INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2025-26

MSQE I 2025-26

Microeconomics I

Date: 21st November 2025 Maximum marks: 60 Duration: 3 hours

Answer all questions rigorously.

1. Recall the model of the producer.

   (a) What is the technology of the producer? [5]

   (b) What is the producer's problem, given a vector of positive prices and the technology? [1]

   (c) Suppose the producer's problem has a unique solution. Show that the solution is increasing and continuous in prices. [4+6]

   (d) State and prove Hotelling's Lemma, articulating all assumptions you are imposing. [4]

2. Let Z be a set of prizes, and $L(Z)$ the set of lotteries over prizes in Z; $L(Z) = \{p \in R_+^{|Z|} | p : Z \to [0,1], \sum_{z \in Z} p(z) = 1\}$. Following the theory of von Neumann and Morgenstern, consider preferences over $L(Z)$.

   (a) Recall Conditions C and C^*.

      i. State Conditions C and C^*. [1+1]

      ii. Show that $C \Rightarrow C^*$. [6]

      iii. Does C^* imply C? [2]

   (a) Recall Conditions I and I^*.

      i. State Conditions I and I^*. [1+1]

      ii. Show that $I \Rightarrow I^*$. [6]

      iii. Does I^* imply I? [2]

3. Suppose there are a total of K commodities. A continuous and monotonic preference relation is said to be *separable* if it can be represented by an additive utility function: $u(x) = \sum_{k=1}^{K} v_k(x_k)$, where x_k is the k^{th} component of bundle x, and $(v_k)_{k=1}^{K}$ are continuous and non-decreasing functions.

   (a) Suppose preferences are separable. Show that for any subset of commodities J and for any bundles a, b, c, d, $(a_J, c_{-J}) \succeq (b_J, c_{-J}) \iff (a_J, d_{-J}) \succeq (b_J, d_{-J})$, where (x_J, y_{-J}) is the vector that takes the components of x for any $k \in J$ and takes the components of y for any $k \notin J$. [10]

   (b) Suppose $K = 2$. For any bundles a, b, c, d, e, f, show that if $(a, b) \succeq (c, d)$ and $(c, e) \succeq (f, b)$, then $(a, e) \succeq (f, d)$, where (x, y) is the vector that takes the component of x for $k = 1$ and takes the component of y for $k = 2$. [10]