of firm i of type s in regime $z (\in \{u, r, m_1, m_2\})$ and let Q_z denote the aggregate equilibrium output in the same regime. Then q_{is}^{z} and Q_z must satisfy the following first-order condition:[1]

$$P(Q_z) + \lambda_{is} P'(Q_z) q_{is}^{z} - C_{is}^{'}(q_{is}^{z}) = 0, \tag{3.2.1}$$

where $\lambda_{is} = \lambda(1)$ if $s = u(r)$. Differentiating (3.2.1) and rearranging we get

$$\frac{dq_{is}^{z}}{dQ_z} = -\frac{P'(Q_z) + \lambda_{is} P''(Q_z) q_{is}^{z}}{\lambda_{is} P'(Q_z) - C_{is}^{''}(q_{is}^{z})} < 0, \tag{3.2.2}$$

where the inequality follows from noting that $P^{'}(Q_z) < 0$, $C_{is}^{''}(q_{is}^{z}) \geq 0$, and $P'(Q_z) + \lambda_{is} P''(Q_z) q_{is}^{z} < 0$ (by Assumption 1). Exploiting this negative association between firm-level output (q_{is}^{z}) and aggregate output (Q_z), Proposition 3.1 shows that there is a sense of middle-ness that still holds collectively for the two mixed regimes.

Proposition 3.1. *Both mixed duopolies cannot be above or below both public and private*

[1] We omit detailed derivation of Cournot equilibrium here as the analysis is similar to the one in Section 2.2.

duopolies in the consumer surplus ranking of regimes. More formally, let CS_u, CS_r and CS_{m_i} denote the consumer surplus in a public duopoly, private duopoly, and mixed duopoly of type i respectively, where mixed duopoly of type i is defined as the i^{th} firm being privatized, $i = 1,2$. In equilibrium,

(i) $\max\{CS_{m_1}, CS_{m_2}\} \geq \min\{CS_u, CS_r\}$, *and*

(ii) $\min\{CS_{m_1}, CS_{m_2}\} \leq \max\{CS_u, CS_r\}$.

Proof. For (i), it suffices to show that $\max\{Q_{m_1}, Q_{m_2}\} \geq \min\{Q_u, Q_r\}$. Suppose not. That is, suppose $Q_{m_1} = \max\{Q_{m_1}, Q_{m_2}\}$, and yet

$$Q_{m_1} < Q_u \text{ and } Q_{m_1} < Q_r.$$

Using the negative association between q^z_{is} and Q_z in (3.2.2), we get

$$Q_{m_1} < Q_u \Rightarrow q^{m_1}_{2u} > q^u_{2u}, \quad Q_{m_1} < Q_r \Rightarrow q^{m_1}_{1r} > q^r_{1r}. \tag{6a}$$

As $Q_{m_1} - Q_r = (q^{m_1}_{1r} - q^r_{1r}) + (q^{m_1}_{2u} - q^r_{2r}) < 0$ and $q^{m_1}_{1r} - q^r_{1r} > 0$, it must be that

$$q^r_{2r} > q^{m_1}_{2u}. \tag{6b}$$

Combining (6b) with the first part of (6a) we get

$$q^r_{2r} > q^{m_1}_{2u} > q^u_{2u} \Rightarrow q^r_{2r} > q^u_{2u}. \tag{6c}$$

By assumption, $Q_{m_1} = \max\{Q_{m_1}, Q_{m_2}\}$ which implies $Q_{m_2} < Q_{m_1}$. As Q_{m_1} is strictly less than Q_u and Q_r, the same holds for Q_{m_2}. Proceeding as above we find that

$$Q_{m_2} < Q_r \Rightarrow q^{m_2}_{2r} > q^r_{2r}, \quad Q_{m_2} < Q_u \Rightarrow q^{m_2}_{1u} > q^u_{1u}. \tag{6d}$$

As $Q_{m_2} - Q_u = (q_{1u}^{m_2} - q_{1u}^{u}) + (q_{2r}^{m_2} - q_{2u}^{u}) < 0$ and $q_{1u}^{m_2} - q_{1u}^{u} > 0$, it must be that

$$q_{2u}^{u} > q_{2r}^{m_2}. \tag{6e}$$

Combining (6e) with the first part of (6d) gives $q_{2u}^{u} > q_{2r}^{m_2} > q_{2r}^{r} \Rightarrow q_{2u}^{u} > q_{2r}^{r}$ which contradicts inequality (6c). This completes the proof of Proposition 3.1 (i). Proof of part (ii) is analogous and hence omitted. □

Proposition 3.1 nests Proposition 2.1. When there is no cost heterogeneity, we have $CS_{m_1} = CS_{m_2}$ ($\equiv CS_m$, say). Together, parts (i) and (ii) of Proposition 3.1 then imply

$$\min\{CS_u, CS_r\} \leq CS_m \leq \max\{CS_u, CS_r\},$$

which is the same as Proposition 2.1. Naturally, for a suitably small degree of heterogeneity, Proposition 3.1 continues to reflect Proposition 2.1, at least in spirit, as both CS_{m_1} and CS_{m_2} continue to lie between CS_u and CS_r unless $CS_u = CS_r$.

The main appeal of Proposition 3.1 is its breadth: it holds for an arbitrary degree of cost heterogeneity. Irrespective of cost differences, the best of the two mixed duopolies (in terms of consumer surplus) cannot be worse than both private and public duopolies. Similarly, the worst of the two mixed duopolies cannot be better than both private and public duopolies. Thus, Proposition 3.1 suggests that the *Goldilocks* feature of a mixed regime—as reflected in Proposition 2.1—is retained under heterogeneity.

That is as far as Proposition 3.1 can take us. To determine whether a mixed duopoly can be at the top or the bottom of CS ranking and escape the middle, we need to examine public and private duopolies more closely. Proposition 3.2 provides an escape route.

Proposition 3.2. *A necessary condition for a mixed duopoly to be uniquely at the top of the consumer surplus ranking, i.e.,* $\max\{CS_{m_1}, CS_{m_2}\} > \max\{CS_u, CS_r\}$*, or uniquely at the* *bottom, i.e.,* $\min\{CS_{m_1}, CS_{m_2}\} < \min\{CS_u, CS_r\}$*, is*

$$\text{sgn}(q^r_{1r} - q^u_{1u}) \neq \text{sgn}(q^r_{2r} - q^u_{2u})$$

where q^z_{is} *denotes output of firm i of type s in regime z, and* $\text{sgn}(x) = \frac{|x|}{x}$ *if* $x \neq 0$ *and* 0 *otherwise.*

Proof. Here we demonstrate that

$$\max\{CS_{m_1}, CS_{m_2}\} > \max\{CS_u, CS_r\} \Rightarrow \text{sgn}(q^r_{1r} - q^u_{1u}) \neq \text{sgn}(q^r_{2r} - q^u_{2u}).$$

We have $\max\{CS_{m_1}, CS_{m_2}\} > \max\{CS_u, CS_r\} \Leftrightarrow \max\{Q_{m_1}, Q_{m_2}\} > \max\{Q_u, Q_r\}$. Without loss of generality, let $Q_{m_1} = \max\{Q_{m_1}, Q_{m_2}\}$. Then

$$Q_{m_1} > Q_u \Rightarrow q^{m_1}_{2u} < q^u_{2u} \Rightarrow q^{m_1}_{1r} > q^u_{1u}, \tag{7a}$$

where the first implication follows from (3.2.2), and the second implication follows from noting that $(Q_{m_1} - Q_u) = (q^{m_1}_{1r} - q^u_{1u}) + (q^{m_1}_{2u} - q^u_{2u}) > 0$ and $q^{m_1}_{2u} - q^u_{2u} < 0$. Similarly, we find that

$$Q_{m_1} > Q_r \Rightarrow q^{m_1}_{1r} < q^r_{1r} \Rightarrow q^{m_1}_{2u} > q^r_{2r}. \tag{7b}$$

Collecting inequalities suitably from (7a) and (7b) and rearranging we get

$$q^r_{2r} < q^{m_1}_{2u} < q^u_{2u}, \text{ implying } q^r_{2r} - q^u_{2u} < 0,$$

and

$$q^r_{1r} > q^{m_1}_{1r} > q^u_{1u}, \text{ implying } q^r_{1r} - q^u_{1u} > 0.$$

Thus $\text{sgn}(q^r_{1r} - q^u_{1u}) \neq \text{sgn}(q^r_{2r} - q^u_{2u})$.

Similarly, we can show that $\min\{CS_{m_1}, CS_{m_2}\} < \min\{CS_u, CS_r\} \Rightarrow \text{sgn}(q^r_{1r} - q^u_{1u}) \neq \text{sgn}(q^r_{2r} - q^u_{2u})$. $\square$

The essence of Proposition 3.2 is as follows. Starting from public ownership, if both firms expand output with full-scale privatization, private duopoly yields the maximum *CS*. Instead, if both firms reduce output with full-scale privatization, public duopoly yields the maximum *CS*. Thus, a necessary condition for a mixed duopoly to deliver the highest *CS* is that one firm expands while the other firm contracts when the regime changes from public to private duopoly. This arises naturally when $Q_u = Q_r$ holds under cost heterogeneity. Except for the special case where $q^r_{1r} = q^u_{1u}$, $q^r_{1r} - q^u_{1u} > (<)0 \Leftrightarrow q^r_{2r} - q^u_{2u} < (>)0$. In other words, when $Q_u - Q_r = 0$ and firms have heterogeneous costs, the sign condition in Proposition 3.2 always holds.

Below we show that, indeed, in a class of environments, a mixed regime tops the *CS* ranking even for arbitrarily small levels of heterogeneity when $Q_u - Q_r = 0$. When $Q_u - Q_r \neq 0$, a mixed regime continues to occupy the top spot in the *CS* ranking, for larger levels of heterogeneity (see Proposition 3.3 for a precise statement). Note, while heterogeneity plays a key role, neither the arguments presented above nor the proofs of Propositions 3.1 and 3.2 use the assumption that firm 1 is less efficient than firm 2. Hereafter, we use this assumption and add some more structure to the model for a sharper characterization. Apart from characterization, this unambiguous labeling—i.e., firm 1 is the inefficient firm and firm 2 is the efficient firm—helps us to relate our findings to the debate on selection of firms for privatization, an issue which we turn to at the end of this section.

3.3 Model: a brief description

There are two firms, firm 1 and firm 2, which can be public or private. For a firm i of type s, cost of producing q_{is} units is

$$C_{is}(q_{is}) = c_{is}q_{is} + \frac{\gamma q_{is}^2}{2}, \tag{3.3.1}$$

where $i = 1, 2$ and $s = u(r)$ when firm i is public (private). We assume $c_{ir} = c_i$ and $c_{iu} = c_i + \Delta$. As in Proposition 2.3, Δ captures the increase in efficiency due to privatization. We assume that γ is strictly positive and suitably large so that $q_{is} > 0$ for $i \in \{1, 2\}$ and $s \in \{u, r\}$.

To capture that firm 2 has innate cost advantage, we assume $c_2 < c_1$. We model the cost differences as mean preserving spread and assume $c_1 = c + h$ and $c_2 = c - h$ where $h \geq 0$.[2] The innate cost differential, $2h$, persists even when both firms are of the same type—public or private. Table 3.1 shows how c_{is} varies with the ownership structure and the identity of firms:

Type s / Firm i	Private (r)	Public (u)
Firm 1	$c+h$	$c+h+\Delta$
Firm 2	$c-h$	$c-h+\Delta$

Table 3.1: Cost heterogeneity (c_{is})

We interpret h as the degree of cost heterogeneity between the two firms and explore how the presence of h impacts the CS ranking.[3]

[2] We assume $h \geq 0$ for simplicity. It allows us to label firm 1 as the high-cost firm and firm 2 as the low-cost firm. If $h < 0$, the opposite is true. Focusing on positive values of h is without loss of generality.

[3] Note that we relax the assumption of identical costs in this chapter but maintain identical λ. Different public firms may have different objective functions. Kala (2019) finds that, in SOEs with earned autonomy, managers value things differently from the managers of SOEs without such autonomous status. One way to

3.4 Cournot equilibrium

With this added structure, the first-order condition in (3.2.1) modifies to

$$P(Q_z) + \lambda_{is} P'(Q_z) q_{is}^z - c_{is} - \gamma q_{is}^z = 0, \tag{3.4.1}$$

where c_{is} is as in Table 3.1 and $\lambda_{is} = 1(\lambda)$ when $s = r(u)$. Each firm i of type s equates its marginal benefit $P(Q_z) + \lambda_{is} P'(Q_z) q_{is}^z$ with its marginal cost $c_{is} + \gamma q_{is}^z$.

In the unique Cournot equilibrium of market regime $z \in \{u, r, m_1, m_2\}$, firm i of type s chooses

$$q_{is}^z = \frac{P(Q_z) - c_{is}}{-\lambda_{is} P'(Q_z) + \gamma},$$

where Q_z uniquely solves

$$Q_z = \frac{P(Q_z) - c_{is}}{-\lambda_{is} P'(Q_z) + \gamma} + \frac{P(Q_z) - c_{kt}}{-\lambda_{kt} P'(Q_z) + \gamma} \tag{3.4.2}$$

with $i, k \in \{1, 2\}$, $k \neq i$ and $s, t \in \{u, r\}$. Substituting appropriate values of λ_{is}, λ_{kt}, c_{is} and c_{kt} in (3.4.2) and rearranging, we get implicit equilibrium values of aggregate output in private and public duopolies respectively:

$$Q_r = \frac{2(P(Q_r) - c)}{-P'(Q_r) + \gamma}, \quad \text{and} \quad Q_u = \frac{2(P(Q_u) - c - \Delta)}{-\lambda P'(Q_u) + \gamma}.$$

Although Q_u and Q_r are independent of h, the same is not true for aggregate outputs in

capture such differences in managerial objectives is to introduce heterogeneity in λ. Here, we assume away such heterogeneity to ensure minimal departure from the symmetric setup, and more importantly to focus on a kind of heterogeneity that is present in all regimes including the purely private one.

mixed duopolies. We have

$$Q_{m_1} = \frac{P(Q_{m_1}) - (c+h)}{-P'(Q_{m_1}) + \gamma} + \frac{P(Q_{m_1}) - (c-h+\Delta)}{-\lambda P'(Q_{m_1}) + \gamma}, \text{ and}$$
$$Q_{m_2} = \frac{P(Q_{m_2}) - (c+h+\Delta)}{-\lambda P'(Q_{m_2}) + \gamma} + \frac{P(Q_{m_2}) - (c-h)}{-P'(Q_{m_2}) + \gamma},$$

where $Q_{m_1}(Q_{m_2})$ denotes the aggregate output of mixed regime with inefficient (efficient) firm $1(2)$ being privatized. Observe that Q_{m_1} and Q_{m_2} vary with h. Lemma 3.1 records how equilibrium aggregate outputs in different duopoly regimes respond to a change in the degree of heterogeneity.

Lemma 3.1. *Let Q_u, Q_r and Q_{m_i} respectively denote the equilibrium values of aggregate output in a public duopoly, private duopoly, and a mixed duopoly where firm i is private.*

(i) *Q_u and Q_r are independent of h.*

(ii) *Q_{m_1} is increasing in h, while Q_{m_2} is decreasing in h.*

(iii) *$Q_{m_1} > Q_{m_2}$ holds for all $h > 0$.*

Proof. See Appendix 3.7.1. □

Irrespective of the duopoly regime, an increase in h lowers inefficient firm 1's output and raises efficient firm 2's output. They cancel each other in both private and public duopolies. In an environment with constant marginal costs and purely profit-maximizing firms, Bergstrom and Varian (1985) have shown that aggregate output in Cournot equilibrium does not vary with a change in unit costs as long as the sum of unit costs remains unchanged. Part (i) of Lemma 3.1 shows that this invariance result can hold in our framework too even though marginal costs are not constant and firms are not necessarily profit-maximizers.

Although firms 1 and 2 do not maximize profits in a public duopoly, they have the same objective. Part (ii) of Lemma 3.1 says that the invariance result does not hold for mixed duopolies where public and private firms have different objective functions. A profit-maximizing private firm i equates marginal cost with its marginal revenue $MR_i(= P(.) + P'(.)q_i)$. On the other hand, a public firm i equates marginal cost with its perceived marginal benefit, $MB_i \equiv P(.) + \lambda P'(.)q_i = MR_i - (1-\lambda)P'(.)q_i$. As q_i increases, both MB_i and MR_i decrease but MB_i decreases less since an increase in CS offsets part of the decline in marginal revenue. Similarly, when q_i decreases, both MB_i and MR_i increase but MB_i increases less. As MB_i changes less than MR_i, a change in marginal cost (via change in h) generates a more aggressive response from a public firm than from a private one. When firm 1 is private, its output reduction (prompted by an increase in h) is more than offset by the increased output of public firm 2 implying that Q_{m_1} is increasing in h. On the other hand, when firm 1 is public, its output reduction outweighs any increase in firm 2's output implying that Q_{m_2} is decreasing in h.

Part (iii) of Lemma 3.1, i.e., $Q_{m_1} > Q_{m_2}$ holds for all $h > 0$, follows from noting that $Q_{m_1} = Q_{m_2}$ when $h = 0$, and $Q_{m_1}(Q_{m_2})$ is strictly increasing (decreasing) in h. As Q is a sufficient statistic for CS, $Q_{m_1} > Q_{m_2} \Rightarrow CS_{m_1} > CS_{m_2}$, implying that if a government privatizes only one of the two firms, privatizing the inefficient firm is better from a consumer surplus perspective. It is also better than at least one of the two—public duopoly or private duopoly—as Proposition 2.5 (i) says that $\max\{CS_{m_1}, CS_{m_2}\}$, which is CS_{m_1} in this case, is strictly greater than $\min\{CS_u, CS_r\}$. Thus, mixed duopoly of type 1—where only inefficient firm 1 is private—beats at least two other regimes in the CS ranking. Can it beat all three and secure the top spot in CS ranking?

3.5 Consumer surplus

In the presence of cost heterogeneity, it is indeed possible that CS is maximum in a mixed duopoly of type 1, and minimum in a mixed duopoly of type 2. The realization of these possibilities depends, among other things, on the magnitude of efficiency gains from privatization (Δ) and the degree of heterogeneity (h). Lemma 3.1 sheds light on how a change in h impacts consumer surplus differently in different duopoly regimes. Lemma 3.2 identifies a critical value of Δ, which, along with h, helps in determining the range of parameterizations for which the two mixed regimes are at the two ends of CS ranking.

Lemma 3.2. *Suppose $h = 0$. There exists a unique $\hat{\Delta} > 0$ such that*

(i) $CS_u \gtreqless CS_r \Leftrightarrow \Delta \lesseqgtr \hat{\Delta}$, *and*

(ii) $CS_u = CS_{m_1} = CS_{m_2} = CS_r$ *holds when* $\Delta = \hat{\Delta}$,

where CS_u, CS_r and CS_{m_i} are as defined in Proposition 3.1.

Proof. See Appendix 3.7.2. □

Although the lemma is stated in terms of CS, we continue the discussion in terms of aggregate output (Q) as Q is a sufficient statistic for CS. Welfare orientation encourages public firms to produce more than their private counterparts, whereas the relative inefficiency of public firms prompts them to produce less. Public duopoly yields higher aggregate output and consequently higher consumer surplus than private duopoly if and only if efficiency gains from privatization, Δ, is smaller than a threshold value $\hat{\Delta}$. When $\Delta = \hat{\Delta}$, public and private duopoly yield the same aggregate output irrespective of the level of heterogeneity.

Figure 3.1 illustrates the relationship between degree of heterogeneity (h) and aggregate output (Q_z) when $\Delta = \hat{\Delta}$. It reflects the findings in Lemmas 3.1 and 3.2. Lemma 3.2 implies that when $\Delta = \hat{\Delta}$, aggregate outputs in all four regimes are the same in the absence of heterogeneity. That is why $Q_u = Q_{m_2} = Q_{m_1} = Q_r$ at $h = 0$. Lemma 3.1 says that as h increases, Q_{m_1} increases, Q_{m_2} declines, but Q_u and Q_r remain the same. Combining the findings from these two lemmas, Figure 3.1 demonstrates how even an arbitrarily small degree of heterogeneity can place the two mixed regimes at two ends of output/consumer surplus ranking when $\Delta = \hat{\Delta}$.

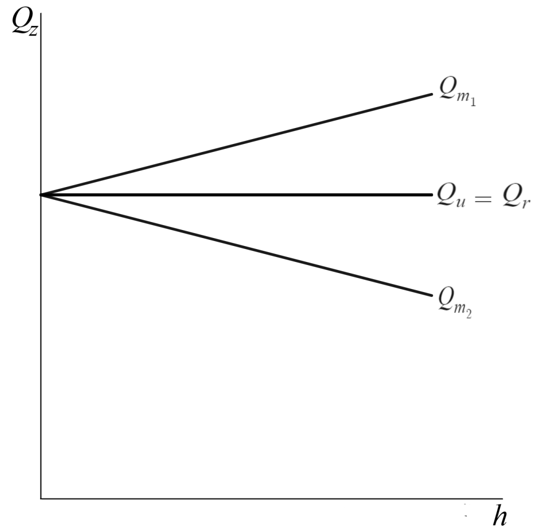
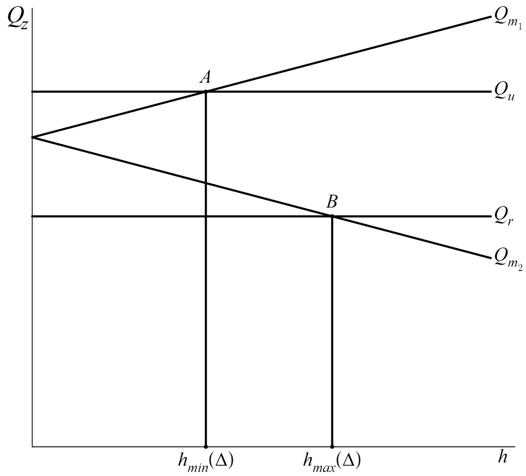


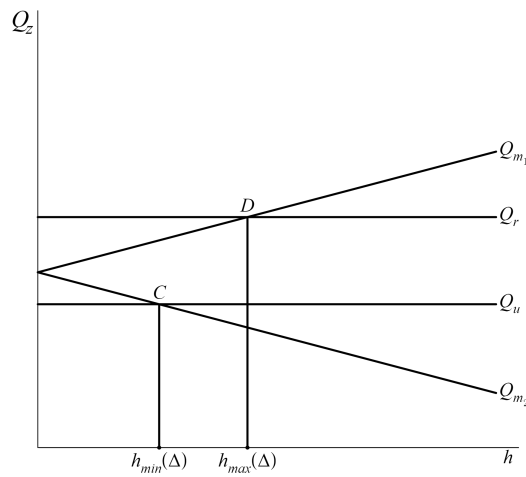
Figure 3.1: Heterogeneity and mixed markets: $\Delta = \hat{\Delta}$

Figure 3.2 illustrates that a mixed duopoly can escape the middle of CS ranking even when $\Delta \neq \hat{\Delta}$. Figure 3.2(a) considers $\Delta < \hat{\Delta}$ for which $Q_u > Q_r$ (by Lemma 3.2 (i)). Given how Q_z differs in terms of (a) their values at $h = 0$, and (b) their responses to a change in h, it follows that Q_{m_1} must exceed Q_u when h is higher than a threshold $h_{min}(\Delta)$ say, and similarly Q_{m_2} must fall below Q_r when h is higher than a threshold, $h_{max}(\Delta)$ say.[4] Similar thresholds also arise for $\Delta > \hat{\Delta}$—see Figure 3.2(b). Only substantive difference with $\Delta < \hat{\Delta}$ case is that $Q_r > Q_u$ instead of $Q_r < Q_u$.

[4]We consider parameter values such that market structure in all regimes is duopoly. That is, the innate cost differences between the two firms are not so large that the inefficient firm exits the market.



(a) $0 < \Delta < \hat{\Delta}$



(b) $\Delta > \hat{\Delta}$

Figure 3.2: Heterogeneity and mixed markets

The discussions above illustrate how the efficiency gains from privatization (Δ) and the level of cost heterogeneity (h) jointly determine the consumer surplus ranking of different regimes. Proposition 3.3 provides a more precise characterization.

Proposition 3.3. *When firms 1 and 2 have innate cost differences unrelated to ownership, a mixed duopoly of type 1(2)—where the inefficient (efficient) firm 1(2) is private—delivers the highest (lowest) consumer surplus for moderate efficiency gains from privatization (Δ) when the level of heterogeneity (h) is higher than a threshold. For low levels of heterogeneity, mixed duopolies continue to lie in the middle of consumer surplus ranking. More formally,*

(i) mixed duopoly of type 1(2)—where only the inefficient (efficient) firm is private—lies at the top (bottom) of consumer surplus ranking even for arbitrarily small level of heterogeneity when $\Delta = \hat{\Delta}$, where $\hat{\Delta}$ is as defined in Lemma 3.2;

(ii) there exist $\underline{\Delta}$ and $\overline{\Delta}$ satisfying $0 < \underline{\Delta} < \hat{\Delta} < \overline{\Delta}$ such that for each $\Delta \in (\underline{\Delta}, \overline{\Delta})$ we have a unique $h_{min}(\Delta)$ and $h_{max}(\Delta)$ such that

(a) $CS_{m_2} < \min\{CS_u, CS_r\} < \max\{CS_u, CS_r\} < CS_{m_1}$ holds for all $h > h_{max}(\Delta)$,

(b) $\min\{CS_u, CS_r\} \leq CS_{m_2} \leq CS_{m_1} \leq \max\{CS_u, CS_r\}$ *holds for all* $h \leq h_{min}(\Delta)$, *and*

both $h_{min}(\Delta)$ *and* $h_{max}(\Delta)$ *are increasing in* $|\Delta - \hat{\Delta}|$. *Furthermore,* $h_{min}(\hat{\Delta}) = h_{max}(\hat{\Delta}) = 0$.

