

On the Choice of Shadow Prices
for Project Evaluation

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To Bharat

TABLE OF CONTENTS

	<u>page:</u>
<u>Acknowledgements</u>	(v)
<u>Introduction : Motivation and summary</u>	(vi)
<u>Footnotes to Introduction</u>	(xvii)
<u>General notational conventions and a simple technical tool</u>	(xviii)
<u>Part 1 : Theoretical findings</u>	
<u>Chapter I : Theories of shadow prices</u>	1.1
Section I.1 The model	1.1
I.2 An example	1.6
I.3 Project evaluation, cost-benefit tests and shadow prices	1.11
I.4 Second-best optimality and decentralization prices	1.18
I.5 Traditional cost-benefit analysis and welfare prices	1.28
I.6 Price-guided reform processes and fictitious prices	1.35
I.7 Elements of synthesis	1.50
I.8 Project evaluation revisited	1.59
I.9 Some useful complementary results	1.68
Footnotes	1.73
References	1.75
<u>Chapter II : Shadow prices in a messy economy</u>	
Section II.1 Outline of the approach	II.1
II.2 Shadow pricing rules	II.7
II.3 An example : Keynesian unemployment	II.20
II.4 Shadow prices and optimal policy rules	II.23
II.5 Concluding remarks	II.30
Footnotes	II.31
Appendix A.2 : Elementary properties of constrained demand and supply functions	II.32
References	II.35

page:

Part 2 : Selected applications (with special reference to India)

Chapter III : Shadow prices and distribution policies in a dual economy

Introduction	III.1
Section III.1 A simple general equilibrium model of a dual economy	III.7
§ III.1.1 General features	III.7
§ III.1.2 Behavioural assumptions	III.11
§ III.1.3 Summary	III.19
§ III.1.4 Wage determination	III.23
Section III.2 Shadow prices in a dual economy	III.27
§ III.2.1 General expressions	III.27
§ III.2.2 The "first-best" environment	III.30
§ III.2.3 Theoretical analysis : the case of rigid money wages	III.31
§ III.2.4 Some numerical simulations	III.42
Section III.3 Income distribution policies: anatomy and evaluation	III.50
§ III.3.1 General features of the Indian transfer system	III.52
§ III.3.2 Case study : the Public Distribution System	III.58
Footnotes	III.65
Appendix A.3.1 : Some formal proofs	III.69
Appendix A.3.2 : The data	III.76
References	III.83

Chapter IV : Shadow prices and tax reform in a two-
factor economy

Section IV.1	Preliminaries : further remarks on shadow prices in a reform context	IV.1
Section IV.2	Tax reform with two scarce factors: theoretical analysis	IV.4
§ IV.2.1	The model	IV.4
§ IV.2.2	Tax reform and shadow prices	IV.9
§ IV.2.3	A simple characterization of desirable tax reforms	IV.13
§ IV.2.4	Optimal taxation	IV.19
Section IV.3	An illustrative application to India	IV.25
§ IV.3.1	Outline of the exercise and the results	IV.25
§ IV.3.2	Diagrammatic interpretation	IV.27
§ IV.3.3	Discussion of the results	IV.33
Section IV.4	Two simple extensions	IV.50
§ IV.4.1	Direct taxation	IV.50
§ IV.4.2	Involuntary unemployment	IV.53
Section IV.5	Concluding comments	IV.55
Footnotes		IV.57
Appendix A.4.1	Some formal proofs	IV.59
Appendix A.4.2	The data	IV.63
References		IV.70
<u>Concluding comments</u>		V.1
<u>Annex 1 : Statistical material</u>		A.1
Section A.I.	Main data sources	A.4
Section A.II	Notational conventions	A.5
Section A.III	The model	A.9

		<u>page:</u>
Section A.IV	The core data set	A.14
§ A.IV.1	Normalization rules	A.14
§ A.IV.2	The input-output table	A.14
§ A.IV.3	Prices and taxes	A.17
§ A.IV.4	Population	A.20
§ A.IV.5	The consumer expenditure matrix	A.22
§ A.IV.6	Imports of consumer goods	A.25
§ A.IV.7	Income distribution	A.26
§ A.IV.8	Average savings rates	A.31
§ A.IV.9	Marginal propensities to save and marginal expenditure shares	A.33
§ A.IV.10	Income and expenditure growth factors	A.34
§ A.IV.11	The consumer price elasticity matrix	A.36
§ A.IV.12	Value added coefficients	A.45
§ A.IV.13	The distribution of value-added	A.46
Section A.V	Aggregation scheme	A.52
Data Tables		A.54
Footnotes		A.74
References		A.76

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Introduction : Motivation and summary

Manohar Thanna is one of the remotest parts of Jhalawar District (Rajasthan, India). In the month of June it offers a striking contrast of scenic beauty and economic destitution. Agricultural activity is at a virtual standstill. The soil is very arid, irrigation practically non-existent, and by then the villagers (many of them tribals) have resolved to wait upon the good will of the rain gods. Hence they have very little to do, or at least so they believe. Some gather wood to sell it in Manohar, walking miles under the scorching sun for a meagre reward, and adding slowly but perceptibly to the already acute deforestation problem in the region. Others cling to traditional and often perishing village crafts, such as rope-making; or perhaps they sit and drink. Many join the rural public works schemes, when these are active. In those lean days, hunger stands at many of their doors. Whether they are happy, however, we do not know; to the passing foreigner they usually manifest a typical blend of amusement and hospitality.

When I went there in the summer of 1980, a great part of the country was suffering from the after-effects of the terrible drought of 1979.

The magnitude of the 1979 drought, and its impact on agricultural production, were of a phenomenal severity. The net per capita availability of foodgrains (after allowing for imports and changes in government stocks) fell by more than 13% compared to the previous year, ^{1/} a rather alarming figure considering the low

amount of fat storing which an average Indian diet permits. And yet there is no evidence of starvation following this disaster, which largely escaped international attention.

How starvation was averted, how the necessary austerity was distributed, why the relative price of foodgrains remained fairly stable, are all challenging questions which will hopefully one day deserve a more carefully written piece of economic history. However, one can scarcely doubt the important role played in the process by the massive Food-for-Work (FFW) programme of rural public works, which in the year 1979-80 provided (according to official statistics) as many as 1 billion man-days of employment at a subsistence wage to the poorest segment of the rural population; ^{2/} enough, in principle, to help 17 million poor households, or about 100 million individuals, through the 60 worst days of the lean season (in practice, of course, the available employment is not so well distributed).

To me, Manohar Thanna in June 1980 offered a very convincing (though definitely not typical) picture of what public works programmes can achieve in such situations. No doubt the prospects for Food-for-Work were particularly favourable in Jhalawar District: mass poverty and unemployment in the lean season combined with a great scope for public works (irrigation works, roads, community buildings such as schools and wells, etc.), and, last but not least, a dedicated and energetic District Collector. In June 1980, most of the works consisted of building small "anicut". An anicut is simply a small wall (perhaps 60 feet long, 10 feet wide, and 12 feet high, in a typical case; though its size can vary very much) built

across a natural dale, which is dry most of the year but battered with rain during the monsoon. The purpose of the anicut is to retain the rains and thereby raise the water table in the surrounding wells (which otherwise often dry up during the summer); besides, the pool it retains can provide drinking water for cattle, irrigation water for plots downstream, and serve other useful purposes such as the breeding of fish. The need for such infrastructure seems particularly acute in Jhalawar District, which, though it tends to benefit from good rainfalls by the standards of Rajasthan as a whole, shows very poor yields in agriculture, partly because of the lack of water resource management.

Most of the important inputs required for the construction of an anicut are available within walking distance of the worksite: sand, stones, unskilled labour, some rudimentary tools, bullocks, and, more critically, water. The noteworthy exceptions to this rule are various kinds of skilled labour, and, more importantly, cement.

Cement is very scarce in India at most times. The total supply usually falls short of demand at the controlled price, and the purchase of cement is licensed.^{3/} Of course a large black market exists, where a much higher price is applicable. Imports of cement are the monopoly of the government. The VIth plan works out tentative import figures on the basis of rough demand-supply projections (which explicitly take into account the requirements of the FFW programme); nevertheless, "the import policies of the Government are adjusted annually primarily depending on the availability of foreign exchange".^{4/} Domestic production, on the other hand, often shows high rates of excess capacity, allegedly due to "the lack of adequate

5/
 infrastructural facilities like coal, power and transport". To summarize, the source of a marginal unit of cement affected to the FFW programme is far from obvious : imports, domestic production, stiffer rationing in other uses are some of the possibilities one may wish to allow for.

Unless the work is not completed in time before the monsoon, in which case the unfinished construction is likely to be washed away by the rains, the output of an anicut construction project can simply be considered as the anicut itself, which is a complex kind of local public good; equivalently, it may be regarded as the services provided in future years by the anicut. Again, a proper identification of these services, and their precise impact on the rural economy, pose difficult theoretical and empirical problems.

6/
 In Jhalawar District the FFW programme is administered entirely through the official panchayat system of local government, and supervised at different levels and with differing degrees of autonomy, by 7/
 the Sarpanch, the Block Development Officer, and finally the District Collector. Further upstream, a Steering Committee consisting of various State Government officials monitors the programme for the whole state of Rajasthan, while being itself accountable to the central Ministry of Rural Reconstruction in Delhi. The latter is responsible for taking major allocation decisions and issuing general guidelines, taking into account the financial constraints imposed by the annual Government Budget (designed essentially by the Finance Minister, and discussed in the Parliament), and, finally, the long-term perspective laid down in the five-year plan. Besides this monumental machinery, various kinds of local, national and international agencies are involved in the

management of the programme in other parts of the country.

The nature, number, location and other features of the many projects finally undertaken under the FFW programme is therefore the outcome of a myriad of decisions taken by a wide collection of agents. A rational coordination of these decisions, and the delineation of operational criteria to guide them, at various levels, are challenging tasks. In this case, the admirable decentralization often made possible by capitalist markets seems bound to fail miserably: since building an anicut is a disastrous operation in terms of financial profitability, such public works would never be undertaken by profit-maximising private entrepreneurs, and a private management of the programme would leave the poor of Manohar Thanna starving. Obvious macro-economic disequilibria (such as massive involuntary unemployment), externalities (e.g. the public goods features of an anicut), and acute distributional distortions (resulting partly from the absence of any direct social security system to cope with natural disasters such as the 1979 drought) seem to account quite convincingly for this market failure. If these distortions are pervasive, then, the management of the FFW programme must rely upon institutional arrangements and decision rules different from those associated with competitive markets.

A great deal of research in economic theory has built upon the premise that a decentralized design and evaluation of public projects can be achieved by promoting the use of shadow prices throughout the public sector. How these shadow prices should be chosen, and therefore, implicitly, which kind of criteria should govern the public decisions involved in the management of such schemes as FFW, are the twin concerns which form the background of the research we have carried out in the past four years at the Indian Statistical Institute (New

Delhi), and the Development Economics Research Centre of the University of Warwick (U.K.). This thesis reports some of our findings and failures.

The plan of our work is as follow.

Chapter I is devoted to an enquiry into the theoretical foundations of shadow pricing. A survey of the literature on project evaluation suggests at least three ways of thinking about public production projects, and motivating the appropriate choice of shadow prices.

According to one school of thought, the design of a FFW programme must be considered as an integral part of the planning problem, which should be solved as any constrained optimization problem after due recognition of the planner's objectives and constraints. The number of anicuts to be built is then in effect determined jointly with the level of cement imports, the charges to be levied on irrigation water, subsidies on private employment, etc. within the set of opportunities available to the planner, and in the light of his objectives. Associated with the second-best optimum is a vector of shadow prices for commodities which represents their social values in the well-defined sense of the marginal contribution which their availability can make to the planner's objective function; these shadow prices permit a decentralized implementation of the optimum, and are appropriate for the evaluation of a marginal public sector project. We shall call this the second-best viewpoint.

An apparently different, but in fact closely related, approach consists of confining one's attention to a particular project, and attempting to identify objectively its total effect on the state of the economy. A normative comparison of "the world with and the world without" the project then provides a conceptually straightforward project evaluation criterion. Under appropriate regularity conditions, there will again exist a set of shadow prices which harmonize this "primal route" to project evaluation with the "dual route" consisting of assessing projects on the basis of their shadow profits. Thus the desirability of undertaking the construction of an anicut will in this case be assessed by considering the actual source of the cement used, the actual water rates one expects to be levied, etc.

In Chapter I, we shall refer to this approach as the traditional cost-benefit analysis viewpoint. We shall also argue that the second-best viewpoint and the traditional CBA viewpoint are neither contradictory nor fundamentally different. Essentially, they boil down to a proper identification of the project evaluator's "area of control". For instance, while a central planning agency entrusted with a wide range of economic policy decisions would naturally draw inspiration from second-best theory, an aid agency sponsoring a particular project may find the traditional CBA viewpoint more relevant. Moreover, the traditional CBA viewpoint may be regarded as dealing with a very special second-best problem, where the only instrument available is a particular project. To that extent, both approaches would recommend that the agent who sets the shadow prices should optimize within his opportunity set if the social value of a project is to be well-defined.

It is sometimes contended, however, that the study of optimal decisions is a largely futile exercise, insofar as their implementation requires the acquisition of information relating to an optimal state which may lie "far away" from the initial state of the economy. A more realistic approach may seem to be to recognize the sub-optimality of the state of the economy, and undertake small reforms towards an optimum, which make use of local information only. Public projects should then be evaluated in the light of the whole set of reforms they make available. In this perspective, which we shall call the reform viewpoint, the construction of an anicut can be undertaken by diverting cement from other uses, or by importing it, or by increasing domestic production, etc; cement has many possible alternative uses, and its actual source is neither objectively well-defined (as in the traditional CBA viewpoint) nor immaterial (as in a second-best optimum where, by a property of the optimum, each alternative source entails the same social opportunity cost).

In a reform situation, where welfare improvements seem to be possible without a project, it seems very difficult to explain precisely what we mean by a "welfare-improving" project. This is the essence of the problem we shall encounter in choosing shadow prices in a reform context. Further progress in this direction seems contingent upon an explicit recognition of the constraints which prevent the attainment of an optimum in the first place.

These ideas and issues form the subject-matter of Chapter I.

In Chapter II, we undertake an explicit discussion of shadow

pricing rules in a specific model which, in spite of its striking simplicity, seems to encompass a wide number of the more influential models in the literature. Its systematic use of quantity-constrained demand and supply functions is a step towards bridging the gap between flex-price second-best models with competitive, unconstrained private agents, and some implicitly fix-price models of the literature on project appraisal in developing countries which, while insisting upon the importance of macro-economic disequilibria, often lack rigorous micro-economic foundations. Not surprisingly, the general impression conveyed by the results of Chapter II is that shadow pricing rules in heavily distorted economies are often very complex to characterize analytically, while their application may involve fantastic informational requirements.

If the allowance for price rigidities and quantity rationing is a positive step towards a better understanding of shadow pricing in macro-economic disequilibrium situations, the simplistic assumption of either rigid or competitive prices, inherent to the model of Chapter II, falls short of a satisfactory theory of price formation. To see the potential importance of the problem, consider an economy with involuntary unemployment, where working hours are fixed, wages are perfectly indexed (i.e. maintain a specific level of utility per employed person) and unemployment benefits are also set in real terms. With a social welfare function of the Bergson-Samuelson type and a determinate distribution of employment, social welfare becomes in effect a monotonic function of the rate of unemployment! This observation suggests that the policy prescriptions associated with a model which correctly specifies wage movements may here dramatically

differ from those emerging under either a competitive (no unemployment) or a fix-price (involuntary unemployment, but exogenous wages) model.

A further exploration of this issue is contained in Chapter III in the context of a simple dual economy model, where labour-market imperfections create a wedge between the marginal productivity of labour in the two sectors, and the sensitivity of the model to different assumptions pertaining to wage movements is further examined. Particular attention is paid to their implications for distributional policies. The discussion is supported with some numerical simulations, and illustrated with reference to the Public Distribution System for foodgrains in India.

The difficulty of giving a precise characterization of shadow pricing rules in distorted economies, points to the potential usefulness of numerical simulations and empirical research. We therefore set out to perform a wide range of empirical exercises based upon applications of the model of Chapter II to the Indian economy in 1979, within the framework of the VIth five-year plan. The results of these exercises are unfortunately still incomplete and we cannot decently attempt their presentation in this already overly thick thesis. A simple example of such exercises is however contained in Chapter IV, which analyses an elementary tax reform problem, both theoretically and empirically. Besides, the tedious task of data construction undertaken for our broader purpose is outlined in Annex 1, although the statistical exercise described there only finds a very marginal use in this thesis itself.

Each chapter, as well as the Annex, has been written at a different time, in a largely self-contained manner. In the concluding comments, we reconsider some of our original preoccupations with the hindsight of four years of research, and try to reflect upon why, after such prolonged efforts, we still find it very difficult to decide whether the value of 1.25 adopted (at the time of writing) by the Indian Planning Commission's Project Appraisal Division as the shadow price of foreign exchange is too low or too high.

Footnotes to Introduction:

- 1/ See the Government of India's 1981-82 Economic Survey (New Delhi, 1982), p.84.
- 2/ See India 1980 (Government of India, New Delhi, 1981), p.255.
- 3/ The picture drawn here refers to the year 1980. Since then, cement policy has undergone a variety of changes.
- 4/ Technical Note to the VIth plan of India 1980-85 (Government of India, New Delhi, 1981), p.31.
- 5/ Sixth five-year plan 1980-85 (Government of India, New Delhi, 1981), p.269.
- 6/ A panchayat is a democratically elected village council.
- 7/ The sarpanch is the head of the panchayat.

General notational conventions and a simple technical tool

(1) Apart from the general conventions mentioned below, the notation will be gradually introduced in the text. Although some effort has been made to achieve consistency across chapters, a certain lack of uniformity remains, for which we would like to apologize.

Annex 1 follows a completely autonomous notation, which is explained in Section A.II.

Indexes. Throughout this thesis, the indexes $i \in I = \{1, \dots, N\}$, $j \in J = \{1, \dots, M\}$, $h \in H$ will refer to consumers, commodities and firms, respectively. Commodities are indexed by subscripts and agents by superscripts.

Partial derivatives. Subscripts attached to symbols denoting functions indicate (vectors of) partial derivatives.

Vector notation. As a rule we shall not need to distinguish between row and column vectors; when the distinction arises at all, the appropriate alternative should be clear from the context (e.g. the need for consistent multiplication). The symbol "0" denotes a vector (of suitable dimension) with zero entries. Vector inequalities should be understood as follows:

If $x = (x_j)$, $y = (y_j) \in R^M$, then

$$(i) \quad x \geq y \text{ iff } \begin{cases} x_j \geq y_j & \forall j \\ x_j > y_j & \text{for some } j \end{cases}$$

$$(ii) \quad x > y \text{ iff } x_j > y_j \quad \forall j$$

(2) In Chapter I we shall occasionally use the following simple technical tool.

Let a be a vector in R^M , and A a set in R^M . The cone of feasible directions of A at a , denoted by $\tilde{k}(A, a)$, is defined as

$$\tilde{k}(A, a) = \left\{ y \in R^M : \exists \bar{\alpha} > 0 \text{ s.t. } (a + \alpha y) \in A \quad \forall \alpha \in]0, \bar{\alpha}[\right\}$$

CHAPTER I : Theories of Shadow Prices

Section I.1 : The model

In this chapter we shall consider an abstract economy where $N + 1$ agents interact, a crucial aspect of their interaction being the exchange of commodities. Commodities are indexed by subscripts $j \in J = \{1, \dots, M\}$. By convention, we shall call the N first agents "private agents", and baptize the last one "the planner", "the public sector" or "the government". For purposes of interpretation we shall think of their identity in the usual manner; however, most of the theory applies equally well if any agent is chosen as a "lay planner".

Private agents behave systematically, in response to a comprehensive vector $s = (s_l)_{l=1, \dots, L}$ of "signals" (e.g. prices, incomes, quantity constraints, etc.) belonging to some subset $\bar{S}$ of L -dimensional Euclidean space. In particular, their net excess demand vector is a function $E(\cdot)$ of s :

$$(1) \quad E : s \rightarrow E(s) \in \mathbb{R}^M.$$

Let $z \in \mathbb{R}^M$ denote the vector of aggregate net supplies of a public firm under the control of the planner, called for simplicity the public production plan. The fixed characteristics of the economy, and the behaviour of private agents, impose objective restrictions on the set of pairs (s, z) which the planner may consider as feasible:

- (2.i) $E(s) - z = 0$ (Scarcity constraints)
 (2.ii) $s \in S^*$ (Side constraints)
 (2.iii) $z \in Z^*$ (Technology constraint)

where S^* is the set of admissible signals and Z^* is the public production set. Strict equality is required in (2.i): free disposal, when it exists, should be regarded as a property of the production set Z^* , and has to be resorted to explicitly if at all.^{2/}

Notice that since z is uniquely related to s by (2.i), it is not restrictive to regard S^* as independent of z : seemingly more general side constraints of the form $(s, z) \in \mathcal{F}$ are equivalent, for our purposes, to (2.ii) with $S^* = \left\{ s : (s, E(s)) \in \mathcal{F} \right\}$.

For the same reason, we may legitimately interpret s from now on as the state of the economy. The normative element of our model simply consists of the planner's objective function, which reflects his (or her)^{3/} preferences over the set of states:

$$(3) \quad W : s \in \bar{S} \rightarrow W(s)$$

By "abus de langage", we shall also speak of W as the social welfare function.

(2.i), (2.ii), (2.iii) and (3) describe the model of the planner. We shall take it to provide an objective and undisputed element of the discussion: disagreements about the meaning or correct choice of shadow prices must therefore lie elsewhere.

Most of the ideas and results to be discussed in this chapter can be stated and proved (under appropriate regularity conditions) in this very general framework, e.g. using the techniques developed by Guesnerie (11) and along the same lines as in Dreze (10), Appendix 1. For clarity of exposition, however, we shall systematically avoid technical chicanery, while maintaining a reasonable degree of rigour by appealing to generous assumptions, e.g. differentiability. Thus we shall assume that (2.ii) is equivalent to

$$(4) \quad F(s) - \delta = 0 \quad \text{and} \quad \delta \in \Delta^*$$

where Δ^* is a rectangular set in R^{K-M} . Letting

$$(5) \quad \omega \equiv \begin{bmatrix} z \\ \delta \end{bmatrix}, \quad G(\cdot) \equiv \begin{bmatrix} E(\cdot) \\ F(\cdot) \end{bmatrix} \equiv \left(G^{(k)}(\cdot) \right)_{k=1, \dots, K}, \quad \Omega^* \equiv Z^* \times \Delta^* \subseteq R^K$$

the model of the planner may be rewritten as

$$(6.i) \quad W = W(s)$$

$$(6.ii) \quad G(s) - \omega = 0$$

$$(6.iii) \quad \omega \in \Omega^*$$

In this formulation the elements of ω will be called "intervention variables", since they potentially enable the planner to relax or tighten the constraints at play. Intervention variables may or may not have a meaningful economic interpretation; when they do not, we can think of them as imaginary "slack variables".

The following assumptions will be maintained throughout:

A.1. The functions $G(\cdot)$ and $W(\cdot)$ are differentiable on $\bar{S}$.

A.2. Ω^* is convex. Z^* allows free disposal, i.e. $Z^* \supseteq Z^* - R_+^M$.

A.3. Let s^0 be the initial state of the economy, with

$$G(s^0) = \omega^0 = \begin{bmatrix} z^0 \\ \delta^0 \end{bmatrix}$$

Further, let $\tilde{\delta}, \tilde{\omega}, \tilde{G}(\cdot), \tilde{F}(\cdot), \tilde{\Delta}^*, \tilde{\Omega}$ be appropriate truncations of $\delta, \omega, G(\cdot), F(\cdot), \Delta^*, \Omega^*$ obtained by deleting from the planner's model all the side constraints which are slack at s^0 ; the g^{th} side constraint being considered to be slack at s^0 if both of the following hold:

$$e_g \in \bar{k}(\Delta^*, \delta^0)$$

$$-e_g \in \bar{k}(\Delta^*, \delta^0)$$

where e_g is the g^{th} unit vector in R^{K-M} . Then the matrix

$$\tilde{G}_s \equiv \frac{\partial \tilde{G}}{\partial s} \text{ at } s^0$$

has full row rank.

Intuitively, A.3 ensures that, for every small perturbation dz of the initial production plan z^0 , there exists at least one small change ds such that $(s^0 + ds, z^0 + dz)$ is an equilibrium in which all the constraints which are binding in the initial state remain tight. Moreover, when such a change ds is unique, we shall say that the model is "deterministic" at s^0 :

Defn.1: The model of the planner is said to be deterministic (at s^0) if the matrix $\tilde{G}_s$ is square and non-singular.

The reader may feel that the model presented here suffers from an excessive degree of abstraction. In the next section, we shall describe a specification of this general model which will be used in later chapters to study shadow pricing rules more explicitly, as well as to carry out empirical exercises. Since abundant applications of our theory appear in the remainder of the text, the present chapter will remain essentially confined to an abstract theoretical exposition.

Section I.2 : An Example

Our example is a straightforward extension of the model analysed in Dreze (10), section III, to which the reader is referred for greater detail.

Private agents are of two types.

Consumers, indexed by superscripts $i \in I = \{1, \dots, N\}$, maximize their utility subject to a budget constraint, and, possibly, quantity constraints; their (direct) demand functions $x^i(q, \bar{x}^i, m^i)$ solve

$$(X) \quad \begin{array}{ll} \text{Max. } U^i(x^i) & \text{s.t.} \left\{ \begin{array}{l} q \cdot x^i = m^i \\ x_j^i = \bar{x}_j^i \quad \text{if } j \in \bar{J}^i \end{array} \right. \\ x^i & \end{array}$$

where q is a vector of consumer prices, $\bar{x}^i = (\bar{x}_j^i)_{j \in \bar{J}^i}$ is a vector of quotas, and m^i denotes i 's lump-sum income. The indirect utility function of the i^{th} consumer is defined as:

$$(7) \quad V^i(q, \bar{x}^i, m^i) \equiv U^i(x^i(q, \bar{x}^i, m^i)).$$

The aggregate net consumer demand vector is denoted by $x \equiv \sum_i x^i$.

Producers, indexed by superscripts $h \in H$, maximize profits subject to technological constraints and, possibly, quantity constraints; their net supply functions $y^h(p, \bar{y}^h)$ solve

$$(Y) \quad \text{Max.}_{y^h} \pi^h \equiv py^h \quad \text{s.t.} \quad \begin{cases} y^h \in Y^h \\ y_j^h = \bar{y}_j^h \quad \text{if } j \in \bar{J}^h \end{cases}$$

where Y^h is a convex production set, p is a vector of producer prices and $\bar{y}^h \equiv (\bar{y}_j^h)_{j \in \bar{J}^h}$ is a vector of quotas. The profit function of the h^{th} producer is defined as

$$(8) \quad \pi^h(p, \bar{y}^h) \equiv py^h(p, \bar{y}^h)$$

The aggregate net supply vector of private producers is denoted by $y \equiv \sum_h y^h$.

Consumer and producer prices differ by a vector of indirect taxes t :

$$(9) \quad t \equiv q - p$$

Private lump-sum incomes $(m^i)_{i \in I}$ consist of direct transfers from the government $(r^i)_{i \in I}$ as well as profit incomes:

$$(10) \quad m^i \equiv r^i + \sum_h a^{ih} \pi^h(p, \bar{y}^h)$$

where $A \equiv ((a^{ih}))_{\substack{i \in I \\ h \in H}}$ is a matrix of profit shares. For each

$h \in H$, $a^{oh} \equiv 1 - \sum_i a^{ih}$ may be interpreted as a profits tax.

Finally, net exports are denoted by e and the net addition to foreign exchange reserves by V^f :

$$(11) \quad V^f \equiv \sum_j p_j^w(e_j) e_j \stackrel{\text{not.}}{=} p^w(e) \cdot e$$

where p_j^w is the world price of commodity j which is assumed to depend solely upon e_j . The tariff revenue $(p^w - p)e$ accrues to the government.

In this specification of the general model the vector s consists of a selection of the components of the following vector (the other components can be considered as fixed characteristics of the economy):

$$(12) \quad s^* = \left((p_j), (t_j), ((\bar{x}_j^i)), ((\bar{y}_j^h), (r^i), ((a^{ih})), (e_j) \right)$$

The function $G(\cdot)$ takes the form

$$(13) \quad G(s) \equiv \begin{bmatrix} \sum_i x^i(p+t, \bar{x}^i, m^i) + e - \sum_h y^h(p, \bar{y}^h) \\ F(s) \end{bmatrix} \equiv \begin{bmatrix} E(s) \\ F(s) \end{bmatrix}$$

with $(m^i)_{i \in I}$ as in (10).

When $K = M$ so that $G(\cdot) \equiv E(\cdot)$, we shall speak of "the model without side constraints" (this terminology implies nothing a priori about the degree of freedom left to the planner in choosing s).

The social welfare function is of the form:

$$(14) \quad W(s) \equiv \psi(V^f, V^1(\cdot), \dots, V^N(\cdot))$$

with V^f and $(V^i(\cdot))_{i \in I}$ as in (11) and (7). The social valuation of foreign exchange reserves is exogenous to the model, but it will be redundant if a side constraint requires foreign payments to balance. Moreover, by Walras' law, if all markets clear and the government balances its foreign books, it balances its overall budget as well:

$$(15) \quad B \equiv pz + tx + \sum_h a^{oh} \pi^h + (p^w - p)e - \sum_i r^i \\ = p^w e \quad \text{when } x + e - y - z = 0 \quad (\text{see Dreze (10) p.36}).$$

For later reference, it is useful to mention an alternative presentation of the model:

$$(16) \quad s^* = \left((p_j), (t_j), ((\bar{x}_j^i)), ((\bar{y}_j^h)), (r^i), ((a^{ih})), (u^i) \right)$$

$$(17) \quad W(s) \equiv \Psi(V^f, u^1, \dots, u^N)$$

$$(18) \quad G(s) \equiv \left[\begin{array}{l} \sum_i \hat{x}^i(p + t, \bar{x}^i, u^i) + e - \sum_h y^h(p, \bar{y}^h) \\ [c^i(p + t, \bar{x}^i, u^i) - \sum_h a^{ih} \pi^h(p, \bar{y}^h) - r^i]_{i \in I} \\ [\bar{G}^{(k)}(s)]_{k=M+N+1, \dots, K} \end{array} \right]$$

with V^f as in (11), and $\hat{x}^i(\cdot)$, $c^i(\cdot)$ denoting respectively i 's compensated demand functions and expenditure (or cost) function:

$$(19) \quad c^i(q, \bar{x}^i, u^i) \equiv q \hat{x}^i(q, \bar{x}^i, u^i) \quad \text{where } \hat{x}^i(\cdot) \text{ solves}$$

$$(\hat{X}) \quad \begin{array}{ll} \text{Min. } q x^i & \text{s.t. } \left\{ \begin{array}{l} U^i(x^i) = u^i \\ x_j^i = \bar{x}_j^i \quad \text{if } j \in J^i \end{array} \right. \\ x^i & \end{array}$$

We shall refer to (12), (13), (14) as the "direct version" of the model, and to (16), (17), (18) as its "indirect version". Both versions are equivalent and they will be used interchangeably, according to convenience.

