Probability and Statistics for MTech (CrS) - I (2025-26): EndSem

Date: 21.11.2025

Time: 180 Minutes | Maximum Marks: 50

Note: We follow usual notations from class lectures and assignments.
Answer all parts of one question together.

1. (a) State the different modes of stochastic convergence.
(b) Prove that convergence in quadratic mean implies convergence in probability. Is the converse true?
(c) Let S_n^2 denote the usual sample variance. Show that $S_n^2 \xrightarrow{P} \sigma^2$ as $n \to \infty$, where $\sigma^2 = Var(X_1)$. [4+(2+1)+5=12]

2. (a) Suppose $X_1, \ldots, X_n$ are independent and identically distributed (i.i.d.) random variables (rvs) from $Uniform[0, \theta]$ with $\theta > 0$. Derive an expression for $Var(X_1)$.
(b) Find an unbiased estimator (based on $\bar{X}_n^2$) for this variance.
(c) How does this estimator compare with the usual sample variance S_n^2?
[3+5+4=12]

3. (a) Let $X_1, \ldots, X_n$ be i.i.d. rvs with $E(X_1) = \mu$. Fix $0 < \alpha < 1$. Under appropriate conditions, construct an approximate $(1 - \alpha)\%$ confidence interval (CI) for μ.
(b) Suppose $X_1, \ldots, X_n$ are i.i.d. $Exponential(\lambda)$ with $\lambda > 0$. Derive the NP test for testing $H_0 : \lambda = 0.04$ against $H_1 : \lambda = 4$. [5+5=10]

4. (a) State an algorithm to generate from the pmf: $x_1, \ldots, x_m$ with probabilities $p_1, \ldots, p_m$ such that $p_i > 0$ and $\sum_i p_i = 1$.
(b) Let $X \sim Uniform[0, 1]$ and $0 < a < b < 1$. Define

$$Y = \begin{cases} 1 & 0 < x < b \\ 0 & \text{otherwise} \end{cases}$$

and

$$Z = \begin{cases} 1 & a < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Are Y and Z independent? Why/why not?
(ii) Find $E(Y \mid Z)$. [*Hint:* What values z can Z take? Now, find $E(Y \mid Z = z)$]
[4+(3+3)=10]

5. (a) For a Markov chain (MC), define the different states like closed, transient, recurrent (null and positive), etc.
(b) Assume that the states of a MC are $\{1, \ldots, 6\}$ with the following transition probability matrix (TPM):

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 & 0 & 0.7 \end{bmatrix}.$$

Classify the states based on this TPM. [5+5=10]

All the best!