Proof. See Appendix 3.7.3. □

Parts (i) and (ii) of Proposition 3.3 translate Figures 3.1 and 3.2 into words. For the first time in this thesis, we find that a mixed duopoly regime can secure the top spot in *CS* ranking. As part (i) of the proposition notes, this possibility appears most prominently when the efficiency gap between public and private firms, i.e., Δ, takes the value $\Delta = \hat{\Delta}$. Any degree of heterogeneity between firm 1 and firm 2, i.e., $h > 0$, no matter how small, puts mixed duopoly of type 1 at the top. The middle-ness of mixed regimes resurfaces when $\Delta \neq \hat{\Delta}$. However, as part (ii) highlights, the possibility of a mixed regime delivering the highest *CS* remains. All that is required is a suitably high level of heterogeneity. The requirement becomes more stringent, i.e., the threshold level of heterogeneity increases, as $|\Delta - \hat{\Delta}|$ increases.

Let us begin with $\Delta - \hat{\Delta} = 0$. Welfare orientation prompts firms to produce more in a public regime, while efficiency plays that role in a private regime. At $\Delta = \hat{\Delta}$ those concerns balance each other yielding $Q_u - Q_r = 0$. This is exactly when a mixed duopoly of type 1 fares the best. With one firm of each ownership type, a mixed duopoly blends welfare orientation (of a public firm) with cost efficiency (of a private firm). This blend works toward higher *CS* when the inefficient public firm is privatized while keeping the efficient firm public. When $|\Delta - \hat{\Delta}|$ increases from zero due to an increase in Δ, efficiency concerns become more salient. As a private duopoly has two efficient firms, while a mixed duopoly has only one, the possibility of a mixed regime topping the *CS* ranking becomes harder. When $|\Delta - \hat{\Delta}|$ increases from zero due to a decrease in Δ, welfare orientation becomes more salient in determining the top spot in the *CS* ranking. As a public duopoly has two welfare-oriented firms, while a mixed duopoly has only one, the possibility of a mixed regime topping the *CS* ranking becomes harder again. In both cases, a mixed regime continues to be at the top of *CS* ranking, but the threshold h—beyond which this holds—increases with an increase in $|\Delta - \hat{\Delta}|$.

The discussion above focuses on a mixed duopoly of type 1 as it delivers the highest *CS* among all regimes—a possibility that did not arise in previous chapter. Proposition 3.3 also records that a mixed duopoly of type 2—where only the low-cost firm is private—can deliver the lowest *CS*. We have shown in Section 2.3 that *CS* can be the lowest in a mixed regime even in the absence of heterogeneity when firms compete in prices. However, when firms compete in quantities, cost heterogeneity is necessary for a mixed regime to rank below all regimes in the *CS* ranking.

3.6 Conclusion

In this chapter, we address the first limitation of the model developed in Chapter 2. That is, not all public or private firms have same costs and firms can have innate cost differences within the same type. Under cost heterogeneity introduced within the same type of firms in this chapter, the number of possible mixed regimes increases. Despite this increase in the number of possible mixed regimes, the essence of Proposition 2.1 can be retained: not all mixed regimes can be strictly better (or worse) than both public and private regimes. However, in addition to this, we find that a mixed regime can deliver the highest consumer surplus. In a duopoly setting, a mixed regime where an inefficient public firm is privatized can be more beneficial to consumers than a purely public or a purely private regime. This result provides partial justification for the existence of mixed markets in various industries across developing as well as developed countries. On the other hand, privatizing the efficient public firm in a mixed regime can put the mixed regime at the bottom of the consumer surplus ranking.

Firm heterogeneity often distorts the true gains from privatization without careful consideration in designing privatization policies. For instance, the Chinese government's preference for "grasping the large and letting go of the small" implies that even if private firms are more productive than the SOEs, privatized firms might not appear equally productive due to negative selection (Chen, Igami, Sawada, and Xiao, 2021). On the other hand, gains from privatization might be overestimated if relatively efficient public firms are privatized first, as some of the gains are attributed to selection. The "best-foot-forward" strategy of privatizing the efficient firms first, often maximizes the proceeds from privatization when efficiencies and/or values of the assets held by the public firms are private information (Chakraborty, Gupta, and Harbaugh, 2006). Our static, full-information model is not well-suited to evaluate the informational argument associated with the sequencing of privatization. However, irrespective of whether we choose the ownership structure of firms all at once or sequentially—say, in a two-period version of our model—privatizing only the efficient firm is never the best option from consumers' perspective. The divergence in interests between consumers and other stakeholders might explain why privatization still generates passionate debate despite the broad acceptance of efficiency gains from privatization.

3.7 Appendix

3.7.1 Proof of Lemma 3.1

Proof. Expanding (3.4.2), we get

$$Q_u = \frac{P(Q_u)-(c+h)}{-\lambda P'(Q_u)+\gamma} + \frac{P(Q_u)-(c-h)}{-\lambda P'(Q_u)+\gamma}, \tag{3.7.1}$$

$$Q_r = \frac{P(Q_r)-(c+h-\Delta)}{-P'(Q_r)+\gamma} + \frac{P(Q_r)-(c-h-\Delta)}{-P'(Q_r)+\gamma}, \tag{3.7.2}$$

$$Q_{m_1} = \frac{P(Q_{m_1})-(c+h-\Delta)}{-P'(Q_{m_1})+\gamma} + \frac{P(Q_{m_1})-(c-h)}{-\lambda P'(Q_{m_1})+\gamma}, \tag{3.7.3}$$

$$Q_{m_2} = \frac{P(Q_{m_2})-(c+h)}{-\lambda P'(Q_{m_2})+\gamma} + \frac{P(Q_{m_2})-(c-h-\Delta)}{-P'(Q_{m_2})+\gamma}, \tag{3.7.4}$$

where Q_z denote the aggregate output in Cournot equilibrium in $z \in \{u, r, m_1, m_2\}$. We use these equations to prove different parts of Lemma 3.1.

(i) Rearranging (3.7.1) and (3.7.2) we get

$$Q_u = \frac{2(P(Q_u)-c)}{-\lambda P'(Q_u)+\gamma}, \quad \text{and} \quad Q_r = \frac{2(P(Q_r)-c+\Delta)}{-P'(Q_r)+\gamma}.$$

Observe that both Q_u and Q_r are independent of h.

(ii) Differentiating (3.7.3) and (3.7.4), and rearranging we get

$$\frac{dQ_{m_1}}{dh} = \frac{-P'(Q_{m_1})(1-\lambda)}{A_1}, \text{ and } \frac{dQ_{m_2}}{dh} = \frac{P'(Q_{m_2})(1-\lambda)}{A_2},$$

where

$$A_1 \equiv (-P'(Q_{m_1})+\gamma)(-\lambda P'(Q_{m_1})+\gamma)\left[1+\left(\frac{-P'(Q_{m_1})}{-P'(Q_{m_1})+\gamma}\right)(1+s_{1r}^{m_1}\eta)+\left(\frac{-P'(Q_{m_1})}{-\lambda P'(Q_{m_1})+\gamma}\right)(1+\lambda s_{2u}^{m_1}\eta)\right],$$

$$A_2 \equiv (-P^{'}(Q_{m_2})+\gamma)(-\lambda P^{'}(Q_{m_2})+\gamma)\left[1+\left(\frac{-P^{'}(Q_{m_2})}{-\lambda P^{'}(Q_{m_2})+\gamma}\right)(1+\lambda s_{1u}^{m_2}\eta)+\left(\frac{-P^{'}(Q_{m_2})}{-\lambda P^{'}(Q_{m_2})+\gamma}\right)(1+s_{2r}^{m_2}\right.$$

and $\eta = \frac{QP''(Q)}{P'(Q)}$, $s_{iu}^z = \frac{q_{iu}^z}{Q_z} \in [0,1]$ and $s_{ir}^z = \frac{q_{ir}^z}{Q_z} \in [0,1]$.

Both $A_1 > 0$ and $A_2 > 0$ as expressions within all parentheses are strictly positive owing to two features (i) $-P'(Q_z) > 0$ and (ii) $1+\eta > 0$ (by Assumption 1).[5] This implies

$$\frac{dQ_{m_1}}{dh} > 0 \text{ and } \frac{dQ_{m_2}}{dh} < 0.$$

(iii) When $h = 0$, (3.7.3) and (3.7.4) reduce to

$$Q_{m_i} = \frac{P(Q_{m_i})-(c-\Delta)}{-P'(Q_{m_i})+\gamma} + \frac{P(Q_{m_i})-c}{-\lambda P'(Q_{m_i})+\gamma}$$

where $i \in \{1,2\}$. As the solution to the above equation is unique, we get $Q_{m_1} = Q_{m_2}$. This equality together with $\frac{dQ_{m_1}}{dh} > 0$ and $\frac{dQ_{m_2}}{dh} < 0$ implies that $Q_{m_1} > Q_{m_2}$ holds for all $h > 0$. □

3.7.2 Proof of Lemma 3.2

Proof. (i) When $h = 0$, $c_{1r} = c_{2r} = c$ and $c_{1u} = c_{2u} = c+\Delta$. Existence and uniqueness of $\hat{\Delta}$ follows from Proposition 2.3 once we make the following substitutions: $n = 2$, $c_r = c$, $c_u = c+\Delta$, and $C(q) = \frac{\gamma q^2}{2}$.

(ii) From the proof of Lemma 2 (iii) we know that $Q_{m_1} = Q_{m_2}$ or equivalently $CS_{m_1} = CS_{m_2}$ when $h = 0$. Note, Proposition 2.1 is effectively the CS ranking when $h = 0$. Applying

[5]Naturally, when $\eta \geq 0$, all expressions of the form $1+s_{is}^z\eta$ or $1+\lambda s_{is}^z\eta$ are strictly positive. When $\eta \in (-1,0)$, the same conclusion holds as $1+s_{is}^z\eta \geq 1+\eta > 0$.

Proposition 2.1 here gives

$$\min\{CS_u, CS_r\} \leq CS_{m_1} = CS_{m_2} \leq \max\{CS_u, CS_r\}.$$

The claim then follows from noting that $\min\{CS_u, CS_r\} = \max\{CS_u, CS_r\}$ when $\Delta = \hat{\Delta}$. $\square$

3.7.3 Proof of Proposition 3.3

Proof. (i) Lemma 3.2 implies that

$$Q_u = Q_{m_1} = Q_{m_2} = Q_r$$

holds at $\Delta = \hat{\Delta}$ when $h = 0$. Keeping Δ fixed at $\hat{\Delta}$, as we increase h from 0, we find that Q_{m_1} increases, Q_{m_2} decreases, while Q_u and Q_r remain the same (Lemma 3.1). Thus,

$$Q_{m_1} > Q_u = Q_r > Q_{m_2},$$

or equivalently,

$$CS_{m_1} > CS_u = CS_r > CS_{m_2}.$$

(ii) We prove this part using four claims. Claims 3.1 and 3.2 focus on the case $\Delta < \hat{\Delta}$, whereas Claims 3.3 and 3.4 focus on the case $\Delta > \hat{\Delta}$. Claim 3.1 establishes the existence of an interval to the left of $\hat{\Delta}$, namely $(\underline{\Delta}, \hat{\Delta}]$, such that m_1 (m_2) regime delivers the highest (lowest) CS for all $\Delta \in (\underline{\Delta}, \hat{\Delta}]$ whenever h exceeds a threshold $h_{max}(\Delta)$. On the other hand, below a threshold $h_{min}(\Delta)$, mixed regimes lie in the middle of the CS ranking. Claim 3.2 establishes uniqueness and monotonicity of $h_{max}(\Delta)$ and $h_{min}(\Delta)$. Claims 3.3 and 3.4 are counterparts of Claims 3.1 and 3.2 respectively for $\Delta > \hat{\Delta}$. Below we state Claims 3.1 - 3.4 precisely but we omit detailed proofs for Claims 3.3 and 3.4 as they are analogous to those of Claims 3.1 and 3.2.

Claim 3.1. *There exists* $\underline{\Delta}$ *satisfying* $0 < \underline{\Delta} < \hat{\Delta}$ *such that for each* $\Delta \in (\underline{\Delta}, \hat{\Delta}]$ *we have* $h_{min}(\Delta)$ *and* $h_{max}(\Delta)$ *such that*

$$CS_{m_2} < \min\{CS_u, CS_r\} < \max\{CS_u, CS_r\} < CS_{m_1} \;\; \textit{holds for all} \;\; h > h_{max}(\Delta),$$

$$\min\{CS_u, CS_r\} \leq CS_{m_2} \leq CS_{m_1} \leq \max\{CS_u, CS_r\} \;\; \textit{holds for all} \;\; h \leq h_{min}(\Delta).$$

Furthermore, $h_{min}(\hat{\Delta}) = h_{max}(\hat{\Delta}) = 0$.

Proof. Let $Q_z(\Delta, h)$ denote the aggregate output in Cournot equilibrium of regime z for given (Δ, h).[6] We drop h from the arguments of $Q_u(.)$ and $Q_r(.)$ since they are independent of h (Lemma 3.1). Define

$$F(\Delta, h) = Q_u(\Delta) - Q_{m_1}(\Delta, h), \quad \text{and} \quad G(\Delta, h) = Q_r(\Delta) - Q_{m_2}(\Delta, h)$$

on an open ball around $(\hat{\Delta}, 0)$. Both $F(\Delta, h)$ and $G(\Delta, h)$ are C^1 since $Q_z(.)$ is C^1 for all $z \in \{u, r, m_1, m_2\}$.

Using Lemmas 3.1 and 3.2, we get

$$F(\hat{\Delta}, 0) = 0 = G(\hat{\Delta}, 0), \;\; \frac{\partial F}{\partial h}(\hat{\Delta}, 0) = -\frac{\partial Q_{m_1}}{\partial h}(\hat{\Delta}, 0) \neq 0, \text{ and } \frac{\partial G}{\partial h}(\hat{\Delta}, 0) = -\frac{\partial Q_{m_2}}{\partial h}(\hat{\Delta}, 0) \neq 0.$$

Applying Implicit Function Theorem, we find that there exists C^1 function $h_i(\Delta)$ defined on an open interval $(\hat{\Delta} - \varepsilon_i, \hat{\Delta} + \varepsilon_i)$ where $\varepsilon_i > 0$, $i = 1, 2$, such that

(a) $F(\Delta, h_1(\Delta)) = 0$, for all Δ in $(\hat{\Delta} - \varepsilon_1, \hat{\Delta} + \varepsilon_1)$, $h_1(\hat{\Delta}) = 0$, and $\frac{dh_1}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial F}{\partial \Delta}(\hat{\Delta}, 0)}{\frac{\partial F}{\partial h}(\hat{\Delta}, 0)}$,

(b) $G(\Delta, h_2(\Delta)) = 0$, for all Δ in $(\hat{\Delta} - \varepsilon_2, \hat{\Delta} + \varepsilon_2)$, $h_2(\hat{\Delta}) = 0$, and $\frac{dh_2}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial G}{\partial \Delta}(\hat{\Delta}, 0)}{\frac{\partial G}{\partial h}(\hat{\Delta}, 0)}$.

[6] We allow h to take both positive and negative values in the proof as it facilitates the use of Implicit Function Theorem. It is easy to check that equation (3.4.2) and $Q_z(\Delta, h)$—which is the implicit solution to the equation—is well defined for both positive and negative values of h.

Define $\underline{\Delta}$ as $\underline{\Delta} = \hat{\Delta} - \min\{\varepsilon_1, \varepsilon_2\}$. Both $h_1(\Delta)$ and $h_2(\Delta)$ exist in the interval $(\underline{\Delta}, \hat{\Delta}]$. Furthermore, for any $\Delta \in (\underline{\Delta}, \hat{\Delta}]$, define $h_{\max}(\Delta)$ and $h_{\min}(\Delta)$ as

$$h_{\max}(\Delta) = \max\{h_1(\Delta), h_2(\Delta)\}, \text{ and } h_{\min}(\Delta) = \min\{h_1(\Delta), h_2(\Delta)\}.$$

From Lemma 3.1 and the definition of $h_i(\Delta)$, it follows that

$$h \gtreqless h_1(\Delta) \Leftrightarrow Q_u(\Delta) - Q_{m_1}(\Delta, h) \lesseqgtr 0 \text{ and } h \gtreqless h_2(\Delta) \Leftrightarrow Q_r(\Delta) - Q_{m_2}(\Delta, h) \gtreqless 0. \quad (3.7.1)$$

Using (3.7.1) and the fact that $Q_u(\Delta) > Q_r(\Delta)$ for $\Delta \in (0, \hat{\Delta})$, we find that

$$h > h_{\max}(\Delta) \Leftrightarrow Q_{m_2}(\Delta, h) < Q_r(\Delta) < Q_u(\Delta) < Q_{m_1}(\Delta, h) \quad (3.7.2)$$

holds in the interval $(\underline{\Delta}, \hat{\Delta}]$. Similarly, using (3.7.1) and the fact that $Q_{m_1}(\Delta, h) \geq Q_{m_2}(\Delta, h)$ for all $h \geq 0$ (from Lemma 3.1), we find that

$$h \leq h_{\min}(\Delta) \Leftrightarrow Q_r(\Delta) \leq Q_{m_2}(\Delta, h) \leq Q_{m_1}(\Delta, h) \leq Q_u(\Delta) \quad (3.7.3)$$

holds for all $\Delta \in (\underline{\Delta}, \hat{\Delta}]$. The first part of Claim 3.1 follows from combining (3.7.2) and (3.7.3), and noting that $Q_z \geq Q_{z'} \Leftrightarrow CS_z \geq CS_{z'}$ for any two regimes $z, z' \in \{u, r, m_1, m_2\}$. The second part of the claim, i.e., $h_{max}(\hat{\Delta}) = h_{min}(\hat{\Delta}) = 0$ follows from noting that $h_i(\hat{\Delta}) = 0$ for $i = 1, 2$—as stated in (a) and (b) in earlier part of the proof. □

Claim 3.2. *Both $h_{min}(\Delta)$ and $h_{max}(\Delta)$ are (i) unique in the interval $(\underline{\Delta}, \hat{\Delta}]$, and (ii) increasing in $|\Delta - \hat{\Delta}|$.*

Proof. (i) Recall, $h_{min}(\Delta) = \min\{h_1(\Delta), h_2(\Delta)\}$ and $h_{max}(\Delta) = \max\{h_1(\Delta), h_2(\Delta)\}$. To prove uniqueness of $h_{\min}$ and $h_{\max}$, it suffices to show that $h_1(\Delta)$ and $h_2(\Delta)$ are unique.

We do that by showing that $h_1(\Delta)$ and $h_2(\Delta)$ changes monotonically with Δ in the interval $(\underline{\Delta}, \hat{\Delta}]$,

In the proof of Claim 3.1, we established that

$$\frac{dh_1}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial F}{\partial \Delta}(\hat{\Delta},0)}{\frac{\partial F}{\partial h}(\hat{\Delta},0)}, \text{ and } \frac{dh_2}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial G}{\partial \Delta}(\hat{\Delta},0)}{\frac{\partial G}{\partial h}(\hat{\Delta},0)}$$

hold for all $\Delta \in (\hat{\Delta} - \varepsilon_i, \hat{\Delta} + \varepsilon_i)$, $i \in \{1,2\}$. We have

$$\frac{\partial F}{\partial h}(\hat{\Delta},0) = -\frac{\partial Q_{m_1}}{\partial h}(\hat{\Delta},0) < 0, \text{ and } \frac{\partial G}{\partial h}(\hat{\Delta},0) = -\frac{\partial Q_{m_2}}{\partial h}(\hat{\Delta},0) > 0.$$

where both inequalities follow from applying Lemma 3.1 (ii). Then

$$\operatorname{sgn}\frac{dh_1}{d\Delta}(\hat{\Delta}) = \operatorname{sgn}\frac{\partial F}{\partial \Delta}(\hat{\Delta},0) = \operatorname{sgn}\left(\frac{\partial Q_u}{\partial \Delta}(\hat{\Delta}) - \frac{\partial Q_{m_1}}{\partial \Delta}(\hat{\Delta},0)\right), \tag{3.7.4}$$

$$\operatorname{sgn}\frac{dh_2}{d\Delta}(\hat{\Delta}) = -\operatorname{sgn}\frac{\partial G}{\partial \Delta}(\hat{\Delta},0) = -\operatorname{sgn}\left(\frac{\partial Q_r}{\partial \Delta}(\hat{\Delta}) - \frac{\partial Q_{m_2}}{\partial \Delta}(\hat{\Delta},0)\right). \tag{3.7.5}$$

When Δ increases, the marginal cost of production increases for public firms. As a consequence, aggregate output declines in all regimes that involve public firms. Aggregate output in a private duopoly does not change. Thus, $\frac{\partial Q_z}{\partial \Delta}(\hat{\Delta}) < 0 \ \ \forall z \in \{u, m_1, m_2\}$ and $\frac{\partial Q_r}{\partial \Delta}(\hat{\Delta}) = 0$ which implies

$$\frac{\partial Q_r}{\partial \Delta}(\hat{\Delta}) - \frac{\partial Q_{m_2}}{\partial \Delta}(\hat{\Delta}) > 0.$$

Then, from (3.7.5) it follows that

$$\frac{dh_2}{d\Delta}(\hat{\Delta}) < 0.$$

Differentiating (3.7.1) and (3.7.3) respectively and rearranging we get

$$\frac{\partial Q_u}{\partial \Delta}(\hat{\Delta}) = -\frac{\frac{2}{-\lambda P^{'}(Q_u)+\gamma}}{1+\frac{-P^{'}(Q_u)}{-\lambda P^{'}(Q_u)+\gamma}(2+\lambda\eta)},$$

$$\frac{\partial Q_{m_1}}{\partial \Delta}(\hat{\Delta}) = -\frac{\frac{1}{-\lambda P^{'}(Q_{m_1})+\gamma}}{1+\frac{-P^{'}(Q_{m_1})}{-P^{'}(Q_{m_1})+\gamma}(1+s_{1r}^{m_1}\eta)+\frac{-P^{'}(Q_{m_1})}{-\lambda P^{'}(Q_{m_1})+\gamma}(1+\lambda s_{2u}^{m_1}\eta)},$$

where η and s_{is}^{z} are as defined in Section 3.7.1. Subtracting $\frac{\partial Q_{m_1}}{\partial \Delta}(\hat{\Delta})$ from $\frac{\partial Q_u}{\partial \Delta}(\hat{\Delta})$, rearranging, and simplifying subsequently we find that

$$\begin{aligned} &\operatorname{sgn}\left(\frac{\partial Q_u}{\partial \Delta}(\hat{\Delta}) - \frac{\partial Q_{m_1}}{\partial \Delta}(\hat{\Delta},0)\right) \\ &= -\operatorname{sgn}\left[1+2(1+s_{1r}^{m_1}\eta)\left(\frac{-P^{'}(Q_u)(1-\lambda)\gamma}{(-P^{'}(Q_u)+\gamma)(-\lambda P^{'}(Q_u)+\gamma)}\right)+(2+\eta)\left(\frac{-\lambda P^{'}(Q_u)}{-\lambda P^{'}(Q_u)+\gamma}\right)\right]. \end{aligned}$$

Assumption 1 implies $1+\eta > 0$ which in turn implies $(1+s_{1r}^{m_1}\eta) > 0$ as well as $(2+\eta) > 0$. We have $-P'(Q_z) > 0$, implying $-\lambda P^{'}(Q_u)+\gamma > 0$ and $-P^{'}(Q_u)+\gamma > 0$. Thus, expressions within all parentheses, i.e., (.), are strictly positive. Consequently, the entire expression within the bracket, i.e., [.], is strictly positive. Thus, we have $\frac{\partial Q_u}{\partial \Delta}(\hat{\Delta}) - \frac{\partial Q_{m_1}}{\partial \Delta}(\hat{\Delta},0) < 0$. Then, from (3.7.4), it follows that

$$\frac{dh_1}{d\Delta}(\hat{\Delta}) < 0.$$

(ii) Since $h_i(\Delta)$ is decreasing in Δ in the interval $\Delta \in (\hat{\Delta}-\varepsilon_i, \hat{\Delta}+\varepsilon_i)$, $i = 1,2$, it is also decreasing in Δ in the sub-interval $(\underline{\Delta}, \hat{\Delta}]$. From Lemmas 3.1 and 3.2, we have $\hat{\Delta}$ is independent of h. It follows that $h_1(\Delta)$ and $h_2(\Delta)$ are increasing in $|\Delta - \hat{\Delta}|$.[7] As $h_1(\Delta)$ and $h_2(\Delta)$ are increasing in $|\Delta - \hat{\Delta}|$, we have $h_{max} = \max\{h_1(\Delta), h_2(\Delta)\}$ and $h_{min} = \min\{h_1(\Delta), h_2(\Delta)\}$ also increasing in $|\Delta - \hat{\Delta}|$ for $\Delta \in (\underline{\Delta}, \hat{\Delta}]$. □

Claim 3.3. *There exists $\overline{\Delta}$ satisfying $0 < \hat{\Delta} < \overline{\Delta}$ such that for each $\Delta \in [\hat{\Delta}, \overline{\Delta})$ we have*

[7]For $\Delta \in (\underline{\Delta}, \hat{\Delta}]$, $|\Delta - \hat{\Delta}| = \hat{\Delta} - \Delta$. Since $\frac{dh_i}{d\Delta}(\hat{\Delta}) < 0$, we have $\frac{d|\Delta-\hat{\Delta}|}{dh_i} = \frac{d(\hat{\Delta}-\Delta)}{dh_i} = -\frac{d\Delta}{dh_i} > 0$, $i = 1,2$.

$h_{min}(\Delta)$ and $h_{max}(\Delta)$ such that

$$CS_{m_2} < \min\{CS_u, CS_r\} < \max\{CS_u, CS_r\} < CS_{m_1} \ \ holds \ \ for\ all \ \ h > h_{max}(\Delta),$$
$$\min\{CS_u, CS_r\} \leq CS_{m_2} \leq CS_{m_1} \leq \max\{CS_u, CS_r\} \ \ holds \ \ for\ all \ \ h \leq h_{min}(\Delta).$$

Furthermore, $h_{min}(\hat{\Delta}) = h_{max}(\hat{\Delta}) = 0$.

Proof. Define

$$K(\Delta, h) = Q_r(\Delta) - Q_{m_1}(\Delta, h), \text{ and } L(\Delta, h) = Q_u(\Delta) - Q_{m_2}(\Delta, h)$$

on an open ball around $(\hat{\Delta}, 0)$. Both $K(\Delta, h)$ and $L(\Delta, h)$ are C^1 since $Q_z(.)$ is C^1 for all $z \in \{u, r, m_1, m_2\}$.