Some elementary properties of the constrained demand and supply functions introduced above are recalled in Appendix A.2 to Chapter II.

Section I.3 : Project evaluation, cost-benefit tests, and shadow prices

It is not very helpful to define shadow prices in meta-physical terms such as "the true value of commodities", and to view their prominent property as that of "possibly differing from market prices" (see e.g. the Penguin Dictionary of Economics (5), 1971, p.406).

Shadow prices are primarily analytical tools useful to the theorist. This is abundantly exemplified in the literature on second-best theory.

They may also become active ingredients of empirical models, as in a project evaluation or national income accounting exercise.

They may even reach the actual status of live economic signals, as when they orient various public decision processes in a mixed economy.

In any case, their meaning and properties can only be sensibly discussed in the explicit light of the particular function they are expected to perform. Although their potential field of application is very wide (analysis of policy reforms, national income accounting, design of a rationing scheme, public sector management, etc.), they are best known in the context of the project evaluation problem, which consists of identifying desirable changes in the public production plan. Let us then take up this problem in

the model outlined in Section I.1, and examine how shadow prices may contribute towards its solution.

Let s^0 be the initial state of the economy, with $\omega^0 \equiv G(s^0) \in \Omega^*$. A finite project is simply a vector $\Delta z \in R_0^M$, interpreted as a proposed modification of the public production plan z^0 . By a small project, on the other hand, we shall understand a vector $\dot{z} \in R_0^M$ indicating a direction of change for z . Two small projects $\dot{z}$ and $\dot{z}'$ such that $\dot{z} = k\dot{z}'$ for some $k > 0$ will therefore be treated as equivalent.

Usually, a small project will be considered to possess a particular property (e.g. feasibility, desirability, etc.) if there exists, near the origin, a continuum of finite projects endowed with the stated property; or in other words, if finite moves undertaken in the stated direction at a sufficiently small scale have the required property. The justification for using differential techniques rests on similar grounds; however, repeated interpretation in terms of finite moves will be avoided, and we shall concentrate most of our attention on small projects, or "projects" for short.

The first sentence of the previous paragraph is illustrated in the following definition:

Defn. 2: A (small) project $\dot{z} \in R_0^M$ is feasible if $\dot{z} \in K(Z^*, z^0)$, i.e. if

$$\exists \bar{\alpha} > 0 : (z^0 + \alpha \dot{z}) \in Z^* \quad \forall \alpha \in [0, \bar{\alpha}]$$

The public production set Z^* need not be fully known to the planner. Indeed, projects can often be interpreted as a discovery of hitherto unknown opportunities in public production. But as a rule issues of project design will not concern us.

The project evaluation problem (PEP), then, consists of deciding whether to "accept" or "reject" a pre-specified, feasible project $\dot{z}$. For the time being it is not necessary to be more precise about the exact implications of "accepting" a project.

If the project evaluation problem always has an unconditional answer, there exists a well-defined set of desirable projects, say Z^+ . Since we do not distinguish between projects which are positive scalar multiples of each other, Z^+ is a cone.^{5/} In addition, it seems reasonable to expect two projects which are separately desirable to be jointly desirable as well:

$$(20) \quad \dot{z}^1, \dot{z}^2 \in Z^+ \Rightarrow (\dot{z}^1 + \dot{z}^2) \in Z^+$$

When (20) holds, Z^+ is a convex cone, and it can be to a great extent characterized by its polar cone Λ where^{6/}

$$(21) \quad \Lambda \equiv \left\{ \lambda \in R_0^M : \lambda \dot{z} \geq 0 \quad \forall \dot{z} \in Z^+ \right\}$$

In particular

$$(22) \quad \begin{cases} \dot{z} \in Z^+ \Rightarrow \lambda \dot{z} \geq 0 \quad \forall \lambda \in \Lambda \\ \lambda \dot{z} > 0 \quad \forall \lambda \in \Lambda \Rightarrow \dot{z} \in Z^+ \end{cases}$$

Furthermore, if Z^+ is an open set:

$$(23) \quad \dot{z} \in Z^+ \Leftrightarrow \lambda \dot{z} > 0 \quad \forall \lambda \in \Lambda$$

These observations underline the potential ability of cost-benefit tests to identify Z^+ , and thus solve the PEP. Cost-benefit tests can be designed in various ways; for the sake of definiteness we shall concentrate on the following one:

Defn.3: The cost-benefit test for project evaluation recommends

- (i) Accepting $\dot{z}$ if $\lambda \dot{z} > 0 \quad \forall \lambda \in \Lambda^*$
- (ii) Rejecting $\dot{z}$ if $\lambda \dot{z} < 0$ for some $\lambda \in \Lambda^*$

where $\Lambda^* \subseteq R_0^M$ is a pre-specified (non-empty) shadow price set.

If neither (i) nor (ii) apply, the test is inconclusive.

The elements of Λ^* are vectors of shadow prices for project evaluation. As the foregoing discussion shows, the adoption of a particular project evaluation criterion (i.e. a description of Z^+), and the specification of a shadow price set permitting accurate CB tests, are dual problems. The search for the set of desirable projects is therefore equivalent (at least when Z^+ is open) to an enquiry into the appropriate choice of shadow prices for project evaluation.

All positive scalar multiples of a shadow price vector can be regarded as equivalent; correspondingly, we shall regard two shadow price sets as equivalent when each element of each set has

an equivalent in the other set, and two CB tests as equivalent when they use equivalent shadow price sets. We shall say that a CB test is simple if Λ^* is equivalent to a singleton, i.e. if

$$(24) \quad \exists \lambda^* \neq 0 : \lambda \in \Lambda^* \Rightarrow \lambda = \alpha \lambda^* \text{ for some } \alpha > 0.$$

Finally, a CB test will be regarded as accurate if it never recommends one of the following mistakes:

(i) Rejecting $\dot{z}$ while $\dot{z} \in Z^+$

(ii) Accepting $\dot{z}$ while $\dot{z} \notin Z^+$

When two simple and accurate CB tests exist, they are obviously equivalent. Without loss of generality, then, we may assert that when a simple and accurate CB test exists, it is unique.

Simple and accurate CB tests are especially attractive when one insists on a decentralized solution to the project evaluation problem. Indeed, when the planner does not fully know Z^* or the technical details of projects, but a simple and accurate CB test exists, he can communicate the vector λ^* of (24) to specialized project evaluators and entrust them with project acceptance decisions. In principle, of course, a decentralized solution to the project evaluation problem is possible as soon as a well-defined set of desirable projects exists; but it is not likely to be very operational without the informational economy inherent in simple and accurate CB tests.

As Defn. 3 and (24) make clear, a simple and accurate CB test exists if and only if the closure of the set of desirable

projects is a half-space in R^M . This feature of Z^+ is particularly plausible when we can construct a smooth social preference relation over the set of public production plans. Such preferences seem naturally defined when a unique state of the economy can be associated with each production plan. We shall refer to this situation as one where the planner has a policy:

Defn. 4: A policy or closing rule is a function 7/

$$\lambda : z \rightarrow \lambda(z) \in R^L \text{ s.t. } E(\lambda(z)) = z \text{ and } \lambda(z) \in S^*$$

Remark 1: It is easy to show that, along with a suitable redefinition of the vector of signals if necessary, a closing rule in effect converts a non-deterministic model into a deterministic one (see Defn. 1 and the comments preceeding it).

Given a policy $\lambda(\cdot)$, then we can define the social value of the production plan z as

$$(25) \quad V(z) \equiv W(\lambda(z))$$

and say that a project $\dot{z}$ is welfare-improving if $\dot{z} \in \bar{k}(\{z : V(z) > V(z^0)\}, z^0)$, i.e. if

$$(26) \quad \exists \bar{\alpha} > 0 : V(z^0 + \alpha \dot{z}) > V(z^0) \quad \forall \alpha \in]0, \bar{\alpha}[$$

If $V(\cdot)$ is differentiable at z^0 , and if "desirable" is understood as synonymous with "welfare-improving", a simple and accurate CB test exists associated with the shadow price vector

$$(27) \quad \lambda^* = V_z \text{ at } z^0$$

provided $\lambda^* \neq 0$ (see Dreze (10) Propn.1, p.9).

As the literature on project appraisal often emphasizes, however, project evaluation and policy design are intimately related problems; project evaluation should not proceed without a proper articulation between public production decisions and planning at large.

At least three major branches of theoretical welfare economics offer a powerful conceptual framework for tackling this issue. They are, in a suggestive decreasing order of intellectual maturity: second-best optimality theory, cost-benefit analysis, and the theory of reform. Correspondingly, each of these theories offers a particular interpretation of the project evaluation problem, and an original perspective on shadow pricing. To be more precise, we shall find that the correct shadow pricing rules may well differ according as to whether projects are interpreted as:

- (i) Building blocks in the design of an optimum (the second-best viewpoint).
- (ii) An isolated perturbation of the initial equilibrium (the traditional CBA viewpoint).
- (iii) An opportunity for reform (the reform viewpoint).

As a first step towards organizing our thoughts, we shall briefly review the three theories mentioned above, and investigate areas of agreement and contrast between the emerging notions of shadow prices.

Section I.4 : Second-best optimality theory and decentralization prices

A typical second-best optimality problem for the planner consists of the following program:

$$(P) \quad \begin{array}{ll} \text{Max. } W(s) & \text{s.t. (i) } E(s) - z = 0 \\ s, z & \text{(ii) } s \in S \\ & \text{(iii) } z \in Z \end{array}$$

where $S \subseteq S^*$ is an opportunity set and Z embodies the current knowledge of the public production set. A state s^* of the economy is optimal relative to (S, Z) if (s^*, z^*) solves (P), with $z^* \equiv E(s^*)$. A state optimal relative to (S^*, Z^*) is a full optimum. A full optimum is assumed to exist.

A large class of problems of the type (P) have been investigated in the literature on second-best theory. The recent work of Guesnerie (11) has convincingly exhibited the common structure underlying many of the more influential models. In particular Guesnerie has studied the conditions of existence and the general properties of a vector $p \neq 0$ which decentralizes an optimal production plan in the sense familiar to economists:

$$(28) \quad pz^* = \max_{z \in Z} pz \quad \text{and} \quad \exists s^* : (s^*, z^*) \text{ solves (P),}$$

When (28) holds, the public production plan z^* will be acceptable to a profit-maximizing "manager" confronted with the price system p .

The existence of a vector ρ satisfying (28) can be asserted as soon as Z is convex and (P) has a solution (s^*, z^*) with z^* on the boundary of Z . Moreover, if Z allows free-disposal of commodities, $\rho \geq 0$, implying (weak) public production efficiency at z^* .

The following assumptions are sufficient to guarantee the existence of an optimum characterized by public production efficiency. Intuitively, they require the instruments at the disposal of the planner to be powerful enough to ensure that a welfare improvement is possible when all the scarcity constraints are simultaneously relaxed.

A*.1. Z is convex and allows free disposal of commodities.

A*.2. A solution (s^*, z^*) of (P) exists such that:

(i) $E(\cdot)$ is continuous at s^*

(ii) For every ϵ -neighbourhood $N_\epsilon(s^*)$ of s^* ,

$$\exists s \in S \cap N_\epsilon(s^*) : W(s) > W(s^*)$$

Propn.1. (Public production efficiency)

If A*.1 and A*.2 are satisfied, there exists a solution (s^*, z^*) of (P) and a vector $\rho \geq 0$ s.t.

$$(29) \quad \rho z^* = \max_{z \in Z} \rho z$$

Proof:

It is sufficient to show that the solution (s^*, z^*) mentioned in A*.2 has z^* on the boundary of Z . Suppose (if possible) that this is not the case. Let $\epsilon > 0$ be given. By A*.2(ii), $\exists s_\epsilon \in N_\epsilon(s^*) : s_\epsilon \in S$ and $W(s_\epsilon) > W(s^*)$. Moreover, since $z^* \in Z$ and $E(\cdot)$ is cts at s^* , $z_\epsilon \equiv E(s_\epsilon) \in Z$ if ϵ is small enough. But then (s_ϵ, z_ϵ) is feasible and $W(s_\epsilon) > W(s_\epsilon^*)$, a contradiction. Thus z^* must be on the boundary of Z . #

Any vector ρ satisfying (28) permits a decentralization of the public production decisions associated with an optimum. Not all of them have equally attractive properties, however, and we shall confine our interest to a particular subset of them, consisting of those which, in addition, allow a decentralization of technological knowledge, in a sense made precise in the following definition.

Defn. 5: A vector $\rho^* \neq 0$ is a decentralization price vector (DP) for a solution (s^*, z^*) of (P) if

$$\rho^* z^* = \max_{z \in \hat{Z}} \rho^* z$$

for every set $\hat{Z} \subseteq R^M$ satisfying:

- (i) $\hat{Z}$ is convex
- (ii) (s^*, z^*) is optimal relative to $(S, \hat{Z})$

Loosely speaking, this definition restricts the set of eligible vectors to those which, at an optimum, can be quoted without

any knowledge of the (local or global) characteristics of Z , other than that it is convex and includes z^* . Figure 1 illustrates this point.

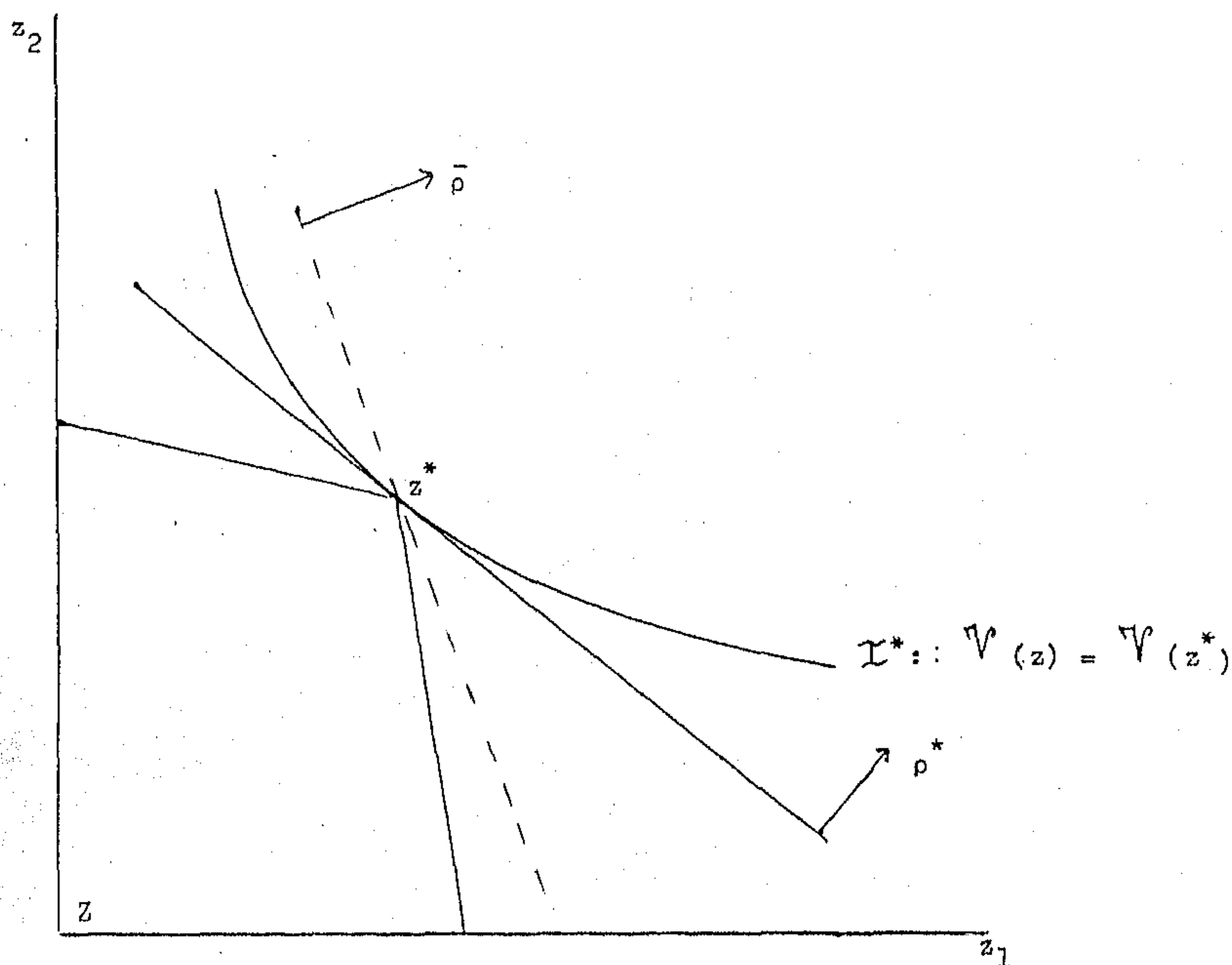


FIGURE 1

On this diagram, I^* represents a well-defined social indifference curve to be derived shortly. The vector p^* supporting I^* at z^* is a DP vector since any set $\hat{Z}$ satisfying the requirements mentioned in Defn.5 must lie below the hyperplane orthogonal to p^* and passing through z^* . By contrast, the vector $\bar{p}$ satisfies (28) but not Defn.5.

We shall now exhibit the most fundamental characterization of decentralization prices, namely as marginal rates of substitution in a social utility function which is well-defined on the commodity space. This social utility function indicates the maximum social welfare attainable for a given public production plan z .

Formally, for any sets (S, Z) such that (P) has a solution, let $W^*(S, Z)$ be the maximum of that program. The potential social value of a production plan z is defined as

$$(30) \quad V(z) \equiv W^*(S, \{z\})$$

when the latter exists. For convenience, we shall assume that, for every set S of interest, Z^* lies in the interior of the domain of $V(\cdot)$, in which case (P) is equivalent to:

$$(P') \quad \underset{Z}{\text{Max.}} V(z) \quad \text{s.t.} \quad z \in Z$$

This formulation of the planner's problem justifies the construction of Figure 1, and makes the content of the next proposition intuitively obvious.

Propn. 2: Let $A^*.1$ hold and $A^*.2$ be satisfied for some (s^*, z^*) . If V is differentiable at z^* , and $V_z(z^*) \neq 0$, then there exists a unique DP vector $\rho^* \geq 0$ associated with (s^*, z^*) and given by

$$(31) \quad \boxed{\rho^* \equiv V_z \text{ at } z^*}$$

Proof:

(i) Existence. Let $\rho^* \in \mathcal{V}_Z$ at z^* and let $\hat{Z}$ be any convex set s.t. $\exists s : (s, z^*)$ is optimal relative to $(S, \hat{Z})$.

Suppose (if possible) that $\exists z' \in \hat{Z}$ s.t. $\rho^* z' > \rho^* z^*$ and let $z_\epsilon = z^* + \epsilon(z' - z^*)$, with $\epsilon \in]0, 1[$.

Since $\hat{Z}$ is convex, $z_\epsilon \in \hat{Z}$; moreover, if ϵ is small enough, z_ϵ belongs to the domain of $\mathcal{V}$ and so:

$$\mathcal{V}(z_\epsilon) = \mathcal{V}(z^*) + \mathcal{V}_Z(z^*)(z_\epsilon - z^*) + o(\|z_\epsilon - z^*\|^2)$$

Hence

$$\begin{aligned} \mathcal{V}(z_\epsilon) &= \mathcal{V}(z^*) + \epsilon \rho^*(z' - z^*) + \|z' - z^*\| o(\epsilon^2) \\ &> \mathcal{V}(z^*) \text{ for small } \epsilon. \end{aligned}$$

But this contradicts the optimality of (s^*, z^*) since (P) is equivalent to (P'). Thus we must have

$$\rho^* z^* = \max_{z \in \hat{Z}} \rho^* z \text{ for every } \hat{Z} \text{ convex and s.t. } \exists s : (s, z^*) \text{ is optimal relative to } (S, \hat{Z}).$$

i.e. ρ^* is a DP vector. Moreover, since A*.1 holds, $\rho^* \geq 0$.

(ii) Uniqueness. Now let ρ be any vector of $\mathbb{R}^K$ which is not colinear with ρ^* . We may assert

$$\begin{aligned} \exists \dot{z} \in \mathbb{R}_0^K : \quad &\rho \dot{z} > 0 \\ &\rho^* \dot{z} < 0 \end{aligned}$$

Consider the non-empty convex set

$$\hat{Z}_\epsilon = \left\{ z : z = z^* + \alpha \dot{z} \text{ for some } \alpha \in [0, \epsilon] \right\} \quad (\epsilon > 0).$$

$$\text{Now } z \in \hat{Z}_\epsilon \Rightarrow \nabla_z V(z^*) \cdot (z - z^*) = \rho^*(\alpha \dot{z}) \quad \text{for some } \alpha \in [0, \epsilon]$$

$$< 0 \quad (\text{unless } z = z^*)$$

and so, for small ϵ :

$$\nabla V(z^*) = \max_{z \in \hat{Z}_\epsilon} \nabla V(z)$$

Therefore $\exists s : (s, z^*)$ is optimal relative to $(S, \hat{Z}_\epsilon)$. However, if $z_\alpha = z^* + \alpha \dot{z}$ and $\alpha > 0$ is small enough:

$$z_\alpha \in \hat{Z}_\epsilon \quad \text{and} \quad \rho z_\alpha = \rho z^* + \alpha \rho \dot{z} > \rho z^*.$$

So ρ cannot decentralize (s, z^*) in the sense of Defn.5. #

As a corollary to Propn.2, we can infer that, when the initial state of the economy is optimal, the DP vector ρ^* allows the design of CB tests which identify directions of change in the public production plan (e.g. made possible by an improved knowledge of the public production set) leading to an improvement in potential or maximum welfare. What about actual welfare? One would normally expect the welfare gains associated with a project to depend upon the actual response of the planner. A remarkable property of the optimum, known in optimization theory as the envelope property, precisely denies this expectation: to the first-order, the welfare gains accompanying a project are the same irrespective of the reaction of the planner. This is shown in Propn. 3 below. Defn.6 recalls and extends Defn.4.

Defn.6 (i) A policy is a function

$$J : z \in R^K \rightarrow J(z) \in R^L \text{ s.t. } E(J(z)) = z \text{ and } J(z) \in S^*$$

(ii) A policy is said to be admissible (relative to S) if

$$J(z) \in S \quad \forall z \in R^K$$

The set of admissible policies is denoted by $\mathcal{P}_S$, or simply $\mathcal{P}$ when no confusion is likely.

(iii) A policy J is optimal at z if it is admissible and

$$W(J(z)) = V(z)$$

Propn.3 (Envelope property)

Let (s^*, z^*) be a solution of (P) and $J(\cdot)$ be any admissible policy satisfying:

(i) $J(\cdot)$ is differentiable at z^*

(ii) $J(z^*) = s^*$

Then, if A.1 and the assumptions of Propn.2 are satisfied:

$$(32) \quad W_{s,z} J = V_z = \rho^* \text{ at } z^*$$

where ρ^* is the unique DP vector associated with (s^*, z^*) .

Proof:

Let $V(z) \equiv W(\lambda(z))$. Since $\lambda(\cdot)$ is an admissible policy,
 $V(z) \leq \bar{V}(z) \forall z$. On the other hand, $V(z^*) = W(\lambda(z^*)) = W(s^*) = \bar{V}(z^*)$.
 Since both V and $\bar{V}$ are differentiable at z^* , we may conclude:

$$V_z = \bar{V}_z \text{ at } z^*, \text{ or } V_z = \rho^*$$

where ρ^* is the unique DP vector associated with (s^*, z^*) and
 introduced in Propn.2. Finally, since W is differentiable at z^*

$$V_z = W_{s\lambda} \text{ at } z^*$$

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The envelope property is roughly illustrated in Figure 2.

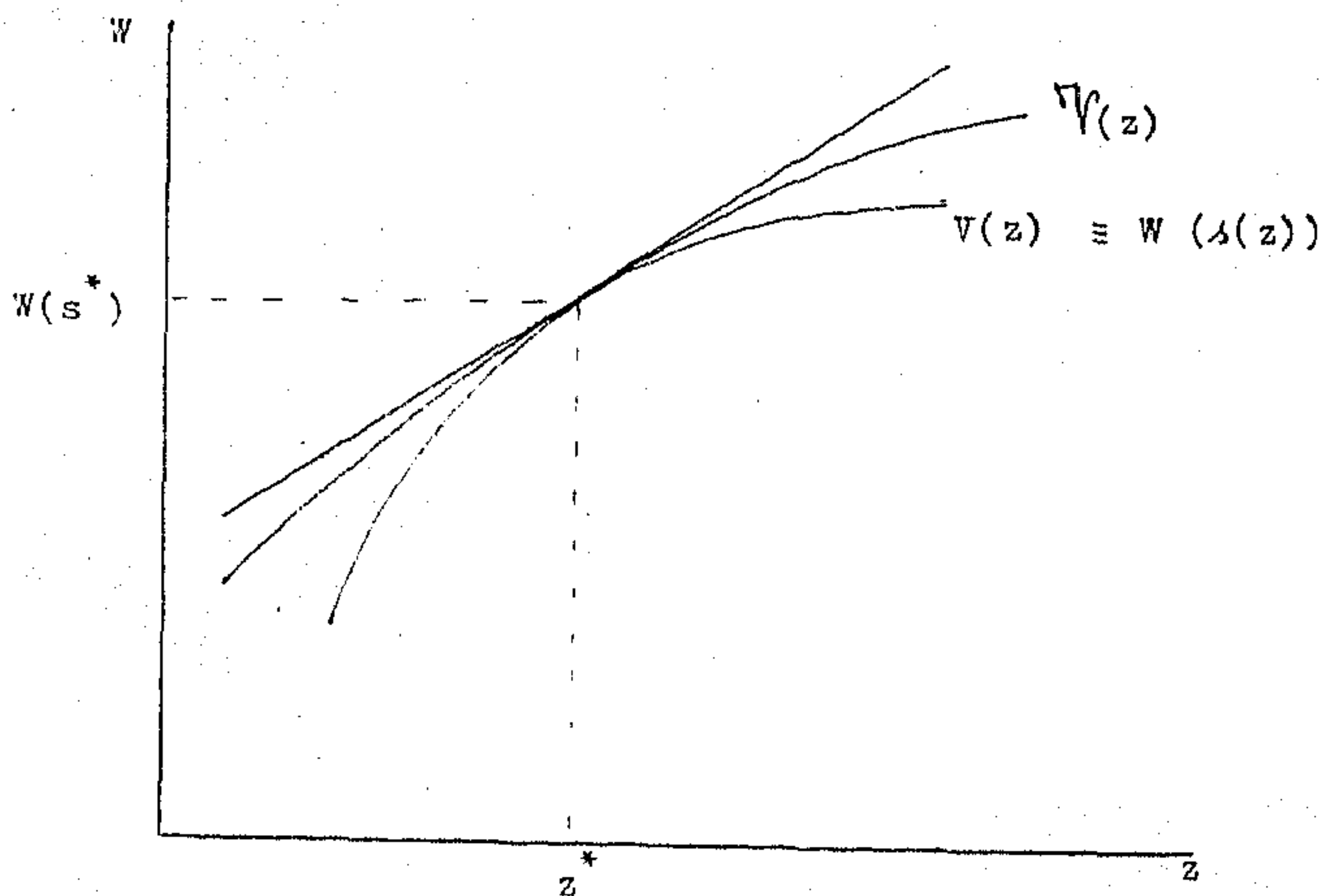


FIGURE 2

Incidentally, (32) illustrates, in a particular context, the equivalence between the primal route and the dual route to project evaluation. The impact of a project on social welfare can equivalently be measured by shadow pricing its net output vector, or by calculating its total effect on the state of the economy and comparing the world with and without the project from the point of view of social preferences. This equivalence is quite general and is indeed an intrinsic property of shadow prices for project evaluation whenever they are meant to measure actual welfare changes.

We conclude this section with a very simple illustration.

Example 1: (Optimal trade)

In the model of Section I.2, suppose that the opportunity set S allows an unrestricted choice of $\bar{e} \equiv (e_j)_{j \in J_1}$, where $J_1 = \{1, \dots, M_1\}$ is an index set covering tradeable commodities. It is then straightforward to verify that

$$(33) \quad \frac{\rho_j}{\rho_k} = \frac{p_j^w \left(1 + \frac{1}{\epsilon_j^w}\right)}{p_k^w \left(1 + \frac{1}{\epsilon_k^w}\right)} \quad \text{if } j, k \in J_1$$

where ϵ^w is a vector of elasticities of world demand (a formal derivation is contained in Chapter II). At an optimum of public production and foreign trade, therefore, relative decentralization prices for tradeable commodities coincide with their relative marginal costs (or revenues) on the world market. This well-known result has been emphasized by Dasgupta-Stiglitz (4), among others.

Section I.5 : Traditional cost-benefit analysis and welfare prices

According to a modern and authoritative text book on cost-benefit analysis, it is important to "emphasize the difference between the theory of the economic optimum, and the theory of the comparison of economic states"; "the theory of the comparison of economic states, generally called *cost-benefit analysis*, attempts (...) to find how to compare any two states of the economy, neither of which need to be an optimum".^{10/}

When cost-benefit analysis is applied to the project evaluation problem, the terms of the comparison are normally the states of the economy resulting alternatively from the adoption or rejection of the contemplated project. This is the fundamental with and without principle of traditional CBA.

The effects of a project generally depend, of course, upon the actual policy of the planner; the latter may be very difficult to identify precisely, especially when the horizon of the with and without principle extends to the infinite future. In particular, since every new project confronts the planner with different circumstances, the planner's policy may fail to be objectively observable. The associated problems have been of some concern to cost-benefit theorists (see e.g. Sen's (23) early discussion of some aspects of the issue). However, the difficulty at this point is, strictly speaking, of a merely informational nature. Provided that the actual behaviour of the planner can somehow be predicted (as when government policies are explicitly laid out in a 5-year Plan), the with and without principle unambiguously recommends evaluating projects on the basis of their

actual impact on social welfare.

