

Indian Statistical Institute

Backpaper examination : (2025-26)

M. Math. II year

Differential Geometry I

Date : 24.12.25 Maximum marks : 100 Duration : 3 hours.

Answer ALL questions.

(1) Compute the Gaussian curvature at an arbitrary point of the ellipsoid S defined as a level surface in $\mathbb{R}^3$ by

$$S = \{(x_1, x_2, x_3) : \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1\},$$

where a, b, c are positive constants. [25]

(2) Compute the signed curvature at every point of the oriented planar curve $\{(x, y) \in \mathbb{R}^2 : f(x, y) \equiv x^2 - y^2 = 1, x > 0\}$, with the usual orientation given by $\frac{\nabla f}{\|\nabla f\|}$. [25]

(3) Compute (with proof) the geodesics of the cylinder $S = \{(x, y, z) : x^2 + y^2 = 1\}$. [25]

(4) Let M be a smooth manifold of dimension n and let TM be the set $\{(x, v) : x \in M, v \in T_x M\}$. Prove the following:

(i) TM is a smooth manifold of dimension $2n$.

(ii) TM is orientable.

[10+15=25]