Indian Statistical Institute
Probability Theory I
B.Stat (hons) 1st year,
1st semestral examination

Date: Nov 14. 2025 Duration: 3hrs.

Attempt all questions. The maximum you can score is 50. This is an open book, open notes examination. You may use your own calculator. Notes and books may not be shared. Justify all your steps. Marks will be deducted for unclear, incomplete or non-rigourous writing. Neater algebraic expressions would be given more credit.

1. State (with justification) whether the following statements are true or false.

   (a) For any two sets C, D we define $C\Delta D = (C \cap D^c) \cup (C^c \cap D)$. Let A_n and A be events in a probability space for $n \in \mathbb{N}$. If $A_n \Delta A \downarrow \phi$, then we must have $P(A_n) \to P(A)$.

   [6]

   (b) For a simple symmetric random walk of length n starting from 0, let the probability of never returning to 0 be denoted by p_n. Then $p_{101} = \frac{1}{2} \times p_{100}$.

   [6]

   (c) Assume that there are 100 men, each of whom is equally likely to be born on any day of the year (assumed to consist of 365 days). Birthdays of different men are independent. Let X_i be the indicator of the i-th day being the birthday of exactly 3 of these men. Then $X_1, \ldots, X_{365}$ are independent random variables.

   [6]

   (d) If X, Y, Z are three jointly distributed random variables, such that X, Y are dependent, and so are Y, Z, then X, Z must be dependent.

   [7]

2. $X \sim Poisson(\lambda)$ and given $X = n$ the conditional distribution of Y is $Binomial(n, p)$ for $p \in (0, 1)$. Show that $Y \sim Poisson(\mu)$ for some μ. Express μ in terms of λ and p.

   [10]

3. God rolls a fair die, and creates as many identical cells as the number shown. After each second each cell splits into i offspring cells with probability p_i, where $p_0 = 0.1, p_1 = 0.4, p_2 = 0.5$ and $p_i = 0$ for $i > 2$. The offsprings cells continue to behave similarly. All the cells behave identically and independently. Find the probability of extinction of this branching process. You may use without proof the relevant results regarding branching processes done in class.

   [10]

4. Consider a simple, symmetric random walk of length 10 starting from 0. Let its position at time i be X_i. Find the conditional covariance between between X_3 and X_{10} given X_6. Also find the conditional probability that $X_{10} = 2$ given that ($X_3 = 3$ and the walk has hit 5 at least once)?

   [3+7]