In the language of our model, then, CBA presumes the existence of a policy, say λ^0 , describing the actual behaviour of the planner (notice that when the model is "deterministic", λ^0 is naturally and uniquely defined, at least around s^0). It is then conceptually straightforward to compute the actual social value of a production plan z , say $V^0(z)$, as

$$(34) \quad V^0(z) \equiv W(\lambda^0(z)).$$

We shall call welfare prices (WP) the shadow prices implicit in this social preference function (see Figure 2):

Defn.7: Let s^0 be the initial state of the economy, with $z^0 \equiv E(s^0) \in Z^*$ and $s^0 = \lambda^0(z^0)$. When the function V^0 defined in (34) is differentiable at z^0 and $V_z^0(z^0) \neq 0$, we call welfare price vector the unique vector $v^0 \in R_0^K$ satisfying

(35)

$$v^0 = V_z^0 \text{ at } z^0$$

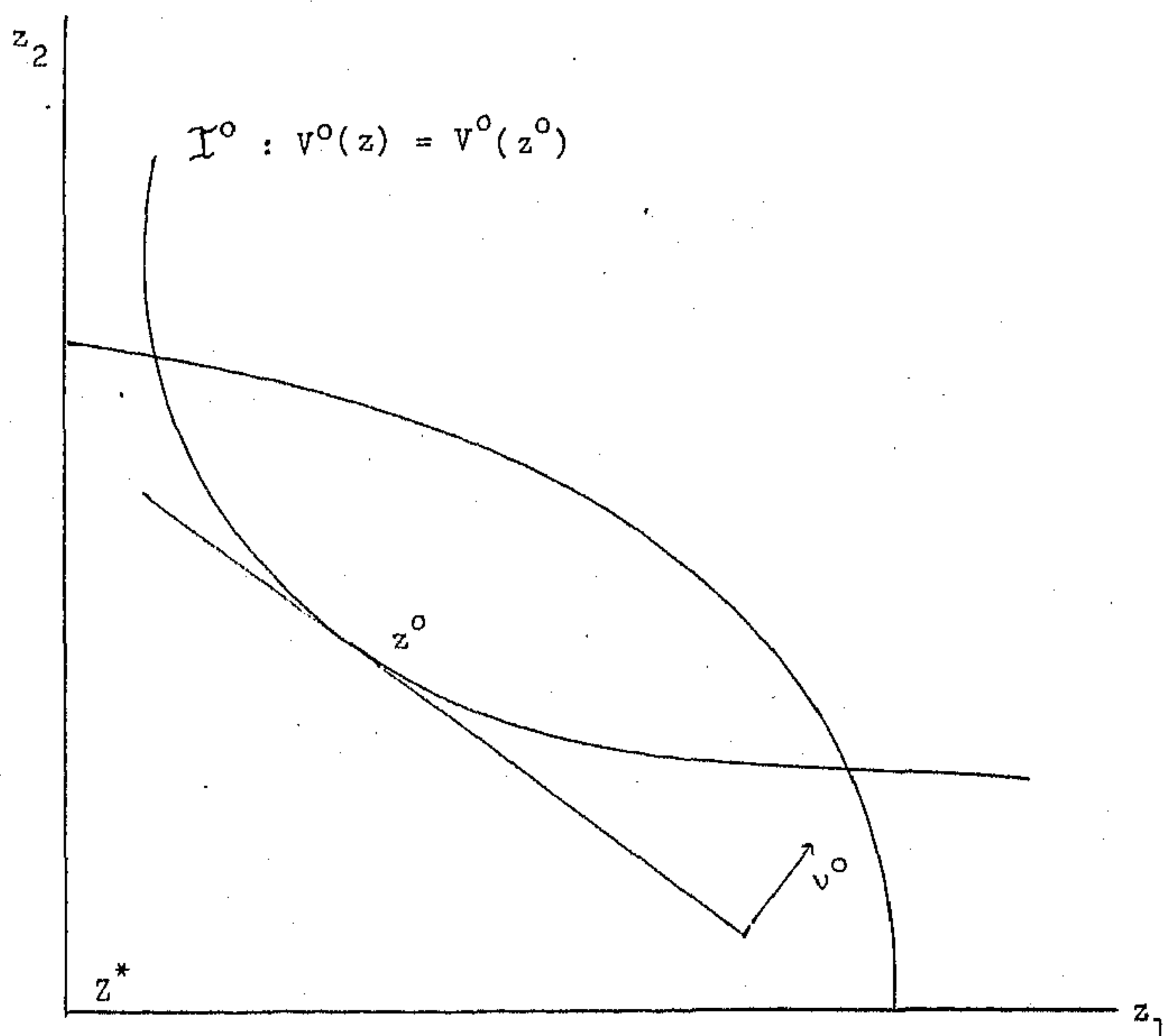


FIGURE 3

As the next proposition confirms, welfare prices detect projects which are desirable according to the actual welfare improvement criterion.

Defn. 8 : Let s^0 be the initial state of the economy, and $z^0 = E(s^0)$. A project $\dot{z}$ is said to be welfare-improving if $\dot{z} \in \bar{k} \left(\left\{ z : W(\lambda^0(z)) > W(z^0) \right\}, z^0 \right)$, i.e. if

$$(36) \quad \exists \bar{\alpha} > 0 : W(\lambda^0(z^0 + \alpha \dot{z})) > W(s^0) \quad \forall \alpha \in]0, \bar{\alpha}[$$

Propn. 4: Let $W(\cdot)$ and $\lambda^0(\cdot)$ be differentiable at s^0 and z^0 , respectively, and $V_z(z^0) \neq 0$. If the set Z^+ of desirable projects consists of all welfare-improving projects, then:

- (i) $\dot{z} \in Z^+ \Rightarrow v^0 \dot{z} \geq 0$
- (ii) $v^0 \dot{z} > 0 \Rightarrow \dot{z} \in Z^+$

where v^0 is the welfare price vector.

Proof:

The result follows immediately from (34), (35), (36) and the notion of differentiability.

#

Corollary: When Z^+ consists of all welfare-improving projects, the welfare price vector (if it exists) defines simple and accurate CB tests.

Remark 2: The vector v^0 can prove useful in carrying out a cost-benefit analysis of policy reforms as well as of public production projects. Loosely speaking, it may be shown that, if a small move $\dot{s}$ satisfies

- (i) $F_s \dot{s} \in \bar{k}(\Delta^*, \delta^0)$
- (ii) $W_s \dot{s} - v^0 E_s \dot{s} > 0$

then the planner can improve his policy (locally) by revising it marginally in a direction indicated by

$$(iii) \quad \dot{s}^* = (I - \lambda_z^0 E_s) \dot{s}$$

Indeed, $W_s \dot{s}^* = (W_s - v^0 E_s) \dot{s}^* > 0$ by (ii) and $\dot{s}^*$ may also be shown to be a feasible direction of policy reform. Since these issues are dealt with in detail in the next section, they will not be pursued further here.

Informational problems apart, the with and without principle of traditional CBA seems to provide an appealing and operational criterion for project evaluation when the planner is committed, explicitly, to following known policies. However, when policies are, for some reason, initially suboptimal, such a commitment need not exist. On the contrary, the planner may look forward to opportunities for improving them. A project provides an opportunity for undertaking joint reforms of the public production plan and other policy decisions, which one would expect to be worth investigating. This possibility, and the potential role of shadow prices in guiding such a reform process, is considered in the next section. We conclude here with a simple illustration, parallel to Example 1 of the previous section.

Example 2 : (Free trade). In the application of Section I.2 without side constraints, let

$$(i) \quad s = ((e_j)_{j \in J_2}, (s_{\ell_j})_{J \setminus J_2})$$

where $J_2 = \{1, \dots, M_2\}$ is an index set covering freely traded commodities and $(s_{\ell_j})_{J \setminus J_2}$ are such that the matrix

$$(iii) \quad \tilde{G}_s \equiv E_s \text{ at } s^0$$

has full rank. The model is then deterministic (see Remark 1 above), and, by the implicit function theorem, a unique policy λ^0 is defined around s^0 and satisfies

$$(iii) \lambda_z^0 = E_s^{-1} \text{ at } s^0$$

Hence

$$(iv) v^0 = W_s \lambda_z^0 = W_s E_s^{-1}$$

Equivalently

$$(v) W_s - v^0 E_s = 0$$

Now the matrix E_s is easily verified to be of the form:

$$(vi) E_s = \begin{bmatrix} I_{M_2} & E_s^{12} \\ 0 & E_s^{22} \end{bmatrix} \text{ for some matrices } E_s^{12}, E_s^{22}.$$

and hence (v) implies

$$(vi) v_j^0 = \mu^f p_j^w \left(1 + \frac{1}{\varepsilon_j^w}\right) \quad \forall j \in J_2$$

where $\mu^f \equiv \frac{\partial \Psi}{\partial V^f}$ is the marginal social value of foreign exchange,

and ε_j^w is as in Example 1. Hence, in the model under discussion, the welfare price of freely traded goods is their marginal cost (or revenue) in world trade, multiplied by the marginal social value of

foreign exchange. This is not surprising, since the general equilibrium effect of a small change in the public production of such a good reduces in this model to a depletion or repletion of foreign exchange reserves amounting to the marginal cost or marginal revenue^{11/} of that good on the world market.

Section I.6 : Price-guided reform processes and fictitious prices

The benefits of a project generally depend upon how its inputs are acquired and how its outputs are disposed of. In a mixed economy these considerations are usually to some extent a matter of policy, within the control of the planner himself.

In Section I.4, we saw how, by the envelope property of welfare optima, the welfare gains from a project are in fact the same (to the first order) for every admissible policy compatible with the initial state, provided that the latter is optimal. As explained in Section I.5, on the other hand, the traditional CBA viewpoint considers the planner as any other observable agent, behaving systematically though not necessarily optimally, and suggests evaluating projects in the light of his actual policies.

If the latter approach is sometimes satisfactory from the viewpoint of an observer, however, it may seem unappealing to the planner himself, and apparently contrasts with the frequent emphasis in cost-benefit literature on the need to harmonize project evaluation and planning. When the planner is neither optimizing nor committed (possibly because of exogenous changes in the environment, or informational failures) this emphasis would suggest evaluating a project in the light of the whole set of reforms it makes possible. This viewpoint has been adopted in a few recent writings on CBA (e.g. Diewert-Hammond (8)); we shall scrutinize it in the model of Section I.1.

For the purposes of this section, it will be convenient to

work with version (6.i)-(6.iii) of the planner's model. Furthermore, since we shall confine our attention to infinitesimal changes in the state of the economy, it is permissible to weed out all the constraints which are slack in the initial state. The planner's model then becomes

$$(37)(i) \quad W = W(s)$$

$$(37)(ii) \quad \tilde{G}(s) - \tilde{\omega} = 0$$

$$(37)(iii) \quad \tilde{\omega} \in \tilde{\Omega}$$

where $\tilde{G}$, $\tilde{\omega}$ and $\tilde{\Omega}$ are appropriate truncations of G , ω and Ω respectively (see A.3). By construction, if

$$(38) \quad \tilde{\omega} \equiv \begin{bmatrix} z \\ \delta \end{bmatrix} \quad \text{and} \quad \tilde{\Omega} \equiv Z^* \times \tilde{\Delta}$$

then $\tilde{\Delta}$ is a rectangle and $\tilde{\delta}$ lies at one of its corners.

The above preliminary construction being understood throughout this section, we shall henceforth omit the symbol " $\sim$ " applying to δ , ω , $F(\cdot)$, $G(\cdot)$, G_s , Δ and Ω .

Defn. 9: Let s^0 be the initial state of the economy, and $\omega^0 = G(s^0)$.

(i) A reform is a vector $\dot{s} \in R^L$, understood as a proposed direction of change for the state of the economy.

(ii) A reform is potentially feasible if:

$$(39) \quad F_s \dot{s} \in \bar{k}(\Delta, \delta^0)$$

It is feasible if, in addition, there exists a feasible project $\dot{z}$ s.t.

$$(40) \quad E_s \dot{s} = \dot{z}$$

(iii) A reform $\dot{s}$ is welfare-improving if

$$(41) \quad \dot{s} \in \bar{k}(\{s : W(s) > W(s^0)\}, s^0)$$

It is possible to provide a rigorous justification for the second clause of Defn. 9. Intuitively, it runs as follows: when a pair $(\dot{s}, \dot{z})$ is feasible according to Defn. 9(ii), and A.3 is satisfied, a small finite move in a direction $\dot{s}'$ arbitrarily close to that indicated by $\dot{s}$ ^{12/} is compatible with the project $\dot{z}$.

With every project $\dot{z}$ we may now associate a whole set of feasible reforms $\mathcal{R}_{\dot{z}}$:

$$(42) \quad \mathcal{R}_{\dot{z}} = \left\{ \dot{s} : E_s \dot{s} = \dot{z} \text{ and } F_s \dot{s} \in \bar{k}(\Delta, \delta^0) \right\}$$

By an "allocation mechanism", say Γ , we shall understand a function indicating, for any project, a feasible way of acquiring its inputs and disposing of its outputs. When using differential techniques, it is not restrictive to require this function to be linear:

Defn. 10: An allocation mechanism is a linear function ^{13/}

$$(43) \quad \Gamma : \dot{z} \in R^M \rightarrow \dot{s} = \Gamma \dot{z} \in R^L$$

satisfying

$$(44) \quad \Gamma \dot{z} \in \mathcal{Q}_z \quad \forall \dot{z} \in R^M$$

The set of all allocation mechanisms at s^0 is denoted by γ^0 .

Before discussing the evaluation of projects and reforms, it is useful to derive a few technical results on allocation mechanisms. This is the purpose of lemma 1 and its corollaries.

Lemma 1: An $L \times M$ matrix Γ is an allocation mechanism iff

$$(45) \quad \begin{aligned} (i) \quad E_s \Gamma &= I \\ (ii) \quad F_s \Gamma &= 0 \end{aligned}$$

Proof:

$\Rightarrow$: Let Γ be an allocation mechanism. Using Defns. 9 and 10:

$$\begin{aligned} (i) \quad E_s \Gamma \dot{z} &= \dot{z} \quad \forall \dot{z} \in R^M \Rightarrow E_s \Gamma = I \\ (ii) \quad &\text{Without loss of generality we may assume (since } \delta^0 \text{ lies at a corner of } \Delta): \delta \in \bar{k}(\Delta, \delta^0) \Leftrightarrow \delta \leq 0. \\ &\text{Then} \end{aligned}$$

$$F_s \Gamma \dot{z} \leq 0 \quad \forall \dot{z} \in R^M \Rightarrow F_s \Gamma = 0$$

$\Leftarrow$: Let Γ be a matrix s.t. $E_S \Gamma = I$, $F_S \Gamma = 0$. Then
 $\forall \dot{z} \in \mathbb{R}^M$:

$$\begin{cases} E_S \Gamma \dot{z} = \dot{z} \\ F_S \Gamma \dot{z} = 0 \quad \in \bar{k}(\Delta, \delta^0) \end{cases}$$

i.e. $\Gamma \dot{z} \in \mathcal{R}_z$. Hence Γ is an allocation mechanism. $\#$

Corollary 1: Let $\lambda(.)$ be a policy s.t. $\lambda(.)$ is differentiable at s^0 . Then λ_z is an allocation mechanism.

Proof: Differentiating the identity $E(\lambda(z)) = z$, we obtain

$$E_S \lambda_z = I. \text{ Furthermore, } \forall \epsilon > 0, \forall \dot{z} \in \mathbb{R}^M : F(\lambda(z^0 + \epsilon \dot{z})) \in \Delta \text{ and} \\ F(\lambda(z^0 + \epsilon \dot{z})) = \delta^0 + \epsilon F_S \lambda_z \dot{z} + o(\epsilon^2)$$

Since δ^0 lies on the boundary of Δ , this is possible only if

$$F_S \lambda_z = 0. \text{ An appeal to lemma 1 completes the proof. } \#$$

Corollary 2: An $L \times M$ matrix Γ is an allocation mechanism iff there exists an $(L \times (K-M))$ "auxiliary" matrix A s.t.

$$(46) \quad G_S^- \equiv [\Gamma \ A]$$

is a right-inverse of G_S . The class of all right-inverses of G_S is denoted by $\mathcal{G}^-$ (this class is non-empty by (A.3)).

Consider now a project $\dot{z}$. A reform $\dot{s} \in \mathcal{R}_z$ will be referred to as an implementation of the project $\dot{z}$. When a project $\dot{z}$ is implemented via an allocation mechanism Γ , its impact on social welfare becomes:

$$(47) \quad \dot{W} = W_s \dot{s} = W_s \Gamma \dot{z} = \phi \dot{z}$$

where ϕ is a shadow price vector designed to harmonize the primal and the dual route:

$$(48) \quad \phi \equiv W_s \Gamma$$

The elements of ϕ will be called "fictitious prices"

(FP) :

Defn.11: Let s^0 be the initial state of the economy. A fictitious price vector is an element ϕ of R^M satisfying (48) for some $\Gamma \in \gamma^0$ where γ^0 is the set of all allocation mechanisms at s^0 . The set of all fictitious price vectors at s^0 is denoted by Φ .

Fictitious prices can prove useful in guiding a systematic process of reform. To see this, let us consider any potentially feasible reform $\dot{s}$. The resource requirement of this reform, say $\dot{e}$ is defined as

$$(49) \quad \dot{e} \equiv E_s \dot{s}$$

The reason why we should at all pay attention to infeasible reforms ($\dot{e} \neq 0$) is that actual reform proposals seldom specify explicitly a direction of change for the whole state of the economy; rather, they usually concentrate on a limited set of instruments (e.g. tax rates, or import licenses), leaving implicit or undefined the total adjustment of the system. For our purposes, such "vague" proposals are simply considered as infeasible reforms.

One way of evaluating a potential reform proposal, then, is to subtract from its direct welfare impact ($W_s \dot{s}$) a social valuation of its resource requirement; when this social valuation is carried out with the price vector ϕ , a measure Π^* of net social value emerges as:

$$(50) \quad \Pi^* = W_s \dot{s} - \phi \dot{e}.$$

Not unexpectedly, Π^* is precisely a first-order measure of the actual welfare impact of the proposed reform when its resource requirement is met via the allocation mechanism implicit in ϕ .

Indeed, if $\phi = W_s \Gamma$, then

$$(i) \quad \dot{s}' = \dot{s} - \Gamma \dot{e} \quad , \quad \text{is a feasible reform since,}$$

using lemma 1:

$$E_s \dot{s}' = E_s (\dot{s} - \Gamma \dot{e}) = E_s \dot{s} - \dot{e} = 0$$

$$F_s \dot{s}' = F_s (\dot{s} - \Gamma \dot{e}) = F_s \dot{s} \in \bar{k}(\Delta, \delta^0) \quad \text{since } \dot{s} \text{ is}$$

potentially feasible.

$$(ii) \quad \dot{s}' \text{ induces the welfare change}$$

The possibility of evaluating projects and potential reforms by formulae such as (47) and (50) can prove useful in at least two ways. Firstly, it is helpful in interpreting disagreement between people who share the same model but hold conflicting views about the desirability of alternative projects and reforms: different shadow prices, and different allocation mechanisms, must underlie their respective viewpoints.

Secondly, once an allocation mechanism is specified, (47) and (50) permit the evaluation of projects as well as of potentially feasible reforms.

Going one step further, one can contemplate the prospect of initiating a systematic reform process by calling forth reform proposals, evaluating them as in (50), and proceeding to meet their resource requirements through the corresponding allocation mechanism. If only these reforms are accepted for which Π^* is positive, this procedure will have the properties of feasibility and monotonicity familiar in the planning literature (see e.g. Malinvaud (22) or Heal (14)). Indeed the reform process just described can be loosely interpreted as a decentralized procedure for planning; it is very clearly illustrated by some contributions to the tax reform literature.

These possibilities raise two important questions for further investigation:

- (i) Are the kinds of reform processes envisaged here a "good" way of approaching the complex problem of reforming a messy economy?

- (ii) If so, which criteria should govern the selection of a fictitious price vector?

Unfortunately neither of these two questions seem to have been elucidated very far in economic theory.

As to the first one, the theory of reform as it exists (developed mostly in the context of the tax reform problem) deals essentially with the task of identifying the set of welfare-improving reforms. The problem of choosing a specific reform in that set is much more complex; its properly economic treatment (as distinct from mathematical algorithms devoid of economic interpretation) seems to require an enrichment of our theoretical framework (e.g. taking into account informational constraints, dynamic considerations, or the costs and risks of change), because our current analytical tools cannot meaningfully discriminate between two welfare-improving reforms, and a fortiori between two monotonic reform processes. This failure will turn out to be strikingly damaging in the present context.

Our inability to give a satisfactory answer to the first of the two questions mentioned above should temper our hopes of treating the second convincingly. This apprehension is confirmed in our next proposition, which shows that, as soon as a welfare improvement is possible without a project, fictitious prices can potentially be chosen arbitrarily.

Propn.5: (Indeterminacy of fictitious prices)

The set ϕ introduced in Defn. 11 is either a singleton or the whole of R^M . Moreover ϕ is a singleton iff

$$(58) \quad \nexists \dot{s} : G_s \dot{s} = 0 \text{ and } W_s \dot{s} \neq 0$$

Proof:

(i) To prove the first part of the proposition, suppose that ϕ contains two distinct elements, say ϕ_1 and ϕ_2 with

$$\left. \begin{aligned} \phi_1 &= W_s \Gamma_1 \\ \phi_2 &= W_s \Gamma_2 \end{aligned} \right\} \Gamma_1, \Gamma_2 \in \gamma^0$$

Notice that, for any $(M \times M)$ matrix B :

$$\Gamma \equiv \Gamma_1 B + \Gamma_2 (I - B) \in \gamma^0$$

(this is immediate from lemma 1).

Premultiplying by W_s :

$$\phi \equiv W_s \Gamma \equiv \phi_1 B + \phi_2 (I - B) = \phi_2 + (\phi_1 - \phi_2)B \in \phi$$

Since $\phi_1 \neq \phi_2$ and B is arbitrary, ϕ is arbitrary, and so $\phi = R^M$. Finally, since G_s has full row rank, ϕ is non-empty;

thus if it is not a singleton ϕ must coincide with the whole of R^M .

(ii) Turning to the first implication of the second part of the proposition, consider the set

$$(59) \quad P = \left\{ p : p = W_S G_S^- \text{ for some } G_S^- \in \mathcal{L}^- \right\} \subseteq R^K$$

By a reasoning analogous to (i) above, P is either a singleton or the whole of R^K ; hence, recalling corollary 2 of lemma 1, P is a singleton iff ϕ is a singleton. Let us suppose then, that ϕ and P are both singletons:

$$\exists p \in R^K : p = W_S G_S^- \quad \forall G_S^- \in \mathcal{L}^-$$

Hence

$$W_S y = p G_S^- y \quad \forall y \in Y = \left\{ y \in R^L : y = G_S^- x, x \in R^K, G_S^- \in \mathcal{L}^- \right\}$$

Now since G_S has full row rank, $\mathcal{L}^-$ is precisely the class of all generalized inverses of G_S , and $Y = R^L$. Thus $W_S = p G_S$ and

$$G_S \dot{s} = 0 \Rightarrow W_S \dot{s} = 0$$

(iii) Finally, suppose that $\exists \dot{s} : G_S \dot{s} = 0, W_S \dot{s} \neq 0$. Then

$$G_S \dot{s} = 0 \Rightarrow W_S \dot{s} = 0$$

$$\text{i.e. } (G_S')^\perp \subseteq (W_S')^\perp$$

where $'$ denotes transposition and $(A)^\perp$ denotes the linear subspace orthogonal to the column span $C(A)$ of a matrix A . Hence

i.e.

$$\exists p \neq 0 : W'_s = G'_s p \text{ or } W_s = p' G_s$$

Thus, $\forall G_s^- \in \mathcal{G}^- : W_s G_s^- = p' G_s G_s^- = p'$, i.e. P is a singleton, and so is ϕ . #

Corollary: ϕ is a singleton $\Leftrightarrow \exists p \neq 0 : W_s - p G_s = 0$. The latter is immediately recognized as a first-order optimality condition. (Notice that it is always fulfilled if the model is deterministic).

The result obtained in Propn. 5 may seem discouraging but it is not at all surprising. To understand why it arises, it is helpful to recall the frequent suggestion of thinking about the social opportunity cost of the j^{th} resource as its marginal social value in the next best alternative use. Unfortunately, when welfare improvements are possible without additional resources, there exists no formal bound on the potential welfare gain (or loss) per unit increase in the availability of the j^{th} resource. In particular, the above social opportunity cost concept, though intuitively appealing, must remain to some extent arbitrary, and rely upon intuitive restrictions imposed upon the class of permissible allocation mechanisms.

As a corollary, it appears that, in an economy where welfare improvements are possible without a project, the evaluation criterion

Accept $\dot{z}$ if there exists a welfare-improving
reform $\dot{s}$ which is feasible with $\dot{z}$

is not operational, again as long as all feasible reforms are permissible.

There are several ways of dealing with the indeterminacy problem evoked in Propn. 5 (e.g. restricting the class of permissible allocation mechanisms, strengthening project acceptance criteria, introducing a metric on the space of signals, etc.) It is arguable, for instance, that at a lower level of abstraction than that of the model analysed here, intuitively appealing arguments exist to rule out a large number of "perverse" allocation mechanisms, e.g. "stability" arguments, or the desire to avoid large adjustments in some signals in response to small exogenous changes in the environment. Alternatively, a natural allocation mechanism may suggest itself, when an objective distinction can be drawn between "endogenous variables" (which adjust autonomously to satisfy the constraints) and "policy instruments" (which are the object of a discretionary choice by the planner), as in a situation of uncertainty where only the latter can be fixed ex ante.

Nevertheless, these intuitions are generally difficult to formalize. Besides, they leave unanswered a more fundamental question whose examination should motivate and orient the theory of reform: why is the planner unable to carry out optimal policies? As long as no explanation is offered as to why welfare improvements are possible without a project, the prospects of defining desirable projects, and of providing convincing theoretical foundations for the choice of shadow prices in a context of reform, will be naturally weakened by the apparently irrational behaviour of the planner.

A thorough examination of these issues is much beyond the scope of our present enquiry. Hopefully, however, some of the empirical exercises of Part 2 will provide us with additional insights into the problem.

We conclude this section with a simple illustration, parallel to Examples 1 and 2.

Example 3: (Joint reforms of public production and trade)

In the model of Section I.2 without side constraints, consider a situation where trade is not optimal, and free trade is not permitted. The planner, knowing that at an optimum public production and foreign trade should be jointly efficient (see Example 1), may wish to contemplate joint reforms of public production and foreign trade. To achieve these, he can instruct project evaluators to use world prices as shadow prices for tradeable commodities, and implement projects by (inter alia) disposing of their net output of tradeables on the world market. More generally, when quotas exist but are not optimal, world trade gives the planner one particular opportunity for meeting the resource requirements in traded goods of any potentially feasible reform; he may choose to adopt it.

Formally, let $s = (\bar{e}, \dots)$ with $\bar{e} = (e_j)_{j \in J_1}$ as in Example 1; an allocation mechanism of the form

$$(i) \quad \Gamma = \begin{bmatrix} I_{M_1} & \Gamma^{12} \\ 0 & \Gamma^{22} \end{bmatrix} \quad \text{for some } \Gamma^{12}, \Gamma^{22}$$

yields the fictitious price vector

$$(ii) \quad \phi = W_s \Gamma = \left[\left(\mu_j^f p_j^w \left(1 + \frac{1}{\epsilon_j^w} \right) \right)_{j \in J_1}, (\phi_j)_{j \in J \setminus J_1} \right]$$

With such an allocation mechanism, then, the relative fictitious prices of tradeable commodities coincide with their relative marginal costs (or revenues) on the world market.

Anticipating somewhat the lessons of our brief survey for project evaluation, we may infer from Examples 1, 2, 3 the three precise circumstances under which, in the model of Section I.2, the relative shadow prices of tradeable commodities are correctly taken to be their relative marginal costs (or revenues) on the world market: ^{14/}

- (i) trade is optimal
- (ii) trade is free
- (iii) the planner is undertaking joint reforms of public production and foreign trade

This classification is sometimes useful for a clear understanding of the controversy surrounding the Little-Mirrlees rule for shadow pricing of tradeable goods (for further discussion see Dreze (10), section III.7).

Section I.7 : Elements of synthesis

The last three sections have emphasized the distinctions between three shadow price concepts : decentralization prices (DP), welfare prices (WP), and fictitious prices (FP). We shall now briefly examine their affinities, and find that DP's and WP's are in some sense polar cases of each other, while both are particular instances of FP's. A precise statement of these relationships may prove useful in deepening lines of synthesis between three important areas of theoretical welfare economics.

Let us first recall the shadow price notions involved, and write, for any $\lambda \in R^M$:

(DP) $\lambda \in \rho[S, Z]$ if λ satisfies Defn.5 for some pair (s^*, z^*) s.t. s^* is optimal relative to (S, Z) and $z^* = E(s^*)$.

(WP) $\lambda \in v[A, z]$ if, for the policy λ :

(i) $V(\cdot) \equiv W(\lambda(\cdot))$ is differentiable at z

(ii) $\lambda = V_z$ at z .

(FP) $\lambda \in \phi[s, \gamma]$ if $\exists \Gamma \in \gamma : \lambda = W_s \Gamma$ at s , where $\gamma \subseteq \gamma_s$ and γ_s is the class of all allocation mechanisms at s .

We shall assume the existence and uniqueness of a full optimum, say s^* , and in order to avoid burdening the text at little gain of insight, we shall restrict our attention to two states of the economy : the initial state s^0 and the full optimum

s^* . As usual, $\omega^0 \equiv G(s^0)$, $\omega^* \equiv G(s^*)$, and λ^0 denotes the actual policy of the planner. We shall assume that λ^0 is differentiable and that the hypotheses of Propn. 2 are verified at (s^*, z^*) .