Similar to the proof of Claim 3.1, using Lemma 3.1 and Lemma 3.2, we find that the conditions for applying the Implicit Function Theorem are satisfied. This in turn implies that there exists C^1 function $h_j(\Delta)$ defined on an open interval $(\hat{\Delta} - \varepsilon_j, \hat{\Delta} + \varepsilon_j)$ where $\varepsilon_j > 0$ $(j = 3, 4)$ such that

(a) $K(\Delta, h_3(\Delta)) = 0$, for all Δ in $(\hat{\Delta} - \varepsilon_3, \hat{\Delta} + \varepsilon_3)$, $h_3(\hat{\Delta}) = 0$, and $\frac{dh_3}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial K}{\partial \Delta}(\hat{\Delta},0)}{\frac{\partial K}{\partial h}(\hat{\Delta},0)}$,

(b) $L(\Delta, h_4(\Delta)) = 0$, for all Δ in $(\hat{\Delta} - \varepsilon_4, \hat{\Delta} + \varepsilon_4)$, $h_4(\hat{\Delta}) = 0$, and $\frac{dh_4}{d\Delta}(\hat{\Delta}) = -\frac{\frac{\partial L}{\partial \Delta}(\hat{\Delta},0)}{\frac{\partial L}{\partial h}(\hat{\Delta},0)}$.

Define $\overline{\Delta}$ as $\overline{\Delta} = \hat{\Delta} + \min\{\varepsilon_3, \varepsilon_4\}$. Both $h_3(\Delta)$ and $h_4(\Delta)$ exist in the interval $[\hat{\Delta}, \overline{\Delta})$. Moreover, for any $\Delta \in [\hat{\Delta}, \overline{\Delta})$, let us define $h_{\max}(\Delta)$ and $h_{\min}(\Delta)$ as

$$h_{\max}(\Delta) = \max\{h_3(\Delta), h_4(\Delta)\}, \ \text{ and } \ h_{\min}(\Delta) = \min\{h_3(\Delta), h_4(\Delta)\}.$$

The remainder of the proof is analogous to that of Claim 3.1 and hence omitted here. □

Claim 3.4. *$h_{min}(\Delta)$ and $h_{max}(\Delta)$ are unique in the interval $[\hat{\Delta}, \overline{\Delta})$. Both $h_{min}(\Delta)$ and $h_{max}(\Delta)$ are also increasing in $|\Delta - \hat{\Delta}|$.*

Proof. Analogous to the proof of Claim 3.2 and hence omitted here. $\square$

Observe that $\underline{\Delta} < \overline{\Delta}$ for all $\varepsilon_i, \varepsilon_j > 0$, $i = 1, 2$, and $j = 3, 4$. Combining the results from Claims 3.1, 3.2, 3.3 and 3.4, it follows that Proposition 3.3(ii) holds in the open interval $(\underline{\Delta}, \overline{\Delta})$ where $0 < \underline{\Delta} < \hat{\Delta} < \overline{\Delta}$. $\square$

Chapter 4

Mixed Markets, Privatization and Consumer Welfare: Vertically Related Markets

4.1 Introduction

Despite the difference in themes across Chapter 2 and Chapter 3—namely, homogeneous products Cournot (2.2), differentiated product Bertrand (2.3), and cost heterogeneity (3.3)—a common feature is that both public and private firms operate in the same sector, and more specifically in downstream, consumer-facing industries producing final goods that are perfect or imperfect substitutes. However, the production of final goods requires intermediate inputs which are produced in upstream industries. In this chapter we explore the desirability of mixed markets when firms operate in distinct but vertically related sectors and produce complementary products.

Around half (in value terms) of all the SOEs in OECD countries are in sectors such as mining, manufacturing, and other energy industries—most of which can be classified as upstream industries (Christiansen, 2011). In China, despite significant privatization in downstream industries (Chen et al., 2021), large SOEs (e.g., Shenhua Group Corporation Limited, China Baowu Steel Group Corporation Limited, China National Chemical Corporation) dominate the upstream sectors producing critical intermediate inputs. Similarly, in India, some of the largest public enterprises (e.g., Coal India Limited, Steel Authority of India Limited) are in the upstream sector.

The distinction between upstream and downstream is not critical for our analysis when one of the sectors is perfectly competitive. In such environments, a single-stage oligopoly model is adequate for capturing the main insights. However, in environments where both downstream and upstream producers have market power, we need an explicit model to capture vertical relationships. Comparing different regimes with multiple oligopolistic firms in both upstream and downstream sectors of production requires a vertical oligopoly framework (see, for example, Salinger, 1988; Ghosh and Morita, 2007). We capture vertical relationships in a two-tier, successive monopoly model where each tier consists of only one firm which could be private or public. In a private (public) regime, private (public) firms operate in both tiers, while in a mixed regime, a private firm operates in only one of the two tiers. How does a mixed regime fare vis-à-vis the other two in the consumer surplus ranking? That is the key question we address in this chapter. We abstract away from vertical oligopoly not only to keep the analysis simple but also to distinguish vertical relationships from mode of competition, and heterogeneity—issues that can be analyzed without invoking a vertical structure.

Upstream firms produce and sell intermediate goods to downstream firms, which, in turn, produce and sell final goods to consumers. The presence of public firms in both sectors implies two layers of inefficiency whereas the presence of private firms in both sectors implies two rounds of markups. Compared to a fully public regime, production efficiency is greater in a mixed regime because one of the two sectors involves a private firm which is more efficient than its public counterpart. Compared to a fully private regime, the scope of exercising market power is limited in a mixed regime, as a public firm does not charge high markup. A mixed regime can deliver the highest consumer surplus by removing one layer of inefficiency and one round of markup. The possibility of a mixed regime securing the top spot in consumer surplus ranking arises when the markup is moderate, but this alone is not enough. If the degree of a public firm's inefficiency is similar in the upstream and downstream sectors of production, mixed regimes either lie in the middle (as shown in Section 2.2) or even secure the bottom spot in the *consumer surplus* ranking (as shown in Section 2.3). Sufficient dissimilarity in the degree of inefficiency in the two sectors, combined with moderate markups, can generate the highest consumer surplus under mixed regimes.

We enrich the model further by introducing competition in both sectors. In particular, we allow arbitrary number of firms in each sector which in turn enables differences in downstream and upstream markups. When competition is weak in both sectors, markups are high, and the public regime is favored. By contrast, when competition is strong, markups are lower, and the private regime becomes more attractive. Mixed regimes emerge as optimal when competition is strong in one sector but weak in the other. A robust finding in the successive monopoly set up is that privatizing the relatively less inefficient public sector—where the public-private efficiency gap is low—is never optimal. For example, if the public-private efficiency gap is (relatively) smaller in the upstream sector, privatizing the upstream sector alone never maximizes consumer surplus in a successive monopoly. However, once we allow for competition in both sectors, and the upstream sector is more competitive than the downstream sector, privatizing the upstream sector alone might be best for consumers.

Extending the literature on mixed oligopolies presented in Chapter 2 and 3 to vertically related industries, Matsumura and Matsushima (2012) provide one of the earliest analyses of mixed markets in a vertical structure. More recent contributions include Chang and Ryu (2016), Wu, Chang, and Chen (2016), Heywood, Wang, and Ye (2021) and Liu, Wang, and Zeng (2022), among others. These studies examine privatization in either the upstream or the downstream sector and adopt welfare as the relevant objective. With the exception of Heywood et al. (2021), all incorporate foreign firms in at least one stage of production. While this feature enriches the strategic environment, it also complicates the welfare analysis, as cross-border profit shifting introduces an additional channel affecting national welfare.

Our contribution differs from this literature by focusing on consumer surplus, allowing for both privatization and nationalization, and embedding ownership choice in a vertical market structure with market power at multiple stages. We must emphasize that the perspective adopted in this chapter represents one of several approaches to studying public ownership and privatization. We do not model privatization as a problem of optimal regulation, vertical contracting, or integration. Rather, taking market structure as given, we focus on how cost inefficiencies—often associated with public ownership—together with ownership-specific objectives, such as profit maximization versus welfare orientation, interact with vertical market power to shape consumer surplus outcomes.

The chapter is organized as follows. Section 4.2 provides a brief description of the model and derives the equilibrium prices. The consumer surplus across regimes are compared and characterized in Section 4.3. Section 4.5 concludes. Detailed proofs are developed in the Appendix.

4.2 Model

Consider two vertically related sectors of production: upstream and downstream. In the upstream sector, a homogeneous intermediate good is produced with a constant marginal cost $c > 0$. In the downstream sector, one unit of intermediate product is transformed into one unit of homogeneous final product with a constant marginal cost $d \geq 0$. Assume that

$c = c_u(c_r)$ for a public (private) upstream firm and $d = d_u(d_r)$ for a public (private) downstream firm. We set $d_r = 0$, and assume $d_u = d > 0$ and $c_u > c_r$ reflecting that public firms are relatively inefficient compared to their private counterparts. Furthermore, to reduce the number of cases, we assume that the relative inefficiency of public firms is higher in the downstream sector compared to the upstream sector, i.e.,

$$d_u - d_r \geq c_u - c_r \equiv \Delta \Rightarrow d \geq \Delta.$$

Our results remain qualitatively the same if we assume the opposite, i.e., $d < \Delta$.[1] For simplicity, we work with

$$Q = P^{-e} \tag{4.2.1}$$

as the demand function where $e > 1$ is the constant elasticity of demand.[2]

We consider a two-stage game. Upstream monopolist sets the input price w in stage 1. Observing that, the downstream monopolist sets price P in stage 2 to maximize its objective function. Irrespective of the sector in which it operates, a private firm maximizes profits, while a public one, for simplicity, is assumed to maximize welfare subject to a non-negative profit constraint.[3] We use the subscript $u(r)$ for public (private) regime where both upstream and downstream firms are public (private). In a mixed regime, only one of the two firms is public. We refer to a regime as a mixed regime of type 1(2), and use the subscript $m_1(m_2)$, if only the downstream (upstream) firm is private.

[1] We chose $d \geq \Delta \equiv c_u - c_r$ as there is some evidence that the productivity gap between private and public firms is larger in consumer-facing industries (Chen et al., 2021).

[2] One advantage of working with downstream monopoly is that it allows us to bypass the question regarding choice of strategic variables (i.e., price or quantity) and focus on the issue at hand: vertical relationships. Constant elasticity demand function simplifies the algebra, significantly.

[3] In previous chapters we assumed that a public firm maximizes a weighted average of welfare and its own profits. Here, allowing for arbitrary weights generates negative profits for public firms. In reality, public firms do incur losses. However, for simplicity, we restrict public firms' profits to be non-negative. Importantly, this assumption is disadvantageous for us as it makes mixed regimes less likely to deliver the highest *consumer surplus*. An alternative would have been to consider the weighted average and restrict the value of λ. However, that restriction would depend on other parameter values, and also on the regime under consideration. Maximizing welfare subject to non-negative profit constraint bypasses these complexities.

Private regime

Private regime refers to the setting where both upstream and downstream firms are profit-maximizing monopolists. The downstream monopolist chooses P to maximize profit, $(P-w)Q=(P-w)P^{-e}$. Profit-maximizing price is

$$P(w)=\frac{w}{1-\frac{1}{e}}=\mu w,$$

where $\mu=\frac{1}{1-\frac{1}{e}}$ denotes the markup.

Substituting $P=P(w)$ in (4.2.1) and noting the one-to-one relationship between input and the final good, we find that demand for intermediate product is $Q(w)=(P(w))^{-e}=(\mu w)^{-e}$. Upstream profit is $(w-c)Q(w)=(w-c_r)(\mu w)^{-e}$, which is maximized at $w=\mu c_r$. Substituting $w=\mu c_r$ in $P(w)$ above yields the equilibrium price of the final good:

$$P_r=\mu^2 c_r. \tag{4.2.2}$$

Equation (4.2.2) captures the essence of a double marginalization problem. If the input were to be produced internally, a monopolist would have applied a markup of μ over c_r and set the price of the final good at μc_r. However, as the input is purchased from an upstream monopolist which applies a monopoly markup of μ over c_r, the equilibrium price of the final good (i.e., $P_r=\mu^2 c_r$) reflects two rounds of markups, imposed by two monopolists—one in each sector.

Mixed regimes

Mixed regimes refer to settings where both public and private firms are present but in different sectors. Suppose the downstream monopolist is a private, profit-maximizing firm. As in the private regime, facing an input price w, the monopolist will set

$$P(w)=\mu w$$

as the price of the final good and demand $Q(w) = (\mu w)^{-e}$ units of intermediate input.

An upstream public firm chooses w to maximize welfare, $\int_0^{Q(w)} P(y)dy - c_u Q(w)$, subject to non-negative profits constraint in the upstream sector, i.e., $w - c_u \geq 0$. In the absence of constraint, the welfare-maximizing value of w is given by $\hat{w}$ which solves $P(\hat{w}) - c_u = 0$ implying $\hat{w} = \frac{c_u}{\mu} < c_u$. Note that, the public firm makes losses at $w = \hat{w}$.[4] Requiring non-negative profits prompts the upstream monopolist to raise the input price from $w = \hat{w}$ to $w = c_u$ which in turn raises the price of the final good from c_u to

$$P_{m_1} = \mu c_u. \tag{4.2.3}$$

The subscript m_1 denotes the first type of mixed regime where a public upstream firm co-exists with a private downstream firm.

Now consider the second type of mixed regime where the downstream firm is public while the upstream firm is private. Facing input price w, the downstream public firm sets P to maximize welfare, $\int_0^{Q} P(y)dy - (w+d)Q$, subject to $P \geq w+d$. The constraint binds in equilibrium which implies that the public firm will set

$$\tilde{P}(w) = w + d$$

as the price of the final good and demand $\tilde{Q}(w) = (\tilde{P}(w))^{-e} = (w+d)^{-e}$ units of intermediate input. Upstream profit is $(w - c_r)(w+d)^{-e}$ which is maximized at $w = \mu c_r + (\mu - 1)d$. Substituting this value of w in $\tilde{P}(w)$ yields an equilibrium price of final good as

$$P_{m_2} = \mu(c_r + d) \tag{4.2.4}$$

[4] We impose a non-negative profit constraint on public firms. While this constraint is typically non-binding in standard single-stage mixed oligopoly models, a welfare-maximizing public firm in a vertically related market may otherwise price below cost and incur losses. Since financing such losses raises public finance issues beyond the scope of this paper, we impose a non-negative profit constraint.

where the subscript m_2 denotes a type 2 mixed regime.

Public regime

Finally, suppose both upstream and downstream firms are public. As in the mixed regime of type 2, facing input price w, a downstream public firm sets

$$\tilde{P}(w) = w + d$$

as the price of the final good and demand $\tilde{Q}(w) = (\tilde{P}(w))^{-e} = (w+d)^{-e}$ units of intermediate input.

The upstream monopolist chooses w to maximize welfare, $\int_0^{\tilde{Q}(w)} P(y)dy - (c_u + d)\tilde{Q}(w)$, subject to non-negative profits constraint. Welfare maximizing input price is the value of w that solves $\tilde{P}(w) = c_u + d$. As $\tilde{P}(w) = w + d$, it follows that $w = c_u$ and equilibrium price of the final good in the public regime is

$$P_u = c_u + d. \tag{4.2.5}$$

Note that $w - c_u = 0$ implies non-negative profit constraint is also satisfied.

Using (4.2.2), (4.2.3), (4.2.4) and (4.2.5), below we list the final good prices in all four market regimes:

$$P_u = c_u + d = c_r + \Delta + d, \quad P_{m_1} = \mu c_u = \mu(c_r + \Delta), \quad P_{m_2} = \mu(c_r + d), \quad \text{and} \quad P_r = \mu^2 c_r, \tag{4.2.6}$$

where P_z refers to the equilibrium price of the final good in regime $z \in \{u, r, m_1, m_2\}$ and $\mu = \frac{1}{1-\frac{1}{e}}$ denotes the markup.

Observe that, equilibrium price in any regime depends on markup and unit costs, both of which vary with the ownership structure in the two sectors. Below, we discuss how markup (μ) and public firms' relative inefficiency in the downstream and upstream sectors—given by d and $\Delta(\equiv c_u - c_r)$ respectively—jointly shape the *consumer surplus* ranking in the presence of vertical relationships.

4.3 Consumer surplus

Consumer surplus in a market regime z, given by

$$CS_z = \int_0^{Q_z} Q^{-\frac{1}{e}} dQ - P_z Q_z = \frac{1}{e} P_z^{1-e},$$

is monotonically decreasing in prices implying that comparison of regimes in terms of prices is sufficient to determine their positions in the consumer surplus ranking.

We start by comparing prices between the two mixed regimes. Both have one layer of markup pricing and one layer of inefficiency. Constant elasticity of demand implies that markup μ is the same and constant irrespective of the sector where the markup is applied. Thus, efficiency concerns govern price comparison. By assumption, public firms are inefficient but more so in the downstream sector. Using $d \geq \Delta$ and (4.2.6), we get

$$P_{m_1} - P_{m_2} = \mu(\Delta - d) \leq 0.$$

As we show below, price comparison between any two regimes—other than m_1 and m_2—depends on both costs and markup.

When demand elasticity (e) is high, i.e., μ is low, production costs play a more prominent role (than markup) in the price ranking of different regimes. As production costs are lowest in the fully private regime, P_r is the lowest among the four prices. On the other hand, P_u is the highest because production costs are highest under the fully public regime. Prices in mixed regimes lie between P_r and P_u, implying

$$P_r \leq P_{m_1} \leq P_{m_2} \leq P_u.$$

When demand elasticity is low, i.e., μ is high, markup considerations play a more prominent role (than production costs). P_u is lowest among all prices because P_u is unaffected by markup while other prices involve at least one round of markup. On the other hand, P_r is highest as it involves two rounds of markup whereas the others involve one at most. Prices in mixed regimes continue to lie in the middle; only P_u and P_r switch positions:

$$P_u \leq P_{m_1} \leq P_{m_2} \leq P_r.$$

Proposition 4.1 translates the last two strings of inequalities in terms of *consumer surplus* and provides a more precise characterization linking markup and the relative inefficiency of public firms in the two sectors when the markup is either too high or too low.

Proposition 4.1. *Mixed regimes lie in the middle of the consumer surplus ranking when the markup μ is sufficiently high or sufficiently low. Denote $\omega \equiv (d, \Delta)$ where d and $\Delta = c_u - c_r$ respectively capture the extent of inefficiency of public firms in the downstream and upstream sectors. Define*

$$\underline{\mu}(\omega) \equiv 1 + \frac{\Delta}{c_r + d}, \quad \text{and} \quad \overline{\mu}(\omega) \equiv 1 + \frac{d}{c_r}.$$

Then for all ω satisfying $(d - \Delta) \geq 0$,

(i) $CS_u \leq CS_{m_2} \leq CS_{m_1} \leq CS_r$ *if and only if* $\mu \leq \underline{\mu}(\omega)$, *and*

(ii) $CS_r \leq CS_{m_2} \leq CS_{m_1} \leq CS_u$ *if and only if* $\mu \geq \overline{\mu}(\omega)$,

where CS_u, CS_r and CS_{m_i} denote the consumer surplus in the public regime, private regime, and the mixed regime of type i respectively, where $i = 1(2)$ if the downstream (upstream) firm is private.

Proof. See Appendix 4.6.3. □

Proposition 4.1 highlights the tradeoff between market power and production efficiency. When the markup μ is sufficiently high—reflecting high market power—consumer surplus is highest in the public regime. On the other hand, when the markup μ is sufficiently low, and market power is less of a concern, consumer surplus is highest in the private regime. Proposition 4.1 is similar in spirit to Proposition 2.1 in the sense that both tell us that mixed regimes secure middle spots in the consumer surplus ranking. However, it does not tell us what happens when the markup is neither too high nor too low. Proposition 4.2 below addresses this question. Before turning to Proposition 4.2, we illustrate diagrammatically how the markup and relative inefficiency of public firms in different sectors shape the consumer surplus ranking.

To facilitate comparison of consumer surplus across different regimes, we work with a markup-deflated price, $p_z \equiv \frac{P_z}{\mu}$, which (like P_z) varies inversely with CS_z. We have

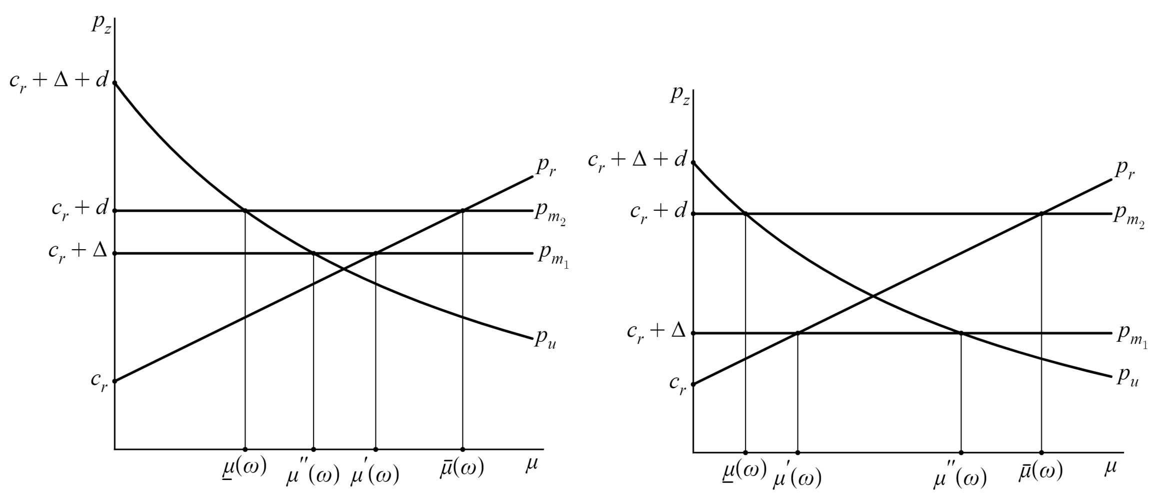
$$p_u = \frac{c_r + \Delta + d}{\mu}, \quad p_{m_1} = c_r + \Delta, \quad p_{m_2} = c_r + d, \quad p_r = \mu c_r. \tag{4.3.1}$$

One advantage of using p_z (instead of P_z) is that it separates public, private, and mixed regimes in terms of response to a change in markup: p_u is strictly decreasing in μ, p_r is strictly increasing in μ, and both p_{m_1} and p_{m_2} are independent of μ. The additional assumption $d \geq \Delta$ gives $p_{m_1} \leq p_{m_2}$.

Figure 4.1 plots markup-deflated prices for different regimes. It captures the essence of Proposition 4.1: consumer surplus in both mixed regimes lies in the middle when $\mu \leq \underline{\mu}(\omega)$ as well as when $\mu \geq \overline{\mu}(\omega)$. Figure 4.1 also illustrates what happens when the markup μ is neither too high nor too low. For moderate markups, the relative efficiency of public firms in the two sectors plays an important role. Figure 4.1(a) shows that, for small values of $d - \Delta$, there exists a moderate range of markup values, $(\mu''(\omega), \mu'(\omega))$, such that prices in both mixed regimes exceed the prices in both private and public regimes, i.e.,

$$p_{m_2} > p_{m_1} > \max\{p_u, p_r\} \geq \min\{p_u, p_r\}.$$

Mixed regimes secure the bottom two places in the consumer surplus ranking. Figure 4.1(b)—applicable for large values of $(d-\Delta)$—makes the case for mixed regime m_1. For suitably large $d-\Delta$, there exists a possible range of moderate markup values, $(\mu'(\omega), \mu''(\omega))$, where p_{m_1} is the lowest among all four markup-deflated prices; consumer surplus is highest when only the downstream sector is privatized. Note that the values of d and Δ that make the mixed regime m_1 most appealing to consumers, also make the other mixed regime m_2 the least appealing option. This again highlights the importance of selection of the sectors for privatization.



(a) $(d-\Delta)$ small

(b) $(d-\Delta)$ large

Figure 4.1: Markup-deflated prices in different regimes

Interestingly, as depicted in Figure 4.1(a), the message of Proposition 3.1, the middleness of mixed regimes, does not survive entirely in the presence of vertical relationships. While part (ii) of Proposition 3.1, i.e., $\min\{CS_{m_1}, CS_{m_2}\} \leq \max\{CS_u, CS_r\}$, holds for all ω, but part (i) $\max\{CS_{m_1}, CS_{m_2}\} \geq \min\{CS_u, CS_r\}$ does not necessarily hold when $(d-\Delta)$

is small.[5] Proposition 4.2 summarizes the discussion above.

Proposition 4.2. *For all $\omega(\equiv (d,\Delta))$, define*

$$\mu'(\omega) \equiv 1 + \frac{\Delta}{c_r}, \text{ and } \mu''(\omega) \equiv 1 + \frac{d}{c_r + \Delta}.$$

Also, $\underline{\mu}(\omega)$ and $\overline{\mu}(\omega)$ are as defined in Proposition 4.1.

(i) For all ω satisfying $(d - \Delta) \in (0, \frac{\Delta^2}{c_r})$,

$$CS_{m_2} < CS_{m_1} < \min\{CS_u, CS_r\} \leq \max\{CS_u, CS_r\} \text{ if and only if}$$
$$\mu \in (\mu''(\omega), \mu'(\omega)),$$

where $\underline{\mu}(\omega) < \mu''(\omega) < \mu'(\omega) < \overline{\mu}(\omega)$.

(ii) For all ω satisfying $(d - \Delta) > \frac{\Delta^2}{c_r}$,

$$CS_{m_2} < \min\{CS_u, CS_r\} \leq \max\{CS_u, CS_r\} < CS_{m_1} \text{ if and only if}$$
$$\mu \in (\mu'(\omega), \mu''(\omega)),$$

where $\underline{\mu}(\omega) < \mu'(\omega) < \mu''(\omega) < \overline{\mu}(\omega)$.

