INDIAN STATISTICAL INSTITUTE

First Semestral Examination : (2024-2025)

PGDBA 1st Year

Stochastic Processes and Applications

PART A

Date: 17. 12. 24 Maximum marks: 50 Time: 90 minutes

Note: Answer as much as you can. Maximum you can score combining Parts A and B is 100. You may use a scientific calculator and any result proved in class after clearly stating the result. **Proofs will not be considered complete unless explanations are given for every step.**

1. (a) Two teams play a series of games: the first team to win 4 games is declared the overall winner. Suppose that both teams have an equal probability 0.5 of winning each game. Find the expected number of games played. [8]

   (b) An urn initially contains one red and one blue ball. At each stage, a ball is randomly chosen and then replaced along with another of the same colour. Let X denote the selection number when the blue ball is selected for the first time. For instance, if the first selection is red and the second blue, then X is equal to 2.

   (i) Find $P\{X > i\}, i \geq 1$.
   (ii) Find $E[X]$. [4+4]

2. The joint density of X and Y is given by:

$$f(x, y) = c(y - x)\, e^{-y}, \quad 0 < y < \infty, \quad -y < x < y.$$

   (a) Find c.

   (b) Find the density function of X.

   (c) Find the density function of Y.

   (d) Find $E(X)$.

   (e) Find $E(Y)$. [3+4+3+4+3]

3. A fair die is rolled 10 times. Find the expected value and the variance of the sum of the 10 rolls. [4+6]

4. Consider a stationary AR(2) process:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t,$$

   where ε_t is white noise with mean 0 and variance σ^2. Let the auto-correlations of lag k be denoted by ρ_k for k = 1,2,....
   (a) Deduce that $\rho_1 = \frac{\phi_1}{1-\phi_2}$.
   (b) Derive the recurrence relation $\phi_k = \phi_1\, \rho_{k-1} + \phi_2\, \rho_{k-2}$ for $k \geq 2$.
   (c) Using (b) or otherwise find ρ_2 and ρ_3. [4+4+4]