Accordingly we may assert the existence of:

- (i) a unique DP vector ρ^* associated with (s^*, z^*) ; we shall call it the "true" DP vector:

$$\rho[s^*, z^*] \equiv \{\rho^*\}$$

- (ii) a unique WP vector reflecting the actual policy of the planner; we shall call it the "true" WP vector:

$$(62) \quad v[\lambda^0, z^0] \equiv \{v^0\}$$

A short list of further symbols and an assumption complete the spadework:

$$(63.i) \quad \bar{z}^0 \equiv \{z^0\} : \text{a simple production set in which } z^0 \text{ is optimal.}$$

$$(63.ii) \quad \bar{s}^0 \equiv \{s : s = \lambda^0(z) \text{ for some } z \in \bar{z}^*\}$$

the set of signals which are compatible with the policy of the planner

$$(63.iii) \quad \bar{\gamma}^0 \equiv \left\{ \begin{matrix} \lambda^0 \\ z \end{matrix} \right\}$$

a singleton containing the allocation mechanism implicit in the policy of the planner

$$(63.iv) \quad \gamma^0 \equiv \gamma_{s^0}$$

the set of all allocation mechanisms at s^0 .

$$(63.v) \quad \gamma^* \equiv \gamma_{s^*}$$

the set of all allocation mechanisms at s^* .

$$(63.vi) \quad \mathcal{P}^* \equiv \left\{ \begin{matrix} \lambda : \lambda(z^*) = s^* & \text{and } \lambda \text{ is} \\ & \text{differentiable} \\ & \text{at } z^* \end{matrix} \right\}$$

the set of policies which are optimal at z^*

- (63 vii) $\tilde{G}_s^*$: a matrix constructed exactly as in A.3 (p.1.4),
but at s^* rather than at s^0 .

A.4. (i) The matrix

$$G_s^* = \frac{\partial G}{\partial s} \text{ at } s^*$$

has full row rank. Hence so has $\tilde{G}_s^*$.

- (ii) $\lambda^0(.)$ is differentiable at z^0 .

Our findings take the form of two simple observations.

The first points to a possible interpretation of the DP tradition and the WP tradition of shadow pricing as two plausible answers to the indeterminacy problem evoked in the previous section. The second brings out the complementarity of these two traditions.

Observation 1 : Let A.1-A.4 hold and the hypotheses of Propn.2 be satisfied at (s^*, z^*) . Then:

- (i) DP's and WP's are both special instances of FP's:

a) $\rho^* \in \Phi[s^*, \gamma^*]$

b) $v^0 \in \Phi[s^0, \gamma^0]$

- (ii) At the optimum, all FP vectors coincide with the true DP vector:

c) $\{\rho^*\} = \Phi[s^*, \gamma^*]$

(iii) At the initial state, and with no constraints on the set of permissible allocation mechanisms, FP's can generally be chosen arbitrarily in R^M ; however, when the allocation mechanism is required to agree with the policy of the planner, the set of FP's reduces to the unique WP vector:

$$d) \quad \{v^0\} = \phi[s^0, \bar{\gamma}^0]$$

Proof:

1) Clearly, if s^* is fully optimal, it solves

$$\text{Max.}_s W(s) \quad \text{s.t.} \quad \tilde{G}(s) = \omega^*$$

Since G_s has full row rank at s^* and $W(\cdot)$ is differentiable, an elementary theorem of optimization theory asserts the existence of a vector $p \stackrel{\text{not.}}{=} [\bar{p} \ \bar{q}]$ s.t.

$$W_s - p \tilde{G}_s^* = 0 \quad \text{at} \quad s^*$$

This defines p uniquely; further, $\forall \Gamma \in \gamma^*$:

$$W_s \Gamma = p \tilde{G}_s^* \Gamma = \rho \quad \text{since} \quad \tilde{G}_s^* \Gamma = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad \text{from lemma 1.}$$

Now let λ^* be any element of ρ^* and define, around z^* :

$$V(z) \equiv W(\lambda^*(z))$$

Then V is differentiable at z^* and

$$V_z = W_s^* z = \rho \quad \text{since } \lambda_z^* \in \gamma^* \text{ by corollary 1 of lemma 1.}$$

Invoking Propn. 3 we conclude

$$\rho = \rho^*$$

which establishes (ii); (i) a) follows trivially.

2) Let $v^o(.) \equiv W(\lambda^o(.))$. The identities

$$v^o \equiv v_z^o \equiv W_s \lambda_z^o \quad \text{at } z^o$$

together with (62), (63-iii) and the definitions of $v[\cdot]$ and $\phi[\cdot]$ immediately imply (i) b) and (iii) d).

3) Finally, Propn. 5 justifies the additional comments in (iii):

$$\phi[s^o, \gamma^o] = R^M \quad \text{unless } \exists p \neq 0 : W_s - pG_s = 0 \quad \text{at } s^o, \quad \text{i.e.}$$

unless the necessary first-order conditions for an optimal choice of s are satisfied at s^o . #

Observation 2: Let A.1-A.4 hold and the hypotheses of Propn.2 be satisfied at (s^*, z^*) . Then:

(i) "True" welfare prices may be interpreted as a very special kind of DP's, by considering the initial state as a trivial optimum constrained by a commitment to the policy of the planner :

$$(64) \quad \{v^o\} = \rho[\bar{s}^o, \bar{z}^o]$$

- (ii) Conversely, "true" decentralization prices may be interpreted as the welfare prices which emerge at the optimum for any policy compatible with the latter:

$$(65) \quad \left\{ \rho^* \right\} = v[\lambda, z^*] \quad \forall \lambda \in \mathcal{P}^*$$

- (iii) Finally, welfare prices and decentralization prices coincide at the optimum:

$$(66) \quad v^0 = \rho^* \quad \text{if} \quad s^0 = s^*$$

Indeed this equality can be interpreted as an optimality condition.

Proof:

1) Let s be s.t. $E(s) = z^0$ and $s = \lambda^0(z)$ for some $z \in Z^*$. Then $z^0 = E(\lambda^0(z)) = z$, and $s = \lambda^0(z^0) = s^0$. In other words, when $S = S^0$ and $Z = \bar{Z}^0$, (P) has the unique and trivial solution (s^0, z^0) , and its maximum value function is

$$v^0(.) \equiv W(\lambda^0(.))$$

Therefore, reasoning exactly as in the proof of Propn.2, we may conclude

$$\rho[\bar{S}^0, \bar{Z}^0] = \left\{ v^0 \right\}_z = \left\{ W_{\lambda^0} \right\}_{s, z} = \left\{ v^0 \right\}$$

establishing (i).

#

The duality relationships holding between DP's and WP'S, and expressed in (64) and (65), potentially allow a rich pooling of many results on shadow pricing from two distinct traditions, as illustrated in Dreze (10).

The table on the next page summarizes our attempted synthesis. Its last row anticipates some lessons for project evaluation; these form the subject-matter of the next section.

Shadow pricing traditions Characteristics	DP (SECOND-BEST THEORY)	WP (TRADITIONAL CBA)	FP (THEORY OF REFORM)
Role of shadow prices	Decentralizing a production plan.	Evaluating a single project by CB test	Guiding a process of reform
Perception of the planner's behaviour	Optimizing	Systematic	Open-minded
Perception of the state of the economy	Optimum	An equilibrium resulting from the planner's policy	An arbitrary equilibrium
Desired property of shadow prices	The emerging production plan is optimal	CB tests detect welfare-improving projects	(i) The reform process is feasible and monotone (ii) (?)
Analytical tools	Optimization theory	Comparative statics	(?)
Shadow pricing rule.	$\rho^* = \nabla_z$ at z^*	$v^o = W_s \lambda_z^o$ at z^o	$\Phi = \{\phi : \phi = W_s \Gamma, \Gamma \in \gamma\}$ for some $\gamma \in \gamma^o$
Scope for indeterminacy	None: all fictitious prices coincide, by the envelope property $\phi = \left\{ \rho^* \right\}$	None: the policy of the planner singles out a unique permissible allocation mechanism $\gamma = \{\lambda_z^o\} \Rightarrow \phi = \{v^o\}$	Generally unlimited if $\gamma = \gamma^o$
Suggested link with the project evaluation problem	Projects are building blocks of optimal plans	A project is an isolated displacement of equilibrium (with and without principle)	A project is an opportunity for reform

Section I.8 : Project evaluation revisited

Shadow prices can be helpful in a variety of contexts, and a prerequisite to a clear understanding of their properties is an agreement on the precise role they are expected to perform. Some of the aims towards which they can contribute are implementing an optimum, evaluating a single project within a well-defined policy frame, or guiding a comprehensive process of reform.

This simple remark is relevant to the problem of choosing shadow prices for project evaluation. Different approaches to the shadow pricing problem may be relevant according to the precise interpretation one gives to the project evaluation exercise, e.g. as dealing with an element of an optimal plan, an isolated perturbation of an initial equilibrium, or an opportunity for reform.

Consider, for instance, the two best-known cookbooks of applied project evaluation, i.e. the Little-Mirrlees Manual (19) and the UNIDO Guidelines (26), and the debate about some of their apparently contrasting recommendations (e.g. about the use of world prices as shadow prices). At the risk of some oversimplification, it is tempting to regard the LM Manual as primarily concerned with helping a planner to harmonize public production decisions with overall policies aiming at the implementation of a constrained optimum, and the UNIDO Guidelines as addressed to an evaluator whose "control area" is limited to the assessment of a single project.^{17/} Their methodologies need therefore not be contradictory.

As we have seen, the choice of shadow prices is in a simple

sense equivalent to the adoption of a project evaluation criterion. Once an operational criterion is accepted, the set of desirable projects is well-defined; and when the latter is a convex cone, its dual cone is a shadow price set. An operational and compelling criterion is often thought to be that a desirable project should lead to a welfare improvement. Many other criteria are encountered in the literature, e.g. compensation criteria (Hicks, Kaldor, Scitovsky), production efficiency criteria (Diewert (6)), Pareto-improvement criteria (Hammond (13)), social choice-theoretical criteria, etc. Whatever their rationale, however, these approaches seem to belong to a different class of models, since they imply either the lack of a preference ordering over the set of states (i.e. $W(\cdot)$ is not defined), or an appeal to considerations hardly lying within the scope of the present model (e.g. the need to resort to rules of thumb to achieve political or computational expediency, informational failures, etc.).

We shall therefore postulate that when a project makes a well-defined contribution to social welfare, it should be assessed in this light, and investigate the circumstances under which the "welfare improvement" criterion provides an operational basis for the choice of shadow prices. In other words, when can the net output vector $\bar{z}$ of a small public sector project be valued at a set of shadow prices which correctly measure its effect on social welfare, and the task of project appraisal be decentralized by means of the corresponding CB test?

Notice that we require shadow pricing to apply specifically to the net output vector of a project. This is a natural condition

to allow for a decentralized evaluation of projects, since a project manager would normally not be expected to be aware of other aspects of government policy. Of course, when some policy change forms an explicit part of a project (which then really becomes, in our terminology, a "reform"), CBA should be able to take it into account. But the desire to decentralize the appraisal of public sector production projects motivates the search for a vector of shadow prices for commodities adequate to detect welfare-improving projects; this indeed is the concern of much of the project evaluation literature.

Generally we should expect the effect of a project on welfare to depend upon how it is "implemented". To take a simple example, the social value of a project which requires foreign exchange would normally vary according to how the planner proposes to restore balance of payments equilibrium, e.g. by manipulating the exchange rate, raising tariffs or stiffening quotas. As the results of the previous sections show, however, there are precisely two situations in which these qualifications are not very damaging.

When the planner is optimizing, no ambiguity need arise. By the envelope property of the optimum, the effect of a small project on welfare is the same irrespective of how it is implemented. Thus if all tariffs, quotas and the exchange rate are chosen optimally within the opportunity set of the planner (however restricted it may be), foreign exchange has the same social opportunity cost whatever its source; a project evaluation need not go into the intricacies of foreign exchange policy to appraise a small public production project.

If the economy is away from an optimum, the welfare effect of a project is still determinate as long as the planner is committed to a well-defined and observable policy. Given the actual behaviour of the planner, the model becomes in effect "deterministic", and again a well-defined vector of shadow prices for commodities exists which detects welfare-improving projects (as well as welfare-improving reforms of the policy of the planner). Thus if balance of payments crises are dealt with by, say, a revision of the exchange rate, there exist clear guidelines to evaluate the social value of foreign exchange.

These two situations are very familiar to the project evaluation literature. Srinivasan's (24) well-known synthesis, for instance, aptly recalls the two traditions of shadow pricing they have inspired:

"One approach to the problem of deriving shadow prices is to assume that the government maximizes social welfare through the use of its policy instruments.(...) An alternative approach is to take as a point of departure an existing equilibrium that reflects possibly nonoptimal public policies and define the shadow price of a good as the change in social welfare resulting from a change in the use of that good by one unit at the existing equilibrium". 18/

Again these two "approaches to shadow pricing" are each relevant in a different context and need not be contradictory or even fundamentally different: the first may appeal to a planner who possesses a significant degree of freedom in designing policies, and the second to an analyst dealing with a deterministic model. Moreover, as we have argued in the last section, a strong complementarity exists between the two approaches.

But what if the planner is neither "optimizing" nor "systematic", and finds himself initially carrying out ad hoc or historically given policies which he would precisely like CBA to teach him to improve upon? This possibility, and the difficulties it precipitates, have recently received increasing attention in some writings on CBA (e.g. Blitzer-Dasgupta-Stiglitz (3), Diewert-Hammond (8), Diewert (7), Hammond (13), Bell-Devaranjan (2) and Johansson (15)). Bell and Devaranjan's statement of the problem is particularly clear:

"For purposes of social cost-benefit analysis, a project may be viewed as a disturbance to the economy, displacing it from some initial equilibrium to a new one. But the new configuration will depend on which particular variables adjust to restore equilibrium. Since there may be more than one admissible form of adjustment, it is natural to ask how - if at all - the corresponding shadow prices for project evaluation depend on the nature of the adjustment. Now, the manner in which the economy equilibrates depends on how the government responds to the disturbance that the project generates (...). These equilibrating mechanisms are specified as different rules for "closing" a general equilibrium system which is initially specified so as not to be fully determined. By taking different combinations of variables to be fixed exogenously, one arrives at different formulations of the way in which the economy adjusts to the introduction of a project. It is then possible to solve for the different variables which enter into the social welfare function, the gradient of which yields the vector of shadow prices for the economy. (...) (we) emphasize that apparently minor changes in the choice of endogenous variables can lead to quite different results". 19/

Quite different results, indeed, since, as Propn. 5 shows, with arbitrary allocation mechanisms the set of potential shadow price vectors covers the whole of R^M . Thus in our previous example of a foreign exchange-using project, the planner now has many options for raising the necessary resource, each of which

has a different social opportunity cost; moreover, by a suitable improvement of policies, there generally exist socially costless ways of acquiring foreign exchange (perhaps, say, by increasing the tariff on alcoholic beverages).

While shedding some light on the relationship between allocation mechanisms and shadow pricing rules, however, the contributions mentioned above seem to take a rather conservative view of the problem of actually evaluating a project in this kind of situation, suggesting that the "actual behaviour" of the planner be taken as the guidelines; e.g. the paper quoted above concludes:

"... a careful scrutiny of past behaviour will usually give the analyst enough clues as to what is the appropriate assumption for deriving shadow prices". 20/

More ambitious attempts at integrating project evaluation with policy reforms (e.g. Diewert-Hammond (8)) do not seem to provide very general solution concepts so far. If the analyst is satisfied, the planner, on the other hand, remains helpless.

Consider then a messy economy, lying away from the planner's optimum; existing policies are of an ad hoc or historical nature but the government is not committed to them. A small public production project is available. How might such a project be evaluated?

Taking first the viewpoint of the analyst, the existing literature seems to be apt to solve his problem. Its recommendations may be summarized as follows. If the analyst is entrusted

with a decision, he has no alternative than to make some inference about the planner's behaviour (after all, the "planner" is only the $N + 1$ st agent of the economy; there may or may not be good a priori reasons for singling out his behaviour as particularly unsystematic or unobservable). The analyst who has the status of an adviser, on the other hand, can confine himself to conditional statements, exhibiting the circumstances under which the contemplated project will be welfare-improving.

Let us next take the viewpoint of the planner, and enquire how he might instruct project evaluators to carry out project appraisal. Two courses of action seem to be available.

If the planner cannot make up his mind as to how an arbitrary project should be implemented (i.e. choose an allocation mechanism), projects can no longer be viewed in isolation. Each project, besides describing a net output vector, must now specify explicitly the reform which will implement it. In our earlier example, the description of the project will be considered as incomplete as long as it does not include an explicit statement as to the relative contribution expected from each alternative possible source of foreign exchange in raising the necessary resource.

It is apparent, however, that the informational requirements involved in designing and evaluating feasible reforms may be very demanding; a decentralization of information and decisions may be much more difficult to achieve in this context. In particular, each public production project now makes available an infinite number of reforms, and the consequent reform design problem is beyond the competence of

either a planner ignorant of the detailed inputs and outputs of the project, or a project evaluator unaware of all aspects of public policy.

If a decentralized evaluation of projects is to remain feasible, then, the planner must somehow choose an allocation mechanism, implying a commitment to implement every project in the same way. On this problem, however, he will probably find little guidance from the theorist, since no natural criteria seem to exist to evaluate allocation mechanisms as such.

As long as no convincing explanation exists as to why the planner cannot achieve optimal policies, then, there seems little hope of harmonizing project evaluation with other public policy decisions outside an optimizing framework (after due allowance for all the relevant constraints). Indeed if the model of the planner really involves all the constraints which limit his choice, why not optimize? Unless the considerations which rule out this option (e.g. the impossibility of obtaining distant information) are explicitly recognized, it is natural to find the prospects of a sound choice of shadow prices for project evaluation to be severely handicapped from the start by the apparently irrational behaviour of the planner. When these considerations are justified, on the other hand, their careful study may point to directions of improvement in our theoretical framework (e.g. appealing to statistical decision theory, as in Varian (27)) adequate to permit a satisfactory solution of the problem.

CBA theorists sometimes distrust the optimization framework on the ground that it would be naive for a project evaluator to regard

21/

the planner as a maximising agent. But it may be equally naive for

a planner to hope to decentralize project evaluation unless he has a well-defined policy. ^{22/} If the planner's existing policies are ad hoc and

he is willing to renounce them, the option of resorting to an optimization framework may take the nature of a logical necessity. This approach need not be unrealistic provided that we learn to go beyond the very special kinds of optima studied in the existing literature on second-best theory, by giving a better account of the institutional and informational constraints binding the decisions of real-world planners. Furthermore, the optimization framework in no way precludes the use of CBA techniques to study local policy improvements made possible by a small relaxation of the constraints currently perceived, including perhaps informational constraints. Finally, remembering that the "optimality" approach includes the "comparative static" approach as a limiting case, it seems difficult to dismiss its claim to provide, in the current state of our knowledge, the most coherent theoretical foundation of project evaluation techniques.

Section 1.9 : Some useful complementary results

Our interest for the project evaluation problem justifies the focus, in this chapter, on scarcity constraints, and on shadow prices for commodities. Clearly, however, a closely similar analysis applies to side constraints, and the notion of "project" can be extended from public production to many other areas of public decision-making. Since the corresponding concepts and results will prove useful in later chapters, they deserve a rough presentation here. In doing so, we shall follow, for convenience, the framework of Section I.6. (again, we omit the symbol " $\sim$ " recalling the deletion of slack constraints).

(1) As corollary 2 of lemma 1 (Section 1.6) states, Γ is an allocation mechanism if and only if there exists an $(L \times (K-M))$ "auxiliary matrix", say A , s.t.

$$(67) \quad \Gamma^* \equiv [\Gamma \ A]$$

is a right-inverse of G_S . By analogy with Defn.10 (Section 1.6) and the comments preceding it, a right-inverse of G_S may be called an equilibrium mechanism: for any perturbation $\dot{\omega}$, it indicates a feasible way of restoring equilibrium, since, if $\dot{s} = \Gamma^* \dot{\omega}$, then $G_S \dot{s} = \dot{\omega}$ and so $\dot{s}$ is a feasible reform. Furthermore, the net effect Π^* of this perturbation on social welfare then becomes

$$(68) \quad \Pi^* = W_S \dot{s} = \phi^* \dot{\omega}$$

where

$$(69) \quad \phi^* \equiv W_s \Gamma^*$$

is an "extended" shadow price vector, covering all intervention variables.

(2) Now let $\dot{s}$ be any reform (feasible or infeasible). By analogy with (49) in Section 1.6, we may define the "impact" of $\dot{s}$ as $\dot{g} \equiv G_s \dot{s}$. Then, if Γ^* is an equilibrium mechanism and ϕ^* the associated extended shadow price vector, we find, much as in (50) and the accompanying comments:

$$(70) \quad \dot{s}' = \dot{s} - \Gamma^* \dot{g} \text{ is a feasible reform since } G_s \dot{s}' = 0$$

$$W_s \dot{s}' = W_s \dot{s} - \phi^* \dot{g}$$

Furthermore,

$$(71) \quad W_s \dot{s}' = \Pi_s^* \dot{s}$$

where

$$(72) \quad \Pi_s^* \equiv W_s - \phi^* G_s$$

is a vector of marginal social values of signals.

(3) If Γ^* is an equilibrium mechanism, and ϕ^* the associated extended shadow price vector, define the corresponding Lagrangian for the planner's model as

$$(73) \quad \mathcal{L}(s, \omega; \phi^*) \equiv W(s) - \phi^* [G(s) - \omega]$$

Then

$$(74) \quad \begin{cases} \phi^* &= \frac{\partial \mathcal{L}}{\partial \omega} \\ \Pi_s^* &= \frac{\partial \mathcal{L}}{\partial s} \end{cases}$$

(4) Since G_s has full row rank by assumption, there exists a partition of s , say $s = (\alpha, \beta)$ such that G_α is square and has full rank. Accordingly, the matrix

$$(75) \quad \begin{bmatrix} G_\alpha^{-1} \\ 0 \end{bmatrix} \text{ not. } \begin{bmatrix} \Gamma^\alpha & A^\alpha \end{bmatrix}$$

defines an equilibrium mechanism. When an equilibrium mechanism has this form, it is natural to call α a vector of "endogenous variables". Correspondingly, we shall speak of the components of β as "policy instruments".

16/

It may be shown that, after a suitable reparametrization of the space of signals around s^0 if necessary, a right-inverse of G_s may always be written in the form (75) for some α . We may therefore regard the specification of an equilibrium mechanism as a choice

of instruments.

By construction, the marginal social value of an endogenous variable is zero:

$$(76) \quad \Pi_{\alpha}^* = W_{\alpha} - \phi^* G_{\alpha} = W_{\alpha} - W_{\beta} \Gamma^* G_{\alpha}$$

$$(77) \quad = 0 \quad \text{when} \quad \Gamma^* = \begin{bmatrix} G_{\alpha}^{-1} \\ 0 \end{bmatrix}$$

By contrast, when the marginal social value of a policy instrument is non-zero (say $\Pi_{\beta_{\ell}}^* > 0$), a feasible and welfare-improving reform can be constructed by manipulating α and β_{ℓ} ; for if

$$(78) \quad \dot{\beta}_{\ell} > 0 \quad \text{and} \quad \dot{\alpha} = - G_{\alpha}^{-1} G_{\beta_{\ell}} \dot{\beta}_{\ell}$$

then

$$(79) \quad (i) \quad G_{\alpha} \dot{\alpha} + G_{\beta_{\ell}} \dot{\beta}_{\ell} = 0$$

$$(80) \quad (ii) \quad \dot{W} = W_{\beta_{\ell}} \dot{\beta}_{\ell} - W_{\alpha} G_{\alpha}^{-1} G_{\beta_{\ell}} \dot{\beta}_{\ell}$$

$$(81) \quad = \left[W_{\beta_{\ell}} - \phi^* G_{\beta_{\ell}} \right] \dot{\beta}_{\ell}$$

$$(82) \quad = \Pi_{\beta_{\ell}}^* \dot{\beta}_{\ell} > 0$$

In particular, a necessary condition for an optimal joint choice of α and β_ℓ is

$$(83) \quad \Pi_{\beta_\ell}^* = 0 \quad .$$

Footnotes : Chapter I

- 1/ When a government is not "monolithic", only one of its decision bodies can be taken as our "planner", the others being included in the "private sector".
- 2/ e.g. Keynesian "public works" policies in an economy with underemployment can sometimes be regarded as a free disposal of labour, but they are obviously not undertaken as a matter of course.
- 3/ Throughout the text, the words "he", "him", "his", etc. should be automatically understood as "he (or she)", "him (or her)", "his (or her)", etc.; the unpleasantness of this formula should be attributed to the sexism of the language rather than that of the author.
- 4/ It is straightforward to extend the model so as to take into account taxes which apply to sales rather than production, as well as to allow for household production.
- 5/ For details on cones and their properties, see e.g. Bazaraa and Shetty (1), Chapter 3.
- 6/ If Z^+ is empty, Λ is interpreted as R_0^M .
- 7/ Often this function will be defined only implicitly, as in a general equilibrium model where the planner's decisions are endogenous.
- 8/ That is, unique up to a positive scalar multiple.
- 9/ Figure 2 is adapted from Dixit (9), p.28-29.
- 10/ J.Lesourne (17), p.3-4.
- 11/ This is for unchanged domestic price of the good, and hence there may be an implicit adjustment in the tariff.
- 12/ e.g. as measured by the distance $\left\| \frac{\dot{s}'}{\|\dot{s}'\|} - \frac{\dot{s}}{\|\dot{s}\|} \right\|$
 where $\|\cdot\|$ denotes the Euclidean norm.
- 13/ We shall not distinguish between the linear function Γ and the associated $L \times M$ matrix.
- 14/ Parallel statements are possible for individual commodities.
- 15/ See Dixit (9), p.7, together with the fact that the multipliers associated with the constraints which are slack at the optimum are zero under our differentiability assumptions.

16/ As in chapter IV, section IV.1, R.5.

17/ Contrast the following quotes from co-authors of the respective treatises:

"... the theoretical rationale of (...) the methods of the (...) Manual fit in with (more or less) optimal planning".
(Little-Mirrlees (20), p.262, emphasis added).

"It does not seem right (...) to recommend that project A be undertaken on the ground that 'if the countries had followed policies which resulted in an optimal industrial development' then R would not in fact have been domestically produced: the question is whether or not R will be domestically produced *in fact*" (Sen (23), p.489).

Sen later concludes:

"What the L.M.M. approach seems to me to do is to assume implicitly that either the project evaluator is very *powerful* so that he can ensure that the rest of the government machinery will accept his decisions in other fields as well (...), or that he is very *stupid* so that he cannot be entrusted to make a realistic appraisal of the likely policies and should be better advised to assume a rule of uniform simplicity". (Sen (23), p.490; our emphasis, but his italics).

18/ Srinivasan (24), p.4-5 (his emphasis).

19/ Bell-Devaranjan (2), p.1-2.

20/ Bell-Devaranjan (2), p.25; emphasis added.

21/ cf. e.g. Blitzer-Dasgupta-Stiglitz (3):

"It is convenient to suppose that the government simultaneously optimizes with respect to all variables within its control, and it is the implication of this supposition that has been explored in much of the recent literature on project evaluation. But as a model of government behaviour it is not persuasive". (p.58).

22/ This was recognised by Little and Mirrlees:

"Now take the case of a Central Office of Project Evaluation (COPE), reporting to a Cabinet Committee, which makes the actual decisions. It would be very odd for a government to set up a COPE and expect it to report on the basis of assuming an irrational government. The very fact that the government has constituted COPE can be taken as a statement of rational intent". (Little-Mirrlees (21), p.159).

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CHAPTER II : Shadow Prices in a Messy Economy

1/

Section II.1. Outline of the Approach

In this chapter we shall study shadow pricing rules more explicitly, using an elementary version of the model presented in Section I.2. (as in (12), (13), (14), on p.1.8). For a more elaborate discussion of the model, its shortcomings and link with the literature, the reader is referred to Dreze (6). We shall neglect side constraints, and hence write the model of the planner as:

$$(1) \quad W = W(s) \equiv \psi(V^F, V^1(\cdot), \dots, V^1(p+t, \bar{x}^1, m^1), \dots, V^N(\cdot))$$

$$(2) \quad E(s) - z = 0 \quad \text{or} \quad \sum_i x_i^1(p+t, \bar{x}^1, m^1) + e - \sum_h y^h(p, \bar{y}^h) - z = 0$$

where $m^1 \equiv r^1 + \sum_h a^{1h} y^h(p, \bar{y}^h)$, $V^F \equiv \sum_j p_j^w(e_j) \cdot e_j$, and s is a selection from the vector s^* spelt out in (12), p.1.8.

Shadow prices are well-defined as soon as we select from s a vector $\alpha = (s_{\ell_j})_{j=1, \dots, M}$ ^{not.} of "endogenous" or market-clearing signals; implicitly, we are specifying an allocation mechanism ^{2/}

$$(3) \quad \Gamma^\alpha = \begin{bmatrix} E^{-1} \\ \alpha \\ 0 \end{bmatrix}$$

The shadow price vector λ then satisfies

$$(4) \quad \lambda = W_\alpha E_\alpha^{-1}$$

Further, if $s = (\alpha, \beta)$, β can be regarded as a vector of policy instruments whose marginal social values form a vector Π_{β}^* satisfying

$$(5) \quad \Pi_{\beta}^* = W_{\beta} - \lambda E_{\beta} = \mathcal{L}_{\beta}$$

where

$$(6) \quad \mathcal{L}(s, z; \lambda) \equiv W(s) - \lambda [E(s) - z]$$

is the Lagrangian of the model (see section I.9).

The shadow price vector λ may alternatively be interpreted as:

- (i) A DP vector, for any version of the second-best optimality problem (P), which allows a free choice of the value of α by the planner provided that (4) is evaluated at the optimum.
- (ii) A WP vector, if the policy of the planner can effectively be described as an exogenous choice of the value of β . (4) should now be evaluated at s^0 .
- (iii) A FP vector, associated with the allocation mechanism r^0 of (3).

Bearing these remarks in mind, we shall loosely refer to λ as a "shadow price vector", and take the choice of instruments as given.

For expositional purposes, and after due recognition of the general equilibrium nature of allocation mechanisms, it is convenient to classify commodities according to the adjustment process at work on each market (in the following, α_j denotes the endogenous variable which, intuitively speaking, adjusts to clear the j^{th} market):

(M1) Price adjustment: this is the most familiar market-clearing mechanism; goods to which it applies are called "competitively exchanged goods" and indexed by $J_P \subseteq J$. Thus

$$\alpha_j = p_j \quad \forall j \in J_P.$$

(M2) Tax adjustment: this may occur if the producer price is fixed (e.g. according to the practice of administered "retention prices" in India; or when a good is exchanged between private producers and the government at a fixed price, and between consumers and the government at a market-clearing price). Since the institutional arrangement most appropriate for the operation of this mechanism is a government monopoly of trade, such goods will be called "procured goods" and indexed by $J_T \subseteq J$; thus $\alpha_j = t_j \quad \forall j \in J_T$.