Proof. See Appendix 4.6.4. □

Proposition 4.2(i) states how different regimes rank in terms of consumer surplus when markups are moderate and the inefficiency parameters d and Δ are not too different. We find that consumer surplus is maximized when both upstream and downstream firms are private, or both are public. Unlike Proposition 2.1, however, it does not imply that mixed regimes necessarily occupy intermediate positions. Consumer surplus under both mixed regimes can fall below that achieved under either fully public or fully private ownership. In such cases, a mixed regime yields the lowest consumer surplus among the four possible

[5] Lemma 4.2 in Appendix 4.6.2 demonstrates why part (ii) of Proposition 3.1 holds for all ω.

ownership structures. This possibility is not unique to vertical market structure though. Proposition 2.5 notes the same possibility under Bertrand competition. Proposition 4.2(ii) and Proposition 3.3 express similar optimism about mixed regimes. Both show that mixed regimes can occupy the top spot in consumer surplus ranking when the cost heterogeneity across stages is sufficiently pronounced. However, heterogeneity in relative inefficiency of public firms across sectors is not enough; market power must also be moderate for a mixed regime to dominate.[6]

It is worth noting that heterogeneity across upstream and downstream sectors can also distort measured gains from privatization. For example, the Chinese government's policy of "grasping the large and letting go of the small" implies that, even if private firms are more productive than SOEs, privatized firms may exhibit lower average productivity due to negative selection (Chen et al., 2021). Conversely, estimated gains from privatization may be overstated if relatively efficient public firms are privatized first, as part of the observed improvement reflects selection rather than ownership change. Although our static framework does not address such sequencing issues, the central implication is robust: whether ownership decisions are made simultaneously or sequentially—for instance, in a two-period extension—privatizing only the more efficient firm in either sector is never optimal from the standpoint of consumer surplus.

4.4 Successive oligopoly

While we have examined two-tier successive monopoly so far, the natural question would be that—how does introducing competition in both upstream and downstream sectors qualify our conclusions? To answer this question, we consider a successive oligopoly setting

[6] One might argue that markup plays such a prominent role in our setup because of monopoly in both tiers of production. Indeed, a two-tier monopoly creates a large gap between the public regime (with no markup) and the private regime (with markup squared) through which the mixed option gets in and delivers the lowest price. However, the presence of competition does not undo that (see Appendix 4.6.5).

with $N(\geq 1)$ downstream firms and $M(\geq 1)$ upstream firms. A new possibility that emerges is that, even when $\Delta < d$, the m_2 regime—where only the upstream firms are privatized—can dominate all other regimes in the CS ranking if the upstream sector is sufficiently more competitive than the downstream sector. In what follows, we describe the successive Cournot setup and systematically examine how *competition* and *public–private efficiency gaps* jointly determine the regime that occupies the top position in the CS ranking, with particular focus on the m_2 regime.

Throughout this section, we assume that all firms competing in the same sector have the same ownership type.[7] First, consider the private regime, where all firms, regardless of their sectoral affiliations, maximize profits. Standard analysis of downstream Cournot competition—yields

$$P = \frac{w}{1-\frac{1}{Ne}} = \mu_D w, \tag{4.4.1}$$

where w is the price of input and $\mu_D = \frac{Ne}{Ne-1}$ is the downstream markup. Using the demand specification, $P = Q^{-\frac{1}{e}}$, substituting $X = Q$ in (4.4.1), and rearranging, we get the inverse demand function for input X:[8]

$$w = \frac{X^{-\frac{1}{e}}}{\mu_D} = w(X).$$

An upstream firm i chooses x_i to maximize profit $(w(X) - c_r)x_i$. Solving the firms' profit maximization problem, and after some algebra, we find that the equilibrium input price is

$$w = \frac{c_r}{1-\frac{1}{Me}} = \mu_U c_r,$$

where $\mu_U = \frac{1}{1-\frac{1}{Me}}$ reflects the upstream markup. Using the value of equilibrium w we can

[7]In reality, public and private firms co-exist in the same sector as well as in the vertically linked sectors. However, to focus on the new insights arising from vertical linkage, we abstract from the co-existence of public and private firms in the same sector. A more comprehensive analysis involving both public and private firms in both sectors is left for future research.

[8]Note that one unit of intermediate input X is required to produce one unit of Q. This allows us to substitute Q with X.

express the equilibrium price of the final good as:

$$P_r = \mu_D \mu_U c_r$$

We derive the equilibrium prices for other regimes in the Appendix 4.6.6. Those equilibrium prices, along with the one in the private regime, are recorded in Lemma 4.1.

Lemma 4.1. *Let P_z denote the equilibrium price of the final good under market regime $z \in \{r, u, m_1, m_2\}$, where r, u, and m_i correspond to private regime, public regime and mixed regime of type i respectively. The equilibrium prices in the four market regimes are:*

$$P_u = c_r + \Delta + d,\ P_{m_1} = \mu_D(c_r + \Delta),\ P_{m_2} = \mu_U(c_r + d),\ \text{and } P_r = \mu_D \mu_U c_r,$$

where $\mu_U \equiv \frac{1}{1-\frac{1}{Me}}$, $\mu_D \equiv \frac{1}{1-\frac{1}{Ne}}$, c_s denotes the marginal cost of the intermediate good for a firm of type $s \in \{u, r\}$, and Δ and d capture the relative inefficiencies of the public firm in the upstream and downstream sectors, respectively.

As in section 4.2, we find that equilibrium price in any regime is increasing in markup and the sum of (relevant) unit costs. Comparing the public and the private regimes, we find that

$$CS_r \gtreqless CS_u \Leftrightarrow P_r \lesseqgtr P_u \Leftrightarrow \mu_U \mu_D \lesseqgtr 1 + \frac{d+\Delta}{c_r}. \tag{4.4.2}$$

When both markups are large, public regime is better for consumers. The private regime is better when the markups are small. Similarly, comparison of two mixed regimes—m_1 and m_2—yields

$$CS_{m_1} \gtreqless CS_{m_2} \Leftrightarrow P_{m_1} \lesseqgtr P_{m_2} \Leftrightarrow \frac{\mu_U}{\mu_D} \gtreqless \frac{c_r + \Delta}{c_r + d}. \tag{4.4.3}$$

Since $\Delta < d$, the threshold value in (4.4.3) is less than unity. Thus, if $\mu_U = \mu_D$, as was true in the successive monopoly, CS is always higher in the m_1 regime.

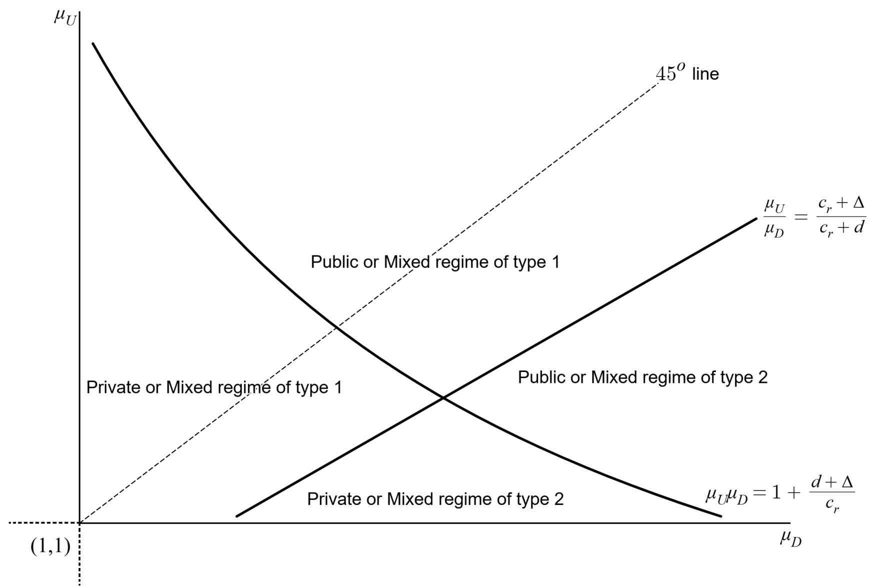


Figure 4.2: Top 2 regimes in CS ranking under competition in both sectors

Figure 4.2 depicts (4.4.2) and (4.4.3), divides the (μ_D, μ_U) space into four regions, and identifies the top two regimes in *CS* ranking for each of these regions. Observe that, irrespective of markups, a mixed regime always appears among the top two ranks. Given that it was always among the bottom two in the case of successive monopoly, m_2 regime—where only the upstream sector is privatized— featuring among the top two might seem surprising. However, note that successive monopoly analysis holds only along the 45-degree line, where $\mu_D = \mu_U = \frac{1}{1-\frac{1}{e}}$. Along that line, m_2 never appears among the top two regimes. A necessary condition for m_2 to feature at the top end of the *CS* ranking is $\mu_U < \mu_D$, which, given $\Delta < d$, requires $M > N$.

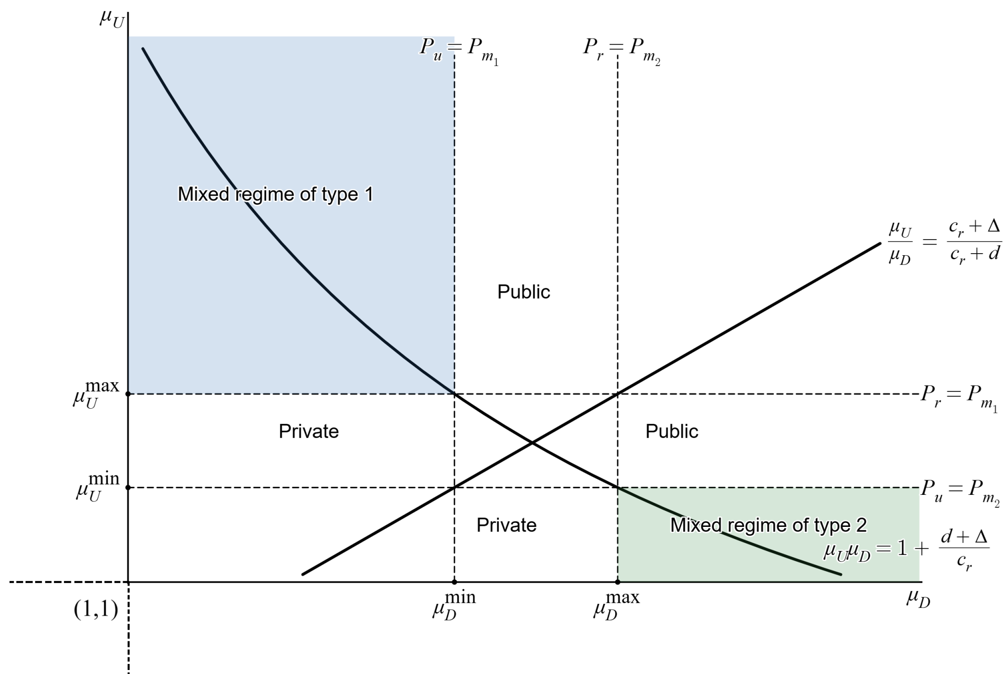


Figure 4.3: Mixed regimes at the top in CS ranking under competition in both sectors

Indeed, m_2 regime can go all the way to the top of CS ranking. From (4.4.3) we know that m_2 beats m_1 when the ratio of the markups, $\frac{\mu_U}{\mu_D}$, is lower than a threshold value, $\frac{c_r+\Delta}{c_r+d}$. For m_2 to secure the top spot, we also need m_2 to deliver higher CS than the public and private regimes. We have

$$CS_{m_2} > CS_u \Leftrightarrow P_u > P_{m_2} \Leftrightarrow \mu_U < 1 + \frac{\Delta}{c_r+d},$$
$$CS_{m_2} > CS_r \Leftrightarrow P_r > P_{m_2} \Leftrightarrow \mu_D > 1 + \frac{d}{c_r}.$$

Thus, a mixed regime of type 2 in which only the upstream firms are privatized delivers the maximum consumer surplus when upstream markup is sufficiently low—which ensures it beats the public regime—downstream markup is sufficiently high—which ensures it beats the private regime.

Figure 4.3 not only shows the region in the (μ_D, μ_U) space for which the mixed regimes secure the top CS ranking, but also shows the top spot in CS ranking for all values of (μ_D, μ_U). Proposition 4.3 translates Figures 4.2 and 4.3 into words. To avoid clutter, we refrain from providing a comprehensive ranking for all regions. The proof is immediate from simple price comparison across regimes and hence omitted.

Proposition 4.3. *Define*

$$\mu_U^{min} = 1 + \frac{\Delta}{c_r + d}, \qquad \mu_U^{max} = 1 + \frac{\Delta}{c_r}, \qquad \mu_D^{min} = 1 + \frac{d}{c_r + \Delta}, \quad \text{and} \quad \mu_D^{max} = 1 + \frac{d}{c_r}.$$

Consumer surplus is uniquely maximum under

(i) private regime, if and only if

$$\mu_D < \mu_D^{max}, \quad \mu_U < \mu_U^{max}, \quad \text{and} \quad \mu_U \mu_D < 1 + \frac{d + \Delta}{c_r}.$$

(ii) public regime, if and only if

$$\mu_D > \mu_D^{min}, \quad \mu_U > \mu_U^{min}, \quad \text{and} \quad \mu_U \mu_D > 1 + \frac{d + \Delta}{c_r}.$$

(iii) mixed regime of type 1, if and only if

$$\mu_D < \mu_D^{min}, \quad \mu_U > \mu_U^{max}, \quad \text{and} \quad \frac{\mu_U}{\mu_D} > \frac{c_r + \Delta}{c_r + d}.$$

(iv) mixed regime of type 2, if and only if

$$\mu_D > \mu_D^{max}, \quad \mu_U < \mu_U^{min}, \quad \text{and} \quad \frac{\mu_U}{\mu_D} < \frac{c_r + \Delta}{c_r + d}.$$

The presence of welfare-maximizing public firms disciplines the market. When firms are privately owned and maximize profits, competition can play a similar disciplinary role. Thus, when M and N are large, competition is intense, markups are low, and consumer surplus is maximized under the private regime. Conversely, when M and N are small, competition is weak, markups are high, and the public regime maximizes consumer surplus. Mixed regimes emerge as optimal when competition is strong in one sector but weak in the other. The public-private efficiency gaps, i.e., d and Δ, further qualify these conclusions. For example, an increase in d—that is, a larger public-private efficiency gap in the downstream sector—raises $\mu_D^{\min}$, enlarging the parameter region for which downstream privatization (i.e., m_1 regime) maximizes consumer surplus. By contrast, the parameter region in which regime m_2 is optimal shrinks.

Overall, the analysis suggests that a *CS*-oriented government must account for two key considerations when evaluating privatization policies: the intensity of competition and the efficiency gap between public and private firms. Privatization is more likely to benefit consumers in sectors where competition is sufficiently strong and the public-private efficiency gap is relatively large.

4.5 Conclusion

In this chapter, we address the second limitation of the model presented in Chapter 2. We introduce vertically related markets—upstream and downstream. The upstream monopolist produces the intermediate input required for the final good and the downstream monopolist produces the final good. The monopolist in either of the sectors can be public or private. In this setting, we find that a mixed regime lies in the middle of consumer surplus ranking when the markup is either sufficiently high or sufficiently low. Even a moderate markup, when combined with similar degrees of inefficiency of public firms in both upstream and downstream sectors of production, leads a mixed regime to the bottom spot in the consumer surplus ranking. Sufficient dissimilarity in the degree of inefficiency in the two sectors along with a moderate markup is required to put the mixed regime, with a public upstream monopolist and a privatized downstream monopolist, at the top of the consumer surplus ranking.

Empirical evidence on privatization suggests that downstream, consumer-facing industries are often privatized earlier than upstream ones.[9] Glaeser and Scheinkman (1996) lends theoretical support to prioritizing downstream privatization. They note that the advantage of private over public ownership lies in more responsiveness to information about consumer demand and input costs. If the retail sector is private, as is often the case, then privatization downstream dominates privatization upstream as privatizing the middle sector (i.e., downstream sector in this case) serves as a more effective conduit for information transmission across the economy.

Whether upstream or downstream privatization is better for consumers matters in our setup too but only to the extent that it enables us to rule one of them out for the top spot in *consumer surplus* ranking. For example, when $d \geq \Delta$—as assumed in our setup—a mixed regime with private upstream firms can no longer secure the top spot. In line with our inquiry in the previous chapters, our primary interest lies in learning whether (and when) a mixed regime, i.e., privatizing just one of the two sectors can do better than all other options including privatizing both sectors as well as keeping both sectors under public ownership. From that perspective, the message is similar to the one obtained under single-stage oligopoly with heterogeneous firms: when relatively more inefficient firms are privatized, *consumer surplus* can be maximized in a mixed regime—a possibility that was missing in symmetric oligopoly environments.

As a first step toward understanding the optimality of mixed markets in a two-tier structure from a consumer-surplus perspective, we adopt a stylized successive-monopoly framework with constant-elasticity demand to isolate the key forces of interest: produc-

[9] See Dinc and Gupta (2011) and Chen et al. (2021) for the privatization experience in the Czech Republic and China, respectively.

tive inefficiency and market power. We then revisit the question—can a mixed regime be optimal?—within richer setting that incorporates competition in both upstream and downstream sectors. The desirability of mixed regimes persists in this alternative specification and, in fact, becomes stronger in the presence of competition.

Beyond the extensions considered here, several important issues remain outside the scope of the analysis. In particular, we do not examine partial privatization of the public monopolist, the coexistence of public and private firms within the same sector, or the optimal extent of privatization when multiple public firms operate within a sector. These features may alter the interaction between productive inefficiency and market power, thereby affecting privatization outcomes. We leave these issues for future research.

4.6 Appendix

4.6.1 Consumer surplus comparison

Using (4.2.2), (4.2.3), (4.2.4) and (4.2.5), we list the final good prices in all market regimes in Table 2.

Market regime (z)	**Price** (P_z)
Public regime (u)	$c_u + d = c_r + \Delta + d$
Mixed regime of type 1 (m_1)	$\mu c_u = \mu(c_r + \Delta)$
Mixed regime of type 2 (m_2)	$\mu(c_r + d)$
Private regime (r)	$\mu^2 c_r$

Table 4.1: Prices in various market regimes

Using the prices listed in Table 4.1, we compare consumer surplus across various market regimes. We also state how μ, d, and Δ affect this comparison.

$$CS_u \gtreqless CS_r \Leftrightarrow P_u \lesseqgtr P_r \Leftrightarrow \mu \gtreqless \sqrt{1+\frac{d+\Delta}{c_r}}, \tag{4.6.1}$$

$$CS_u \gtreqless CS_{m_1} \Leftrightarrow P_u \lesseqgtr P_{m_1} \Leftrightarrow \mu \gtreqless 1+\frac{d}{c_r+\Delta}, \tag{4.6.2}$$

$$CS_u \gtreqless CS_{m_2} \Leftrightarrow P_u \lesseqgtr P_{m_2} \Leftrightarrow \mu \gtreqless 1+\frac{\Delta}{c_r+d}, \tag{4.6.3}$$

$$CS_r \gtreqless CS_{m_1} \Leftrightarrow P_r \lesseqgtr P_{m_1} \Leftrightarrow \mu \lesseqgtr 1+\frac{\Delta}{c_r}, \tag{4.6.4}$$

$$CS_r \gtreqless CS_{m_2} \Leftrightarrow P_r \lesseqgtr P_{m_2} \Leftrightarrow \mu \lesseqgtr 1+\frac{d}{c_r}. \tag{4.6.5}$$

In a similar vein (as above) we can show that $CS_{m_1} \gtreqless CS_{m_2} \Leftrightarrow P_{m_1} \lesseqgtr P_{m_2} \Leftrightarrow d \gtreqless \Delta$. However, by assumption, $d \geq \Delta$. Thus, we always have

$$CS_{m_1} \geq CS_{m_2}. \tag{4.6.6}$$

Using this inequality and the rest of the pairwise comparisons between different regimes we state and prove some claims which we use later in the proofs of Propositions 4.1 and 4.2.

Claim 4.1. *Suppose consumer surplus is lowest in the public regime. Then consumer surplus must be highest in the private regime. That is,*

$$CS_u \leq \min\{CS_{m_1}, CS_{m_2}, CS_r\} \Rightarrow CS_r > \max\{CS_{m_1}, CS_{m_2}, CS_u\}$$

Proof. We have

$$CS_u \leq CS_{m_2} \Rightarrow \mu \leq 1+\frac{\Delta}{c_r+d} \Rightarrow \mu < 1+\frac{\Delta}{c_r} \Rightarrow CS_r > CS_{m_1}$$

where the first and third implications follow from (4.6.3) and (4.6.4) respectively, and the second implication is due to $d > 0$. The claim follows from combining $CS_r > CS_{m_1}$ with

$$CS_{m_1} \geq CS_{m_2} \geq CS_u$$

where the first weak inequality is due to (4.6.6) and the second is due to the supposition that CS is lowest in the public regime. $\square$

Claim 4.2. *Suppose consumer surplus is lowest in the private regime. Then consumer surplus must be highest in the public regime. That is,*

$$CS_r \leq \min\{CS_{m_1}, CS_{m_2}, CS_u\} \Rightarrow CS_u > \max\{CS_{m_1}, CS_{m_2}, CS_r\}$$

Proof. We have

$$CS_r \leq CS_{m_2} \Rightarrow \mu \geq 1 + \frac{d}{c_r} \Rightarrow \mu > 1 + \frac{d}{c_r + \Delta} \Rightarrow CS_u > CS_{m_1}$$

where the first and third implications follow from (4.6.5) and (4.6.2) respectively, and the second implication is due to $\Delta > 0$. The claim follows from combining the finding $CS_u > CS_{m_1}$ with

$$CS_{m_1} \geq CS_{m_2} \geq CS_r$$

where the first weak inequality is due to (4.6.6) and the second one is due to the supposition that CS is lowest in the private regime. $\square$

Claim 4.3. *Suppose consumer surplus is highest in mixed regime 1 where only the downstream firm is private. Then consumer surplus must be lowest in mixed regime 2 where only the upstream firm is private. That is,*

$$CS_{m_1} \geq \max\{CS_{m_2}, CS_u, CS_r\} \Rightarrow CS_{m_2} \leq \min\{CS_{m_2}, CS_u, CS_r\}.$$

Proof. We have

$$CS_{m_1} > CS_u \Rightarrow \mu \leq 1 + \frac{d}{c_r + \Delta} \Rightarrow \mu < 1 + \frac{d}{c_r} \Rightarrow CS_{m_2} < CS_r$$

where the first and third implications follow from (4.6.2) and (4.6.5) respectively, and the second implication is due to $\Delta > 0$. Similarly, we have

$$CS_{m_1} > CS_r \Rightarrow \mu \geq 1 + \frac{\Delta}{c_r} \Rightarrow \mu > 1 + \frac{\Delta}{c_r + d} \Rightarrow CS_{m_2} < CS_u$$

where the first and third implications follow from (4.6.4) and (4.6.3) respectively, and the second implication is due to $\Delta > 0$. Finally, (4.6.6) states that $CS_{m_1} \geq CS_{m_2}$ which can be rewritten as $CS_{m_2} \leq CS_{m_1}$. Therefore, when $CS_{m_1} \geq \max\{CS_u, CS_r, CS_{m_2}\}$ holds true, the claim follows from combining the two inequalities obtained above, i.e., $CS_{m_2} < CS_r$ and $CS_{m_2} < CS_u$, with (4.6.6). □

Claims 4.1-4.3 help us to determine the *CS* ranking of regimes. These claims also help us to rule out some possibilities. Lemma 4.2 presents one such possibility.

4.6.2 Proof of Lemma 4.2

Lemma 4.2. *Both mixed regimes cannot be above both public and private regimes in the consumer surplus ranking. In other words, for all $\omega \equiv (d, \Delta)$,*

$$\min\{CS_{m_1}, CS_{m_2}\} \leq \max\{CS_u, CS_r\}.$$

Proof. Suppose not. Then $\min\{CS_{m_1}, CS_{m_2}\} > \max\{CS_u, CS_r\}$. Combining this inequality with $CS_{m_1} \geq CS_{m_2}$ we get

$$CS_{m_1} \geq CS_{m_2} > \max\{CS_u, CS_r\} \geq \min\{CS_u, CS_r\}.$$

This contradicts Claim 4.3.

∎

4.6.3 Proof of Proposition 4.1

Proof. This proof draws on equations (4.6.1)-(4.6.6) and Claims 4.1-4.3 presented in Section 4.6.1.

(i) We have

$$CS_u \leq \min\{CS_{m_1}, CS_{m_2}, CS_r\} \Rightarrow CS_r > \max\{CS_{m_1}, CS_{m_2}, CS_u\}$$

from Claim 4.1, and

$$CS_{m_2} \leq CS_{m_1}$$

from (4.6.6). Thus,

$$CS_u \leq CS_{m_2} \leq CS_{m_1} \leq CS_r \Leftrightarrow CS_u \leq CS_{m_2} \text{ and } CS_u \leq CS_r.$$

We have

$$CS_u \leq CS_{m_2} \Leftrightarrow \mu \leq 1 + \frac{\Delta}{c_r + d} \text{ [from (4.6.3)], and}$$
$$CS_u \leq CS_r \Leftrightarrow \mu \leq \sqrt{1 + \frac{d + \Delta}{c_r}} \text{ [from (4.6.1)].}$$

Combining the last three two-way implications (i.e., $\Leftrightarrow$) we get

$$CS_u \leq CS_{m_2} \leq CS_{m_1} \leq CS_r \Leftrightarrow \mu \leq \min\left\{1 + \frac{\Delta}{c_r + d}, \sqrt{1 + \frac{d + \Delta}{c_r}}\right\}.$$

Then Proposition 4.1(i) follows from defining $\underline{\mu}(\omega) \equiv 1 + \frac{\Delta}{c_r + d}$ and noting that

$$\min\left\{1+\frac{\Delta}{c_r+d}, \sqrt{1+\frac{d+\Delta}{c_r}}\right\} = 1+\frac{\Delta}{c_r+d}$$

as

$$1+\frac{\Delta}{c_r+d}-\sqrt{1+\frac{d+\Delta}{c_r}} = -\left(\frac{\sqrt{c_r+d+\Delta}}{\sqrt{c_r}(c_r+d)(c_r+d+\sqrt{c_r(c_r+d+\Delta)})}\right)(d^2+(d-\Delta)c_r)<0$$

where the inequality is due to our assumption $d \geq \Delta$.

