Indian Statistical Institute
End Semestral Examination: 2025-26

Course Name: M. Math, 1st year
Subject Name : Analysis of several variables
Date: 24.11.2025, Maximum Marks: 50, Duration: Three hours

You are allowed to consult lecture notes.

1. State and prove Green's theorem for the set

$$S = \{(x, y) \in \mathbb{R}^2 : y \geq 0, 0 \leq x^2 + y^2 \leq 1\}.$$

The statement should clearly mention the orientations and the parametrizations involved. **10**

2. Show that

$$-x_2 \frac{\partial}{\partial x_1} + x_1 \frac{\partial}{\partial x_2} - x_4 \frac{\partial}{\partial x_3}$$

is a nowhere vanishing tangent vector field on S^3. **5**

3. Let $\Omega = \{(x, y, 0) \in \mathbb{R}^3 : x^2 + y^2 < 1, x > 0, y > 0\}$ equipped with the orientation form $dx \wedge dy$. Compute

$$\int_\Omega (dx \wedge dy + dy \wedge dz + dz \wedge dx). \textbf{ 10}$$

4. Suppose $V \subseteq \mathbb{R}^n$ is open, $\omega \in \Omega^k(V)$ and X a smooth vector field on V. Prove that the contraction of ω by the smooth vector field X is a smooth $k-1$-form on V. **10**

5. Consider the set $M = \{(x_1, x_2, x_3, x_4, 0) \in \mathbb{R}^5 : x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1\}$. Construct an orientation form on M and compute the volume form of M with respect to your orientation form. **3 + 7 = 10**

6. Using the method of Lagrange multipliers, find the maximum value of the function $g(x, y, z) = 2x + 2y - z$ on the set $x^2 + y^2 + z^2 = 2$. **10**

7. Let $O(n) = \{A \in M_n(\mathbb{R}) : A^T A = I_n\}$. If $A \in O(n)$, prove that there exists an open subset U of $\mathbb{R}^{\frac{n(n-1)}{2}}$ and a smooth one-one map $\psi : U \to M_n(\mathbb{R})$ such that $\psi(U)$ is an open subset of $O(n)$ containing A and $D\psi(u)$ has full rank for all $u \in U$. **10**