(M3) Trade adjustment: this applies to "fully traded goods" which are indexed by $J_W \subseteq J$: $\alpha_j = e_j \quad \forall j \in J_W$. The domestic price of these goods is fixed.

(M4) Consumer rationing: obvious examples of consumer rationing from the Indian economy include involuntary unemployment, rationing of imported consumer goods, temporary shortages of essential commodities sold at administered prices, as well

as the controlled side of a "dual market" (the description of which is readily obtained here by a suitable indexation of goods). The index sets J_R^i apply to these "C-rationed goods" where i denotes the "marginal consumer" in a rationing scheme, that is, the consumer whose quota will be revised in the event of a small change in supply; thus

$$\alpha_j = \bar{x}_j^i \quad \forall j \in J_R^i.$$

- (M5) Producer rationing: apart from the familiar phenomena of input scarcities, power failures, preferential allocation of credit, licensing of capacity, shortage of skilled personnel, etc., we may also mention the possibility of effective demand constraints creating conditions of excess supply for some outputs. The existence of non-decreasing returns would also render quantity constraints an indispensable feature of any equilibrium, given our description of producer behaviour. Goods which are rationed on the producer side (at the margin) will be called "P-rationed goods", and indexed by J_R^h when firm h 's ration is revised in the event of a small change in supply. Thus $\alpha_j = \bar{y}_j^h$ when $j \in J_R^h$.

The class of rationing schemes which are allowed for in this framework may not seem very wide; (M4) and (M5) confine themselves to identifying a "marginal agent" whose ration is adjusted when an excess demand or supply develops. However, this formulation is motivated solely by expositional clarity and it is easy to show that a suitable manipulation of the model can accomodate any rationing

scheme of the form:

$$(7) \quad \bar{x}_j^i = \phi_j^i \left[z, (x_k^{i'}(.))_{\substack{i' \in I \\ k \in J}}, (y_k^h(.))_{\substack{h \in H \\ k \in J}}; \beta \right] (i \in I, j \in J)$$

$$(8) \quad \bar{y}_j^h = \phi_j^h \left[z, (x_k^{i'}(.))_{\substack{i' \in I \\ k \in J}}, (y_k^{h'}(.))_{\substack{h' \in H \\ k \in J}}; \beta \right] (h \in H, j \in J)$$

for some rationing functions $\phi_j^i(.), \phi_j^h(.)$ ($i \in I, j \in J, h \in H$).

In other words, these are rationing schemes which depend upon:

- (i) The fixed "characteristics" of each agent
- (ii) All exogenous variables and policy instruments
- (iii) The demands and supplies of all agents as we have defined them (see (X) and (Y) in Section I.2).

but not upon "effective demands" of, say, the Benassy variety (see Benassy (1)), which are allowed to violate the quotas applying on the market where they are expressed.

Many rationing schemes satisfy our requirements: e.g. uniform rationing, hierarchical rationing, many coupons systems, rationing of employment by seniority rights or caste systems, etc. Others do not: e.g. proportional rationing, licensing procedures sensitive to applications, many forms of credit and foreign exchange rationing, etc. The latter can in principle be studied after suitable amendments of the basic model, but since their introduction substantially modifies the shadow pricing rules and raises very special problems for empirical estimation, they are better left for a separate investigation.

We now have, according to (M1) to (M5):

$$J = J_P \cup J_T \cup J_W \cup \left(\bigcup_i J_R^i \right) \cup \left(\bigcup_h J_R^h \right)$$

In the remainder of this chapter, we shall investigate the shadow pricing rules (4) and (5) and their implications. Of course, it is hopeless to try to invert analytically the matrix E_α in order to solve (4) explicitly. However, much insight into shadow pricing rules can be obtained by concentrating on the equations

$$(9) \quad \mathcal{L}_\alpha = 0 \quad \text{or} \quad W_\alpha - \lambda E_\alpha = 0$$

which are of course equivalent to (4). We shall call (9) the shadow price equations. Their analysis forms the subject matter of the next section.

Section II.2 : Shadow pricing rules

To facilitate the interpretation of the shadow price equations, it is useful to define the marginal social value of transfer to consumer i , say θ^i , as

$$(10) \quad \theta^i \equiv \pi_{r^i}^* = \cancel{\lambda}_{r^i} = \frac{\partial \Psi}{\partial V^i} \frac{\partial V^i}{\partial m^i} - \lambda \frac{\partial x^i}{\partial m^i} \quad \text{not. } \mu^i \cdot b^i$$

The net effect on social welfare of a marginal lump-sum transfer to consumer i thus consists of two elements.

The first one, μ^i , is the social valuation of i 's utility increase. It is called a welfare weight in some cost-benefit analysis writings (see e.g. Stern (13)).

The second component of θ^i , namely b^i , reflects the social cost of i 's additional consumption requirements. We shall call it i 's consumption conversion factor.

Using (10), and recalling (X4) and (Y1) ^{3/}, the shadow price equations (9) can be written explicitly as:

$$(11) \quad - \sum_i \mu^i x_j^i - \lambda \left[\frac{\partial x}{\partial q_j} - \frac{\partial y}{\partial p_j} \right] + \sum_{i,h} \theta^i a^{ih} y_j^h = 0 \quad (j \in J_P)$$

$$(12) \quad - \sum_i \mu^i x_j^i - \lambda \frac{\partial x}{\partial q_j} = 0 \quad (j \in J_T)$$

$$(13) \quad \frac{\partial \Psi}{\partial V^j} \left[p_j^w + e_j \frac{\partial p_j^w}{\partial e_j} \right] - \lambda_j = 0 \quad (j \in J_W)$$

$$(14) \quad \lambda \frac{\partial y^h}{\partial y_j^h} + \sum_{i,h} \theta^i a^{ih} \frac{\partial \pi^h}{\partial y_j^h} = 0 \quad (j \in J_R^h)$$

$$(15) \quad \frac{\partial \Psi}{\partial V^i} \frac{\partial V^i}{\partial x_j^i} - \lambda \frac{\partial x^i}{\partial x_j^i} = 0 \quad (j \in J_R^i)$$

Similarly, (5) becomes:

$$(16) \quad \pi_{r^i}^* = \theta^i$$

$$(17) \quad \pi_{p_j}^* = - \sum_i \mu^i x_j^i - \lambda \left[\frac{\partial x}{\partial q_j} - \frac{\partial y}{\partial p_j} \right] + \sum_{i,h} \theta^i a^{ih} y_j^h$$

$$(18) \quad \pi_{t_j}^* = - \sum_i \mu^i x_j^i - \lambda \frac{\partial x}{\partial q_j}$$

$$(19) \quad \pi_{e_j}^* = \frac{\partial \Psi}{\partial V^f} \left[p_j^w + e_j \frac{\partial p_j^w}{\partial e_j} \right] - \lambda_j$$

$$(20) \quad \pi_{y_j^h}^* = \lambda \frac{\partial y^h}{\partial y_j^h} + \sum_{i,h} \theta^i a^{ih} \frac{\partial \pi^h}{\partial y_j^h}$$

$$(21) \quad \pi_{\bar{x}_j^i}^* = \frac{\partial \Psi}{\partial V^i} \frac{\partial V^i}{\partial \bar{x}_j^i} - \lambda \frac{\partial x^i}{\partial \bar{x}_j^i}$$

$$(22) \quad \pi_{a^{ih}}^* = - \theta^i \pi^h$$

Under our assumptions, equations (10) to (15) uniquely determine λ and $(\theta^i)_{i \in I}$, while (16) to (22) indicate the marginal social value of each exogenous variable or policy instrument. In discussing the shadow price equations, we shall consider each type of commodity in turn.

(i) Competitively exchanged goods ($j \in J_p$)

A few definitions and elementary manipulations will allow us to reduce (11) to a formula of a familiar type.

First, the net excess demand of consumer i for good j, $-z_j^i$, is defined as

$$(23) \quad -z_j^i \equiv x_j^i - \sum_h a^{ih} y_j^h$$

Indeed, if each firm, instead of distributing its profits to shareholders, entitles each of them to a share of its net supply vector, $-z_j^i$ becomes the net market purchases of good j by consumer i.

Next, we express the divergence between welfare prices and market prices as vectors of fictitious "social taxes" called C-taxes (t_c^*) and P-taxes (t_p^*) respectively:

$$(24) \quad t_c^* \equiv q - \lambda \quad t_p^* \equiv p - \lambda$$

We may now rewrite (11) as:

$$(25) \quad 0 = - \sum_i u^i x_j^i - \lambda \sum_i \left(\frac{\partial x^i}{\partial q_j} - x_j^i \frac{\partial x^i}{\partial m^i} \right) + \lambda \frac{\partial y}{\partial p_j} + \sum_{i,h} \theta^i a^{ih} y_j^h$$

(using (X2))

$$(26) \quad = \sum_i \theta^i z_j^i - \lambda \frac{\partial x}{\partial q_j} + \lambda \frac{\partial y}{\partial p_j} \quad (\text{using (10), (23)})$$

$$(27) \quad = \sum_i \theta^i z_j^i - \lambda \frac{\partial x_j}{\partial q} + \lambda \frac{\partial y_j}{\partial p} \quad (\text{using (X3), (Y2)})$$

$$(28) \quad = \sum_i \theta^i z_j^i + t_c^* \frac{\partial x_j}{\partial q} - t_p^* \frac{\partial y_j}{\partial p} \quad (\text{using (X3), (Y2), (24)})$$

Finally:

$$(29^*) \quad \frac{t_c^* \frac{\partial \hat{x}_j}{\partial q} - t_p^* \frac{\partial y_j}{\partial p}}{x_j} = - \sum_i \theta^i \frac{z_j^i}{x_j} \quad (j \in J_P)$$

The right-hand side of this equation is the net distributional characteristic of good j ^{5/}; it will be higher, the higher the proportion of the net private excess demand for good j which is expressed by individuals whose income has a high marginal social value. More explicitly, letting $\theta^0 = 0$, $z_j^0 = z_j + \sum_h a^{oh} y_j^h$, $I^* = I \cup \{0\}$ and $\theta^h = \sum_i a^{ih} \theta^i$, the net distributional characteristic, say d_j , of good j , can also be written:

$$(30) \quad d_j = - \frac{\text{cov}((\theta^i)_{i \in I^*}, (z_j^i)_{i \in I^*})}{x_j} \quad (N+1)$$

or again

$$(31) \quad d_j = \sum_i \theta^i \frac{x_j^i}{x_j} - \frac{y_j}{x_j} \sum_h \theta^h \frac{y_j^h}{y_j}$$

Thus d_j will be higher the more it is consumed by "transfer-deserving" individuals ($\theta^i > 0$), and supplied by "tax-deserving" firms ($\theta^h < 0$).

The left hand side of (29*) is known as the "fictitious degree of discouragement" of good j (e.g. see Guesnerie (7)); it can be interpreted as a linear approximation to the percentage change in (compensated) excess demand resulting from the existence of the "social taxes" t_c^* and t_p^* ; alternatively, it gives the percentage

change in excess demand which would follow a small "intensification" of the system in social taxes, i.e. small changes dt_c^* and dt_p^* proportional to t_c^* and t_p^* respectively. When cross-price elasticities are negligible and $x_j > 0$, a relatively high fictitious degree of discouragement implies relatively low social taxes on good j , i.e. λ_j is relatively high.

(29*) can now be stated as:

Rule 1: The shadow price vector should be such that the fictitious degree of discouragement for competitively exchanged goods equals their distributional characteristic.

Equations of the type (29*) are not unfamiliar in second-best theory, and they may be said to belong to the "Ramsey-Boiteux tradition" (see Guesnerie (8)). In fact, this equation is already known to hold, in the same kind of economy, for decentralization prices (provided it is evaluated at the optimum), and in view of the results of Section I, it is no surprise to find a direct analogue to hold for "welfare prices" or "fictitious prices".

"Ramsey-Boiteux" pricing rules of the kind (29*) and many of its special cases have been abundantly analysed in the literature on optimal taxation, optimal pricing and optimal public production. We shall therefore not linger on them, except to note two of their best-known special cases.

First, we easily verify that, in a perfectly competitive economy with optimal lump-sum transfers, shadow prices coincide with market

prices. Indeed, let $\theta^i = 0 \forall i$, $p = q$, $J = J_p$, and $\mu^i = \mu \forall i$, with $\mu = 1$ w.l.o.g. Then $\lambda = \gamma p (\gamma > 0)$ solves (29*) for each j (using (X3) and (Y2)). Moreover, from $\theta^i = 0$, (10) and (X1), $\gamma = 1$. Hence shadow prices coincide with market prices.

Second, suppose there are optimal transfers ($\theta^i = 0 \forall i$), and an untaxed commodity, say the first ($p_1 = q_1$). If all the cross-price derivatives appearing in (29*), except possibly those with respect to p_1 and q_1 , are zero, we obtain, under the normalization rule $p_1 = q_1 = \lambda_1$:

$$(32) \quad (q_j - \lambda_j) \frac{\partial \hat{x}_j}{\partial q_j} - (p_j - \lambda_j) \frac{\partial y_j}{\partial p_j} = 0$$

or

$$(36) \quad \lambda_j = q_j \frac{\partial \hat{x}_j / \partial q_j}{\partial \hat{x}_j / \partial q_j - \partial y_j / \partial p_j} + p_j \frac{-\partial y_j / \partial p_j}{\partial \hat{x}_j / \partial q_j - \partial y_j / \partial p_j}$$

In other words, the j^{th} shadow price becomes a weighted average of the consumer and the producer price, the weights being in the same ratio as the own-price derivatives of (compensated) consumer demand and producer supply. This simple rule has several applications in the literature, e.g. the Sandmo-Dreze (14) formula for the social discount rate.

(ii) Procured goods ($j \in J_T$) (12), being a special case of (11) or (29*) (applying when $H = \emptyset$, i.e. there are no private firms), is just another variety of Ramsey-Boiteux pricing rules. More precisely, let us recall the many-person Ramsey rule for optimal taxation (see

Diamond (2)):

$$(34) \quad t \frac{\partial \hat{x}_j}{\partial q} = \sum_i \theta^i x_j^i \quad \text{if } t_j \text{ is optimal.}$$

Then, writing (12) as

$$(35*) \quad \boxed{t_c^* \frac{\partial \hat{x}_j}{\partial q} = \sum_i \theta^i x_j^i} \quad (j \in J_T)$$

we obtain:

Rule 2: The shadow price vector should be such that C-taxes for procured goods follow the many-person Ramsey tax rule.

Formula (35*) has been considered by Hammond (9) and Mirrlees (10) (in economies with fixed or no producer prices) among others.

(iii) Fully traded goods ($j \in J_W$):

Let ϵ_j^W be the elasticity of world demand for good j :

$$(36) \quad \epsilon_j^W = \left(\frac{\partial p_j^W}{\partial e_j} \right)^{-1} \frac{p_j^W}{e_j}.$$

The marginal social value of foreign exchange is $\frac{\partial \psi}{\partial v^f}$ not μ^f .
 For our purposes, μ^f is a parameter. ^{7/} (13) now asserts:

$$(37*) \quad \boxed{\lambda_j = \mu^f p_j^W \left(1 + \frac{1}{\epsilon_j^W} \right)} \quad (j \in J_W)$$

In other words:

Rule 3: The shadow price of a fully traded good is its marginal cost (or revenue) on the world market, multiplied by the marginal social value of foreign exchange.

This rule confirms the examples of Chapter I and requires no further elaboration here.

(iv) P-rationed goods ($j \in J_R^h$ for some h)

Let us assume w.l.o.g. that $y_j^h > 0$. The results presented below, and in particular rule 4, have direct counterparts when $y_j^h < 0$.

Recall, from (Y3), the notion of marginal cost of good j in firm h :

$$(39) \quad \phi_j^h = - \sum_{k \neq j} p_k \frac{\partial y_k^h}{\partial y_j^h}$$

By contrast, the marginal social cost of good j in firm h is defined as:

$$(39) \quad \phi_j^h(*) = - \sum_{k \neq j} \lambda_k \frac{\partial y_k^h}{\partial y_j^h}$$

This is the cost, at shadow prices, of producing an additional unit of good j in firm h , using the technique of the firm's choice, which is of course the cost-minimizing technique at producer prices.

Finally, call $(p_j - \phi_j^h)$ the profit margin of good j in firm h . Normally the profit margin will be non-negative, unless producers can be forced to supply more than they wish. It is zero if firm h can freely exchange on the j^{th} market.

These notions will help us to understand (14):

$$(40) \quad 0 = \lambda \frac{\partial y^h}{\partial y_j^h} + \sum_i \theta^i a^{ih} \frac{\partial \pi^h}{\partial y_j^h}$$

$$(41) \quad = \lambda_j + \sum_{k \neq j} \lambda_k \frac{\partial y_k^h}{\partial y_j^h} + \theta^h (p_j - \phi_j^h)$$

or

$$(42^*) \quad \boxed{\lambda_j = \phi_j^h(*) - \theta^h (p_j - \phi_j^h)} \quad (j \in J_R^h)$$

Rule 4: The shadow price of a P-rationed good for which the rationed firm is a net supplier is equal to its marginal social cost, corrected for the social value of the private profit margin (if any).

A similar reasoning applies when $y_j^h < 0$ and yields:

Rule 5: The shadow price of a P-rationed good for which the rationed firm is a net buyer, is equal to its marginal social revenue product, corrected for the social value of the private profit margin.

The logic behind rules 4 and 5 is very straightforward, since a small change in the public sector supply of a P-rationed good is a corresponding modification of the quota of the rationed firm.

A particularly important application of rule 4 concerns firms producing under constant returns to scale. Suppose good k is produced by a firm, say h_k , whose production set exhibits constant returns to scale (in the variable, i.e. non-constrained, inputs and outputs, as well as k); then, in equilibrium, firm h_k must be producing according to demand, i.e. $k \in J_R^{h_k}$. If, in addition, $p_k = \phi_k^{h_k}$ (firm h_k is then said to be competitive), we obtain:

Rule 6: The shadow price of a good which is produced by a competitive, CRS firm is its marginal social cost. Equivalently, the social value (i.e. the value at shadow prices) of the production plan of such a firm is zero.

Rule 6 generalizes the well-known result of Diamond-Mirrlees (4), which was derived specifically for decentralization prices. It is interesting to observe that the intuitive argument these authors offer in support of their results relies heavily on the optimality of the public production plan, which is not invoked here.

(v) C-rationed goods ($j \in J_R^i$ for some i)

Consider finally (15):

$$(43) \quad 0 = \frac{\partial \Psi}{\partial V^i} \left(\frac{\partial \Pi^i}{\partial x_j^i} - q_j \frac{\partial V^i}{\partial m^i} \right) - \lambda_j - \sum_{k \neq j} \lambda_k \left(\frac{\partial x_k^i}{\partial x_j^i} - q_j \frac{\partial x_k^i}{\partial m^i} \right)$$

(using (R1) and (R3))

$$(44) \quad \frac{\partial \bar{V}^i}{\partial V^i} \frac{\partial U^i}{\partial x_j^i} - \lambda_j - \sum_{k \neq j} \lambda_k \frac{\partial x_k^i}{\partial x_j^i} = q_j \theta^i \quad (\text{using (10)})$$

Finally, using (R1) again:

$$(45*) \quad \lambda_j = \left[\mu^i \phi_j^i - \sum_{k \neq j} \lambda_k \frac{\partial x_k^i}{\partial x_j^i} \right] - q_j \theta^i \quad (j \in J_R^i)$$

Moreover, if i 's utility function is weakly separable between good j and others:

$$(46) \quad \lambda_j = \mu^i \phi_j^i - q_j \theta^i = q_j b^i + \mu^i (\phi_j^i - q_j) \quad (j \in J_R^i)$$

(see (R3) and (10)).

(45*) and (46) are most easily interpreted if we think of the effect of a small change in the public production of a C-rationed good as two-fold:

(i) A physical transfer occurs : the quota of the rationed consumer is adjusted. This is reflected in the bracketed term in (45*): its first component measures the social valuation of the increase in the consumer's utility (i.e. the marginal willingness to pay multiplied by his welfare weight), while the second accounts for the social cost of the change in his consumption of other goods through substitution effects. This second component vanishes when the utility of the rationed consumer is weakly separable between good j and others.

(ii) A financial transfer takes place as i is charged the price q_j . This transfer is valued as any lump-sum transfer: indeed, the payment made against the purchase of a good bought under quota is really an

implicit lump-sum tax. This gives the second term in (45*).

The simpler version (46) of (45*) is stated as rule 7:

Rule 7: The shadow price of a C-rationed good which is weakly separable from other goods in the utility function of the rationed consumer, is the social valuation of the marginal willingness to pay of the latter, corrected for the marginal social efficiency of the financial transfer involved in its sale or purchase.

A particularly important case of (46) occurs when j indexes the labour services of an individual i who is involuntarily unemployed. Then, if λ_{ℓ}^i is the shadow price of i 's labour, q_{ℓ}^i his wage, ϕ_{ℓ}^i his reservation wage, and if i 's utility is separable between goods and leisure, we obtain a shadow wage formula:

$$(47*) \quad \lambda_{\ell}^i = \mu^i \phi_{\ell}^i - q_{\ell}^i \theta^i = q_{\ell}^i b^i - \mu^i (q_{\ell}^i - \phi_{\ell}^i)$$

This simple and intuitively appealing expression for the shadow wage of course only covers one extreme type of unemployment: namely, involuntary unemployment at a fixed money wage, with no alternative occupation. However, more realistic expressions, taking into account self-employment, migration, wage movements, etc. can be derived in a similar framework.

Much more juice can be squeezed from equations (10) to (15), and the shadow pricing rules (29*), (35*), (37*), (42*), (45*), (47*); their systematic study will not be attempted here. Special cases of them appear in many contributions to cost-benefit

analysis and second-best theory. An example of the full solution to (10) - (15) in a very special situation is provided in the next section.

8/

Section II.3 : An example : Keynesian unemployment

As an elementary but instructive illustration of the results obtained above, I shall consider a specification of the model which corresponds roughly to the "textbook-Keynesian" framework:

- (i) $I = H = \{1\}$: there is a single consumer and a single producer; superscripts can therefore be omitted.
- (ii) $J = \{1, 2\}$: there are two goods, the second one being labour services.
- (iii) $t = 0$: no indirect taxation.
- (iv) $a^{01} = 1$: all profits accrue to the government.
- (v) $U(.)$ is separable between goods and leisure.

Notation:

- a) $y_1 = y, x_1 = x, p_1 = p, \lambda_1 = \lambda$
- b) $-y_2 = L, -x_2 = \ell, p_2 = \omega, \phi_2 = \phi_\ell$

In a situation of Keynesian unemployment, both the labour market and the goods market clear by rationing of supply:

$$(vi) \quad a_1 = \bar{y} \quad \text{and} \quad p > \omega \frac{\partial L}{\partial \bar{y}}$$

$$(vii) \quad a_2 = \bar{\ell} \quad \text{and} \quad \omega > \phi_\ell$$

To find shadow prices in this economy, we have to solve

(10) - (15), which here simply reduce to:

$$(viii) \quad \theta = \mu - \frac{\lambda}{p} \quad \left(\text{see (10), with } \frac{\partial x}{\partial m} = \frac{1}{p} \right)$$

$$(ix) \quad \lambda = \lambda_\ell \frac{\partial L}{\partial \bar{y}} \quad (\text{see (42*) with } \theta^h = 0 \text{ since } a^{oh} = 1)$$

$$(x) \quad \lambda_\ell = \mu \phi_\ell - \omega \theta \quad (\text{see (47*)}).$$

The solution of (viii), (ix), (x) for θ , λ_ℓ , λ is easily found:

$$(xi) \quad \lambda_\ell = \mu K (\phi_\ell - \omega) < 0$$

$$(xii) \quad \lambda = \mu K (\phi_\ell - \omega) \frac{\partial L}{\partial \bar{y}} < 0$$

$$(xiii) \quad \theta = \mu K \left(1 - \frac{\phi_\ell}{p} \frac{\partial L}{\partial \bar{y}} \right) > 0$$

where

$$K = \left(1 - \frac{\omega}{p} \frac{\partial L}{\partial \bar{y}} \right)^{-1} \text{ is the Keynesian multiplier.}$$

Observe in particular the following:

(xiv) $\lambda_\ell < 0$: it is socially desirable to increase public employment, even if its marginal product is zero.

(xv) $\lambda < 0$: it is socially desirable to increase public consumption of the produced good, even if public consumption has no social utility as such.

(xvi) $\theta > 0$: it is socially desirable to reduce lump-sum taxes.

All three of these conclusions hold because of the multiplier effects generated by the contemplated policies. They are in close agreement with textbook recommendations about public policies

under Keynesian unemployment.

Notice finally that the shadow prices derived here, being negative, cannot be interpreted as decentralization prices if the public production set allows free disposal : in this simple model, a Keynesian regime is inconsistent with optimal public production.

Section II.4 : Shadow prices and optimal policy rules

In this section we shall assume shadow prices to be known, and see how they often permit a neat characterization of optimal policy decisions.

Let β_ℓ be any policy instrument. Under a mild regularity assumption, the necessary first-order condition for an optimal manipulation of β_ℓ is that its marginal social value $\Pi_{\beta_\ell}^*$ should reduce to zero. If this condition is also sufficient, the optimal policy rules in this model are the following:

(i) Optimal taxation : The tax t_j on good j is optimal when $\Pi_{t_j}^* = 0$, or, using (18), (12) and (35*):

$$(48) \quad t_c^* \frac{\partial \hat{x}_j}{\partial q} = \sum_i \theta^i x_j^i$$

This may be called the generalized Ramsey rule for optimal taxation. It reduces to the conventional Ramsey rule (34) when shadow prices are proportional to producer prices (as in the Diamond-Mirrlees model (3)).

(ii) Optimal transfers : $\Pi_{r^i}^* = 0$ is equivalent to (see (16), (10)):

$$(49) \quad 0 = \theta^i \equiv u^i - b^i \equiv \frac{\partial w}{\partial y^i} \frac{\partial v^i}{\partial m^i} - \lambda \frac{\partial x^i}{\partial m^i}$$

This straightforward rule requires equating the social valuation of i 's marginal utility of income (i.e. his welfare weight

μ^i) with the social cost of his marginal consumption basket.

(iii) Optimal trade : $\Pi_{e_j}^* = 0$ if and only if

$$(50) \quad \lambda_j = \mu^f p_j^w \left(1 + \frac{1}{\epsilon_j^w}\right) \quad ((19), (13) \text{ and } (37*))$$

Under optimal trade, the shadow price of the j^{th} good coincides with the social valuation of its marginal foreign exchange cost on the world market.

(iv) Optimal rationing : when the government can impose a quota $\bar{y}_j^h$ on the net supply of good j in firm h , it should do so according to the rule:

$$(51) \quad \lambda_j = \phi_j^h(*) - \theta^h(p_j - \phi_j^h) \quad ((20), (14) \text{ and } (42*))$$

In particular, if it is controlling the distribution of a rationed producer good, an optimal rationing policy consists of equating its social marginal cost or marginal social revenue product (corrected for the marginal social valuation of profit margins) in each firm.

Similarly, the quota of a good j allocated to consumer i is optimal when (45*) is satisfied.

(v) Optimal pricing : When p_j is a policy instrument, its optimal value is found by solving (29*) (see (17) and (71)). We shall illustrate the implications of this rule in two very simple examples.

Example 1 : Let $j \in \bar{J}^1 \forall i \in I, j \in \bar{J}^h \forall h \in H$, i.e. each agent is rationed on the j^{th} market. Substituting $\frac{\partial \hat{x}_j}{\partial q} = \frac{\partial y_j}{\partial p} = 0$ in (23*), we obtain

$$d_j = 0$$

This pricing rule stresses the similarity between a lump-sum transfer and the payment made for the purchase of a good bought under quota. In India the distributional considerations involved in policy decisions regarding procurement prices for foodgrains, issue prices in ration shops, irrigation water rates, bus and train fares etc. are widely recognized.

Example 2 : Let $j \in \bar{J}^1 \forall i \in I$, and $\alpha_j = \bar{x}_j^1$ (j is a C-rationed good). According to (46), when j is separable from other goods in l 's utility function:

$$(i) \quad \lambda_j = u^1 \phi_j^1 - q \theta_j^1$$

If p_j is optimal, (29*) holds and

$$(ii) \quad -\sum_i \theta_i^1 z_j^i = (\lambda - p) \frac{\partial y_j}{\partial p} \quad (\text{since } \frac{\partial \hat{x}_j}{\partial q} = 0)$$

$$= \lambda \frac{\partial y}{\partial p_j} \quad (\text{see (Y2)})$$

$$\begin{aligned}
&= \sum_h \frac{\partial y_j^h}{\partial p_j} \left(\lambda_j + \sum_{k \neq j} \lambda_k \frac{\partial y_k^h}{\partial y_j^h} \right) \quad \left(\text{since } \frac{\partial y_k^h}{\partial p_j} = \frac{\partial y_k^h}{\partial y_j^h} \frac{\partial y_j^h}{\partial p_j} \right) \\
&= \sum_h \frac{\partial y_j^h}{\partial p_j} \left[\lambda_j - \phi_j^h(*) \right] \quad (\text{see (39)}) \\
&= \frac{\partial y_j^1}{\partial p_j} \left[\lambda_j - \phi_j(*) \right]
\end{aligned}$$

where $\phi_j(*) = \sum_h \frac{\partial y_j^h / \partial p_j}{\partial y_j^1 / \partial p_j} \phi_j^h(*)$ is a weighted average of the marginal social cost (revenue product) of good j in each firm. Combining (i) and (ii):

$$(iii) \quad \phi_j(*) = \mu^1 \phi_j^1 - (q_j \theta^1 - \sum_i \theta^i \frac{z_j^1}{\partial y_j^1 / \partial p_j})$$

The left-hand side of (iii) is the marginal social cost of good j , when an increase in output is achieved by a change in its price. Similarly, the right-hand side of (iii) can be interpreted as the marginal social benefit of a small increase in the private production of good j induced by a price manipulation; its first term is the social valuation of the marginal willingness-to-pay of the rationed consumer, while the second reflects the distributional effects of small variations of the j^{th} price. An optimal pricing policy brings these marginal social costs and social benefits into agreement.

(vi) Optimal public production : Recalling Observation 1 of Section I.7, optimal public production requires:

$$(52) \quad \frac{\lambda_j}{\lambda_k} = MRS_{jk}^0 \quad \forall j, k \in J$$

where MRS_{jk}^0 is the marginal rate of substitution between goods j and k in public production (at the optimum, of course).