(ii) We have

$$CS_r \leq \min\{CS_{m_1}, CS_{m_2}, CS_u\} \Rightarrow CS_u > \max\{CS_{m_1}, CS_{m_2}, CS_r\}$$

from Claim 4.2 and $CS_{m_2} \leq CS_{m_1}$ from (4.6.6). Thus

$$CS_r \leq CS_{m_2} \leq CS_{m_1} \leq CS_u \Leftrightarrow CS_r \leq CS_{m_2} \text{ and } CS_r \leq CS_u.$$

We have

$$CS_r \leq CS_{m_2} \Leftrightarrow \mu \geq 1+\frac{d}{c_r} \text{ [from (4.6.5)], and}$$
$$CS_r \leq CS_u \Leftrightarrow \mu \geq \sqrt{1+\frac{d+\Delta}{c_r}} \text{ [from (4.6.1)].}$$

Combining the last three two-way implications (i.e., $\Leftrightarrow$) we get

$$CS_r \leq CS_{m_2} \leq CS_{m_1} \leq CS_u \Leftrightarrow \mu \geq \max\left\{1+\frac{d}{c_r}, \sqrt{1+\frac{d+\Delta}{c_r}}\right\}.$$

Proposition 4.1(ii) follows from defining $\overline{\mu}(\omega) \equiv 1 + \frac{d}{c_r}$ and noting that

$$\max\left\{1+\frac{d}{c_r}, \sqrt{1+\frac{d+\Delta}{c_r}}\right\} = 1+\frac{d}{c_r}$$

as

$$1+\frac{d}{c_r} - \sqrt{1+\frac{d+\Delta}{c_r}} = \frac{d^2+(d-\Delta)c_r}{c_r(c_r+d+\sqrt{c_r(c_r+d+\Delta)})} > 0$$

where the inequality is due to our assumption $d-\Delta \geq 0$. $\square$

4.6.4 Proof of Proposition 4.2

Proof. The proof draws on equations (4.6.1) - (4.6.6) and Claims 4.1-4.3 presented in Section 4.6.1. To facilitate the proof, we record all relevant threshold levels of markups which are defined in the statement of Propositions 4.1 and/or Proposition 4.2:

$$\underline{\mu}(\omega) = 1+\frac{\Delta}{c_r+d},\ \mu''(\omega) \equiv 1+\frac{d}{c_r+\Delta},\ \mu'(\omega) \equiv 1+\frac{\Delta}{c_r}, \text{ and } \overline{\mu}(\omega) = 1+\frac{d}{c_r}.$$

(i) We have

$$\begin{aligned}
\underline{\mu}(\omega) - \mu''(\omega) &= (1+\frac{\Delta}{c_r+d}) - (1+\frac{d}{c_r+\Delta}) = -\frac{(d-\Delta)(c_r+d+\Delta)}{(c_r+d)(c_r+\Delta)} < 0,\\
\mu''(\omega) - \mu'(\omega) &= (1+\frac{d}{c_r+\Delta}) - (1+\frac{\Delta}{c_r}) = \frac{(d-\Delta)-\frac{\Delta^2}{c_r}}{c_r+\Delta} < 0, \quad \text{and}\\
\mu'(\omega) - \overline{\mu}(\omega) &= (1+\frac{\Delta}{c_r}) - (1+\frac{d}{c_r}) = -\frac{d-\Delta}{c_r} < 0,
\end{aligned}$$

where the first and third inequalities are due to $d-\Delta > 0$, and the second follows from noting that $d-\Delta < \frac{\Delta^2}{c_r}$. Combining the three inequalities yields

$$\underline{\mu}(\omega) < \mu''(\omega) < \mu'(\omega) < \overline{\mu}(\omega)$$

for all $(d-\Delta) \in (0, \frac{\Delta^2}{c_r})$.

Now let us turn to consumer surplus. Since $CS_{m_2} \leq CS_{m_1}$, both mixed regimes lie at the bottom of the CS ranking if and only if $CS_{m_1} < \min\{CS_u, CS_r\}$. We have

$$CS_{m_1} < CS_u \Leftrightarrow \mu > 1 + \frac{d}{c_r + \Delta} \Leftrightarrow \mu > \mu''(\omega) \tag{4.6.1}$$

where the first $\Leftrightarrow$ follows from (4.6.2) and the second $\Leftrightarrow$ follows from the definition of $\mu''(\omega)$. Similarly,

$$CS_{m_1} < CS_r \Leftrightarrow \mu < 1 + \frac{\Delta}{c_r} \Leftrightarrow \mu < \mu'(\omega) \tag{4.6.2}$$

where the first $\Leftrightarrow$ follows from (4.6.4) and the second $\Leftrightarrow$ follows from the definition of $\mu'(\omega)$. Combining (4.6.1) and (4.6.2) yields:

$$CS_{m_2} < CS_{m_1} < \min\{CS_u, CS_r\} \leq \max\{CS_u, CS_r\} \text{ if and only if } \mu \in \left(\mu''(\omega), \mu'(\omega)\right),$$

where, as established earlier in this proof, $\underline{\mu}(\omega) < \mu''(\omega) < \mu'(\omega) < \overline{\mu}(\omega)$ holds for all $(d-\Delta) \in (0, \frac{\Delta^2}{c_r})$.

(ii) We have

$$\begin{aligned}
\underline{\mu}(\omega) - \mu'(\omega) &= (1 + \frac{\Delta}{c_r + d}) - (1 + \frac{\Delta}{c_r}) = -\frac{d\Delta}{c_r(c_r + d)} < 0, \\
\mu'(\omega) - \mu''(\omega) &= (1 + \frac{\Delta}{c_r}) - (1 + \frac{d}{c_r + \Delta}) = -\frac{(d-\Delta) - \frac{\Delta^2}{c_r}}{c_r + \Delta} < 0, \text{ and} \\
\mu''(\omega) - \overline{\mu}(\omega) &= (1 + \frac{d}{c_r + \Delta}) - (1 + \frac{d}{c_r}) = -\frac{d\Delta}{c_r(c_r + \Delta)} < 0.
\end{aligned}$$

where the first and third inequalities are obvious, and the second inequality follows from noting that $(d-\Delta) > \frac{\Delta^2}{c_r}$. Combining the three inequalities we find that

$$\underline{\mu}(\omega) < \mu'(\omega) < \mu''(\omega) < \overline{\mu}(\omega)$$

holds for all $(d-\Delta) > \frac{\Delta^2}{c_r}$.

Now let us turn to CS. From Claim 4.3 we know that when CS_{m_1} is the highest, CS_{m_2} is the lowest. This in turn implies

$$CS_{m_1} > \max\{CS_u, CS_r\} \geq \min\{CS_u, CS_r\} > CS_{m_2} \Leftrightarrow CS_{m_1} > \max\{CS_u, CS_r\}.$$

We have

$$CS_{m_1} > CS_u \Leftrightarrow \mu < 1 + \frac{d}{c_r + \Delta} \Leftrightarrow \mu < \mu''(\omega) \tag{4.6.3}$$

where the first $\Leftrightarrow$ follows from (4.6.2) and the second $\Leftrightarrow$ follows from the definition of $\mu''(\omega)$. Similarly,

$$CS_{m_1} > CS_r \Leftrightarrow \mu > 1 + \frac{\Delta}{c_r} \Leftrightarrow \mu > \mu'(\omega) \tag{4.6.4}$$

where the first $\Leftrightarrow$ follows from (4.6.4) and the second $\Leftrightarrow$ follows from the definition of $\mu'(\omega)$.

Combining (4.6.3) and (4.6.4) yields:

$$CS_{m_1} > \max\{CS_u, CS_r\} \geq \min\{CS_u, CS_r\} > CS_{m_2} \text{ if and only if } \mu \in \big(\mu'(\omega), \mu''(\omega)\big)$$

where, as established earlier in this proof, $\underline{\mu}(\omega) < \mu'(\omega) < \mu''(\omega) < \overline{\mu}(\omega)$ holds for all $\omega \equiv (d, \Delta)$ that satisfies $(d-\Delta) > \frac{\Delta^2}{c_r}$. □

4.6.5 Competition in the downstream sector

Suppose there are N downstream firms competing in quantities. We continue to assume that private firms maximize profits and public firms maximize welfare subject to non-negative profit constraints. In regimes where downstream firms maximize profits—i.e., m_1 and r—the equilibrium price of the final good declines with competition (i.e., with an increase in N) as is immediate from the following expressions:

$$P_{m_1} = \frac{\mu c_u}{\mu - \frac{1}{N}(\mu - 1)} \quad \text{and} \quad P_r = \frac{\mu^2 c_r}{\mu - \frac{1}{N}(\mu - 1)}.$$

In regimes with welfare-maximizing downstream firms and constant marginal costs, the price of the final good remains unchanged:[10]

$$P_u = c_u + d, \quad \text{and} \quad P_{m_2} = \mu(c_r + d).$$

Competition makes it more likely (at least weakly) for type 1 mixed regime to deliver the lowest price as P_{m_1} declines while P_u and P_{m_2} remain unchanged. While equilibrium price in the private regime also declines, a mixed regime's appeal vis-à-vis a private regime does not change with competition as the ratio $\frac{P_{m_1}}{P_r} = \frac{c_u}{\mu c_r}$ does not vary with N. Competition disciplines the downstream sector and lowers overall markup when downstream firms are private profit-maximizers, while no separate disciplinary device is needed in case of public firms. The presence of a welfare maximizing firm imposes discipline although it comes at the expense of higher cost. A mixed regime continues to deliver the lowest price in the presence of competition for a range of parameterizations.

[10]This is indeed a special case that arises with a purely welfare-maximizing firm and constant marginal costs. Instead, if public firm's objective function is a weighted average of welfare and profits, equilibrium price responds to a change in N. The values of P_u and P_{m_2} reported in the text might be considered as limiting prices arising from maximization of a weighted objective function where the weight on profit approaches zero. Qualitatively, the key result that a mixed option can deliver the lowest prices continues to hold.

4.6.6 Successive oligopoly

Using the demand function given by (4.2.1), we derive the equilibrium prices in all four regimes.

Private regime

In a private regime, there are M private firms and N downstream firms in upstream and downstream sector respectively. Starting with private firm i's profit maximization problem in the downstream sector:

$$\max_{q_i} \pi_i = (P(Q) - w)q_i$$

The first-order condition is given by

$$P(Q) + q_i P'(Q) = 0.$$

Since firms are all symmetric within a sector, we have $q_i = \frac{Q}{N}$. Substituting this into the first-order condition, we get

$$\begin{aligned}
&P(Q) + \frac{Q}{N}P'(Q) = w \\
\Rightarrow &P(Q)\left(1 + \frac{1}{N}\frac{QP'(Q)}{P(Q)}\right) = w \\
\Rightarrow &P(Q)(1 - \frac{1}{Ne}) = w \\
\Rightarrow &P(Q) = \frac{w}{1 - \frac{1}{Ne}} = \mu_D w.
\end{aligned}$$

Using the demand specification, we substitute $X = Q$ in $P = Q^{-\frac{1}{e}}$, and get the inverse demand function for the upstream firm as

$$w(X) = \frac{X^{-\frac{1}{e}}}{\mu_D}$$

Private firm j in upstream sector chooses x_j to maximize $(w(X)-c_r)x_j$. The first-order condition is given by

$$w(X)+x_j w'(X)=c_r.$$

Using the analogous argument as before, in symmetric equilibrium, the first order equation can be rearranged as

$$w(X)\left(1-\frac{1}{Me}\right)=c_r \Rightarrow w(X)=\frac{c_r}{\left(1-\frac{1}{Me}\right)}=\mu_U c_r.$$

Substituting w into the expression for price of final good in private regime, we get

$$P_r=\mu_D w=\mu_U \mu_D c_r.$$

Mixed regimes: Type 1 **and Type** 2

We start with mixed regime of type 1 where there are M public firms in the upstream sector and N private firms in the downstream sector. From the profit maximization problem of each private firm, we get

$$P=\mu_D w$$

as before similar to the private regime. However, unlike private regime, in the upstream sector, each public firm j chooses x_j to maximize

$$W=\int_0^X P(y)dy-(c_r+\Delta)X$$

subject to

$$w - (c_r + \Delta) \geq 0,$$

where W denotes welfare.

In the absence of the constraint, the welfare is maximized at $P(\hat{w}) = (c_r + \Delta)$. However, from the downstream optimization problem, we have $\hat{w} = \frac{c_r + \Delta}{\mu_d} < c_r + \Delta$. Therefore, the constraint is not satisfied at $\hat{w}$. Therefore, we must set $w = (c_r + \Delta)$. Substituting this into the price of the final good in mixed regime of type 1, we find that

$$P_{m_1} = \mu_D w = \mu_D (c_r + \Delta).$$

Next, we have mixed regime of type 2 where there are M private firms and N public firms in upstream and downstream sector respectively. In the downstream, each public firm i chooses q_i to maximize

$$W = \int_0^Q P(y)dy - (w + d)Q,$$

subject to

$$\text{subject to } P \geq (w + d),$$

where W denotes welfare. The constraint binds in equilibrium implying that each public firm will set

$$P(w) = w + d.$$

Each private firm j in upstream sector chooses x_j to maximize its profits, i.e.,

$$(w(X) - c_r)x_i$$

where $w(X) = P(X) - d$. Using analogous argument as before, in symmetric equilibrium, the first-order condition can be rearranged and price of the final good can be derived as

$$P_{m_2} = \frac{c_r + d}{1 - \frac{1}{Me}} = \mu_U(c_r + d).$$

Public regime

In a public regime, there are M public firms and N public firms in the upstream and downstream sectors, respectively. Similar to the mixed regime of type 2, in the downstream sector, public firms' welfare maximization problem yields,

$$P = w + d.$$

In the upstream sector, public firm j chooses x_j to maximize welfare subject to a non-negative profit constraint, $w - c_u \geq 0$. In equilibrium, public firms choose output such that $P - c_u - d = 0$. As $P = w + d$, it follows that $w = c_u$ and the equilibrium price of the final good in the public regime is

$$P_u = c_u + d = c_r + \Delta + d.$$

Note that $w - c_u = 0$ implies that the non-negative profits constraint for the upstream public firm is also satisfied.[11]

[11] It is worth noting that since marginal cost in both upstream and downstream sectors is constant, and price equals marginal cost in the equilibrium of the public regime, market structure does not impact P_u. This invariance—P_u not responding to q_u—goes away once we assume quadratic cost functions. The conclusion obtained here continues to hold as long as the coefficient of the quadratic term in the cost function is not too large.

Below, we list final good prices in all four regimes:

$$P_r = \mu_U \mu_D c_r,\ P_{m_1} = \mu_D(c_r+\Delta), P_{m_2} = \mu_U(c_r+d), \text{ and } P_u = c_r+\Delta+d. \tag{4.6.1}$$

Next, we derive thresholds by pairwise comparison of prices given by (4.6.1). That is,

$$P_u \gtreqless P_{m_2} \Leftrightarrow c_r+\Delta+d \gtreqless \mu_U(c_r+d) \Leftrightarrow \mu_U \lesseqgtr 1+\frac{\Delta}{c_r+d}, \tag{4.6.2}$$

$$P_u \gtreqless P_r \Leftrightarrow c_r+\Delta+d \gtreqless \mu_U \mu_D c_r \Leftrightarrow \mu_U \mu_D \lesseqgtr 1+\frac{d+\Delta}{c_r}, \tag{4.6.3}$$

$$P_u \gtreqless P_{m_1} \Leftrightarrow c_r+\Delta+d \gtreqless \mu_D(c_r+\Delta) \Leftrightarrow \mu_D \lesseqgtr 1+\frac{d}{c_r+\Delta}, \tag{4.6.4}$$

$$P_r \gtreqless P_{m_1} \Leftrightarrow \mu_U \mu_D c_r \gtreqless \mu_D(c_r+\Delta) \Leftrightarrow \mu_U \gtreqless 1+\frac{\Delta}{c_r}, \tag{4.6.5}$$

$$P_r \gtreqless P_{m_2} \Leftrightarrow \mu_U \mu_D c_r \gtreqless \mu_U(c_r+d) \Leftrightarrow \mu_D \gtreqless 1+\frac{d}{c_r}, \tag{4.6.6}$$

$$P_{m_2} \gtreqless P_{m_1} \Leftrightarrow \mu_U(c_r+d) \gtreqless \mu_D(c_r+\Delta) \Leftrightarrow \frac{\mu_U}{\mu_D} \gtrless \frac{c_r+\Delta}{c_r+d}. \tag{4.6.7}$$

Chapter 5

Public versus Private Provision in Congested Markets

5.1 Introduction

For several products and services, as a firm serves more customers, the utility declines for each individual customer. We refer to this phenomenon as congestion disutility, which is evident in sectors such as healthcare, education, telecommunications, public transport, banking, and many others. Larger class sizes in a school contribute to reduced learning outcomes (Card and Krueger, 1992; Angrist and Lavy, 1999; Duflo, Dupas, and Kremer, 2015; Muralidharan and Sundararaman, 2015). More patients in a hospital lead to longer waiting times (Siciliani and Hurst, 2005; Dixon and Siciliani, 2009; Kreindler, 2010). Heavier traffic on roads results in longer and more costly delays (Schrank and Lomax, 2007). An increased number of internet users slows down connectivity for everyone (Kim, Chan, and Gupta, 2007).

These congestion-prone sectors are often served by both public and private firms (Christiansen, 2011). Welfare-oriented public firms tend to charge lower prices and serve more consumers. However, a larger number of consumers raises congestion disutility. In contrast, profit-orientation prompts private firms to serve fewer consumers, resulting in lower congestion disutility. But this often comes at the cost of higher prices. How does the presence of congestion affect the choice between public and private provision? How does an increase in market size, or an increase in competition affect this choice? These are some of the questions we address in this chapter.

Two assumptions are maintained throughout the chapter. Firstly, guided by predominant findings in the empirical literature (see, for example, Megginson and Netter, 2001), we assume that public firms are inefficient, i.e., they have higher costs compared to their private counterparts. Secondly, we assume that private firms maximize profits while public firms maximize welfare. This aligns well with the theoretical literature on mixed markets. Despite profit orientation, private provision can deliver higher welfare (or even higher consumer surplus or lower prices) if inefficiency of the public firm exceeds a threshold. Our work explores how this threshold varies in the presence of congestion under the consumer surplus metric.

There exists a vast theoretical literature focusing on congestion concerns in different research fields—industrial organization (Reitman, 1991), public economics (Scotchmer, 1997; Grilo, Shy, and Thisse, 2001), operation research (Acemoglu and Ozdaglar, 2007). Additionally, positive network externalities have been widely explored in the literature (Katz and Shapiro, 1985, 1986, 1992; Farrell and Saloner, 1985; Economides, 1989, 1996; Economides and Salop, 1992; Choi, 1994; Kristiansen, 1998). These papers, however, have only considered private, profit-maximizing providers. However the sectors where consumers are more sensitive to congestion disutility are often served by both public and private firms. While the literature on mixed oligopoly shows that private provision can improve production efficiency and enhance welfare (De Fraja and Delbono, 1989; Cremer et al., 1991; Matsumura and Matsushima, 2004; Anderson et al., 1997), it overlooks the implications of congestion concerns in mixed markets. In the broader context of choice between private and public provision, the role of congestion is understudied and deserves more attention. Our research revisits the debate on public versus private provision in the presence of congestion.

The economics of privatization in network industries has long emphasized that the relative merits of public and private ownership depend on market structure, production technology, and demand characteristics. For example, network externalities can substantially weaken the case for privatization, with public ownership yielding higher social welfare when network effects are strong or efficiency gains from privatization are limited (Willner, 2006; Wang and Chiou, 2015). Empirical evidence from the telecommunications sector similarly suggests that privatization and competition are complementary, with the largest performance gains occurring when privatization is accompanied by greater market competition (Li and Xu, 2004).

A growing body of literature has also examined strategic interaction in congested service markets, where firms jointly determine prices, output, or capacity, while consumers experience congestion costs. This literature shows that congestion fundamentally alters firms' investment incentives, market outcomes, and welfare relative to conventional oligopoly models (Brueckner, 2002; Johari, Weintraub, and Roy, 2010; Perakis and Sun, 2014; Harks and Schedel, 2019). More broadly, these studies highlight the importance of accounting for congestion and capacity constraints when evaluating market performance and regulatory policies.

Our comparison of public versus private provision considers oligopoly—an imperfectly competitive market structure. We assume that each firm, public or private, can optimally choose investment to expand its capacity to reduce the perceived congestion disutility. A common theme emerges from these analyses: the threshold level of cost difference, beyond which private provision fares better than public in terms of consumer surplus, decline in the presence of congestion compared to a no-congestion scenario. In other words, the case for private provision is strengthened in the presence of congestion.

Beyond finding support for private provision, our analysis of congested markets uncovers some important insights. First, at the consumer surplus threshold where aggregate output, consumer surplus, and hedonic prices are equal across public and private provision regimes, the prices charged by public and private firms differ. Specifically, consumers served by the public firm continue to experience higher congestion disutility compared to those served by the private firm. To compensate for this higher disutility, the public firm charges a lower price relative to the private firm. Second, in the presence of congestion, the consumer surplus threshold declines with an increase in market size. That is, with the rise in potential consumers in the presence of congestion, private provision is favored more than public provision. Finally, contrary to conventional wisdom, increased competition does not necessarily promote private provision in the presence of congestion. While competition improves allocative efficiency, as in standard models, and reduces congestion disutility through smaller firm sizes, it does not alleviate inefficiencies stemming from differences in production efficiency. Private provision is more likely to generate higher consumer surplus when gains from improving production efficiency outweigh the benefits of improved allocation. This occurs when the initial level of competition is low. Thus, in the presence of congestion, competition facilitates private provision only when initial competition is limited.

Our work contributes to the existing literature in three ways. Firstly, as congestion lowers the perceived quality and satisfaction of consumers, it significantly influences the choice between public and private providers for consumers. Therefore, incorporating congestion concerns along with endogenous capacity choices into the public versus private provision debate widens our understanding of the design and the intended goal of private provision to address these concerns and improve the quality of service. Secondly, the literature on the debate around privatization has always revolved around production efficiency, profitability, prices, and welfare. Consumer concerns were absent from this debate. In light of the distributional concerns of privatization gaining center stage around the world (Estrin and Pelletier, 2018), consumer surplus emerges as a useful metric to measure the effect of private provision. Thirdly, we explore the crucial role of market size and competition influencing the debate of public versus private provision in the context of congested markets.

The plan of the chapter is as follows. Section 5.2 sets up the model. Section 5.3 describes the equilibria under public and private provision. Section 5.4 compares public and private provision using the consumer surplus as welfare metric and examines the change in the threshold in response to the presence of congestion as well as the change in market size and competition. Section 5.5 concludes. Detailed proofs are developed in the Appendix.

5.2 The Model

We consider a market consisting of a continuum of consumers of measure M who want to purchase either one unit of a homogeneous product or nothing at all. Consumers are differentiated on the basis of their willingness to pay for the product, θ. We assume that θ follows a general distribution function F with support $[\underline{\theta}, \overline{\theta}]$.

In the presence of congestion, along with price, the total number of consumers served by the firm also affects the choice of consumers. For example, there can be a longer queue, a larger class size, a longer waiting time for a kidney transplant, etc. We refer to this as *congestion disutility*—that is, quality of the product or service decreases as the number of consumers purchasing from the same provider increases. Let q_i^e denote the expected number of consumers purchasing from the same provider i.[1] Then congestion disutility increases with q_i^e. However, investment in capacity by firm i, k_i, mitigates the negative effects

[1] As each consumer buys only one unit of the product, number of consumers is the same as the amount of output provided by the firm.

of congestion on consumers. Expansion of capacity by firms to serve more consumers is a well-adopted strategy worldwide. The increase in capacities of firms reduces the waiting time, and makes the service less congested and more exclusive for the consumers. Hence the congestion disutilities experienced/ perceived by the consumers go down with the increase in the firm's capacity.

Putting the above two observations together, we postulate that congestion disutility is an increasing function of the expected number of consumers relative to the firm's capacity, i.e., an increasing function of the expected output-capacity ratio ($\frac{q_i^e}{k_i}$): $\alpha v(x_i^e)$, where $x_i^e = \frac{q_i^e}{k_i}$. The parameter $\alpha \geq 0$ captures the strength of congestion disutility, with $\alpha = 0$ standing for a complete absence of congestion disutility for all $x_i^e \geq 0$. We assume that $v(0) = 0$. Additionally, $v(x_i^e)$ is twice differentiable with $v'(x_i^e) > 0$ and $v''(x_i^e) \geq 0$ for all $x_i^e \geq 0$. That is, congestion disutility is strictly increasing and convex in expected output-capacity ratio. Thus, $\alpha v(x_i^e)$ is the extra discomfort that a consumer has to bear while making a consumption decision in the presence of congestion. Then the utility of a consumer with valuation θ is

$$u = \theta - p_i - \alpha v\left(\frac{q_i^e}{k_i}\right), \tag{5.2.1}$$

if she purchases the product paying a price p_i from firm i with expected number of consumers q_i^e and capacity k_i, and 0 otherwise.[2]

We define $\rho_i^e \equiv p_i + \alpha v\left(\frac{q_i^e}{k_i}\right)$ as the expected *hedonic price* or 'quality-adjusted price' of the product. This is a composite measure of the cost for the product to the consumer and depends not only on the price charged by the firm but also on the expected number of consumers served by it. Each consumer incurs this cost and thus faces this quality-adjusted price rather than merely the price when making her purchase decision.

[2]We assume that consumer heterogeneity enters utility only through the valuation parameter θ, while congestion disutility depends solely on the expected quantity-capacity ratio. This specification is appropriate for services in which congestion is a common quality attribute experienced similarly by all users, whereas consumers differ primarily in their willingness to pay. Examples include transportation, electricity distribution, broadband networks, and healthcare systems, where congestion affects service quality in a similar manner across consumers.

A consumer with valuation θ prefers to buy from firm i if $\forall j \neq i$

$$\theta - p_i - \alpha v\left(\frac{q_i^e}{k_i}\right) \geq \max\left\{\theta - p_j - \alpha v\left(\frac{q_j^e}{k_j}\right), 0\right\}.$$

Observe that if a consumer with valuation θ prefers buying from firm i over firm j, then every consumer prefers buying from firm i over firm j. It follows that, in equilibrium, for all the n firms to be active, it must be the case that

$$p_i + \alpha v\left(\frac{q_i^e}{k_i}\right) = p_j + \alpha v\left(\frac{q_j^e}{k_j}\right) \equiv \rho \text{ (say).}$$

That is, while firms i and j might set different prices and serve different number of consumers, their hedonic prices (as perceived by consumers) must be the same (Burdett, Shi, and Wright, 2001). Otherwise, it cannot be the case that both firms are active in the market.