This completes our short overview of optimal policy rules. An alternative but closely related exercise consists of using (16) - (22) to study welfare-improving policy reforms. We shall unfortunately neglect this interesting topic.

To conclude this section, we shall illustrate the use of the shadow pricing and optimality rules just derived by recalling three well-known results of second-best optimality theory.

First, let $J = J_p$, $a^{ih} = 0 \quad \forall i, h$, and all indirect taxes be optimal. Collecting (48) and (29*) we immediately obtain:

$$(53) \quad \lambda \frac{\partial y_j}{\partial p} = 0 \quad \forall j$$

The latter implies:

$$(54) \quad \lambda = \gamma p \quad \text{for some } \gamma > 0$$

if the matrix $\frac{\partial y}{\partial p}$ has rank $(M-1)$. Thus shadow prices are colinear with producer prices, and, as a corollary, optimal public production implies aggregate production efficiency. A simple set of circumstances under which shadow prices coincide with producer prices therefore amounts to optimal taxation in a

Diamond-Mirrlees economy (see Diamond-Mirrlees (3)). More general conditions for aggregate production efficiency to hold at a second-best optimum have been studied by Guesnerie (7).

Next, suppose that all taxes and transfers are optimal, though all goods need not be competitively exchanged. Recalling (48) and (49) we find

$$(55) \quad \lambda \frac{\partial \hat{x}_j}{\partial p} = 0 \quad \forall j$$

and thus

$$(56) \quad \lambda = \gamma q \quad \text{for some } \gamma > 0$$

if the matrix $\frac{\partial \hat{x}}{\partial q}$ has rank $(M-1)$. Shadow prices now coincide with consumer prices. This result illustrates the potential C-C efficiency property of second-best optima studied in a general framework by Guesnerie (7).

Finally, under optimal trade, shadow prices for traded goods coincide with their marginal cost (or revenue) in international trade (multiplied by the social value of foreign exchange), as confirmed directly by (50).

The contrast between the last three results underline the futility of arguing in a vacuum about the relative merits of producer prices, consumer prices, world prices (or some convex combination of them) as shadow prices; only a careful study of

how the economy operates, an explicit statement of the constraints and objectives relevant to the decisions of the planner, and an agreement about the expected role of shadow prices, can give meaning to such issues.

Section II.5 : Concluding remarks

The model analysed in this chapter recalls many well-known contributions of the literature on second-best theory as well as cost-benefit analysis (for a more detailed account of its relation to the literature, see Dreze (6), pp.57-59). To a great extent, the ability of the model for synthesis is due both to its solid microeconomic foundations (elaborated in the second-best tradition) and to its allowance for quantity restrictions (regarded as essential in the CBA literature).

However, our treatment of rationing is still far from satisfactory, both on account of its failure to describe many realistic rationing schemes (see the comments following (8)), and because it rules out an endogenous determination of the prices of rationed goods, e.g. through cost mark-ups etc. A description of wage and price movements would require the introduction of additional constraints besides the scarcity constraints, an extension which poses no new conceptual problem but gradually increases the complexity of welfare pricing rules as well as their operational viability.

These shortcomings provide challenging directions of future research. They will be discussed again from an empirical viewpoint in Part 2 of this thesis.

Footnotes : Chapter II

- 1/ Most of the present chapter is extracted or adapted from Section III of Dreze (6); minor discrepancies of presentation may therefore appear as compared with the previous chapter, and the author wishes to apologize for any consequent inconvenience in reading.
- 2/ The appropriate rank condition is assumed to be satisfied for the choice of α under study.
- 3/ Throughout the text, the symbols $(X1), (X2), \dots, (Y1), (Y2), \dots$, and $(R1), (R2), \dots$ refer to the properties of constrained demand and supply functions mentioned in the Appendix to this Chapter.
- 4/ This terminology is due to Guesnerie.
- 5/ The distributional characteristic is more usually written as $\sum_i \theta^i \frac{x_j^i}{x_j}$, an expression which is appropriate when all profits are zero or taxed away (a feature of many second best models). More generally the net excess demands $(-z_j^i)_{i \in I}$ have to be taken into account.
- 6/ $-\theta^h$ is readily interpreted as the marginal social value of the h^{th} profit tax, for fixed shares in after-tax profits.
- 7/ Alternatively, we could have imposed a constraint of balance of payments (i.e. $V^f = 0$), and obtained μ^f endogenously as the "shadow price" of foreign exchange, after endowing the system with a mechanism to satisfy this constraint: e.g. adding the exchange rate, or, say, the level of oil imports, to the list of endogenous variables.
- 8/ For a more thorough analysis of this topic, see Roberts (11).

Appendix A.2 : Elementary properties of constrained demand and supply functions

(i) Let $x^i(q, \bar{x}^i, m^i)$, $\hat{x}^i(q, \bar{x}^i, u^i)$ and $\tilde{x}^i(q, \bar{x}^i, \tilde{m}^i)$ solve respectively:

$$(X) \quad \begin{array}{ll} \text{Max. } U^i(x^i) \\ x^i \end{array} \quad \text{s.t.} \quad \begin{cases} qx^i = m^i \\ x_j^i = \bar{x}_j^i \quad \text{if } j \in \bar{J}^i \end{cases}$$

$$(\hat{X}) \quad \begin{array}{ll} \text{Min. } qx^i \\ x^i \end{array} \quad \text{s.t.} \quad \begin{cases} U^i(x^i) = u^i \\ x_j^i = \bar{x}_j^i \quad \text{if } j \in \bar{J}^i \end{cases}$$

$$(\tilde{X}) \quad \begin{array}{ll} \text{Max. } U^i(x^i) \\ x^i \end{array} \quad \text{s.t.} \quad \begin{cases} \sum_{j \in \bar{J}^i} q_j x_j^i = \tilde{m}^i \\ x_j^i = \bar{x}_j^i \quad \text{if } j \in \bar{J}^i \end{cases}$$

The maximum (or minimum) value functions of these programmes are denoted by $V^i(q, \bar{x}^i, m^i)$, $c^i(q, \bar{x}^i, u^i)$ and $\tilde{V}^i(q, \bar{x}^i, \tilde{m}^i)$ respectively.

The marginal willingness to pay of consumer i for commodity j , ϕ_j^i , is defined as:

$$\phi_j^i \equiv \frac{\partial U^i / \partial x_j^i}{\partial V^i / \partial m^i}$$

Some elementary properties of the functions introduced above are the following (the appropriate regularity conditions

are assumed to hold):

$$(X1) \quad q \frac{\partial x^1}{\partial m^1} = 1$$

$$(X2) \quad \frac{\partial \hat{x}_j^1}{\partial q_k} = \frac{\partial x_j^1}{\partial q_k} + x_k^1 \frac{\partial x_j^1}{\partial m^1} \stackrel{\text{not}}{=} \sigma_{j,k}^1 \quad (= 0 \text{ if } j \in \bar{J}^1)$$

(X3) The matrix $\Sigma^1 = (\sigma_{j,k}^1)$ is symmetric, negative semi-definite and $\Sigma^1 q = 0$.

$$(X4) \quad \frac{\partial V^1}{\partial q} = - \frac{\partial V^1}{\partial m^1} x^1$$

$$(R1) \quad \frac{\partial V^1}{\partial \bar{x}_j^1} = \frac{\partial U^1}{\partial x_j^1} - q_j \frac{\partial V^1}{\partial m^1} = \frac{\partial V^1}{\partial m^1} (\phi_j^1 - q_j)$$

$$(R2) \quad \frac{\partial c^1}{\partial \bar{x}_j^1} = q \frac{\partial \hat{x}_j^1}{\partial x_j^1} = q_j - \phi_j^1$$

$$(R3) \quad \frac{\partial x_j^1}{\partial x_k^1} = - q_k \frac{\partial x_j^1}{\partial m^1} + \frac{\partial \bar{x}_j^1}{\partial x_k^1}$$

$= -q_k \frac{\partial x_j^1}{\partial m^1}$ if goods j and k are weakly separable in i 's utility function.

(ii) Now let $y^h(p, \bar{y}^h)$ solve

$$(Y) \quad \begin{array}{l} \text{Max. } \pi^h \equiv p y^h \text{ s.t. } \left\{ \begin{array}{l} y^h \in Y^h \\ y_j^h = \bar{y}_j^h \text{ if } j \in \bar{J}^h \end{array} \right. \end{array}$$

The maximum value function of (Y) is denoted by $\pi^h(p, \bar{y}^h) \equiv p y^h(p, \bar{y}^h)$. The marginal cost (if $y_j^h > 0$) or marginal revenue product (if $y_j^h < 0$) of good j in firm h , ϕ_j^h , is defined as

$$\phi_j^h \equiv - \sum_{k \neq j} p_k \frac{\partial y_k^h}{\partial y_j^h}$$

Under appropriate regularity conditions, the following properties hold:

$$(Y1) \quad \frac{\partial \pi^h}{\partial p} = y^h$$

$$(Y2) \quad \text{The matrix } \Sigma^h = \left(\left(\frac{\partial y_j^h}{\partial p_k} \right) \right) \text{ is symmetric, positive semi-definite and } \Sigma^h p = 0.$$

$$(Y3) \quad \frac{\partial \pi^h}{\partial y_j^h} = p_j - \phi_j^h$$

(iii) All the properties listed above can easily be derived using the elementary tools of optimization theory (see e.g. Dixit (5)). For a more rigorous treatment see e.g. Neary-Roberts (11).

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CHAPTER III : Shadow prices and distribution
policies in a dual economy

Introduction

The economic theory of developing economies displays a healthy concern for the proper recognition of macroeconomic disequilibria. Unfortunately, its distaste for the paradigm of perfect competition often strips it of rigorous micro-economic foundations. Such is clearly the case, for instance, with the fashionable dual economy models of development theory. The mainstream of welfare economics, on the other hand, has evolved in the wake of Walrasian general equilibrium theory, insisting upon a consistent description of the behaviour of rational individual agents, but only recently weaning itself from the hypothesis of instantaneous market-clearing by price adjustment.

While both approaches can offer useful insights into the problems of short-run public policy in developing economies, they tend to stand uncomfortably apart.

In the last decade, however, a significant progress has been expected in the integration of the micro and macroeconomic traditions with the emergence of the concept of temporary equilibrium with rationing, fruitfully exploited in particular in a rapidly growing number of fix-price models.

This chapter draws much inspiration from the recent work in this area in an attempt to provide a theoretical framework for the discussion of short-run economic policy in a simple dual economy, where macroeconomic disequilibrium will be detected, in a precise sense, in the existence of a substantial wedge between

the marginal productivity of labour in the two aggregate sectors.

Temporary general equilibrium theory is sometimes criticized for its failure to incorporate a satisfactory theory of price formation. Indeed, once the polar assumptions of competitive or rigid prices are discarded, many alternative hypotheses can sustain an endogenous treatment of prices. They are prone to arbitrariness as well as complexity, especially when the collective behaviour of agents comes into play.

A similar uneasiness pervades much of the dual economy literature concerning the determination of the wage rate in an economy where, by hypothesis, the labour market is not competitive. Fixed subsistence wage in terms of food, fixed money wages, fixed wages in terms of industrial output, average product in agriculture, complex migration equilibrium conditions are some of the assumptions occasionally invoked to deal with the problem.

The approach adopted here is rather to recognize the poverty of our knowledge about the kinds of wage determination processes which actually take place in developing economies, and allow for a variety of alternative assumptions while attempting to understand the possible sensitivity of policy recommendations to any chosen specification. In particular, in the context of a well-defined problem of income distribution in a dual economy we shall notice the importance of a correct modelling of wage and price movements: a direct income transfer to a given agent should be evaluated quite differently according as to whether or not we expect the recipient to modify his (or her) wage claims as a

consequence.

Thus, if fix-price theory is a precious analytical tool for improving our understanding of macroeconomic disequilibrium phenomena, it may fall short of providing a satisfactory framework for a realistic assessment of policy decisions. A convincing theory of prices, perhaps drawing more skilfully from the lessons of empirical studies, will often be an essential element of such a framework.

The structure of this chapter is as follows: Section III.1 contains a detailed presentation of a simple dual economy model. Given the wage rate, industrial output and employment is the outcome of standard profit-maximizing behaviour on the part of urban capitalists. The rural sector is composed of surplus farming enterprises operating along capitalist lines, and peasant plots absorbing surplus labour. Thus the labour market adjusts through quantities; by contrast commodity markets clear by price adjustment. "Dualism" is represented by a divergence in the marginal productivity of labour in the two sectors. The delicate issue of wage determination is treated separately and a variety of hypotheses are allowed for. At one extreme lies the assumption of fixed money wages conform to the spirit of fix-price analysis. At another we shall formalize the idea of a migration-equilibrium preserving wage underlying much of the dual economy literature.

The model is oriented towards the comparative static analysis of short-run policy decisions. When the social welfare function is individualistic, the existence of welfare-improving

policy reforms will be seen to be attributable in a precise sense to both dualism and a concern for equity. Roughly speaking, the first allows short-run increases in aggregate output, while the second leaves scope for improved distribution.

Section III.2 is devoted to an analysis of shadow pricing in the dual economy, and draws some preliminary implications for distribution policies. In § III.2.1, we recall the version of the general shadow pricing rules of Chapter II applicable to our model. In § III.2.2, we show that, in a "first-best" environment where dualism is absent and no concern for equity exists, shadow prices coincide with market prices while transfer policies are neutral at the margin. The characteristic features displayed by the shadow price system once these restrictive assumptions are relaxed are discussed in § III.2.3. which concentrates for simplicity and definiteness on the case of fixed money wages. It is argued that with fixed money wages we should expect social prices to revalue commodities against labour, and agricultural against industrial output (as compared to market prices); while the set of welfare-improving transfers includes wage subsidies and progressive transfers. § III.2.4 presents some numerical calculations, and examines the sensitivity of the results to alternative wage assumptions.

Although fairly extensive sensitivity exercises have been performed, pressing deadlines forbids their detailed presentation here. However, a brief account of their main lessons, and some justification for their omission, are contained in § III.2.4.

Section III.3 undertakes a more detailed analysis of income distribution policies, with special reference to India. After recalling the illusory character of unconstrained lump-sum transfers, §III.3.1 draws attention to the objective constraints imposed upon income distribution policies by the imperfect flexibility of the particular institutional vehicles which carry them, such as preferential trading arrangements between the government and a specific category of consumers or producers. In India, for instance, the Public Distribution System permits, very roughly speaking, indiscriminate per head subsidies to the urban population. As an illustrative example of the analysis of feasible transfer policies, §III.3.2 envisages, in our simple dual economy model, a marginal diversion of public funds from such a scheme towards a programme specifically directed at supporting the incomes of the rural poor (such as some rural public works programmes). The suggested reform apparently deserves approval since it is essentially a progressive transfer. As a more careful analysis reveals, however, an informed evaluation hinges crucially upon a correct modelling of wage movements. More precisely, a sharp dichotomy appears in the set of available hypotheses, according as to whether or not they imply an immediate modification of wage claims in response to lump-sum transfers. When the wage bargaining process is relatively insulated from the transfer system, the reform seems desirable conditional upon a mild degree of inequality aversion. By contrast, when implicit subsidies induce a moderation of wage claims, the desire to promote industrial employment may motivate an urban bias in transfer policies which, under conditions of strong dualism and mild inequality aversion legitimizes an obstruction to the proposed reform.

Finally, a word of caution. The modelling exercise undertaken in the next section, and underlying the whole chapter, would assuredly fail to satisfy the pioneers of the dual economy literature. For a start, no reliable consensus seems to exist concerning the "distinguishing features" of dual economies. Moreover, the theory of dual economies has primarily sought to address the important problems of growth and development, rather than the more conventional issues of public economics. In this respect, our summary neglect of intertemporal considerations may appear particularly cavalier. As the sympathetic reader will recognize, however, simple and widely acknowledged features of dual economies are also relevant for the kind of short-run policy issues we shall discuss. Besides, the model seems amenable to the necessary
1/
improvements.

Section III.1 : A simple general equilibrium model of a dual economy

§ III.1.1 : General Features

The model presented in this section is a simple variant^{2/} of the synthetic model discussed in Section I.2 and Chapter II. In this chapter we shall use its "indirect version" (see Section I.2), rewritten as follows:

$$(1) \quad W = \psi(\dots u^i \dots)$$

$$(2) \quad \sum_i \hat{x}^i(p+t, \bar{x}^i, u^i) - \sum_h y^h(p, \bar{y}^h) - z = 0 \quad (v)$$

$$(3) \quad c^i(p+t, \bar{x}^i, u^i) - \sum_h a^{ih} \pi^h(.) - r^i = 0 \quad (e^i)$$

$$(4) \quad \xi(s) = 0 \quad (o)$$

where now $s^* \equiv (p, t, (u^i)_{i \in I}, (\bar{x}^i)_{i \in I}, (\bar{y}^h)_{h \in H})$ (international trade is exogenous), and $\xi(.)$ describes a set of "side constraints".

In the specification of this section, we shall endeavour to arrive at a simple but fair representation of the salient features of a so-called "dual economy".^{3/} The most important of these is the division of the economy into two contrasting sectors, and the recognition of a labour-market imperfection driving a substantial wedge between the marginal productivity of labour in the two sectors. We shall understand this as the essential meaning of "dualism"; but other less important assumptions will as far as possible also follow the dual economy literature.

Our economy, then, consists of two major sectors, the urban-industrial sector and the rural-agricultural sector. There are four goods : agricultural output (or "food" for short), industrial output, urban labour and rural labour; they are indexed by 1, 2, m, l and exchanged on their respective markets against money. Thus $J = \{1, 2, m, l\}$. For simplicity urban and rural labour services will be considered as homogeneous (e.g. yielding the same disutility); but the necessity to distinguish them persists because typically they will command different wages.

Consumers are divided into four broad and homogeneous economic groups, distinguished by their savings and consumption patterns as well as by their endowments of labour services and productive assets. The four classes are identified and indexed as: rural labourers-cum-peasants (R), urban workers (U), urban capitalist entrepreneurs (C), and surplus capitalist farmers (F). Thus $I = \{R, U, C, F\}$. Each class sells labour services, saves and spends. Besides, urban capitalists manage the industrial sector and receive its profits, while capitalist farmers have a similar position in surplus farms. In addition to hiring out their labour services when they can, rural labourers operate subsistence plots.

Thus, production activities are carried out within three aggregate enterprises, which are conveniently indexed in the same way as the class to which their owners belong : the industrial enterprise (C), the peasant farming enterprise (R), and the capitalist farming enterprise (F). Hence $H = \{R, C, F\}$ and

$$(5) \quad a^{ih} = 0 \quad (i \neq h)$$

$$(6) \quad a^{ih} = 1 - a^{oh} \quad (i = h)$$

Since peasant farms are self-managed enterprises, we shall naturally take $a^{oR} = 0$. Further:

$$(7) \quad y_2^R = y_2^F = y_1^C = y_m^R = y_m^F = y_\ell^C = 0, \quad y_2^C = y_2, \quad y_m^C = y_m, \quad y_1 = y_1^R + y_1^F$$

Non-labour inputs being fixed, the technology is described by

$$(8) \quad y_1^R = f(-y_\ell^R)$$

$$(9) \quad y_1^F = h(-y_\ell^F)$$

$$(10) \quad y_2^C = g(-y_m^C)$$

where the production functions f, g, h , are assumed to satisfy the following :

A*.1 f, g and h are concave and $f(0) = 0$.

In accordance with the spirit of dual economy models, the urban work force is drawn, at the margin, from the rural population. A correct representation of this assumption would strictly speaking demand a formalization of the phenomenon of migration. This refinement introduces minor technical difficulties, however, while it has been observed to have a quantitatively negligible impact in the numerical simulations. For our purposes, then,

little insight will be lost by assuming the marginal suppliers of industrial labour to remain in the class of rural labourers. The latter are then formally allowed to supply the two types of labour (in the short-run; in the longer-run, we may imagine migration to take place). Other classes supply a single type of labour.

Finally, and more importantly, labour supplies are fixed^{5/} for each class and denoted by $(L^i)_{i \in I}$. A convenient implication of this assumption is the legitimacy of dispensing completely with an explicit consideration of the labour services of the two entrepreneurial classes (C and F), since their quantity is invariant^{6/} while the associated wage payments reduce to intra-class transfers. Accordingly, we shall simply consider them as separate commodities, and neglect them throughout. Thus

$$(11) \quad \begin{cases} x_{\ell}^U = x_j^i = 0 & (j = m, \ell; i = F, C) \\ x_{\ell}^R + x_m^R = -L^R & (x_m^R = 0 \text{ in the initial state}) \\ x_m^U = -L^U \end{cases}$$

By a suitable choice of units for labour services L^i can also be interpreted as the total population of each class.

§III.1.2 : Behavioural assumptions

1. Consumers

(i) Labour supply. As explained earlier, labour supplies are fixed. In addition, rural labourers are rationed in their supply of urban labour. Hence

$$(12) \quad \hat{x}_m^i(.) = \bar{x}_m^i \quad \text{where} \quad \bar{x}_m^C = \bar{x}_m^F = 0, \quad \bar{x}_m^U = -L^U$$

$$(13) \quad x_\ell^i(.) = -L^i - \bar{x}_m^i \quad (i = R) \\ = 0 \quad (i \neq R)$$

(ii) Savings. As a first approximation, savings are taken to be a function of total income, where total incomes $(M^i)_{i \in I}$ include wage payments, dividends and direct transfers:

$$(14) \quad M^i = r^i + \sum_h a_{ih}^h \pi^h - q_m^i x_m^i - q_\ell^i x_\ell^i$$

$$(15) \quad e^i = q_1^i x_1^i + q_2^i x_2^i$$

$$(16) \quad e^i = e^i(M^i)$$

where e^i is i 's consumption expenditure. Marginal savings rates are denoted by $(\gamma^i)_{i \in I}$:

$$(17) \quad \gamma^i = 1 - \frac{\partial e^i}{\partial M^i}, \quad \gamma^i \in]0, 1[\quad \text{by assumption.}$$

(iii) Consumption expenditure. Since labour supplies are fixed, utility is derived from consumption expenditure only, and the standard expenditure minimization and utility maximization programs yield compensated demand functions $(\hat{x}_j^1(.))_{\substack{i \in I \\ j=1,2}}$ and uncompensated demand functions $(x_j^1(.))_{\substack{i \in I \\ j=1,2}}$ of the form:

$$(18) \quad x_j^i = \hat{x}_j^i(q_1, q_2, u^i) = x_j^i(q_1, q_2, e^i) \quad (i \in I, j = 1, 2)$$

In particular we may assert:

$$(18) \quad \frac{\partial \hat{x}_j^i}{\partial \bar{x}_k^i} = \frac{\partial \hat{x}_j^i}{\partial q_k} = 0 \quad \forall i \quad (j = 1, 2; k = 2, m)$$

There is no rationing on commodity markets:

$$(20) \quad \bar{J}^i \subset \{m, 2\} \quad \forall i \in I.$$

The i^{th} expenditure or cost function $c^i(q, \bar{x}^i, u^i)$ is defined implicitly by the identity:

$$(21) \quad e^i(c^i - q_m \bar{x}_m^i(.) - q_\ell \bar{x}_\ell^i(.)) - q_1 \bar{x}_1^i(.) - q_2 \bar{x}_2^i(.) = 0$$

or, equivalently:

$$(22) \quad u^i = v^i(q_1, q_2, e^i(c^i - q_m \bar{x}_m^i(.) - q_\ell \bar{x}_\ell^i(.)))$$

where $v^i(.)$ is the standard indirect utility function. If q^0 is the initial consumer price vector, it is convenient to cardinalize each utility function according to the "money-metric" utility

7/
rule :

$$(23) \quad v^i(q^0, e^i) = e^i$$

Some of the useful properties of the functions introduced above are stated below without proof:

$$(24) \quad (X1) \quad \frac{\partial c^i}{\partial u^i} = (1 - \gamma^i)^{-1} \quad \forall i \in I$$

$$(25) \quad (X2) \quad q \frac{\partial \hat{x}^i}{\partial q_j} = q \frac{\partial \hat{x}_j^i}{\partial q} = 0 \quad \forall i \in I, \forall j \in J$$

$$(26) \quad (X3) \quad \frac{\partial \hat{x}_j^i}{\partial q_k} = \frac{\partial \hat{x}_k^i}{\partial q_j} \quad \forall i \in I, \forall j, k \in J$$

$$(27) \quad (X4) \quad \frac{\partial c^i}{\partial q_j} = x_j^i \quad \forall i \in I, j = m, \ell$$

$$(28) \quad (X5) \quad \frac{\partial c^i}{\partial q_j} = \frac{x_j^i}{1 - \gamma^i} \quad \forall i \in I, j = 1, 2$$

$$(29) \quad (X6) \quad \frac{\partial c^i}{\partial x_m^i} = q_m - q_\ell \quad (i = R)$$

$$(30) \quad (X7) \quad q \frac{\partial \hat{x}^i}{\partial u^i} = 1 \quad \forall i \in I$$

2. Producers 9/

In capitalist enterprises ($h = C, F$), the production plan is determined along the standard profit-maximization line, without quantity constraints, i.e. y^h solves

$$(Y) \quad \text{Max.}_{y^h} \quad p y^h \text{ s.t. } y^h \in Y^h \quad (h = C, F)$$

where Y^h is defined implicitly in (8)-(10) above. Thus

$$\bar{J}^h = \phi \text{ for } h = F, C.$$

Peasant farms, on the other hand, absorb surplus labour.

For the peasant enterprise, then, the labour input is a datum,

$\bar{y}_\ell^R$; this implies

$$(31) \quad y_j^R(.) \equiv \bar{y}_\ell^R \quad (j = \ell)$$

$$(32) \quad y_j^R(.) \equiv 0 \quad (j = 2, m)$$

$$(33) \quad y_j^R(.) \equiv f(-\bar{y}_\ell^R) \quad (j = 1)$$

and $\bar{J}^h = \{\ell\} \quad (h = R).$

If $\pi^h(p)$ is the maximum value function of (Y)

$(h = C, F)$, and $\pi^R(p, \bar{y}_\ell^R) \equiv p_1 f(-\bar{y}_\ell^R) + p_\ell \bar{y}_\ell^R$, the usual relationships hold:

$$(Y1) \quad p \frac{\partial y_j^h}{\partial p_j} = p \frac{\partial y_j^h}{\partial p} = 0 \quad \forall j \in J, h \in H$$

$$(Y2) \quad \frac{\partial \pi^h}{\partial p} = y^h \quad \forall h \in H$$

$$(Y3) \quad \frac{\partial \pi^R}{\partial \bar{y}_\ell^R} = p_\ell - p_1 f'$$

Two polar assumptions about the agrarian structure induce interesting special cases of the model:

A.2 The peasant rural economy : $r^F \equiv L^F \equiv x_j^F \equiv y_k^F \equiv 0 \quad \forall j, k \in J$.

A.3 The capitalist rural economy : $f(.) \equiv 0$

This typology encompasses the well-known models of Jorgenson (10) and Fei-Ranis (4) for the rural sector (see e.g. Dixit (3) p.347).

Notice that when the rural economy is of the peasant type, rural labourers are net suppliers of food, while the reverse holds when it is of the capitalist type. This matter will turn out to be potentially important for policy purposes.

Another feature distinguishing the two types of agrarian structure results from the more intensive use of labour one should expect on peasant plots, since, when the disutility of labour is negligible, labour is costless to an otherwise unemployed peasant. Finally, an important characteristic of the rural peasant economy as we have defined it is the lack of price-responsiveness in food supply. In general the price elasticity of food supply, say η_1 , can be computed as:

$$(34) \quad \eta_1 \equiv \frac{\partial y_1}{\partial p_1} \frac{p_1}{y_1} = \frac{\partial y_1^F}{\partial p_1} \frac{p_1}{y_1^F} = \eta_1^F \frac{y_1^F}{y_1}$$

where η_1^F is the price-elasticity of food supply in farm enterprises, and the ratio y_1^F/y_1 can be interpreted as a simple index of agrarian structure, indicating the size of the share of food supply originating in each sub-sector of the rural economy.

From (34) we may infer three types of circumstances

under which the price elasticity of food supply may be expected to be rather low in this model:

- 1) The rural economy is largely of the peasant type.
- 2) The technology is such as to allow little profitable response to price movement, e.g. rigid coefficients exist between labour and other fixed inputs (typically, land).
- 3) The modelling of rural producers' behaviour is incorrect, e.g. non-labour variable inputs are important, or farmers have peculiar objective functions (or even none at all).

Many writings on the dual economy literature hold a fair degree of sympathy with one or other of these assumptions. While the third is, of course, ruled out a priori here, we shall often postulate that the first two hold to a sufficient degree as to make the price elasticity of food supply negligible. When the same is precluded in the industrial sector, we have:

$$\underline{A^* 4.} \quad \frac{\partial y_1}{\partial p} = 0, \quad \frac{\partial y_2}{\partial p} \neq 0$$

In view of the controversy surrounding the issue of supply responses in agriculture, it may be helpful to point out that this assumption:

- does not presuppose any kind of irrational behaviour
- does not rule out, at a more disaggregated level,
a significant degree of price responsiveness in the
choice of cropping patterns
- does not imply an inelastic marketed surplus.

10/

3. Prices and wages

Commodity prices adjust to clear commodity markets. They are normalized to unity in the initial state :

$$(35) \quad p_1^0 = p_2^0 = 1$$

Moreover, for simplicity we shall assume that no indirect taxes exist initially:

$$\underline{A^* 5.} \quad t^0 = 0$$

Labour markets, on the other hand, clear by quantity adjustment. The important issue of wage determination will be dealt with in detail shortly. At this stage, the presentation of the model is completed by the abstract wage-fixing formulae:

$$(36) \quad \xi^U(s) = 0$$

$$(37) \quad \xi^R(s) = 0$$

for urban and rural wages, respectively.

The only a priory constraints applying throughout on the values of p_m , p_ℓ are the following:

$$\underline{A^* 6.} \quad (i) \quad p_1 f' \leq \min(p_m, p_\ell)$$

$$(ii) \quad p_\ell > p_1 f' \Rightarrow p_m > p_1 f'$$

$A^* 5$ (i) rules out imposing forced labour upon rural workers; $A^* 5$ (ii) will hold if urban workers are granted the option of returning to the village when they find it in their best interest.