Given ρ, it follows from (5.2.1) that only consumers with valuation $\theta \geq \rho = p_i + \alpha v\left(\frac{q_i^e}{k_i}\right)$ will buy the product; others will not buy it. We now impose fulfilled expectations, i.e., $q_i^e = q_i$.[3] Since each consumer buys only one unit and there is a measure M of consumers, it follows that the aggregate quantity demanded is given by

$$Q = M(1 - F(\rho)),$$

[3] We apply the notion of rational or fulfilled expectations equilibrium in our model (Kreps, 1977; Katz and Shapiro, 1985). However, following Griva and Vettas (2011), there is an alternative approach for using fulfilled expectations where consumers' expectation q_i^e aligns with firms' choice q_i only in the equilibrium and firms cannot directly control q_i^e by setting q_i. Our model follows the method of fulfilled expectations where consumer expectations are not influenced by firms' strategic choices directly. That is, firms can indeed control q_i^e by adjusting q_i. This approach aligns well with rational expectation equilibrium where the agents act as if they are too small to shift the aggregate expectations. Compared to the alternative approach where firms' choices can shape the aggregate expectations and eventually the market demand, this approach simplifies the supply side while focusing only on the effect of negative externality of congestion concerns from the demand side on the equilibrium market outcomes.

so that the *hedonic demand* is

$$\rho\left(\frac{Q}{M}\right) = F^{-1}\left(1-\frac{Q}{M}\right). \tag{5.2.2}$$

Note that $\rho'(z) = -\frac{1}{F'(\rho)} < 0$ where $z = \frac{Q}{M}$, ensuring that the hedonic demand curve is downward sloping.[4] The hedonic demand function $\rho\left(\frac{Q}{M}\right)$ is also twice-differentiable and we assume strict log-concavity of demand functions or equivalently hedonic demand functions which satisfy

Assumption 1: $\rho'(z) + \rho''(z)z < 0$, for all $z > 0$ where $z = \frac{Q}{M}$.

Given the hedonic demand, $\rho\left(\frac{Q}{M}\right)$, the inverse demand faced by firm i is given by

$$p_i = \rho\left(\frac{Q}{M}\right) - \alpha v\left(\frac{q_i}{k_i}\right) = F^{-1}\left(1-\frac{Q}{M}\right) - \alpha v\left(\frac{q_i}{k_i}\right). \tag{5.2.3}$$

On the supply side, the industry consists of n firms producing the homogeneous product and competing in quantities. We assume that m (out of n) firms are private and the remaining $n-m$ firms are public, where $0 \leq m \leq n$ and $n \geq 2$. Without loss of generality, we assume that the first m firms are private. Let q_{r_i} and q_{u_j}, respectively, denote the amounts of output produced by a private firm i and a public firm j, where $i \in \{1,2,...,m\}$ and $j \in \{m+1, m+2, ..., n\}$. Firms also choose to invest in capacities, denoted by k_{r_i} and k_{u_j}, respectively, for a private firm i and a public firm j.

The constant marginal cost of production for each private firm i is given by c_r, while the cost of investing in k_{r_i} units of capacity by private firm i is given by $\gamma_r f(k_{r_i})$. Similarly, the constant marginal cost of production of each public firm j is c_u, whereas the cost of investing in k_{u_j} units of capacity by public firm j is $\gamma_u f(k_{u_j})$. We assume that $f(0) = 0$. Moreover, $f(.)$ is twice differentiable with $f'(.) > 0$ and $f''(.) \geq 0$. To capture the relative

[4] Note that $\frac{d}{dQ}\rho\left(\frac{Q}{M}\right) = -\frac{1}{MF'(\rho)} < 0$ and $\frac{d}{dM}\rho\left(\frac{Q}{M}\right) = \frac{Q}{M^2F'(\rho)} > 0$. That is, hedonic demand is decreasing in aggregate output and increasing in market size.

inefficiency of public firms, we assume that the marginal cost of production and, for any $k > 0$, the total as well as the marginal costs of investment in capacities are higher for public firms, that is, $c_u > c_r$ and $\gamma_u > \gamma_r$.

Firms face the hedonic demand $\rho\left(\frac{Q}{M}\right)$, where $Q = \sum_{i=1}^{m} q_{r_i} + \sum_{j=m+1}^{n} q_{u_j}$. Assumption 1 implies that quantities are strategic substitutes. Profits of a private firm i and a public firm j are given by

$$\pi_{r_i} = (p_{r_i} - c_r)q_{r_i} - \gamma_r f(k_{r_i}) = \left(\rho\left(\frac{Q}{M}\right) - \alpha v\left(\frac{q_{r_i}}{k_{r_i}}\right) - c_r\right) q_{r_i} - \gamma_r f(k_{r_i}) \tag{5.2.4}$$

and

$$\pi_{u_j} = (p_{u_j} - c_u)q_{u_j} - \gamma_u f(k_{u_j}) = \left(\rho\left(\frac{Q}{M}\right) - \alpha v\left(\frac{q_{u_j}}{k_{u_j}}\right) - c_u\right) q_{u_j} - \gamma_u f(k_{u_j}), \tag{5.2.5}$$

respectively. Consumer surplus (CS) and welfare (W) respectively are:

$$CS = M\left(\int_0^{\frac{Q}{M}} \rho(z)dz - \rho\left(\frac{Q}{M}\right)\frac{Q}{M}\right), \tag{5.2.6}$$

$$\begin{aligned} W &= CS + \sum_{i=1}^{m} \pi_{r_i} + \sum_{j=m+1}^{n} \pi_{u_j}, \\ &= M\int_0^{\frac{Q}{M}} \rho\left(z\right)dz - \left(\sum_{i=1}^{m} \alpha v\left(\frac{q_{r_i}}{k_{r_i}}\right) q_{r_i} + \sum_{j=m+1}^{n} \alpha v\left(\frac{q_{u_j}}{k_{u_j}}\right) q_{u_j}\right) - \sum_{i=1}^{m}\left(c_r q_{r_i} + \gamma_r f(k_{r_i})\right) \\ &\qquad - \sum_{j=m+1}^{n}\left(c_u q_{u_j} + \gamma_u f(k_{u_j})\right). \end{aligned} \tag{5.2.7}$$

5.3 Cournot Equilibrium

Firms compete in quantities. Each private firm i chooses its output q_{r_i} and capacity k_{r_i} to maximize its own profit given by (5.2.4). A public firm, however, chooses q_{u_j} and k_{u_j} to maximize the welfare given by (5.2.7). Suppose all firms produce strictly positive amounts of output. The first-order conditions for private firm i, $\frac{\partial \pi_{r_i}}{\partial q_{r_i}} = 0$ and $\frac{\partial \pi_{r_i}}{\partial k_{r_i}} = 0$, are

$$(q_{r_i}) : \rho\left(\frac{Q}{M}\right) + \rho'\left(\frac{Q}{M}\right)\frac{q_{r_i}}{M} - \alpha\left(v(x_{r_i}) + v'(x_{r_i})x_{r_i}\right) = c_r$$

$$\Rightarrow \rho\left(\frac{Q}{M}\right) + \rho'\left(\frac{Q}{M}\right)\frac{q_{r_i}}{M} = c_r + \alpha\left(v(x_{r_i}) + v'(x_{r_i})x_{r_i}\right), \tag{5.3.1}$$

$$(k_{r_i}) : \alpha x_{r_i}^2 v'(x_{r_i}) = \gamma_r f'(k_{r_i}), \tag{5.3.2}$$

where $x_{r_i} = \frac{q_{r_i}}{k_{r_i}}$. The quantity choice of a profit maximizing private firm, as given in the first step of equation (5.3.1), equates marginal revenue $\rho\left(\frac{Q}{M}\right) + \rho'\left(\frac{Q}{M}\right)\frac{q_{r_i}}{M} - \alpha\left(v(x_{r_i}) + v'(x_{r_i})x_{r_i}\right)$ with marginal cost c_r. In the presence of congestion, firms perceive marginal congestion disutility, $\alpha\left(v(x_{r_i}) + v'(x_{r_i})x_{r_i}\right)$, as a loss of revenue and internalizes this as part of its cost. This allows us to refer to $c_r + \alpha\left(v(x_{r_i}) + v'(x_{r_i})x_{r_i}\right)$—the sum of marginal production cost and marginal congestion disutility—as *internalized marginal cost*. Clearly, $\rho\left(\frac{Q}{M}\right) + \rho'\left(\frac{Q}{M}\right)\frac{q_{r_i}}{M}$ is the firm's hedonic marginal revenue. Thus, the second step of equation (5.3.1) says that the quantity choice of a private firm is determined by the equality of its hedonic marginal revenue with internalized marginal cost. Equation (5.3.2), on the other hand, suggests that the capacity choice of a private firm requires the marginal gain from investing in capacity (in terms of reduction in congestion disutility) to be equal to the marginal cost of investment.

Similarly, the first-order conditions for public firm j, $\frac{\partial W}{\partial q_{u_j}} = 0$ and $\frac{\partial W}{\partial k_{u_j}} = 0$, are

$$(q_{u_j}) : \rho\left(\frac{Q}{M}\right) = c_u + \alpha\left(v(x_{u_j}) + v'(x_{u_j})x_{u_j}\right), \tag{5.3.3}$$

$$(k_{u_j}) : \alpha x_{u_j}^2 v'(x_{u_j}) = \gamma_u f'\left(k_{u_j}\right), \tag{5.3.4}$$

where $x_{u_j} = \frac{q_{u_j}}{k_{u_j}}$. Equation (5.3.3) says that a welfare maximizing public firm sets its optimal quantity at a point where the hedonic price is equal to its internalized marginal cost. This is an important contrast with a private firm which equates its hedonic marginal revenue with its internalized marginal cost. As we will see later, this difference will become very important in our understanding of public versus private provision in congested markets. The capacity choice given by equation (5.3.4), reflects that marginal gain from investment in capacity must be equal to the marginal cost of investment in capacity, similar to the condition for capacity choice of a private firm.

Recall that m denotes the number of private firms in a n-firm oligopoly. For notational convenience, we will slightly abuse the notation by using the subscript m to refer both to a mixed oligopoly and to a mixed oligopoly with m private firms. The intended meaning will be clear from the context. Unless the cost differences between public and private firms are too large, there exists a unique, interior Cournot equilibrium where each private firm i chooses quantity $q_{r_i}(Q_m^*)$ and capacity $k_{r_i}(q_{r_i}(Q_m^*))$, and each public firm j chooses quantity $q_{u_j}(Q_m^*)$ and capacity $k_{u_j}(q_{u_j}(Q_m^*))$, and aggregate output $Q = Q_m^*$ uniquely solves the following:

$$Q = \sum_{i=1}^{m} q_{r_i}(Q) + \sum_{j=m+1}^{n} q_{u_j}(Q).$$

In equilibrium, all m $(n-m)$ private (public) firms produce the same amount of output and choose the same level of investment in capacity.

To have a precise characterization of the equilibrium, we proceed with some simplifying steps. We assume that the following elasticities are constant,

$$\varepsilon_v = \frac{xv'(x)}{v(x)}, \text{ and } \varepsilon_f = \frac{kf'(k)}{f(k)}.$$

Substituting these elasticities into the first-order conditions of a private firm, equations

(5.3.1) and (5.3.2), and rearranging, we get

$$\rho\left(\frac{Q_m}{M}\right)+\rho'\left(\frac{Q_m}{M}\right)\frac{q_r}{M}=c_r+\alpha v(x_r)(1+\varepsilon_v), \tag{5.3.5}$$

$$\alpha v(x_r)\varepsilon_v=\frac{\gamma_r}{q_r}\varepsilon_f f\left(\frac{q_r}{x_r}\right). \tag{5.3.6}$$

Using (5.3.6), given any $\gamma_r>0$, we can express x_r as an implicit function of q_r, i.e.,

$$x_r=x(q_r;\gamma_r),$$

with $\frac{\partial x_r(.)}{\partial q_r}>0.$[5] Substituting $x_r=x(q_r;\gamma_r)$ into (5.3.5), we combine the two first-order conditions into one as

$$\rho\left(\frac{Q_m}{M}\right)+\rho'\left(\frac{Q_m}{M}\right)\frac{q_r}{M}=c_r+\alpha v(x(q_r;\gamma_r))(1+\varepsilon_v), \tag{5.3.7}$$

for each private firm i, $i=1,2,...,m$. Using similar arguments as above, given γ_u we express x_u as an implicit function of q_u, i.e.,

$$x_u=x(q_u;\gamma_u),$$

with $\frac{\partial x_u(.)}{\partial q_u}>0$. Substituting $x_u=x(q_u;\gamma_u)$ into (5.3.3), we combine the two first-order conditions into one:

$$\rho\left(\frac{Q_m}{M}\right)=c_u+\alpha v(x(q_u;\gamma_u))(1+\varepsilon_v), \tag{5.3.8}$$

for all public firm j, $j=m+1,m+2,..,n$. Figure 5.1 below illustrates the equilibrium for

[5] We define $G(x,q)=\alpha v(x)\varepsilon_v-\frac{\gamma}{q}\varepsilon_f f\left(\frac{q}{x}\right)$. Since $v(.)$ and $f(.)$ are $\mathbb{C}^2$ functions, we have $G(.)$ is also a $\mathbb{C}^2$ function. Then for any $(x,q)\in\mathbb{R}^2_{++}$, we have $G(x,q)=0$ from (5.3.6). Additionally, we have $G_x=\alpha v'(x)\varepsilon_v+\frac{\gamma}{x^2}\varepsilon_f f'\left(\frac{q}{x}\right)>0$. Therefore, we can apply the implicit function theorem and express x as a $\mathbb{C}^2$ function of q. It also follows that $\frac{dx(q,\gamma)}{dq}=-\frac{G_q}{G_x}>0$ where $G_q=-\frac{\gamma\varepsilon_f(\varepsilon_f-1)f\left(\frac{q}{x}\right)}{q^2}<0$.

the special case where $m = n-1$, that is, when there are $n-1$ private firms and 1 public firm.

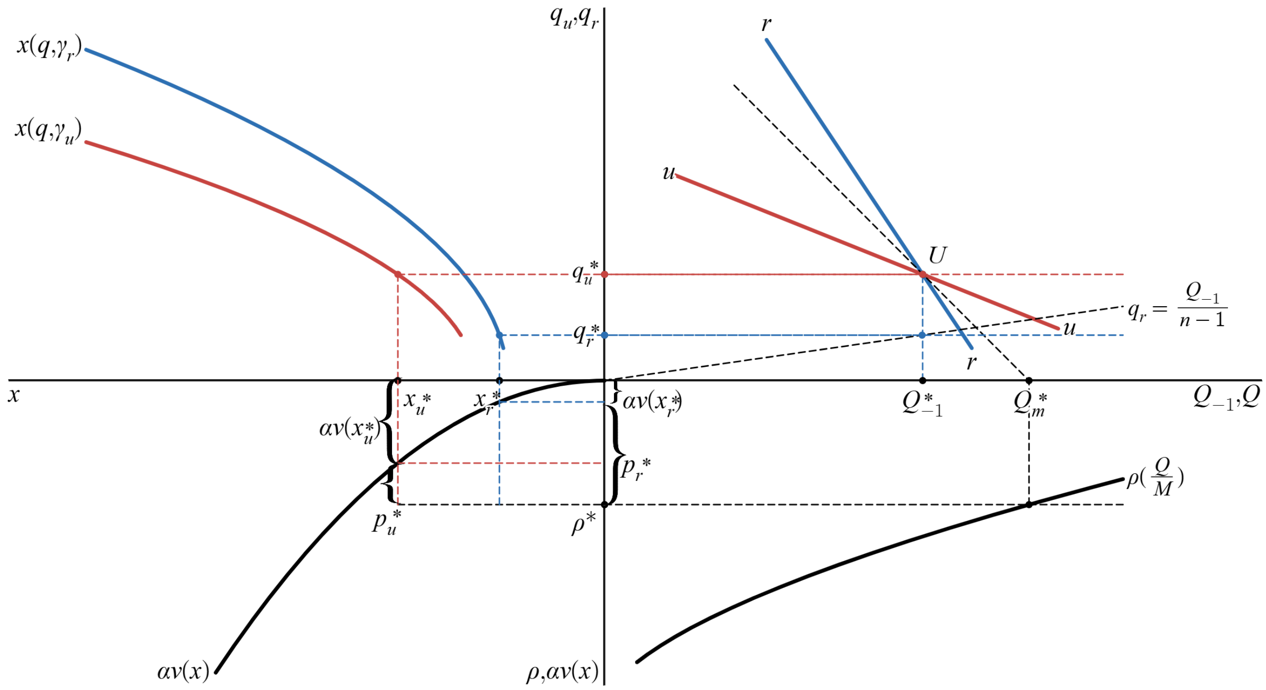


Figure 5.1: Cournot equilibrium ($n-1$ private firms, 1 public firm)

In Figure 5.1, we denote the aggregate private firm output by Q_{-1} so that individual output of each private firm q_r is given by $q_r = \frac{Q_{-1}}{n-1}$. Therefore, the aggregate output under the mixed oligopoly is given by $Q_m = q_u + (n-1)q_r = q_u + Q_{-1}$. In the first quadrant of Figure 5.1, the vertical axis shows the individual outputs of each public and private firm, i.e., q_u and q_r, respectively, while the horizontal axis represents the aggregate private firm output Q_{-1} as well as the aggregate output of all the n firms Q. The curves, rr and uu, depict the first-order conditions

$$\rho\left(\frac{q_u+Q_{-1}}{M}\right)+\rho'\left(\frac{q_u+Q_{-1}}{M}\right)\frac{Q_{-1}}{(n-1)M}=c_r+\alpha v\left(x\left(\frac{Q_{-1}}{n-1};\gamma_r\right)\right)(1+\varepsilon_v),$$
$$\rho\left(\frac{q_u+Q_{-1}}{M}\right)=c_u+\alpha v\left(x\left(q_u;\gamma_u\right)\right)(1+\varepsilon_v),$$

of a representative private firm and the public firm, respectively. When $n=2$ and $m=1$, rr and uu are the standard reaction functions of a private firm and a public firm, respectively. While the reaction function interpretation does not strictly hold when $n>2$, we still use rr and uu for the ease of diagrammatic illustration of the equilibrium.

Both rr and uu are downward sloping and rr is steeper than uu.[6] Point U where rr and uu intersects, gives the equilibrium public firm output, q_u^*, and aggregate private firm output, Q_{-1}^*. The horizontal axis intercept of the 45° line passing through the intersection point of rr and uu denotes the aggregate output in mixed oligopoly, Q_m^*. Also, the representative private firm output, q_r^*, is determined by mapping Q_{-1}^* onto the straight line given by the equation $q_r=\frac{Q_{-1}}{n-1}$. Given the equilibrium firm-level outputs q_u^* and q_r^*, we move on to demonstrate the determination of other equilibrium outcomes in the other three quadrants.

In the second quadrant of Figure 5.1, we have the output capacity ratio x on the horizontal axis and individual firm-level outputs, q_u and q_r, on the vertical axis. It follows from the properties of the implicit output-capacity ratio function $x(q;\gamma)$ that the output-capacity ratio $x(q;\gamma)$ is monotonically increasing in individual output q. The output-capacity ratios of the public firm and the representative private firm, $x(q;\gamma_u)$ and $x(q;\gamma_r)$, respectively, are represented by the two upward sloping curves in the second quadrant. Also, given

[6]Using the above two equations we derive the slopes of rr and uu as $\left.\frac{dq_u}{dQ_{-1}}\right|_{rr}=-\frac{n\rho'(.)+\frac{Q_{-1}}{M}\rho''(.)-\alpha M(1+\varepsilon_v)v'(x)x_q(.;\gamma_r)}{(n-1)\rho'(.)+\frac{Q_{-1}}{M}\rho''(.)}<0$ and $\left.\frac{dq_u}{dQ_{-1}}\right|_{uu}=-\frac{\rho'(.)}{\rho'(.)-\alpha M(1+\varepsilon_v)v'(x)x_q(.;\gamma_u)}<0$, where the signs follow from $\rho'(.)<0$, Assumption 1, $v'(x)>0$ and $x_q>0$. We also find that $\left|\frac{dq_u}{dQ_{-1}}\right|_{rr}>1$ and $\left|\frac{dq_u}{dQ_{-1}}\right|_{uu}<1$ since $\rho'(.)<0$, Assumption 1, $v'(x)>0$ and $x_q>0$. That is, not only the absolute slope of rr is greater than 1 and the absolute slope of uu is less than 1 but also the slope of rr is steeper than slope of uu.

any $q > 0$, output-capacity ratio is higher for higher γ since $x_\gamma > 0$.[7] Since $\gamma_u > \gamma_r$ by assumption, the output-capacity ratio curve for the public firm, $x(q,\gamma_u)$, lies below the output-capacity ratio curve for the representative private firm, $x(q,\gamma_r)$. By mapping q_u^* and q_r^* derived from the first quadrant onto the respective output-capacity ratio curves, we obtain the equilibrium output-capacity ratios for private and public firms, i.e., x_r^* and x_u^*, respectively.

Moving on to the third quadrant of Figure 5.1, the horizontal axis represents the output-capacity ratio x and the vertical axis represents the congestion disutility $\alpha v(x)$. The congestion disutility is increasing in output-capacity ratio x since $v'(x) > 0$. Therefore, we have an upward sloping congestion disutility curve as a function of output-capacity ratio in the third quadrant. By mapping the equilibrium output-capacity ratios x_u^* and x_r^* onto the congestion disutility curve, we find the equilibrium congestion disutilities of the representative private firm and the public firm—$\alpha v(x_r^*)$ and $\alpha v(x_u^*)$, respectively.

Finally, in the fourth quadrant of Figure 5.1, we have aggregate output Q on the horizontal axis and hedonic price ρ on the vertical axis. Given the market size M, hedonic price is decreasing in aggregate output since $\rho'(.) < 0$. This is represented by the downward sloping hedonic demand curve as a function of aggregate output in the fourth quadrant. Thus, equilibrium hedonic price ρ^* is determined by mapping Q_m^* derived in the first quadrant onto the hedonic demand curve. Then, given the equilibrium hedonic price ρ^*, the differences between the hedonic price and congestion disutilities of the respective firms determine the equilibrium prices charged by the representative private firm and the public firm—p_r^* and p_u^*, respectively. Together, using all four quadrants, Figure 5.1 demonstrates the determination of all variables in the equilibrium under mixed oligopoly for the special case where $m = n-1$.

Next, we define two types of provision: private provision and public provision.

[7] It follows from the properties of $x(q;\gamma)$ that $x_\gamma = -\frac{G_\gamma}{G_x} > 0$, where $G_\gamma = -\frac{\varepsilon_f f\left(\frac{q}{x}\right)}{q} < 0$ and $G_x = \alpha v'(x)\varepsilon_v + \frac{\gamma}{x^2}\varepsilon_f f'\left(\frac{q}{x}\right) > 0$.

Private provision

We define private provision as the market setting in which all firms are privately owned. That is, $m = n$ in our setup. The combined first-order condition for each private firm under private provision is then given by

$$\rho\left(\frac{Q^*}{M}\right) + \rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M} = c_r + \alpha v(x(q^*;\gamma_r))(1+\varepsilon_v), \tag{5.3.9}$$

for all private firm $i = 1, 2, ..., n$. In equilibrium, $Q^* = nq^*$. Therefore, we rearrange equation (5.3.9) to solve for Q^*:

$$\rho\left(\frac{Q^*}{M}\right) + \rho'\left(\frac{Q^*}{M}\right)\frac{Q^*}{nM} = c_r + \alpha v\left(x\left(\frac{Q^*}{n};\gamma_r\right)\right)(1+\varepsilon_v).$$

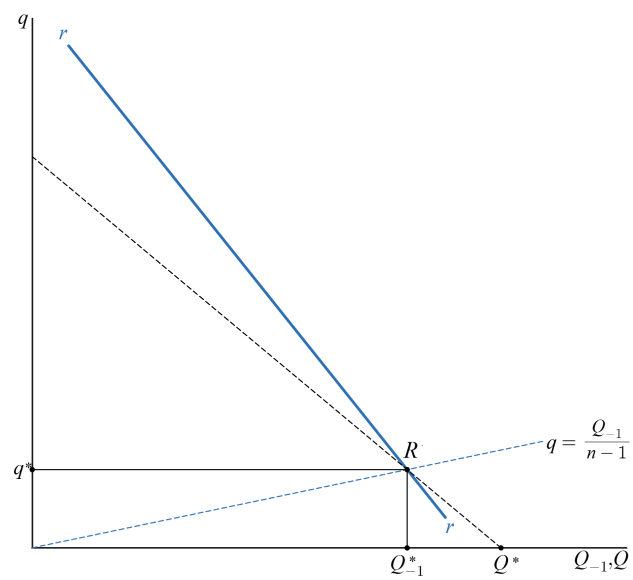


Figure 5.2: Equilibrium under Private Provision

Figure 5.2 depicts the equilibrium under private provision with $m = n$. The rr curve presents the first-order condition of a representative private firm as in Figure 5.1. Point R where the rr curve intersects with the straight line representing equation $q_r = \frac{Q_{-1}}{n-1}$, yields the equilibrium firm-level output q^*. Similarly, the horizontal intercept of the 45° line passing through the intersection point denotes the aggregate output under private provision, Q^*. The determination of other equilibrium outcomes, such as output-capacity ratio, congestion disutility, hedonic price, and price follows from q^* and Q^* as presented in quadrants 2, 3 and 4 of Figure 5.1, and are omitted so as not to clutter the figure.

Public provision

We define a public provision as the market setting in which at least one firm is publicly owned. Without loss of generality, we assume $m = n-1$ in our setup. That is, hereafter, we refer to the public provision as the market structure in which there is a single public firm and $n-1$ private firms. Equations (5.3.7) for each private firm $m = 1, 2, \ldots, n-1$ and (5.3.8) for a public firm jointly determine the equilibrium under public provision. The equilibrium under public provision is illustrated in Figure 5.1.

5.4 Public versus Private Provision

Having defined the equilibria under public and private provisions, we now turn to a comparative analysis between public and private provision with a focus on consumer surplus as the primary welfare metric. In Cournot competition, aggregate output serves as a sufficient statistic for consumer surplus.[8] Therefore, comparing total output across the two regimes provides a direct comparison of consumer surplus outcomes.

To better understand the trade-off between public and private provision, we introduce a welfare-efficiency benchmark in which a social planner maximizes welfare with the costs correspond to the costs of efficient private firms. This benchmark combines the virtues of public and private firms, i.e., welfare maximizing objective of the public firm with ef-

[8] Consumer surplus is increasing in aggregate output since $\frac{\partial CS}{\partial Q} = -\frac{Q}{M}\rho'\left(\frac{Q}{M}\right) > 0$. Therefore, we focus on the comparison of aggregate output to infer the comparison of consumer surplus.

ficiency of the private firm. The planner's combined first-order condition for each firm is

$$\rho\left(\frac{Q}{M}\right) = c_r + \alpha v(x(q, \gamma_r))(1 + \varepsilon_v).$$

Note that The internalized marginal costs under both welfare-efficiency benchmark and private provision are the same. However, the equilibrium choices differ as private firms equate the hedonic marginal revenue with the internalized marginal cost under private provision, i.e.,

$$\rho\left(\frac{Q}{M}\right) + \frac{Q}{nM}\rho'\left(\frac{Q}{M}\right) = c_r + \alpha v(x(q, \gamma_r))(1 + \varepsilon_v),$$

whereas social planner chooses the output at the firm-level by equating hedonic price with internalized marginal cost under the welfare-efficiency benchmark. This leads to lower aggregate output as well as consumer surplus under private provision relative to the benchmark, i.e.,

$$Q^s > Q_r \Leftrightarrow CS^s > CS_r.$$

Similarly, the welfare-efficiency benchmark and public provision differ only in production efficiency, i.e., $c_u > c_r$. This combined with $x_\gamma > 0$ and $\gamma_u > \gamma_r$ implies a higher internalized marginal cost under public provision. Given the first-order condition determining aggregate output under public provision,

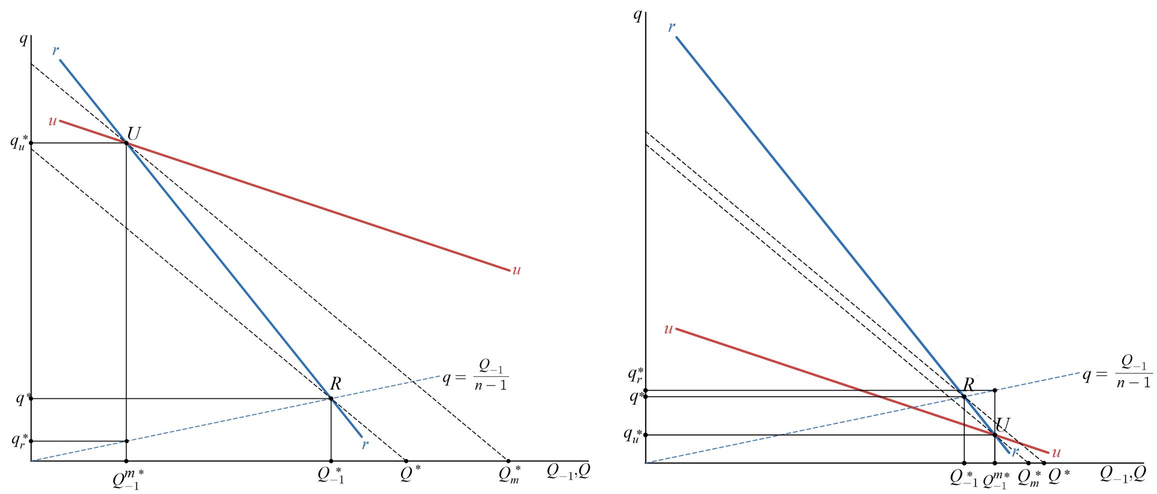
$$\rho\left(\frac{Q}{M}\right) = c_u + \alpha v(x(q, \gamma_u))(1 + \varepsilon_v),$$

the equilibrium hedonic price would be higher under public provision relative to the benchmark. Since $\rho'\left(\frac{Q}{M}\right) < 0$, it follows that switching to public provision from welfare-efficiency benchmark, therefore, leads to lower aggregate output as well as consumer surplus, i.e.,

$$Q^s > Q_u \Leftrightarrow CS^s > CS_u.$$

The welfare-efficiency benchmark therefore disentangles the two forces underlying the ownership comparison. Relative to private provision, it isolates the effect of replacing profit maximization with welfare maximization while holding production efficiency fixed. Relative to public provision, it isolates the effect of productive efficiency while holding the objective function fixed. When both forces are present simultaneously, however, their interaction makes the comparison between public and private provision non-trivial.

The central question we address is: Which provision yields a higher consumer surplus—public or private? The answer, as we shall see, is nuanced. The underlying distinction between the two regimes lies in the objectives and efficiencies of the two types of firm—private and public. Private firms are assumed to be efficient but profit-maximizing, whereas public firms are inefficient but welfare-maximizing. When both types of firms are equally efficient, i.e., they differ only in their objectives, the presence of a welfare-maximizing public firm under public provision raises aggregate output and, consequently, consumer surplus relative to private provision. However, as the inefficiency of the public firm increases—either due to higher production costs or higher costs of investment in capacities—the advantage of public provision diminishes. For sufficiently large inefficiencies of the public firm, private provision delivers higher per-capita output and consumer surplus.



(a) $c_u = c_r$ and $\gamma_u = \gamma_r$

(b) $c_u > c_r$ and $\gamma_u > \gamma_r$

Figure 5.3: Comparison of aggregate output under public and private provisions

Figure 5.3 illustrates the comparison between public and private provisions presenting the equilibria under the two provisions in the same figure in (Q_{-1}, q) space, where q and Q_{-1} denote firm-level output and aggregate output of $n-1$ firms, respectively. It is easy to see from the first-order conditions representing the private and public provisions that, other things remaining the same, if either c_u or γ_u or both increase, then the reaction function of the public firm, uu, shifts to the left. We use this feature to illustrate the comparison between public and private provisions in Figure 5.3.

In the absence of inefficiency of the public firm, i.e., $c_u = c_r$ and $\gamma_u = \gamma_r$, as shown in Figure 5.3(a), the presence of the welfare-maximizing public firm along with $n-1$ private firms under public provision strictly yields a higher aggregate output as well as a higher consumer surplus compared to private provision with n private firms, i.e., $Q_m^* > Q^*$ and $CS_m > CS^*$. However, with an increase in inefficiency, i.e., $c_u > c_r$ and $\gamma_u > \gamma_r$, the role of efficient private firms becomes stronger. As a result, aggregate output and consumer surplus under public provision fall behind aggregate output and consumer surplus under private provision, i.e., $Q_m^* < Q^*$ and $CS_m < CS^*$. Figure 5.3(b) presents the equilibrium outcomes under inefficiencies of the public firm. As illustrated in Figures 5.3(a) and 5.3(b), there is a clear trade-off between public and private provision in an oligopoly market structure. For sufficiently higher level of inefficiencies, private provision can indeed outperform public provision. Therefore, there exists a threshold level of relative inefficiency of the public firm beyond which private provision yields a higher consumer surplus.

To formalize this comparison, let us revisit the first-order conditions for private and public firms under the two provisions. In private provision, the first-order condition of each private firm is

$$\rho\left(\frac{Q^*}{M}\right)+\frac{q^*}{M}\rho'\left(\frac{Q^*}{M}\right)=c_r+\alpha v(x(q^*;\gamma_r)(1+\varepsilon_v),$$

where Q^* and q^*, respectively, denote aggregate output and individual output of each private firm in equilibrium. Under public provision, the corresponding first-order conditions of private and public firms are

$$\begin{aligned}\rho\left(\frac{Q_m^*}{M}\right)+\frac{q_r^*}{M}\rho'\left(\frac{Q_m^*}{M}\right)&=c_r+\alpha v(x(q_r^*;\gamma_r))(1+\varepsilon_v),\\ \rho\left(\frac{Q_m^*}{M}\right)&=c_u+\alpha v(x(q_u^*;\gamma_u))(1+\varepsilon_v),\end{aligned}$$

where Q_m^* denotes the aggregate output, and q_r^* and q_u^*, respectively, are the individual outputs of private and public firms in equilibrium under public provision. Given c_r and γ_r, let Q^* satisfy the first-order condition above for a private firm under private provision. For the equality of consumer surplus under both private and public provision, we need $Q_m^*=Q^*$. Using the first-order conditions above for each private firm under private and public provision we find that $Q_m^*=Q^*$ implies that each private firm produces the same level of output irrespective of the type of provision, i.e.,

$$Q_m^*=Q^*\Rightarrow q_r^*=q^*.$$

This further implies that the public firm, under public provision, also produces the same level of output as the other private firms when aggregate output under both provisions are equal, i.e.,

$$Q_m^* = Q^* \Rightarrow q_u^* + (n-1)q_r^* = q^* + (n-1)q^* \Rightarrow q_u^* = q^* = q_r^*.$$

Using $Q_m^* = Q^*$ and $q_u^* = q^* = q_r^*$, under public provision, we can subtract the first-order condition for a private firm from that of a public firm to derive

$$\Delta_c + \alpha(1+\varepsilon_v)\left[v(x(q^*;\gamma_r+\Delta_\gamma)) - v(x(q^*;\gamma_r))\right] = -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M},$$

where $\Delta_c = c_u - c_r$ and $\Delta_\gamma = \gamma_u - \gamma_r$ respectively denote the relative production inefficiency and relative inefficiency in cost of investment in capacities of the public firm compared to its private counterpart. That is, for $Q_m^* = Q^*$ to satisfy the first-order conditions for public provision, we need the relative production inefficiency of the public firm, Δ_c, to be such that the above equality holds. It follows that

$$CS_m \gtreqless CS^*, Q_m^* \gtreqless Q^* \Leftrightarrow \Delta_c \lesseqgtr -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M} - \alpha(1+\varepsilon_v)\left[v(x(q^*;\gamma_r+\Delta_\gamma)) - v(x(q^*;\gamma_r))\right].$$

Proposition 5.1 states the central result on consumer surplus comparison between public and private provisions.

Proposition 5.1. *Let Δ_c and Δ_γ, respectively, define the relative production inefficiency and relative inefficiency in cost of investment in capacities between public and private firms. For any given $\Delta_\gamma \geq 0$, there exists a production inefficiency threshold of public firm, $\Delta_c^{CS}(\Delta_\gamma)$, such that private provision yields higher consumer surplus and aggregate output and lower hedonic price if and only if*

$$\Delta_c \geq \Delta_c^{CS} \equiv -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M} - \alpha(1+\varepsilon_v)\left[v(x(q^*;\gamma_r+\Delta_\gamma)) - v(x(q^*;\gamma_r))\right]. \tag{5.4.1}$$

Furthermore, $\frac{d\Delta_c^{CS}}{d\Delta_\gamma} < 0$*, i.e., an inefficiency in production cost could be compensated by an efficiency in the cost of investment in capacities.*

Proof. See the above discussion for the derivation of the threshold. It also follows that

$$\frac{d\Delta_c^{CS}}{d\Delta_\gamma} = -\alpha(1+\varepsilon_v)v'(x(q^*;\gamma_r+\Delta_\gamma))x_\gamma(q^*;\gamma_r+\Delta_\gamma) < 0,$$

where the sign follows from $v'(.) > 0$ and $x_\gamma(.) > 0$. □

Note that in the absence of a relative inefficiency of the public firm in the cost of investment in capacities, i.e., $\Delta_\gamma = 0$, the threshold simplifies to:

$$\Delta_c^{CS} = -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M}.$$

In contrast, in the presence of $\Delta_\gamma > 0$, the threshold decreases and strengthens the case for private provision since

$$\Delta_c^{CS} = -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M} - \alpha(1+\varepsilon_v)\left[v(x(q^*;\gamma_u)) - v(x(q^*;\gamma_r))\right] < -\rho'\left(\frac{Q^*}{M}\right)\frac{q^*}{M},$$

where $v(x(q^*;\gamma_r+\Delta_\gamma)) > v(x(q^*;\gamma_r))$ whenever $\Delta_\gamma > 0$, since $x_\gamma > 0$. This also implies that, at the consumer surplus equality threshold, although each individual firm produces an identical level of output under the two provisions, the public firm chooses to invest less in capacity expansion due to higher marginal investment costs. As a result, it operates with a higher output-to-capacity ratio compared to private firms.

At the consumer surplus equality threshold, per-capita output, $\frac{Q}{M}$, is equal across regimes, leading to identical hedonic prices. However, this does not necessarily imply equality of prices. Since the public firm operates with a higher output-capacity ratio, thereby passing on higher congestion disutility to its customers, it charges a lower price to offset the adverse impact of congestion disutility experienced by its customers, i.e.,

$$p_u + \alpha v(x(q^*; \gamma_r + \Delta_\gamma)) = p_r + \alpha v(x(q^*; \gamma_r)) \Rightarrow p_u < p_r,$$

since $v(x(q^*; \gamma_r + \Delta_\gamma)) > v(x(q^*; \gamma_r))$. Given $\Delta_\gamma \geq 0$, does there exist a production-efficiency threshold at which prices under public and private provisions are equalized? We examine this question below.

The weighted average prices under public and private provisions at the consumer surplus threshold are

$$P_u = \frac{p_u q^* + (n-1) p_r q^*}{Q^*}, \text{ and } P_r = \frac{p_r q^* + (n-1) p_r q^*}{Q^*},$$

respectively. The difference in the weighted average prices reduces to

$$P_u - P_r = \frac{(p_u - p_r)}{n} < 0,$$

i.e., the difference between prices charged by public and private firms under public provision at $\Delta_c = \Delta_c^{CS}$. Given $\Delta_\gamma > 0$, the equality in prices p_u and p_r, thereby, can be restored by raising p_u through raising Δ_c.[9] Let $\Delta_c^p(\Delta_\gamma)$ denote the production efficiency threshold at which *prices* under public and private provisions are equalized, that is, $P_u = P_r$ for a given $\Delta_\gamma \geq 0$. It follows immediately that the price threshold exceeds the consumer surplus threshold, i.e., $\Delta_c^{CS}(\Delta_\gamma) \leq \Delta_c^p(\Delta_\gamma)$. Consequently, for any $\Delta_\gamma \geq 0$, there exists an interval

[9] Differentiating p_u with respect to c_u, we find that $\frac{\partial p_u}{\partial c_u} = \frac{\rho'\left(\frac{Q}{M}\right)}{M} \frac{\partial Q}{\partial c_u} - \alpha v'(x(q, \gamma_u)) x_q \frac{\partial q}{\partial c_u} > 0$, since $x_q > 0$, $v'(x) > 0$ $\rho'\left(\frac{Q}{M}\right) < 0$, $\frac{\partial Q}{\partial c_u} = \frac{1}{\frac{\rho'\left(\frac{Q}{M}\right)}{M}} < 0$ and $\frac{\partial q}{\partial c_u} = \frac{1}{\frac{\rho'\left(\frac{Q}{M}\right)}{M} - \alpha(1+\varepsilon_v) v'(x(q, \gamma_u)) x_q} < 0$, where the last two follows from differentiating equation (5.3.8).

$(\Delta_c^{CS}(\Delta_\gamma), \Delta_c^p(\Delta_\gamma))$ over which private provision yields higher consumer surplus relative to public provision (i.e., $CS_r > CS_u$) but it comes at higher prices (i.e., $P_r > P_u$).

Next, we turn to compare the welfare under public and private provisions at the consumer surplus threshold, $\Delta = \Delta_c^{CS}(\Delta_\gamma)$. At the consumer surplus threshold, firms produce identical outputs irrespective of their ownership type $q_u^* = q^* = q_r^*$, but the public firm invests less in capacity relative to private firm, i.e., $k_u^* < k^* = k_r^*$. Consequently, welfare differs only through production costs and investment cost in capacity. Specifically, welfare difference at the consumer surplus threshold is given by:[10]

$$W_u^* - W_r^* = -(c_u - c_r)q^* - ((\gamma_r + \Delta_\gamma) f(k_u^*) - \gamma_r f(k^*)) < 0,$$

implying that welfare under public provision is lower relative to that under private provision even though both regimes generate the same consumer surplus. Then, given $\Delta_\gamma \geq 0$, the only way to restore the equality of welfare under both provisions, i.e., $W_u^* = W_r^*$, is by raising W_u^* through reducing Δ_c.[11] Let $\Delta_c^W(\Delta_\gamma)$ denote the production efficiency threshold at which *welfare* under public and private provisions are equalized, that is, $W_u^* = W_r^*$ for a given $\Delta_\gamma \geq 0$. It follows immediately that the welfare threshold lies strictly below the consumer surplus threshold, i.e., $\Delta_c^W(\Delta_\gamma) \leq \Delta_c^{CS}(\Delta_\gamma)$. The difference between the two thresholds arises from how they treat allocative and productive efficiency—loosely speaking, how they weigh prices versus costs. Allocative efficiency considerations matter for both consumer surplus and welfare, whereas productive efficiency matters only for the welfare. Because productive efficiency is highest under private provision—that is, total production costs and investment costs in capacity are lowest—the private provision performs relatively better when allocative efficiency is achieved, i.e., $\Delta_c = \Delta_c^{CS}(\Delta_\gamma)$.

[10] Differentiating the first-order conditions of firms with respect to k, we get $\frac{\partial k}{\partial \gamma} = -\frac{1}{\alpha \frac{\varepsilon_v(\varepsilon_v+1)}{\varepsilon_f} \frac{v(x)}{f(k)} + \gamma \frac{\varepsilon_f - 1}{k}} < 0$ and $|\frac{\partial k}{\partial \gamma}| < 1$. It follows that a marginal increase in γ lowers equilibrium capacity k by less than proportionately, implying that $\gamma_u f(k_u^*) > \gamma_r f(k_r^*)$ when $\gamma_u > \gamma_r$.

[11] Differentiating the expression of W_u with respect to c_u, we get $\frac{\partial W_u}{\partial c_u} = -q_u < 0$.

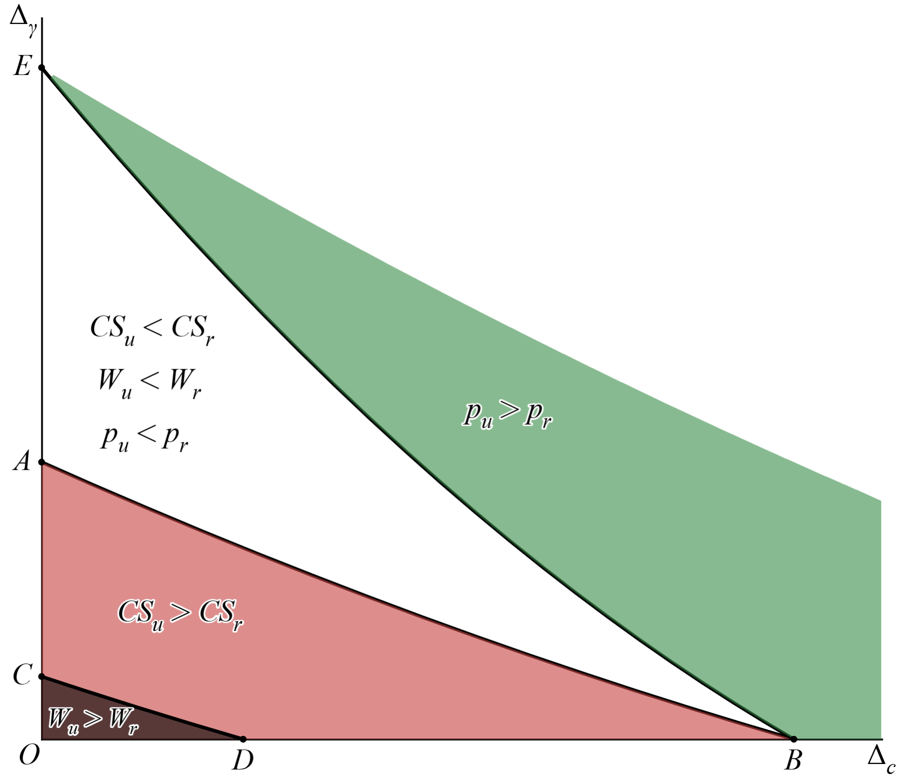


Figure 5.4: Public versus private provision (consumer surplus, prices and welfare)

Figure 5.4 summarizes the comparison between public and private provisions using three performance measures: consumer surplus, welfare, and prices. The curves *AB*, *CD*, and *EB* represent the consumer-surplus, welfare, and price thresholds, respectively. Public provision yields higher consumer surplus, higher welfare, and lower prices in the regions below the corresponding threshold curves, while the reverse holds above each curve. The figure also highlights a region between the consumer-surplus threshold (*AB*) and the price threshold (*EB*) in which private provision generates both higher consumer surplus and higher welfare despite charging higher prices. More generally, the welfare criterion is the most favorable to private provision, whereas the price criterion is the most favorable to public provision, with consumer surplus lying between the two.

Although we have characterized the production efficiency thresholds under the welfare, consumer surplus, and price criteria and established their relative ranking, the remainder of the analysis focuses on the threshold based on consumer surplus. Having identified the parameter space (the space identified by the inefficiency parameters Δ_c and Δ_γ, following equation (5.4.1)) where private provision dominates public provision form the view point of the consumers, or vice versa, we next examine how alternative market structures and market conditions affect this comparison. In what follows, we explore in particular, given any $\Delta_\gamma \geq 0$, how the production inefficiency threshold using consumer surplus standard, i.e., $\Delta_c^{CS}(\Delta_\gamma)$ responds to the presence of congestion (relative to the no-congestion scenario), changes in market size, and variations in the degree of market competition. For this purpose, it is essential to first examine how aggregate outputs under the two provisions, public and private, respond to the changes in these market structures and conditions. Thus, in order to pave the way for the comparison, we first conduct some comparative static analysis under private and public provisions in the next subsection. These comparative static results then form the basis for analyzing how the production inefficiency threshold varies with congestion, market size, and changes in the level of competition in the following subsection.

Comparative static analysis

We focus on the change in equilibrium outcomes under public and private provisions with respect to the presence of congestion, changes in market size, and competition. Consider private provision first. Starting with congestion, compared to a no-congestion economy, i.e., $\alpha = 0$, in the presence of congestion, the internalized marginal cost of each private firm is higher. As a result, each private firm reduces its output. Thus, aggregate output is lower in the presence of congestion. This leads to a fall in per-capita output, and since the hedonic price is inversely related to per-capita output, the hedonic price is higher.

An increase in market size M, however, reflects the growth in the number of potential consumers in the market, affecting the output and capacity decisions of firms in equilibrium. In the absence of congestion, as market size increases, the hedonic demand curve swivels outward, resulting in an increase in market demand. Each private firm expands their output in response to increased demand, resulting in higher aggregate production in the market. However, this increase in aggregate output fully matches the increase in market size since the internalized marginal cost is constant, leaving the per-capita output, hedonic price, as well as price charged by private firms unchanged. In contrast, in the presence of congestion, while a similar expansionary effect can be seen on firm-level output as well as aggregate output, that is not the case for per-capita output. In the presence of congestion, the increasing internalized marginal cost dampens the output response. As a result, the increase in aggregate output fails to match the increase in market size, leading to a decline in per-capita output and consequently an increase in the hedonic price.

Finally, we examine the effects of increased competition on equilibrium outcomes. In the absence of congestion, an increase in the number of firms leads to a reduction in firm-level outputs for each private firm reflecting the standard business-stealing effect. However, aggregate output rises. Consequently, per-capita output rises and hedonic price as well as the price fall. Even in the presence of congestion, the effects on firm-level output and aggregate output are the same. This leads to a higher per-capita output and lower hedonic prices.

Lemma 5.1 summarizes the changes in equilibrium outcomes, under private provision, with respect to presence of congestion (relative to a no-congestion economy), changes in market size and varying degree of market competition.

Lemma 5.1. *Under private provision with n private firms, we have the following comparative statics results.*

(a) In the presence of congestion, while firm-level output, aggregate output, and per-capita output fall, hedonic price rises relative to a no-congestion economy.

(b) Increase in market size M leads to higher firm-level output, higher aggregate output,

lower per-capita output and higher hedonic price.

(c) Increase in competition, i.e., an increase in the number of firms n, results in lower firm-level output, higher aggregate output, higher per-capita output and lower hedonic price.

Proof. See Appendix 5.6.1. □

Consider next the public provision. We start with the role of congestion. Compared to a no-congestion economy, i.e., $\alpha = 0$, as congestion intensifies, the internalized marginal costs for both public and private firms increase. This results in inward shifts of the reaction functions for both types of firms. However, this may not necessarily lead to a reduction in firm-level output by either public or private firms in equilibrium. Both types of firms reduce their outputs as a response to the increasing internalized marginal cost in the presence of congestion. At the same time, since quantities are strategic substitutes, both public and private firms also have the incentive to increase their outputs due to strategic considerations. The net effect, therefore, depends on how inefficient the public firm is. However, at least one type of firm, public or private, will reduce its output in equilibrium. Nevertheless, aggregate output declines, leading to a fall in per-capita output. Since the hedonic price is inversely related to per-capita output, the hedonic price rises.

An increase in market size M affects the output and capacity decisions of public and private firms in equilibrium. In the absence of congestion, as market size M expands, the hedonic demand swivels outward, resulting in an increase in market demand. Both public and private firms expand their outputs in response to the increased demand, resulting in a higher aggregate production in the market. However, this increase in aggregate output would fully match the increase in market size due to constant marginal cost, leaving the per-capita output, hedonic price, as well as prices charged by public and private firms unchanged. In contrast, in the presence of congestion, firm-level outputs as well as aggregate output increase, but the per-capita output does not. In the presence of congestion, the increase in aggregate output fails to match the increase in market size due to increasing internalized marginal cost for both public and private firms, resulting in a decline in per-capita output and, consequently, an increase in hedonic price.