The importance of a correct modelling of rural wage behaviour should be expected to diminish as the share of the peasant sector in the rural economy increases; in particular, under the peasant rural economy hypothesis, (37) is redundant. Even when a substantial capitalist farming sector exists, rural wage shifts can already be seen to reduce, when $A^* 4$ holds, to inter-class transfers within the rural sector, while urban wage movements will directly affect aggregate output; we shall therefore be naturally led to devote relatively more attention to urban than to rural wages.

§ III.1.3 : Summary

Putting now together the pieces of the puzzle, we may rewrite (1)-(4) explicitly as:

- (38) (M0) $W = \Psi (...u^1...)$
- (39) (M1) $\sum_i \hat{x}_1^i(p+t, u^1) - y_1^F(p) - y_1^R(p, \bar{y}_\ell^R) - z_1 = 0 \quad (v_1)$
- (40) (M2) $\sum_i \hat{x}_2^i(p+t, u^1) - y_2^C(p) - z_2 = 0 \quad (v_2)$
- (41) (M3) $-L^U + \bar{x}_m^R - y_m^C(p) - z_m = 0 \quad (v_m)$
- (42) (M4) $-L^R - \bar{x}_m^R - y_\ell^F(p) - \bar{y}_\ell^R - z_\ell = 0 \quad (v_\ell)$
- (43) (M5) $c^R(p+t, \bar{x}_m^R, u^R) - \pi^R(p, \bar{y}_\ell^R) - r^R = 0 \quad (\theta^R)$
- (44) (M6) $c^U(p+t, u^U) - r^U = 0 \quad (\theta^U)$
- (45) (M7) $c^C(p+t, u^C) - (1-a^{oC})\pi^C(p) - r^C = 0 \quad (\theta^C)$
- (46) (M8) $c^F(p+t, u^F) - (1-a^{oF})\pi^F(p) - r^F = 0 \quad (\theta^F)$
- (47) (M9) $\xi^U(s) = 0 \quad (\sigma^U)$
- (48) (M10) $\xi^R(s) = 0 \quad (\sigma^R)$

As usual, the fulfilment of (38)-(48) imply the balance
11/
of the public budget:

$$(49) \quad B \equiv pz + tx + \sum_h a^{oh} r^h + \sum_i (M^i - e^i) - \sum_i r^i$$

= 0 when (M1)-(M10) hold.

As will appear clearly below, when A^{*} 5 holds, the existence of welfare-improving policies in the economy just described can in a precise sense be attributed to two specific violations of a "first-best" environment.

The first is the allocational inefficiency implied by the phenomenon of dualism itself, as reflected by a divergence between the marginal productivity of labour in the two sectors. We shall retain the latter as a convenient index of dualism, D:

$$(50) \quad D = \frac{p_1 f'}{p_2 g'}$$

In view of the "marginal productivity debate" surrounding the dual economy literature, it is worth pointing out that the marginal productivity of rural labour appearing in our index of dualism specifically refers to subsistence plots, rather than to an "average" marginal productivity in agriculture. Indeed, in a capitalist rural economy, it is quite possible to have $D = 0$ and yet $p_1 h' = p_2 g'$! Although "dualism" is sometimes understood to mean $D = 0$, we shall only assume:

A^{*} 7. Dualism: $D \leq 1$.

A common rationalization for the existence of dualism argues for the need, in order to drag a rural labourer out of subsistence farming activities, to pay him (or her) a wage at least equal to the average product on a peasant plot. When this is the case we should find:

$$(51) \quad p_2 g' = p_m \text{ and } p_m, p_\ell \geq \frac{p_1 y_1^R}{-y_\ell^R} \geq p_1 f'$$

i.e. $D \leq 1$.

Dualism, according to this interpretation, can be regarded as the outcome of a labour-market imperfection connected with the organization and reward structure of the peasant enterprise.^{12/} But it is compatible with competitive wages, as well as with various forms of wage rigidities.

The second departure from optimality arises out of a concern for equity. In order to give this a precise, if only conventional, meaning, we shall use the term distributional indifference to refer by contrast to a set of value judgements according to which changes in social welfare can always be assessed from a knowledge of changes in the aggregate consumption of commodities, irrespective of their distribution.

A.8. Distributional indifference: $\exists \mu > 0 : \mu^i = \mu \quad \forall i \in I$, where $\mu^i \equiv \frac{\partial \Psi}{\partial u^i}$

The agreement between (A.8) and the preceding verbal definition is apparent from the identities:

$$(52) \quad dW \equiv \sum_i \mu^i du^i$$

$$(53) \quad = \sum_i \mu^i (de^i - x^i dq) \quad \text{since } u^i \equiv V^i(q, e^i) \text{ and } \frac{\partial V^i}{\partial e^i} = 1$$

$$(54) \quad = \sum_i \mu^i \tilde{q} dx^i \quad \text{since } e^i \equiv \tilde{q} x^i$$

$$(55) \quad = \mu \tilde{q} dx + \sum_i (\mu^i - \mu) \tilde{q} dx^i \quad \text{for any } \mu$$

where " $\sim$ " indicates truncation of a vector to its first two components and $x = \sum_1 x^1$.^{13/}

Notice that distributional indifference as we have defined it need not imply the neutrality of lump-sum transfers, since these generally affect both the distribution and composition of the aggregate consumption bundle (in the notation of Chapter II, $\mu^1 = \mu \forall i \Rightarrow \theta^1 = \theta \forall i$).

§III.1.4 : Wage determination

In the traditional models of welfare economics, prices are usually flexible and competitive; their value is the outcome of market-clearing conditions. This assumption is inadequate when markets clear by quantity adjustment in the short-run, and more recent models, drawing much inspiration from temporary general equilibrium theory, allow for the possibility of quantity rationing by considering some (money) prices as fixed exogenously. This approach sometimes attracts criticism, however, on account of its rather evasive theory of prices. An endogenous treatment of price determination, no longer resting safely on market-clearing conditions, is indeed complex, and prone to arbitrariness, especially when it has to appeal to collective behaviour. A major purpose of our exercise will be to examine the sensitivity of the model to alternative assumptions regarding the fixation of wages.

Some possible specifications of (M9) and (M10) are listed below, and subsequently discussed; in each case the constants $\bar{\xi}^R$, $\bar{\xi}^U$ have to be given an appropriate initial value.

$$(56) \quad H_U^O : \xi^U(.) \equiv p_m + t_m - \bar{\xi}^U \quad \left. \vphantom{\begin{matrix} (56) \\ (57) \end{matrix}} \right\} \quad \text{(fixed money wages)}$$

$$(57) \quad H_R^O : \xi^R(.) \equiv p_\ell + t_\ell - \bar{\xi}^R$$

$$(58) \quad H_U^1 : \xi^U(.) \equiv (p_m + t_m)/(p_1 + t_1) - \bar{\xi}^U \quad \left. \vphantom{\begin{matrix} (58) \\ (59) \end{matrix}} \right\} \quad \text{(fixed food wages)}$$

$$(59) \quad H_R^1 : \xi^R(.) \equiv (p_\ell + t_\ell)/(p_1 + t_1) - \bar{\xi}^R$$

$$\begin{aligned}
 (60) \quad H_U^2 : \xi^U(.) &\equiv (p_m + t_m)/Q^U(p_1 + t_1, p_2 + t_2) - \bar{\xi}^U \\
 (61) \quad H_R^2 : \xi^R(.) &\equiv (p_\ell + t_\ell)/Q^R(p_1 + t_1, p_2 + t_2) - \bar{\xi}^R
 \end{aligned}
 \left. \vphantom{\begin{aligned} (60) \\ (61) \end{aligned}} \right\} \text{(price-linked wages)}$$

$$\begin{aligned}
 (62) \quad H_U^3 : \xi^U(.) &\equiv u^U - \bar{\xi}^R \\
 (63) \quad H_R^3 : \xi^R(.) &\equiv u^R - \bar{\xi}^R
 \end{aligned}
 \left. \vphantom{\begin{aligned} (62) \\ (63) \end{aligned}} \right\} \text{(utility-preserving wages)}$$

$$\begin{aligned}
 (64) \quad H_4^U : \xi^U(.) &\equiv (u^U/L^U)/(u^R/L^R) - \bar{\xi}^U \\
 (65) \quad H_R^4(a) : \xi^R(.) &\equiv p_\ell + t_\ell - p_1 f'(.) - \bar{\xi}^R \\
 H_R^4(b) : \xi^R(.) &\equiv p_\ell + t_\ell - \frac{p_1 y_1^R(.)}{-y_\ell^R(.)} - \bar{\xi}^R
 \end{aligned}
 \left. \vphantom{\begin{aligned} (64) \\ (65) \end{aligned}} \right\} \text{(competitive wages)}$$

The simplest alternative to the competitive wage assumption is that of fixed money wages, stated in H_U^0 , H_R^0 . We shall consider it as the "reference case".

The idea of a fixed institutional or "subsistence" wage which permeates some of the dual economy literature, however, is often translated as a rigidity of wage relative to a suitable numeraire commodity such as food. H_U^1 and H_R^1 therefore impose fixed food wages. They seem particularly suitable when wages are paid in kind and the terms of labour contracts are rigid. Alternatively, we may interpret them as rough indexation formulae. More generally, then, we may allow for price-linked wages as in H_U^2 and H_R^2 , where $Q^U(.)$ and $Q^R(.)$ are cost-of-living price indices, e.g.

$$(66) \quad Q_*^U(q_1, q_2) = \frac{q_1^0 x_1^{U_0} + q_2^0 x_2^{U_0}}{q_1^0 x_1^{U_0} + q_2^0 x_2^{U_0}} \quad \text{for some } q^0, x^{U_0}.$$

Formally, H_U^0 , H_R^0 , H_U^1 , H_R^1 are special cases of H_U^2 , H_R^2 .

As we shall see, an important consideration for the evaluation of transfer policies is the extent to which transfers can be expected to be compensated by wage adjustments. No such compensation would occur when simple price-indexation formulae such as H_U^2 , H_R^2 apply. At another extreme, a "full" compensation would take place when wages follow a perfect indexation formula which aims at defending the real utility of workers. Such a formula, implicit in H_U^3 , H_R^3 , sustains utility-preserving wages.

The utility-preserving wage formula has the shortcoming of preventing workers from sharing in overall welfare gains emanating from improved public policies. In H_U^4 , this drawback is avoided by requiring urban wages to maintain pre-determined utility differentials. In a dual-economy context, H_U^4 is usefully interpreted as a simple condition of migration equilibrium. In particular when the rural economy is of the peasant type and transfers are negligible, one verifies easily that H_U^4 reduces to the more familiar hypothesis of equality between the urban wage and the average product in agriculture.

Under this interpretation of H_U^4 , urban wages may be regarded as competitive given the environment, and a corresponding assumption may be sought in the rural sector. What, then, is the supply price of rural labour to capitalist farms? Logically, when rural labourers do not have free access to the urban labour market,

while the produce of subsistence farming is shared within the whole rural labour class, it should be the marginal product of labour on peasant plots, as in $H_R^4(a)$. Alternatively, if rural wage labourers lose their share in the produce of subsistence farming when accepting outside employment, the average product seems more adequate, as in $H_R^4(b)$. Intermediate cases are of course also possible, but will not be considered explicitly. For convenience we shall refer to H_R^3 , H_U^3 and H_R^4 , H_U^4 collectively as utility-wage hypotheses.

Between the two extremes of fixed money wages (H_R^0 , H_U^0) corresponding to the fix-price methodology, and competitive wages (H_R^4 , H_U^4) corresponding to some dual economy models, we now find a wide variety of possible assumptions. None of them seems totally satisfactory, insofar as they neglect the dynamic aspects of wage bargaining processes (in particular, the extent and swiftness with which wages adapt in the face of price increases). On the other hand, their multiplicity (compounded by the possibility of combining different assumptions for each sector) endangers the clarity of the model; our exercise should therefore be regarded as a modest step towards a more synthetic treatment of wage behaviour.

Section III.2 : Shadow prices in a dual economy

§III.2.1 : General Expressions

Letting

$$(67) \quad s \equiv ((p_j)_{j \in J}, (t_j)_{j \in J}, (r^i)_{i \in I}, (u^i)_{i \in I}, \bar{x}_m^R, \bar{y}_l^R)$$

$$(68) \quad \omega \equiv (z, \delta) \quad \text{where } \delta \equiv (\delta^R, \delta^U, \delta^C, \delta^F, \bar{\xi}^U, \bar{\xi}^R) \quad \text{is a vector of slack variables}$$

$$(69) \quad \Omega \equiv Z^* \times \Delta^*$$

$$(70) \quad \Delta^* \equiv \{\delta^0\},$$

the dual economy described in (38)-(48) fits the abstract models of section 1.1 ((6.i)-(6.iii) on p.1.3) and section 1.2 ((16)-(18) on p.1.9).

The interpretation of the shadow prices derived in this chapter should be along the same lines as in section II.1. We shall here choose the vector α of endogenous variables as

$$(71) \quad \alpha \equiv ((p_j)_{j \in J}, (u^i)_{i \in I}, \bar{x}_m^R, \bar{y}_l^R)$$

This specification will be appropriate for a planner who believes that, in a project evaluation exercise, fiscal policy (in the limited sense of taxes and transfers) should be considered as exogenous. The theory of Chapter I suggests at least three circumstances under which this assumption would be justified:

- a
- (i) Fiscal policy is optimal, and the envelope property applies.
 - (ii) Fiscal policy is predetermined.
 - (iii) Fiscal policy is neither optimal nor predetermined; but the demands of decentralization make it difficult to coordinate project evaluation and fiscal reforms.

Strictly speaking, however, (i) cannot be invoked here, since we assume zero initial taxes.

With $G(s)$ as in (39)-(48), then, the shadow price vector ϕ can be written

$$(72) \quad \phi \equiv W_{\alpha} G_{\alpha}^{-1} \text{ not. } (v_1, v_2, v_m, v_\ell, \theta^R, \theta^U, \theta^C, \theta^F, \sigma^U, \sigma^R)$$

where v_j is the shadow price of the j^{th} commodity and θ^i is the marginal social value of transfer to the i^{th} consumer (the interpretation of σ^U, σ^R depends upon the wage hypotheses adopted).

It is of course unreasonable to attempt at an analytical inversion of the "Jacobian" matrix G_α , and at an explicit expression for ϕ according to (72). However much insight can be gained from the equivalent "shadow price equations":

$$(73) \quad \frac{\partial \mathcal{L}}{\partial \alpha} = 0 \quad \text{where}$$

$$(74) \quad \mathcal{L}(s, \omega; \phi) \equiv W(s) - \phi [G(s) - \omega]$$

is the Lagrangian of the model. The shadow price equations permit a useful discussion of the general features of shadow prices in a dual economy, and will be analysed below under the reference hypothesis of fixed money wages. Explicitly, they read:

$$(75) \quad (p_1) \quad v \left[\frac{\partial \hat{x}}{\partial q_1} - \frac{\partial y}{\partial p_1} \right] - \sum_i \theta^i \frac{x_1^i}{1-\gamma^i} + \theta^R y_1^R + \theta^F (1-a^{OF}) y_1^F - \frac{\partial \xi^*}{\partial p_1} = 0$$

$$(76) \quad (p_2) \quad v \left[\frac{\partial \hat{x}}{\partial q_2} - \frac{\partial y}{\partial p_2} \right] - \sum_i \theta^i \frac{x_2^i}{1-\gamma^i} + \theta^C (1-a^{OC}) y_1^C - \frac{\partial \xi^*}{\partial p_2} = 0$$

$$(77) \quad (p_m) \quad v \frac{\partial y}{\partial p_m} - \sum_i \theta^i x_m^i + \theta^C (1-a^{OC}) y_m^C - \frac{\partial \xi^*}{\partial p_m} = 0$$

$$(78) \quad (p_\ell) \quad v \frac{\partial y}{\partial p_\ell} - \sum_i \theta^i x_\ell^i + \theta^R y_\ell^R + \theta^F (1-a^{OF}) y_\ell^F - \frac{\partial \xi^*}{\partial p_\ell} = 0$$

$$(79) \quad (u^i) \quad \mu^i - v \frac{\partial \hat{x}^i}{\partial u^i} - \theta^i (1-\gamma^i) - \frac{\partial \xi^*}{\partial u^i} = 0 \quad (i \in I)$$

$$(80) \quad (\bar{x}_m^R) \quad v_m + v_\ell - \theta^R (p_m - p_\ell) - \frac{\partial \xi^*}{\partial \bar{x}_m^R} = 0$$

$$(81) \quad (\bar{y}_\ell^R) - v_1 f' + v_\ell + \theta^R (p_\ell - p_1 f') - \frac{\partial \xi^*}{\partial \bar{y}_\ell^R} = 0$$

where $x \equiv \sum_i x^i$, $y \equiv \sum_h y^h$ and

$$(82) \quad \xi^*(s) \equiv \sigma^U \xi^U(s) + \sigma^R \xi^R(s)$$

Except for the last element of each line, which reflects the endogenous treatment of wages, (75)-(81) are simple special cases of the welfare pricing rules studied in Chapter II. (75)-(78) amount to the familiar Ramsey-Boiteux expressions for competitively exchanged goods (29*, Chapter II); (79) defines the marginal social value of lump-sum transfers and reduces easily to (10) of Chapter II; (80) and (81) are the appropriate formulae for P-rationed and C-rationed goods (see (42*) and (45*) in Chapter II). 14/

We shall not undertake a detailed analysis of the shadow pricing rules for arbitrary wage-fixing assumptions. Instead, we shall first concentrate, in § III.2.3, on the reference case of fixed money wages, and later suggest how the results may have to be modified when adopting different hypotheses. Before this, we should briefly mention a general result which holds independently of the precise wage formulae.

§III.2.2 : The "first-best" environment

As we argued earlier, our economy potentially suffers from two major distortions: dualism, and an inequitable distribution. Our first theorem confirms the coincidence of shadow prices with market prices when both of these are absent.

Theorem 1 : Let $p^* = (p_1, p_2, p_m, p_\ell, 0, 0, 0, 0, 0, 0)$. Then, when A^*1 , A^*5 , A^*6 and A^*7 hold:

$$\phi = p^* \Leftrightarrow \begin{cases} D = 1 & \text{(No dualism)} \\ \mu^i = 1 \quad \forall i \in I & \text{(No concern for equity)} \end{cases}$$

Proof: A^*6 implies, along with $p_m = p_2 g'$ and $p_\ell = p_1 h'$:

$$D = 1 \Leftrightarrow p_1 f' = p_2 g' = p_1 h'$$

$\Leftarrow$ Let $\mu^i = 1 \quad \forall i$ and $D = 1$. Then p^* is easily verified to be a solution of (75)-(81), by recalling (X2), (X7), (Y1) and the equalities

$$p_m = p_2 g' = p_1 h' = p_1 f'$$

Since ϕ is the unique solution of (75)-(81), $\phi = p^*$.

$\Rightarrow$ Conversely let $\phi = p^*$. From (79) and (X7), $\mu^i = 1 \quad \forall i$. Moreover, $p_1 f' = p_1 h' = p_2 g'$ results from (75), (76) and the equalities

$$p_m = p_2 g', \quad p_\ell = p_1 h'$$

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§III.2.3 : Theoretical analysis : the case of rigid money wages

When money wages are fixed, H_U^0 and H_R^0 hold, implying, together with (82)

$$(83) \quad \begin{aligned} \frac{\partial \xi^*}{\partial s_\ell} &= 0, & \text{if } s_\ell \neq p_m, p_\ell \\ &= \sigma^U & \text{if } s_\ell = p_m \\ &= \sigma^R & \text{if } s_\ell = p_\ell \end{aligned}$$

Substituting (83) in (75)-(81), one immediately obtains the appropriate version of the shadow price equations

$$(84) \quad (p_j)_{j=1,2} \quad v \left[\frac{\partial x_j}{\partial q} - \frac{\partial y_j}{\partial p} \right] = -d_j x_j \quad (j=1,2)$$

$$(85) \quad (p_m) \quad \sigma^U = -d_m x_m + \frac{\partial y_m}{\partial p_m} (v_m - v_2 g')$$

$$(86) \quad (p_\ell) \quad \sigma^R = -d_\ell x_\ell + \frac{\partial y_\ell}{\partial p_\ell} (v_\ell - v_1 h')$$

$$(87) \quad (\bar{y}_\ell^R) \quad v_\ell = v_1 f' - \theta^R (p_\ell - p_1 f')$$

$$(88) \quad (\bar{x}_m^R) \quad v_m = v_1 f' - \theta^R (p_m - p_1 f')$$

$$(89) \quad (u^i) \quad \theta^i = (1-\gamma^i)^{-1} (\mu^i - b^i)$$

where, much as before: 15/

(i) b^i is i 's consumption conversion factor :

$$(90) \quad b^i \equiv v \frac{\partial \hat{x}^i}{\partial u^i} = \sum_{j=1,2} v_j \frac{\partial x_j^i}{\partial e^i}$$

(ii) d_j is the j^{th} distributional characteristic:

$$(91) \quad d_j \equiv - \frac{\sum_i \theta^i z_j^i}{x_j}$$

where now

$$(92) \quad \begin{cases} -z_j^i = x_j^i - \sum_h a^{ih} y_j^h & (j=2,m) \\ -z_j^i = (1-\gamma^i)^{-1} x_j^i - \sum_h a^{ih} y_j^h & (j=1,2) \end{cases}$$

It is possible, but cumbersome and hardly instructive, to solve (84)-(89) explicitly. However, these equations may prove useful in guiding our intuition about the outstanding features of shadow pricing in a dual economy. As an illustration, we shall try to identify the circumstances under which three a priori plausible conjectures appear to be correct. The latter suggest:

C1) A devaluation of labour relatively to commodities in the shadow price system, as compared with the market price system.

C2) An incentive to subsidize wages, at least when the ensuing inflation of industrial profits can be taxed away.

C3) A bias in favour of food relatively to industrial output in the shadow price system as against the market price system.

Put in a nutshell, the rationale behind these conjectures is the following. C1) is motivated by the presumably low marginal productivity of labour in the peasant sector ($f' \ll g' = p_m$), together with the simple shadow wage formulae (87) and (88). The likelihood of a devaluation of labour in the shadow price system underlies C2), which, as we shall see, is justified when the social opportunity cost of labour falls short of the social value of its marginal product in the industrial sector. C3) is based on similar grounds: if the supply of food is relatively inelastic (as in A 4), food can only be made available by reducing consumption, whereas industrial output is amenable to greater production, which is socially relatively cheap. Distributional considerations should reinforce this argument, since food carries an especially large weight in the consumption basket of the poor. If the agrarian structure is largely of the peasant type, however, distributional considerations may lend support to an opposite conclusion, since then a large part of the poor will be net suppliers of food.

In order to arrive at more precise statements, it is helpful to mention first a few auxiliary results and expressions

derived from (84)-(89). They are established formally in Appendix A.3.1 to this chapter, assuming, quite reasonably, that $\min(v_1, v_2) > 0$ and $\mu^R > 0$.

$$(R1) \quad \frac{v_m}{v_1 p_m} = D - \frac{\theta^R}{v_1} (1-D) < 1 \Leftrightarrow \frac{\theta^R}{v_1} > -1 \text{ and } D < 1.$$

$$(R2) \quad \frac{v_m}{v_2 p_m} = \frac{v_1}{v_2} D - \frac{\theta^R}{v_2} (1-D) < 1 \Leftrightarrow \frac{\theta^R}{v_2} > \frac{\frac{v_1}{v_2} D - 1}{1-D} \text{ or } D = 1 \text{ and } v_1 < v_2$$

$$(R3) \quad \left. \begin{array}{l} v_1 \geq v_2 \Rightarrow \theta^R > -v_1 \\ v_2 \geq v_1 \Rightarrow \theta^R > -v_2 \end{array} \right\} \text{ In particular } \theta^R > \min.(-v_1, -v_2)$$

$$(R4) \quad \text{Let } \sigma_1 \equiv -d_1 x_1 + \frac{\partial y_1}{\partial p_1} \left(v_1 - \frac{v_2}{p_1} \right)$$

$$\sigma_2 \equiv -d_2 x_2 + \frac{\partial y_2}{\partial p_2} \left(v_2 - \frac{v_m}{p_m} \right)$$

If A^* holds, then

$$(i) \quad \max(d_1, d_2) > 0 \text{ unless } v_1 = v_2. \text{ If } v_1 = v_2, d_1 = 0 \leq d_2.$$

$$(ii) \quad \sigma_1 + \sigma_2 = 0; \text{ in particular } \sigma_1 \text{ and } \sigma_2 \text{ have opposite signs.}$$

$$(ii) \quad v_1 > v_2 \Leftrightarrow \sigma_1 < 0 \Leftrightarrow \frac{\sigma_1}{x_1} < \frac{\sigma_2}{x_2}$$

(R5) Under fairly general conditions (which may be expected to be sensibly weaker when dualism is strong), we may assert the existence of a weighted average of the θ^i 's which is positive:

$$\exists \left\{ w^i \right\}_{i \in I} : \sum_{i \in I} w^i \theta^i > 0, \quad \sum_{i \in I} w^i = 1$$

Moreover, each weight is positive when the marginal propensity to save in the capitalist classes and/or profit taxes are high enough. In such case:

$$(93) \quad \max_i \theta^i > 0 \quad ; \quad \text{hence} \quad \max_i (\mu^i - b^i) > 0$$

Intuitively, (R5) detects an inflationary bias in the set of welfare improving policies. When money wages are fixed, price increases reduce real wages and stimulate industrial output. Appropriately distributed gifts of money to the public should therefore increase welfare, especially if dualism is rampant. Unless inequality aversion is fairly weak, we would then confidently conclude from (93) that:

$$(94) \quad c^R > 0$$

We shall now state some formal results corroborating our previous discussion.^{16/} From now on, and in order to avoid a tedious duplication and qualification of results, "social valuation of labour" will refer to industrial labour only.

Our first proposition outlines one possible set of simple sufficient conditions under which each of our three conjectures is indeed verified.

Propn.1. C1), C2) and C3) are true if A*1, A*4, A*5, A*6, A*7 hold and both commodities have identical distributional characteristics, i.e.

$$(95) \quad d_1 = d_2 \neq 0^{17/}$$

Proof: Let $d_1 = d_2 \neq 0$ and $\frac{\partial y_1}{\partial p_1} = 0$. Then

(i) From (R4) (i), $d_1 = d_2 > 0$. Hence $\sigma_1 < 0$ and $\sigma_2 > 0$, using (R4) (ii). By (R4) (iii), $v_1 > v_2$.

(ii) If $d_2 > 0$ and $\sigma_2 > 0$, then $v_2 > \frac{v_m}{p_m}$. Hence $MSV_{t_m}^c > 0$ where $MSV_{t_m}^c$ is the marginal social value of a wage subsidy when appropriately "compensated" by a tax on industrial profits (see "wage subsidization" below).

Thus

$$\left\{ \begin{array}{l} \text{C1)} \quad v_m < \min. (v_1 p_m, v_2 p_m) \\ \text{C2)} \quad MSV_{t_m}^c > 0 \\ \text{C3)} \quad v_1 > v_2 \end{array} \right.$$

#

The essential hypothesis of Propn.1, expressed in (95) is satisfied in particular when all individuals have identical marginal propensities to spend and save and identical welfare weights.

We shall now examine each conjecture in turn. In the following, A^*1, A^*5, A^*6, A^*7 , are assumed throughout.

(1) Social valuation of labour

Propn.2. Let $D < 1$ and $v_1 > 0, v_2 > 0$. Then:

- (i) (Industrial) labour is devalued in the shadow price system against one commodity at least:

$$(96) \quad v_m < \max. (v_1 p_m, v_2 p_m)$$

- (ii) Labour is devalued against food $(v_m < v_1 p_m)$ iff

$$(97) \quad \theta^R > -v_1$$

A sufficient condition for (97) to hold is:

$$\theta^R > 0; \text{ another is } v_1 \geq v_2.$$

- (iii) Labour is devalued against industrial output

$$(v_m > v_2 p_m) \text{ iff}$$

$$(98) \quad \theta^R > \frac{v_1 D - v_2}{1-D}$$

A sufficient condition for (98) to hold is: $v_1 \leq v_2$.

Proof: The first parts of (ii) and (iii) are just restatements of (R1) and (R2). Now if $v_1 \geq v_2$, then $\theta^R > -v_1$, so $v_m < v_1 p_m$ (see (R1), (R3)). And if $v_2 \geq v_1$, then $\theta^R > -v_2 = v_2 \frac{D-1}{1-D} \geq \frac{v_1 D - v_2}{1-D}$, so $v_m < v_2 p_m$ (see (R2), (R3)). Therefore $v_m < \max.(v_1 p_m, v_2 p_m)$, establishing (i). The above reasoning together with the equivalences stated in the first parts of (ii) and (iii), confirms their second part. #

It is difficult to synthesize usefully the conditions which guarantee a devaluation of labour against both commodities, i.e. the simultaneous satisfaction of (97) and (98). Intuitively, we can expect it when dualism is strong while the relative shadow prices of commodities do not differ too significantly from their relative market prices, e.g. sufficient conditions for both (97) and (98) to hold are

$$(99) \quad \theta^R > 0 \quad \text{and} \quad D < \frac{v_2}{v_1}$$

(2) Wage subsidization

The marginal social value of an urban wage subsidy is defined, in accordance with the previous theory, as the total impact on social welfare of a small perturbation of the parameter t_m :

$$(100) \quad MSV_{t_m} = \frac{\partial \mathcal{L}}{\partial t_m} = - \theta^U x_m^U - \sigma^U = \frac{\partial y_m}{\partial p_m} (v_2 g' - v_m) - \theta^C (1 - a^{OC}) y_m^C$$

using (85).

By the marginal social value of a compensated wage subsidy, on the other hand, we shall understand the effect of the same perturbation when the resulting inflation of industrial profits is compensated by a suitable lump-sum tax on capitalists, i.e. when $dr^C = (1 - a^{OC}) y_m^C dt_m$:

$$(101) \quad MSV_{t_m}^C \stackrel{\text{def.}}{=} \frac{\partial \mathcal{L}}{\partial t_m} \bigg|_{dr^C} = (1 - a^{OC}) y_m^C dt_m = \frac{\partial y_m}{\partial p_m} (v_2 g' - v_m)$$

The social desirability of a wage subsidy thus appears to hinge crucially upon the discrepancy between the social cost of labour (v_m) and its marginal social product ($v_2 g'$). Insofar as it clarifies the social valuation of labour, then, Propn.2. also elucidates the circumstances favourable to an industrial wage subsidy. In particular, (101) and Propn.2 (iii) imply immediately:

Propn.3. Let A^* 4 hold and $D < 1$, while $v_1, v_2 > 0$. Then a compensated wage subsidy is desirable unless

$$(102) \quad v_1 > v_2 \text{ and } \theta^R \leq \frac{v_1 D - v_2}{1 - D}$$

The motivation behind wage subsidization is clear: since labour is more productive in the industrial sector, industrial employment should be encouraged, except when industrial output is substantially less socially valuable than food, presumably for distributional reasons.