Finally, we consider the effects of increased competition. In the absence of congestion, an increase in the number of firms leads to a reduction in firm-level output for only the public firm leaving the firm-level output for each private firm as well as the aggregate output unchanged. Consequently, there is no change in per-capita output, hedonic prices, or the prices charged by either public or private firms. In contrast to this, in the presence of congestion, both public and private firms reduce their outputs reflecting the standard business-stealing effect. However, aggregate output increases with increased competition. This leads to a higher per-capita output and a lower hedonic price.

Lemma 5.2 presents the comparative static results of the equilibrium under public provision with respect to presence of congestion (relative to a no-congestion economy), changes in market size and varying degree of market competition.

Lemma 5.2. *Under public provision with* 1 *public firm and* $n-1$ *private firms, we have the following comparative statics results.*

(a) In the presence of congestion, aggregate output falls and there is an ambiguous change in firm-level outputs by both public and private firms relative to a no-congestion economy. However, per-capita output falls, and the hedonic price rises.

(b) Increase in market size M leads to higher firm-level outputs for both public and private firms, as well as higher aggregate output, lower per-capita output and higher hedonic price.

(c) Higher competition, i.e., an increase in the number of firms n, results in lower firm-level outputs for both public and private firms but higher aggregate output, higher per-capita output, and lower hedonic price.

Proof. See Appendix 5.6.2. □

Public versus private provision: Role of congestion, market size and competition

Having derived the results from the comparative static analysis, we now proceed to show how presence of congestion, along with changes in market size and competition shifts the production inefficiency threshold and, consequently, tilts the balance toward private provision. We start with the role of congestion. The presence of congestion plays a pivotal role in shaping the choice between public and private provision. As evident from the threshold itself, in the absence of congestion, i.e., $\alpha = 0$, the threshold simplifies to

$$\Delta_c^{CS} = -\rho' \left(\frac{Q^*}{M} \right) \frac{q^*}{M}$$

which is strictly higher than the production inefficiency threshold derived in the presence of congestion, i.e., $\alpha > 0$ as stated in Proposition 5.1. Compared to a no-congestion economy, the introduction of congestion disutilities reduces aggregate outputs under both private and public provisions due to the increase in internalized marginal costs for all firms, as stated in Lemmas 5.1 and 5.2 above. However, the declines are not symmetric. Under private provision, each firm reduces output by aligning *hedonic marginal revenue* with the higher internalized marginal cost. Under public provision, the output choice of the public firm is determined by equalizing the *hedonic price* with the higher internalized marginal cost. Thus, the public firm responds more aggressively as compared to a private firm, leading to a sharper decline in its output. This results in a lower aggregate output under public provision. Since the aggregate output Q_m is decreasing in Δ_c,[12] to restore consumer surplus

[12] It follows from differentiating the first-order conditions of private and public firms under public provision given by equations (5.3.7) and (5.3.8), respectively, and using Cramer's rule that $\frac{dQ_m}{d\Delta_c} = \frac{M\left[\rho'(.) - M\alpha(1+\varepsilon_v)v'(x)x_q(q_r;\gamma_r)\right]}{\rho'(.)\left[\rho'(.) - M\alpha(1+\varepsilon_v)v'(x)x_q(q_r;\gamma_r)\right] - M\alpha(1+\varepsilon_v)v'(x)x_q(q_u;\gamma_u)\left[n\rho'(.) + (n-1)\frac{q_r}{M}\rho''(.) - M\alpha(1+\varepsilon_v)v'(x)x_q(q_r;\gamma_r)\right]} < 0$, where the sign

equivalence, the production inefficiency threshold—Δ_c—must fall. Therefore, the presence of congestion itself increases the likelihood of the domination of private provision. Proposition 5.2 summarizes this result.

Proposition 5.2. *The production inefficiency threshold—Δ_c^{CS}—falls in the presence of congestion disutilities relative to a no-congestion economy. That is, the presence of congestion facilitates private provision.*

Proof. See Appendix 5.6.3. □

A similar pattern emerges in the behavior of the production inefficiency threshold as market size M increases. An increase in market size increases demand under both provisions as explained by Lemmas 5.1 and 5.2 above. Public and private firms respond to the increase in demand by increasing their productions, leading to an increase in aggregate output under both private and public provisions. However, the magnitudes of the increase differ. The presence of the public firm under public provision along with its inefficiencies—not only in terms of higher marginal production costs, but also in terms of higher costs of investment in capacities—makes the expansion in aggregate output more restrained compared to private provision with n equally efficient private firms. Consequently, aggregate output increases more under private provision than under public provision. Since Q_m is decreasing in Δ_c, to maintain the consumer surplus equivalence, a larger market size necessitates a lower threshold Δ_c^{CS}. This implies, once again, that a larger market size strengthens the case for private provision. Proposition 5.3 presents the comparative static result of the effects on the production inefficiency threshold with respect to changes in market size.

Proposition 5.3. *An increase in market size M reduces the production inefficiency threshold Δ_c^{CS}. That is, as the market expands, private provision becomes more likely to yield a higher consumer surplus than public provision in the presence of congestion.*

follows from Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$.

Proof. See Appendix 5.6.3. □

In contrast to the effects of the presence of congestion or an increase in market size, an increased competition, that is, an increase in the number of firms n, does not unambiguously lead to a decline in the production inefficiency threshold Δ_c^{CS} in the presence of congestion. In the absence of congestion, as the number of firms increases, the aggregate output under public provision stays unchanged while aggregate output under private provision increases, as presented in Lemmas 5.1 and 5.2 above. As a result, Δ_c^{CS} always declines with an increase in n when congestion is absent.

The presence of congestion adds to the internalized marginal costs of both public and private firms, with the public firm incurring an additional component of increased cost due to its inefficiency in investing in capacities. Then, although an increase in competition raises aggregate output under both public and private provisions, the relative magnitudes of these increases differ. The relative magnitudes, in fact, crucially depend on the level of competition already present in the market. When competition is limited, as the number of firms increases, the aggregate output under public provision increases less than the aggregate output under private provision. This is because insufficient competition fails to offset the output-restraining effects of public firms' inefficiencies and heightened disutility to congestion under public provision. In other words, the gains from allocative efficiency under public provision are outweighed by the losses from production inefficiency when competition is limited. In this scenario, Δ_c^{CS} declines with an increase in n. But with higher competition, the competitive pressure becomes strong enough to mitigate the inefficiencies of the public firm. In fact, the public firm's welfare-maximizing objective, when reinforced by adequate competition, leads to a greater expansion of aggregate output under the public provision relative to the private provision. This results in an increase in Δ_c^{CS} with the rise in competition. Therefore, there exists a threshold level of competition, i.e., number of firms $\overline{n}$, below which private provision yields higher consumer surplus. That is, Δ_c^{CS} declines with an increase in n if and only if $n < \overline{n}$.

This ambiguity regarding the effect of competition production inefficiency threshold arises only in the presence of congestion. In the absence of congestion, as explained above, the production inefficiency threshold unambiguously decreases with an increase in competition. Even in the presence of congestion, the threshold decreases with competition when there is too little competition in the market. However, when n is large, an increase in competition can facilitate public provision. Proposition 5.4 states the change in the production inefficiency threshold with respect to the degree of competition.

Proposition 5.4. *An increase in the number of firms n has an ambiguous effect on the production inefficiency threshold Δ_c^{CS}. Greater competition facilitates private provision in the presence of congestion if and only if the initial level of competition is low enough, i.e., for $n < \overline{n}$.*

Proof. See Appendix 5.6.3. □

5.5 Conclusion

Congestion concerns are prevalent in healthcare, education, telecommunication, public transport and several other industries which are served by both public and private firms. Incorporating congestion disutility in an imperfectly competitive setting, we examine the effect of congestion under both public and private provisions. Our key finding is that presence of congestion concerns favors private provision. We also show that in the presence of congestion, an expansion in market size strengthens the case for private provision. However, the effect of increased competition is more nuanced. Specifically, greater competition promotes private provision only when the initial level of competition is limited.

In this chapter, we have relied primarily on the consumer surplus metric to compare public and private provision. Although this is a key measure, it is not only one relevant to policy. Policymakers are often equally concerned with outcomes such as prices and overall welfare associated with each form of provision. Moreover, the optimal choice between public and private provision may also depend on the degree of income inequality present in the economy. We plan to investigate these issues in future research.

5.6 Appendix

5.6.1 Comparative static analysis under private provision

Proof of Lemma 5.1:

(a) Differentiating equation (5.3.9), we find that

$$\frac{dQ^*}{d\alpha} = \frac{nMv(x^*)(1+\varepsilon_v)}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)} < 0,$$
$$\frac{dq^*}{d\alpha} = \frac{1}{n}\frac{dQ^*}{d\alpha} < 0,$$

where the signs follow from Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$. Evaluating at $\alpha = 0$, we have

$$\left.\frac{dQ^*}{d\alpha}\right|_{\alpha=0} = \frac{nMv(x^*)(1+\varepsilon_v)}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)} < 0,$$
$$\left.\frac{dq^*}{d\alpha}\right|_{\alpha=0} = \frac{1}{n}\left.\frac{dQ^*}{d\alpha}\right|_{\alpha=0} < 0.$$

The changes in per-capita output and hedonic prices are given by

$$\frac{d}{d\alpha}\left(\frac{Q^*}{M}\right) = \frac{1}{M}\frac{dQ^*}{d\alpha} < 0, \text{ and } \frac{d}{d\alpha}\rho\left(\frac{Q^*}{M}\right) = \frac{1}{M}\rho'\left(\frac{Q^*}{M}\right)\frac{dQ^*}{d\alpha} > 0,$$

since $\rho'(.) < 0$. Evaluated at $\alpha = 0$, the signs remain same.

(b) Differentiating equation (5.3.9), we have

$$\frac{dQ^*}{dM} = \frac{\frac{Q^*}{M}\left[(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)\right]}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)} > 0,$$

$$\frac{dq^*}{dM} = \frac{1}{n}\frac{dQ^*}{dM} > 0,$$

where the signs follow from Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$. Changes in per-capita output and hedonic price are given by

$$\frac{d}{dM}\left(\frac{Q^*}{M}\right) = \frac{1}{M}\left(\frac{dQ^*}{dM} - \frac{Q^*}{M}\right) = \frac{\frac{Q^*}{M}\alpha(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)} < 0,$$

$$\frac{d}{dM}\rho\left(\frac{Q^*}{M}\right) = \rho'\left(\frac{Q^*}{M}\right)\frac{d}{dM}\left(\frac{Q^*}{M}\right) > 0,$$

where the signs follow from Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$.

(c) Differentiating equation (5.3.9), we have

$$\frac{dQ^*}{dn} = \frac{\frac{Q^*}{n}\left[\rho'\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)\right]}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)} > 0,$$

$$\frac{dq^*}{dn} = -\frac{\frac{Q^*}{n^2}\left[n\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)\right]}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - \alpha M(1+\varepsilon_v)v'(x^*)x_q(q^*;\gamma_r)} < 0,$$

where the signs follow from Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$. The changes in per-capita output and hedonic price are given by

$$\frac{d}{dn}\left(\frac{Q^*}{M}\right) = \frac{1}{M}\frac{dQ^*}{dn} > 0, \text{ and } \frac{d}{dn}\rho\left(\frac{Q^*}{M}\right) = \rho'\left(\frac{Q^*}{M}\right)\frac{d}{dn}\left(\frac{Q^*}{M}\right) < 0,$$

since $\rho'(.) < 0$.

5.6.2 Comparative static analysis under public provision

Proof of Lemma 5.2:

Proof. (a) Differentiating equations (5.3.7) and (5.3.8), we have

$$A\begin{pmatrix}\frac{dq_u}{d\alpha}\\ \frac{dq_r}{d\alpha}\end{pmatrix}=B,$$

where

$$A=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix},\ \text{and}\ B=\begin{pmatrix}M(1+\varepsilon_v)v(x_r)\\ M(1+\varepsilon_v)v(x_u)\end{pmatrix}$$

where $a_{11}=\rho'\left(\frac{Q_m}{M}\right)+\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)$, $a_{12}=n\rho'\left(\frac{Q_m}{M}\right)+(n-1)\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)-\alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)$, $a_{21}=\rho'\left(\frac{Q_m}{M}\right)-\alpha M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)$, and $a_{22}=(n-1)\rho'\left(\frac{Q_m}{M}\right)$. x_u and x_r are given by $x_u=x(q_u;\gamma_u)$ and $x_r(q_r;\gamma_r)$ respectively. The determinant of A is as follows:

$$detA\Big|_{\alpha=0}=-\rho'\left(\frac{Q_m}{M}\right)^2<0,$$

since $\rho'(.)<0$. Following Cramer's rule we find that

$$\frac{dq_u}{d\alpha}\Big|_{\alpha=0}=\frac{M(1+\varepsilon_v)\left[-(n-1)\rho'\left(\frac{Q_m}{M}\right)(v(x_u)-v(x_r))-v(x_u)\left(\rho'\left(\frac{Q_m}{M}\right)+(n-1)\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)\right)\right]}{detA}$$

$$\frac{dq_r}{d\alpha}\Big|_{\alpha=0}=\frac{M(1+\varepsilon_v)\left[\rho'\left(\frac{Q_m}{M}\right)(v(x_u)-v(x_r))+\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)v(x_u)\right]}{detA},$$

$$\frac{dQ_m}{d\alpha}\Big|_{\alpha=0}=-\frac{M(1+\varepsilon_v)v(x_u)\rho'\left(\frac{Q_m}{M}\right)}{detA}<0.$$

The sign of $\left.\frac{dQ_m}{d\alpha}\right|_{\alpha=0}$ follows from $\rho'(.) < 0$. The signs of $\frac{dq_u}{dn}$ and $\frac{dq_r}{d\alpha}$, however, are not unambiguous under the assumptions of the model. The changes in per-capita output and hedonic price are given by

$$\left.\frac{d}{d\alpha}\left(\frac{Q_m}{M}\right)\right|_{\alpha=0} = \left.\frac{1}{M}\frac{dQ_m}{d\alpha}\right|_{\alpha=0} < 0, \text{ and } \frac{d}{d\alpha}\rho\left(\frac{Q_m}{M}\right) = \left.\frac{1}{M}\rho'\left(\frac{Q_m}{M}\right)\frac{dQ_m}{d\alpha}\right|_{\alpha=0} > 0.$$

(b) Differentiating equations (5.3.7) and (5.3.8), we find that

$$A\begin{pmatrix}\frac{dq_u}{dM}\\ \frac{dq_r}{dM}\end{pmatrix} = D,$$

where

$$A = \begin{pmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}, \text{ and } D = \begin{pmatrix}\frac{1}{M}\left(Q_m\left(\rho'\left(\frac{Q_m}{M}\right)+\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)\right)+q_r\rho'\left(\frac{Q_m}{M}\right)\right)\\ \frac{Q_m}{M}\rho'\left(\frac{Q_m}{M}\right).\end{pmatrix},$$

where $a_{11} = \rho'\left(\frac{Q_m}{M}\right)+\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)$, $a_{12} = n\rho'\left(\frac{Q_m}{M}\right)+(n-1)\frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)-\alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)$, $a_{21} = \rho'\left(\frac{Q_m}{M}\right)-\alpha M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)$ and $a_{22} = (n-1)\rho'\left(\frac{Q_m}{M}\right)$. x_u and x_r are given by $x_u = x(q_u;\gamma_u)$ and $x_r(q_r;\gamma_r)$ respectively. The determinant of matrix A is

$$\begin{aligned} detA = &-\rho'(.)[\rho'(.)-\alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)]\\ &+\alpha M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)\left[n\rho'(.)+(n-1)\frac{q_r}{M}\rho''(.)-\alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)\right] < 0,\end{aligned}$$

where the sign follows from $\rho'(.) < 0$, Assumption 1,, $v'(x) > 0$ and $x_q(q;\gamma) > 0$.

Therefore, following Cramer's rule, we find that

$$\frac{dq_u}{dM} = \frac{-\frac{Q_m}{M}\rho'\left(\frac{Q_m}{M}\right)\left(\rho'\left(\frac{Q_m}{M}\right) - \alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)\right) + \frac{(n-1)q_r}{M}\rho'\left(\frac{Q_m}{M}\right)^2}{detA} > 0,$$
$$\frac{dq_r}{dM} = \frac{-\frac{q_r}{M}\rho'(.)^2 + \alpha(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)\left(Q_m(\rho'(.) + \frac{q_r}{M}\rho''(.)) + q_r\rho'(.)\right)}{detA} > 0,$$

and

$$\frac{dQ_m}{dM} = \frac{-\frac{Q_m}{M}\rho'(.)[\rho'(.) - M\alpha(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)] + (n-1)\alpha(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)[Q_m(\rho'(.)+\frac{q_r}{M}\rho''(.)) + q_r\rho'(.)]}{detA} > 0.$$

The signs of $\frac{dq_u}{dM}$, $\frac{dq_r}{dM}$ and $\frac{dQ_m}{dM}$ follow from $\rho'(.) < 0$, Assumption 1, $v'(x) > 0$ and $x_q(q;\gamma) > 0$.

The changes in per-capita output and hedonic price are given by

$$\frac{d}{dM}\left(\frac{Q_m}{M}\right) = \frac{\alpha(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)(-q_u\rho'(.) + Q_m M\alpha(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r))}{MdetA} < 0,$$
$$\frac{d}{dM}\rho\left(\frac{Q_m}{M}\right) = \rho'\left(\frac{Q_m}{M}\right)\frac{d}{dM}\left(\frac{Q_m}{M}\right) > 0.$$

(c) Differentiating equations (5.3.7) and (5.3.8), we find that

$$A\begin{pmatrix}\frac{dq_u}{dn} \\ \frac{dq_r}{dn}\end{pmatrix} = G,$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}, \text{ and } G = \begin{pmatrix} -q_r\left(\rho'\left(\frac{Q_m}{M}\right) + \frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)\right) \\ -q_r\rho'\left(\frac{Q_m}{M}\right)\end{pmatrix}$$

where $a_{11} = \rho'\left(\frac{Q_m}{M}\right) + \frac{q_r}{M}\rho''\left(\frac{Q_m}{M}\right)$, $a_{12} = n\rho'\left(\frac{Q_m}{M}\right) + (n-1)\rho''\left(\frac{Q_m}{M}\right) - \alpha M(1+$

$\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)$, $a_{21} = \rho'\left(\frac{Q_m}{M}\right) - \alpha M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)$ and $a_{22} = (n-1)\rho'\left(\frac{Q_m}{M}\right)$. x_u and x_r are given by $x_u = x(q_u;\gamma_u)$ and $x_r(q_r;\gamma_r)$ respectively. We know that the determinant of matrix A is negative from part (b) of Lemma 5.2. Therefore, applying Cramer's rule, we find that

$$\begin{aligned}
\frac{dq_u}{dn} &= \frac{q_r\rho'(.)(\rho'(.) - \alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r))}{detA} < 0,\\
\frac{dq_r}{dn} &= -\frac{q_r\left(\rho'(.) + \frac{q_r}{M}\rho''(.)\right)\alpha M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)}{detA} < 0,\\
\frac{dQ_m}{dn} &= \frac{\alpha q_r M(1+\varepsilon_v)v'(x_u)x_q(q_u;\gamma_u)[\rho'(.) - \alpha M(1+\varepsilon_v)v'(x_r)x_q(q_r;\gamma_r)]}{detA} > 0.
\end{aligned}$$

The signs follow from $\rho'(.) < 0$, $v'(x) > 0$ and $x_q(q;\gamma) > 0$. The changes in per-capita output and hedonic price are given by

$$\frac{d}{dn}\left(\frac{Q_m}{M}\right) = \frac{1}{M}\frac{dQ_m}{dn} > 0, \text{ and } \frac{d}{dn}\rho\left(\frac{Q_m}{M}\right) = \rho'\left(\frac{Q_m}{M}\right)\frac{d}{dn}\left(\frac{Q_m}{M}\right) < 0,$$

since $\rho'(.) < 0$ and $\frac{dQ_m}{dn} > 0$.

□

5.6.3 Public versus Private Provision: Role of congestion, market size and competition

Proof of Proposition 5.2:

Proof. Differentiating (5.4.1) with respect to α and evaluating at $\alpha = 0$, we find that

$$\left.\frac{d\Delta_c^{CS}}{d\alpha}\right|_{\alpha=0} = -\frac{1}{Mn}\left(\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)\right)\left.\frac{dQ^*}{d\alpha}\right|_{\alpha=0} - (1+\varepsilon_v)\left[v(x(q^*;\gamma_r+\Delta_\gamma)) - v(x(q^*;\gamma_r))\right] < 0,$$

where the sign follows from Assumption 1, $x_\gamma > 0$ and proof of part (a) of Lemma 5.1. □

Proof of Proposition 5.3:

Proof. Differentiating (5.4.1) with respect to M, we find that

$$\frac{d\Delta_c^{CS}}{dM} = -\frac{1}{n}\left(\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)\right)\frac{d}{dM}\left(\frac{Q^*}{M}\right) + \frac{\alpha(1+\varepsilon_v)}{n}\left[v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma) - v'(x_r)x_q(q^*;\gamma_r)\right]\frac{\partial Q^*}{\partial M} < 0,$$

where the sign follows from Assumption 1, $v'(x) > 0$,

$$\frac{\partial^2 x(q;\gamma)}{\partial\gamma\partial q} = \frac{\alpha v'(x)\varepsilon_v\varepsilon_f(\varepsilon_f - 1)f\left(\frac{q}{x}\right)}{q^2\left(\alpha v'(x)\varepsilon_v + \frac{\gamma}{x^2}\varepsilon_f f'\left(\frac{q}{x}\right)\right)^2} > 0,$$

along with proof of part (b) of Lemma 5.1. □

Proof of Proposition 5.4:

Proof. Taking total derivative of (5.4.1) with respect to n and substituting for $\frac{dQ^*}{dn}$ and $\frac{dq^*}{dn}$ from proof of part (c) of Lemma 5.1, we get

$$\frac{d\Delta_c^{CS}}{dn} = \frac{q^*\rho'\left(\frac{Q^*}{M}\right)}{M}\frac{A}{(n+1)\rho'\left(\frac{Q^*}{M}\right) + \frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right) - M\alpha(1+\varepsilon_v)v'(x_r)x_q(q^*;\gamma_r)},$$

where

$$A = \rho'\left(\frac{Q^*}{M}\right) + M\alpha(1+\varepsilon_v)(v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma) - v'(x_r)x_q(q^*;\gamma_r))\left(1+\frac{\eta}{n}\right) + M\frac{\eta}{n}\alpha(1+\varepsilon_v)v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma)$$

and $\eta = \frac{\frac{Q^*}{M}\rho''\left(\frac{Q^*}{M}\right)}{\rho'\left(\frac{Q^*}{M}\right)}$. The denominator is negative under Assumption 1, $\rho'(.) < 0$, $v'(x) > 0$

and $x_q(q;\gamma) > 0$. Since $\rho'\left(\frac{Q^*}{M}\right) < 0$,

$$\frac{d\Delta_c^{CS}}{dn} < 0 \Leftrightarrow \rho'\left(\frac{Q^*}{M}\right) + M\alpha(1+\varepsilon_v)(v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma) - v'(x_r)x_q(q^*;\gamma_r))\left(1+\frac{\eta}{n}\right)$$
$$+M\frac{\eta}{n}\alpha(1+\varepsilon_v)v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma) < 0.$$

Observe that in the absence of congestion, i.e., $\alpha = 0$, $\frac{d\Delta_c^{CS}}{dn} < 0$. In the presence of congestion, i.e., $\alpha > 0$, with a linear hedonic demand, i.e., $\eta = 0$ and $\Delta_\gamma = 0$, we again have $\frac{d\Delta_c^{CS}}{dn} < 0$. However, in the presence of congestion along with $\Delta_\gamma > 0$, the condition needed to be satisfied becomes

$$\rho'\left(\frac{Q^*}{M}\right) + M\alpha(1+\varepsilon_v)(v'(x_u)x_q(q^*;\gamma_r+\Delta_\gamma) - v'(x_r)x_q(q^*;\gamma_r)) < 0.$$

Let us consider a linear hedonic demand $\rho\left(\frac{Q}{M}\right) = 1 - \frac{Q}{M}$ with $\rho'\left(\frac{Q}{M}\right) = -1 < 0$. Moreover, consider $v(x) = x$ and $f(k) = \frac{k^2}{2}$. This implies that $\varepsilon_v = \frac{xv'(x)}{v(x)} = 1$ and $\varepsilon_f = \frac{kf'(k)}{f(k)} = 2$. Using the first-order condition with respect to capacity for any firm, public or private, we have

$$\alpha x = \frac{\gamma q}{x^2} \Rightarrow x(q,\gamma) = \left(\frac{\gamma}{\alpha}\right)^{\frac{1}{3}} q^{\frac{1}{3}}.$$

This implies that

$$x_q(q,\gamma) = \frac{1}{3}\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{3}} q^{-\frac{2}{3}}.$$

Substituting this into the above condition, we find that

$$-1 + \frac{2M}{3}\alpha^{\frac{2}{3}}\left[(\gamma_r+\Delta_\gamma)^{\frac{1}{3}} - \gamma_r^{\frac{1}{3}}\right] {q^*}^{-\frac{2}{3}} < 0 \Leftrightarrow q^* > \left[\frac{2M}{3}\alpha^{\frac{2}{3}}((\gamma_r+\Delta_\gamma)^{\frac{1}{3}} - \gamma_r^{\frac{1}{3}})\right]^{\frac{3}{2}}.$$

Therefore, Δ_c^{CS} declines with increase in n if and only if $q^* > \underline{q}$ where

$$\underline{q} = \left[\frac{2M}{3}\alpha^{\frac{2}{3}}((\gamma_r + \Delta_\gamma)^{\frac{1}{3}} - \gamma_r^{\frac{1}{3}})\right]^{\frac{3}{2}}.$$

Observe that $\underline{q} > 0$ in the presence of congestion if $\Delta_\gamma > 0$. Since $\frac{dq^*}{dn} < 0$ from the proof of part (c) Lemma 5.1, $q^*(n)$ is invertible, and we can invert the inequality in terms of n and find that

$$q^* > \underline{q} \Rightarrow n < \overline{n}.$$

That is, Δ_c^{CS} declines with an increase in n if and only if $n < \overline{n}$. $\square$

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