(3) Social valuation of food

Propn.4. If (A^{*}4) holds, then:

$$(103) \quad v_1 > v_2 \quad \text{iff} \quad d_1 - d_2 > -\eta_2 \left(\frac{v_2}{p_2} - \frac{v_m}{p_m} \right) \frac{y_2}{x_2}$$

where $\eta_2 > 0$ is the own-price elasticity of supply of industrial output.

A sufficient condition for (103) to hold is that food has the highest distributional characteristic of both commodities:

$$(104) \quad d_1 > d_2 \Rightarrow v_1 > v_2$$

Proof: The first part of the proposition follows immediately from (R4) (iii) and the definitions of σ_1 , σ_2 .

Now, if $d_1 > d_2$, then $d_1 > 0$ from (R4)(i).

Hence $\sigma_1 < 0$, and, from (R4) (iii), $v_1 > v_2$.

#

Propn. 4 confirms that, distributional considerations apart (i.e. when $d_1 = d_2$), the low social cost of labour in a dual economy tends to induce a premium on the social value of food insofar as it is in relatively inelastic supply; by contrast, an exogenous pressure on the market for industrial output can to some extent be met by greater, and socially cheaper, production. One would expect this premium to be accentuated by distributional considerations, unless the agrarian structure is essentially of the peasant type so that a large part of the poor are net suppliers of food. If ambiguity arises, it can only be settled by empirical measurement.

§III.2.4. : Some numerical simulations

Some numerical simulations of the model have been carried out, based on the data set described in Annexure 1 and relating to the Indian economy. The necessary modifications, additional assumptions, and aggregation exercises involved are mentioned in Appendix A.3.2 to this chapter. Table 1 below displays the main elements of the data along with a few necessary comments. As far as possible, when reliable information was not available, tentative values have been assigned to the relevant parameters, and sensitivity exercises performed. But few of these exercises will be presented here, partly because, quite strikingly, a wide range of qualitative conclusions are fairly robust with respect to most parametric variations. As a matter of fact, sensitivity analysis indicates the determinant role played by two parameters, which precisely reveal and measure the two distortions discussed in §III.1.3:

- (i) The index of dualism, D .
- (ii) An index of concern for equity, μ^C , described below.

This phenomenon is not very surprising a posteriori; indeed, when there is neither dualism nor concern for equity, theorem 1 establishes the coincidence of shadow prices with market prices independently of the values of other parameters. The robustness property of shadow pricing rules in a "first-best" environment thus survives to some extent in disequilibrium situations. Accordingly, and for our purposes, we may to a large

TABLE 1 : Parameter Estimates

a) Consumer parameters

	L^i	$\frac{e^i}{L^i}$	α_1^i	α_2^i	β_1^i	β_2^i	APS^i	γ^i
R	0.505	0.646	0.73	0.27	0.71	0.29	0.08	0.20
U	0.108	0.700	0.64	0.36	0.65	0.35	0.08	0.20
C	0.122	1.713	0.47	0.53	0.47	0.53	0.28	0.44
F	0.265	1.466	0.59	0.41	0.47	0.53	0.24	0.41

b) Producer parameters

	y_1^h	y_2^h	y_g^h	y_m^h	a^{oh}
R	(*)	0	(*)	0	0
C	0	0.388	0	-0.108	0.2
F	(*)	0	(*)	0	0

Total 0.612 0.388 -0.505 -0.108

c) Commodity parameters

	$\hat{e}_{jj}$	η_{jj}	p_j
1	-0.227	0	1
2	-0.358	(*)	1
g	0	0	(*)
m	0	(*)	0.7

d) Reference values of unobservable parameters (when relevant)

Parameter	η_{mm}	p_g	y_1^R/y_1	f'
Value	-0.5	0.7	0.2	0.4

cont'd....

Notational conventions

$\alpha_j^i \equiv \frac{q_j x_j^i}{e^i}$: share of commodity j in i 's expenditure ($i \in I, j = 1, 2$).

$s_j^i \equiv q_j \frac{\partial x_j^i}{\partial e^i}$: marginal share of commodity j in i 's budget ($i \in I, j = 1, 2$).

APS^i : average propensity to save in class i ($i \in I$).

γ^i : marginal propensity to save in class i ($i \in I$).

$\hat{\epsilon}_{jj}$: compensated own-price elasticity of the aggregate consumer demand for commodity j ($j \in J$).

η_{jj} : own-price elasticity of the aggregate net producer supply of commodity j ($j \in J$).

Explanatory notes

- (1) The symbol (*) indicates an unobserved parameter. The reference values of unobserved parameters follow from part d) of the table and the equalities :

$$(i) \quad \eta_{22} = \frac{p_m y_m}{p_2 y_2} \eta_{mm}$$

$$(ii) \quad y_1^R + y_1^F = y_1$$

$$(iii) \quad y_\ell^R + y_\ell^F = y_\ell$$

$$(iv) \quad p_1 y_1^R - p_\ell y_\ell^F = \frac{e^R}{1 - APS^R} \quad (\text{with } r^R = 0).$$

- (2) Units are chosen so that:

$$(i) \quad \sum_i L^i = 1$$

$$(ii) \quad \sum_i e^i = 1$$

In particular, average per capita consumer expenditure amounts to unity.

extent accomodate ourselves of "plausible" values for unobservable parameters, without fearing to obscure the most interesting features of the model.

At this point, therefore, we shall concentrate our attention to the implications of varying degrees of dualism and inequality aversion for shadow pricing, first with fixed money wages and then under alternative wage determination formulae.

In the shadow pricing formula

$$(105) \quad \phi \equiv W_{\alpha} G_{\alpha}^{-1}$$

the matrix G_{α}^{-1} , to be called here the "inverted Jacobian" of the model, indicates the total effect on the equilibrium value of α of an infinitesimal perturbation of the vector of "intervention variables" ω :

$$(106) \quad d\omega \Rightarrow d\alpha^* = G_{\alpha}^{-1} d\omega$$

Furthermore,

$$(107) \quad \phi = \frac{dW}{d\omega} = \sum_i \mu^i \frac{du^i}{d\omega} = \sum_i \mu^i \phi^i$$

where $\{\phi^i\}_{i \in I}$ may be interpreted as vectors of personnalized shadow prices, measuring the total effect on each individual utility level of small changes $d\omega$; with our normalization rules, $\phi^i d\omega$ is nothing but i 's marginal-willingness-to-pay for a small move $d\omega$.

The matrix G_a^{-1} (and hence the vectors $\{\phi^i\}_{i \in I}$) is presented in Table 2 for the "reference" values of all parameters, assuming fixed money wages. This matrix can be extremely helpful whenever one wishes to trace in detail the general equilibrium effects of a specific policy move. Such instructive exercises will, however, be left to the interested reader.

As (107) makes clear, when the social welfare function is "individualistic", the shadow price vector is a "welfare weighted" sum of the personalized shadow prices $\{\phi^i\}_{i \in I}$. It has become customary, in applied cost-benefit analysis, to constrain welfare weights to lie on an iso-elastic curve, and interpret the elasticity of this curve as an indicator of inequality aversion. In our case, the fair constancy, across sectors, of the estimated per capita expenditure levels of capitalist and working classes seemed to justify a computationally attractive alternative consisting of attributing identical weights, across sectors, to individuals of the same status, i.e.

$$\text{A.9.} \quad \begin{cases} \mu^C = \mu^F \\ \mu^R = \mu^U = 1 \end{cases}$$

The last equality is a harmless normalization rule. A.9 implies

$$(108) \quad \phi = \mu^C \phi^O + (1 - \mu^C) \phi^1$$

where

$$(109) \quad \phi^O \equiv \sum_i \phi^i = \phi \quad \text{if} \quad \mu^C = 1$$

$$(110) \quad \phi^1 \equiv \phi^R + \phi^U = \phi \quad \text{if} \quad \mu^C = 0$$

TABLE 2 : Inverted Jacobian

G_{α}^{-1}	(z_1)	(z_2)	(z_m)	(z_l)	(δ^R)	(δ^U)	(δ^C)	(δ^F)	$(\bar{\xi}^U)$	$(\bar{\xi}^R)$
(p_1)	-3.62	-0.60	-2.11	-2.11	2.11	2.06	1.13	1.19	-0.00	0.33
(p_2)	-0.86	-4.00	-0.77	-0.77	1.42	1.56	1.42	1.49	0.20	-0.02
$(\bar{x}_m^R)$	-0.04	0.21	1.04	0.04	-0.07	-0.08	-0.07	-0.08	0.06	0.00
$(\bar{y}_l^R)$	-0.04	-0.21	-1.04	-1.04	0.07	0.08	0.07	0.08	-0.06	-0.00
(u^R)	0.56	0.38	0.11	0.11	0.39	-0.40	-0.26	-0.27	0.08	0.21
(u^U)	0.19	0.13	0.12	0.12	-0.14	0.65	-0.09	-0.09	0.08	-0.01
(u^C)	0.29	-0.19	0.15	0.15	-0.12	-0.10	0.54	-0.02	-0.03	-0.03
(u^F)	-0.08	0.60	-0.00	-0.00	-0.09	-0.12	-0.16	0.42	-0.03	-0.17
(p_m)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
(p_l)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Cont'd.....

Explanatory notes

- (1) The i, j^{th} element of the above matrix is

$$\frac{d\alpha_i^*}{d\omega_j}$$

where α^* is the equilibrium value of α and

$$\alpha \equiv (p_1, p_2, \bar{x}_m^R, \bar{y}_\ell^R, u^R, u^U, u^C, u^F, p_m, p_\ell)$$

$$\omega \equiv (z_1, z_2, z_m, z_\ell, \delta^R, \delta^U, \delta^C, \delta^F, \bar{\xi}^U, \bar{\xi}^R)$$

- (2) In particular, rows (5)-(8) of this matrix are precisely the "personalized shadow price vectors" $\{\phi^i\}_{i \in I}$ introduced in the text.

- (3) The above estimates assume fixed money wages, and use the "reference values" of unobserved parameters (see Table 1).
A value of 0.4 is assumed for f' , implying $D \approx 0.6$.

In this particular model, then, the shadow price vector is just a weighted average of two vectors which are conveniently interpreted as the shadow price vectors corresponding respectively to utilitarian (ϕ^0) and Rawlsian or maximin (ϕ^1) welfare weights.

Table 3 displays the vectors ϕ^0 and ϕ^1 for a selection of values of D (with fixed money wages), as well as a vector ϕ^* obtained by the more usual CBA technique for estimating μ^C , assuming a "standard" value of unity for the elasticity of the marginal utility of expenditure; ^{19/} the latter implies the value of 0.425 for μ^C with our data. In addition, Table 3 presents the marginal social value of an uncompensated wage subsidy, MSV_{t_m} .

The empirical results seem to corroborate the theory outlined in the previous sections. In particular, when money wages are fixed:

(i) With distributional indifference and no dualism ($\mu^C = 1 = D$), shadow prices coincide with market prices, and lump-sum transfers are neutral at the margin (theorem 1).

(ii) Dualism (with distributional indifference) introduces:

a) An incentive towards inflationary policies :

$$\phi^1 > 0 \quad \forall i \quad (R5).$$

b) A devaluation of labour against both commodities, to the extent of a negative shadow wage when extreme

$$\text{dualism prevails: } v_m < \min(v_1 p_m, v_2 p_m),$$

$$D = 0 \Rightarrow v_m < 0 \quad (C1).$$

TABLE 3 : Shadow prices with fixed money wages

		v_1	v_2	v_m	v_ℓ	θ^R	θ^U	θ^C	θ^F	σ^U	σ^R	MSV_{t_m}
$f' = 0$	$\mu^C = 1$	0.97	0.85	-0.04	-0.04	0.05	0.06	0.05	0.06	-0.05	-0.00	0.053
	$\mu^C = 0.425$	0.84	0.61	-0.13	-0.13	0.18	0.19	-0.16	-0.17	-0.01	0.12	0.028
	$\mu^C = 0$	0.75	0.43	-0.19	-0.19	0.28	0.29	-0.33	-0.34	0.02	0.20	0.009
$f' = 0.2$	$\mu^C = 1$	0.98	0.89	0.18	0.18	0.04	0.04	0.04	0.04	-0.03	-0.00	0.038
	$\mu^C = 0.425$	0.85	0.65	0.09	0.09	0.17	0.17	-0.18	-0.19	0.01	0.12	0.013
	$\mu^C = 0$	0.76	0.48	0.02	0.02	0.26	0.27	-0.34	-0.36	0.03	0.20	-0.005
$f' = 0.4$	$\mu^C = 1$	0.99	0.94	0.39	0.39	0.02	0.03	0.02	0.02	-0.02	-0.00	0.022
	$\mu^C = 0.425$	0.86	0.70	0.30	0.30	0.15	0.16	-0.20	-0.21	0.02	0.12	-0.003
	$\mu^C = 0$	0.77	0.52	0.23	0.23	0.24	0.25	-0.36	-0.38	0.05	0.20	-0.021
$f' = 0.6$	$\mu^C = 1$	1.00	0.98	0.60	0.60	0.01	0.01	0.01	0.01	-0.01	-0.00	0.008
	$\mu^C = 0.425$	0.87	0.74	0.51	0.51	0.13	0.14	-0.21	-0.22	0.03	0.12	-0.017
	$\mu^C = 0$	0.78	0.57	0.44	0.44	0.23	0.24	-0.37	-0.39	0.06	0.20	-0.035
$f' = 0.7$	$\mu^C = 1$	1.00	1.00	0.70	0	0	0	0	0	0	0	0
	$\mu^C = 0.425$	0.87	0.76	0.61	0.61	0.13	0.13	-0.22	-0.23	0.04	0.12	-0.011
	$\mu^C = 0$	0.78	0.59	0.55	0.55	0.22	0.23	-0.38	-0.40	0.07	0.20	-0.043

Explanatory notes

- (1) The three lines of each row correspond to the successive values of 0, 1 and ∞ for the elasticity of the marginal utility of expenditure.
- (2) The index of dualism D varies from 0 to 1 along the rows of the table since

$$D \equiv \frac{p_1 f'}{p_2 g'} = \frac{f'}{0.7}$$

$$(3) \quad MSV_{t_m} \equiv (v_2 g' - v_m) \frac{\partial y_m}{\partial p_m} - y_m (1 - a^0 C) \theta^0$$

Remark: The coincidence of the urban and rural shadow wages is due to the particular "reference" value chosen for the rural money wage (i.e. equal to both the average product on peasant plots and the urban wage); it is not a general feature of the model.

- c) An incentive to subsidize urban wages:
 $MSV_{t_m}, MSV_{t_m}^C > 0. (C2).$
- d) A premium in favour of food in the shadow price system : $v_1 > v_2 (C3).$
- (iii) The concern for equity, on the other hand:
- a) Accentuates the social preference for food relative to industrial output: $\mu^C = 0 \Rightarrow v_1 \gg v_2.$
- b) Discriminates sharply between progressive and regressive transfers: $\mu^C = 0 \Rightarrow \theta^R, \theta^U > 0$ but $\theta^C, \theta^F < 0.$
- c) Hence, it reduces the attractiveness of uncompensated wage subsidies.
- d) Mostly as a result of a), it tends to confirm the devaluation of labour against food, but not necessarily against industrial output if dualism is mild. In the latter case it reduces the desirability of even compensated urban wage subsidies.
- (iv) As a rule, horizontal transfers (i.e. transfers between either the capitalist or the working classes) are fairly neutral, while progressive transfers are desirable even with a weak concern for equity.

As a theoretical fiction, the assumption of fixed money wages is certainly a useful alternative to the competitive wages hypothesis, insofar as it allows a simple treatment of labour-market disequilibrium; however as a suggestive description of the functioning of the economy it suffers from a similar lack of realism.

It is important, therefore, to investigate the sensitivity of the model, and especially the versatility of policy recommendations, to alternative wage hypotheses. We shall now attempt a few steps in this direction.

Table 4 summarizes some of the major shadow price relationships under most possible combinations of the wage indexation hypotheses introduced in §III.1.4 above, and for the polar cases of no dualism ($D = 1$) and extreme dualism ($D=0$). In its light we shall review briefly the conjectures discussed in the previous section.

(i) The phenomenon of undervaluation of labour in the shadow price system of the dual economy seems fairly robust. The only important qualification concerns the relative price of labour and industrial output when dualism is mild while rural wages are fixed. In these circumstances the social value of labour in terms of food is fairly close to its market value (since $\frac{v_m}{p_m} = v_1 D - \theta^R(1-D) = v_1$); but industrial output is devalued against food for distributional reasons, and therefore, in the industrial sector, the social cost of labour now exceeds its marginal social product: $v_m > v_2 p_m = v_2 g'$.

As a direct consequence, a wage subsidy will then cease to be desirable.

(ii) The introduction of wage indexation renders the suggested premium on food in the shadow price system quite volatile. For instance, such a premium is definitely reversed when inequality aversion is strong and rural wages are fixed in terms of food, since then food price increases benefit the greatest part of the

TABLE 4 : Major shadow price relationships under alternative wage hypotheses

a) Relative shadow prices of commodities : $\frac{v_1}{v_2}$

D=0				
	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	1.14(1.74)	1.14(0.24)	1.14(1.37)	1.14(0.31)
H_U^1	1.27(1.60)	1.30(-0.03)	1.27(0.88)	1.27(1.17)
H_U^2	1.16(1.73)	1.20(0.05)	1.17(1.29)	1.17(0.77)
H_U^3	1.17(1.75)	1.19(-0.01)	1.18(1.33)	1.19(0.83)
H_U^4	1.14(1.74)	1.26(-0.01)	1.18(1.31)	1.19(0.83)

D=1				
	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	1(1.32)	1(0.23)	1(0.91)	1(0.23)
H_U^1	1(1.11)	1(-0.23)	1(0.23)	1(-0.23)
H_U^2	1(1.33)	1(-0.14)	1(0.60)	1(-0.13)
H_U^3	1(1.32)	1(-0.24)	1(0.41)	1(-0.24)
H_U^4	1(1.30)	1(-0.25)	1(0.43)	1(-0.25)

cont'd....

b) Shadow wage with food numeraire : $\frac{v_m}{v_l}$

$D=0$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	-0.04(-0.25)	-0.04(- 5.73)	-0.04(-1.65)	-0.03(-39.25)
H_U^1	0.01(-0.32)	0.03(46.36)	0.03(-3.13)	0.08(- 8.59)
H_U^2	0.01(-0.30)	0.02(-36.43)	0.02(-2.61)	0.06(-10.57)
H_U^3	0.02(-0.31)	0.04(130.00)	0.03(-3.00)	0.07(- 9.13)
H_U^4	0.05(-0.32)	0.09(137.50)	0.08(-3.06)	0.20(- 8.93)

$D=1$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0.7(0.71)	0.7(0.70)	0.7(0.69)	0.7(0.70)
H_U^1	0.7(0.71)	0.7(0.73)	0.7(0.70) ^(*)	0.70(0.70)
H_U^2	0.7(0.69)	0.7(0.70) ^(*)	0.7(0.70) ^(*)	0.7(0.70)
H_U^3	0.7(0.69)	0.7(0.75)	0.7(0.71)	0.7(0.70)
H_U^4	0.7(0.70)	0.7(0.70) ^(*)	0.7(0.67) ^(*)	0.7(0.70)

(*) approximate

cont'd.....

c) Shadow wage with industrial output numeraire : $\frac{v}{2}$

$D=0$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	-0.04(-0.44)	-0.04(-4.52)	-0.04(-2.26)	-0.04(-13.08)
H_U^1	0.01(-0.51)	0.03(-1.54)	0.03(-2.76)	0.10(-9.73)
H_U^2	0.01(-0.51)	0.02(-1.67)	0.02(-3.36)	0.07(-8.22)
H_U^3	0.02(-0.55)	0.04(-1.73)	0.03(-4.00)	0.09(-7.3)
H_U^4	0.06(-0.55)	0.11(-1.72)	0.10(-4.00)	0.24(-7.4)

$D=1$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0.7(0.93)	0.7(0.16)	0.7(0.63)	0.7(0.15)
H_U^1	0.7(0.78)	0.7(-0.17)	0.7(0.15)	0.7(-0.17)
H_U^2	0.7(0.92)	0.7(-0.08)	0.7(0.40)	0.7(-0.08)
H_U^3	0.7(0.91)	0.7(-0.18)	0.7(0.29)	0.7(-0.18)
H_U^4	0.7(0.91)	0.7(-0.18)	0.7(0.29)	0.7(-0.18)

cont'd....

d) Marginal social value of transfer to the rural poor : θ^R

D=0

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0.05(0.28)	0.05(0.68)	0.05(0.61)	0.05(0.75)
H_U^1	-0.02(0.31)	-0.04(0.73)	-0.04(0.68)	-0.04(0.67)
H_U^2	-0.01(0.31)	-0.03(0.72)	-0.03(0.67)	-0.03(0.68)
H_U^3	-0.03(0.31)	-0.05(0.74)	-0.05(0.68)	-0.05(0.66)
H_U^4	-0.07(0.33)	-0.14(0.79)	-0.13(0.74)	0.11(0.61)

D=1

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (0.22)	0 (0.64)	0 (0.57)	0 (0.64)
H_U^1	0 (0.33)	0 (0.76)	0 (0.71)	0 (0.76)
H_U^2	0 (0.32)	0 (0.74)	0 (0.69)	0 (0.74)
H_U^3	0 (0.34)	0 (0.77)	0 (0.72)	0 (0.77)
H_U^4	0 (0.41)	0 (0.90)	0 (0.87)	0 (0.90)

cont'd....

e) Marginal social value of (uncompensated) urban subsidy : MSV_{t_m}

$D=0$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0.05(0.01)	0.05(0.04)	0.05(0.03)	0.05(0.12)
H_U^1	0.05(0.01)	0.05(0.04)	0.05(0.04)	0.04(0.11)
H_U^2	0.05(0.01)	0.05(0.05)	0.05(0.04)	0.05(0.12)
H_U^3	0.05(0.01)	0.05(0.06)	0.05(0.04)	0.04(0.12)
H_U^4	0.05(0.01)	0.04(0.07)	0.04(0.04)	0.03(0.11)

$D=1$

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (-0.04)	0 (0.00)	0 (-0.01)	0 (0.01)
H_U^1	0 (-0.03)	0 (0.02)	0 (-0.01)	0 (0.02)
H_U^2	0 (-0.04)	0 (0.01)	0 (-0.00)	0 (0.01)
H_U^3	0 (-0.03)	0 (0.02)	0 (-0.00)	0 (0.02)
H_U^4	0 (-0.04)	0 (0.02)	0 (-0.00)	0 (0.02)

cont'd...

f) Marginal social value of compensated wage subsidy : $(v_2 g' - v_m) \frac{\partial y_m}{\partial p_m}$

D=0

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0.05(0.04)	0.05(0.06)	0.05(0.04)	0.05(0.12)
H_U^1	0.05(0.04)	0.05(0.06)	0.05(0.05)	0.04(0.12)
H_U^2	0.05(0.04)	0.05(0.06)	0.05(0.04)	0.05(0.13)
H_U^3	0.05(0.04)	0.05(0.06)	0.05(0.04)	0.04(0.13)
H_U^4	0.04(0.04)	0.04(0.06)	0.04(0.05)	0.03(0.12)

D=1

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (-0.01)	0 (0.02)	0 (0.00)	0 (0.02)
H_U^1	0 (-0.00)	0 (0.03)	0 (0.00)	0 (0.03)
H_U^2	0 (-0.01)	0 (0.02)	0 (0.01)	0 (0.02)
H_U^3	0 (-0.01)	0 (0.02)	0 (0.01)	0 (0.03)
H_U^4	0 (-0.01)	0 (0.03)	0 (0.01)	0 (0.03)

cont'd...

g) Marginal social value of a vertical transfer : $\theta^R - \theta^C$

D=0

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	-0.00(0.61)	0(0.80)	0 (0.73)	0 (0.79)
H_U^1	-0.04(0.62)	-0.05(0.83)	-0.05(0.78)	-0.05(0.76)
H_U^2	-0.02(0.62)	-0.03(0.82)	-0.03(0.78)	-0.04(0.77)
H_U^3	-0.03(0.61)	-0.04(0.83)	-0.04(0.76)	-0.04(0.76)
H_U^4	-0.11(0.65)	-0.14(0.88)	-0.13(0.83)	0.11(0.70)

D=1

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (0.60)	0 (0.80)	0 (0.74)	0 (0.80)
H_U^1	0 (0.66)	0 (0.86)	0 (0.80)	0 (0.86)
H_U^2	0 (0.64)	0 (0.84)	0 (0.78)	0 (0.84)
H_U^3	0 (0.64)	0 (0.85)	0 (0.79)	0 (0.85)
H_U^4	0 (0.77)	0 (0.99)	0 (0.95)	0 (0.99)

cont'd...

h) Marginal social value of a horizontal transfer : $\theta^R - \theta^U$

D=0

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	-0.01(-0.02)	-0.01(0.02)	-0.01(0.00)	-0.01(0.00)
H_U^1	-0.01(-0.01)	-0.01(0.01)	-0.01(0.00)	-0.01(0.00)
H_U^2	-0.01(-0.01)	-0.01(0.01)	-0.01(0.00)	-0.01(0.00)
H_U^3	-0.48(0.21)	-0.48(0.29)	-0.49(0.34)	-0.45(-0.40)
H_U^4	-0.50(0.21)	-0.52(0.31)	-0.53(0.37)	-0.19(-0.16)

D=1

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (-0.01)	0 (0.02)	0 (0.01)	0 (0.02)
H_U^1	0 (0.00)	0 (0.02)	0 (0.01)	0 (0.02)
H_U^2	0 (0.00)	0 (0.02)	0 (0.01)	0 (0.02)
H_U^3	0 (0.67)	0 (0.62)	0 (0.73)	0 (0.62)
H_U^4	0 (0.78)	0 (0.73)	0 (0.88)	0 (0.73)

cont'd...

Explanatory notes

- (1) Each pair of tables corresponds to the two extreme values of our measure of dualism :
- $D = 0$ (extreme dualism) and
- $D = 1$ (no dualism).
- (2) Within a table, each entry corresponds to a specific pair of wage assumptions, as indicated by the headings attached to each row and column (see §III,1.4 for details)
- (3) The numbers outside brackets are calculated under the assumption of distributional indifference ($\mu^C = 1$). Those inside brackets assume maximum welfare weights ($\mu^C = 0$).

poor. When both wages follow some indexation formula, the picture can become quite complex since rural wage increases may be welcome while urban ones are detrimental.

The precise indexation formulae applying in each sector, as well as the exact degrees of dualism and inequality aversion (besides other parameters such as the wage elasticity of urban employment and the index of agrarian structure) will then all exert an important influence upon the relative social price of food and industrial output.

(iii) When urban wages are indexed, the incentive towards inflationary policies typically disappears. Direct transfers from the government to consumers, even to the rural poor, may become suspect insofar as they adversely affect aggregate output by driving up prices and wages.

(iv) As a corollary, progressive vertical transfers may cease to be desirable, since they are typically inflationary. Quantitatively, however, a mild degree of inequality aversion is generally sufficient to overcome this adverse feature of progressive transfers; the only notable exception to this rule emerges under the "utility wage" hypotheses (H^3 , H^4), which imply an immediate adjustment of urban wages in the event of a transfer to the rural poor.

(v) More importantly, when direct transfers find some compensation in wage adjustments, horizontal transfers may become distinctly non-neutral. Indeed, when either H_U^3 or H_U^4 hold, direct transfers from rural

to urban labourers become socially desirable in a non-negligible range of the parameter space (displaying relatively high dualism, and low inequality aversion). Thus, when urban workers are likely to react to direct transfers by moderating their wage claims, an urban bias may be justified in the transfer system of dualistic economies. The bias arises both on account of the contrasting nature of the urban and rural wage bargaining processes, as well as from the higher marginal productivity of labour in the industrial sector.

In general equilibrium models, the evaluation of specific public policies always bear the stamp of policy failures in other areas. Accordingly, the above observations are of potential importance for an informed discussion of transfer policies in a dual economy where wages cannot be easily taxed or subsidized. This issue is pursued in the next section.

Section III.3 : Income distribution policies : anatomy and evaluation

The policy reforms that have been occasionally considered in the previous sections are of an "unconstrained" type: they consist of enquiring whether such policy decisions as subsidizing the urban wage, or supporting the income of a particular socio-economic group, are desirable, irrespective of their actual feasibility. In particular, with this convention, we argued that in a dual economy with excessive income disparities, progressive transfers are consistently desirable, while the evaluation of horizontal transfers often requires great care in the specification of the model.

This type of analysis, however, offers a rather incomplete clue for an understanding of the actual policies adopted in real-world dual economies such as India. It fails to explain, for instance, why in that country the urban rich are allowed to share very substantially in the benefits of an implicit transfer scheme such as the Public Distribution System, or why high agricultural incomes are consistently left out of the income-tax net, or again why social security schemes directed selectively to the poorest sections of the community has so far proved rather erratic.. The contrast partly arises because transfer policies do not take place in an institutional vacuum. A proper understanding of the behaviour of the planner, then, and a useful discussion of the reforms available, must take a careful account of:

- (i) the functioning of the economy, and in particular the

distortions or opportunities motivating the intervention of the planner

- (ii) the general constraints limiting the set of actions available to the planner (e.g. the flexibility of indirect taxation, the flexibility of wage subsidies, ..) and determining the expected achievements of the transfer system
- (iii) the constraints bearing upon the transfer system itself.

So far, the first point has been recognized in our endeavour to construct a plausible model of the dual economy. The second is illustrated by our assumption of exogenous indirect taxation. We shall now pay closer attention to the third.

§ III.3.1 : General features of the Indian transfer system

As is now well recognized, the possibility of carrying out lump-sum transfer payments is severely limited in real economies. In many so-called "developed" economies, however, reasonable substitutes for lump-sum transfers exist and provide powerful tools of income distribution policy, e.g. sophisticated income tax schedules, unemployment allowances, discriminatory income support schemes, public goods with exclusion, etc. In India, by contrast, even these traditional tools of income distribution policy are practically non-existent: income taxation only affects a small fraction of the (urban) population, unemployment benefits not only do not exist, the provision of public goods is often indiscriminate, land reform and minimum wage legislation are largely unenforced, etc. Besides a possible lack of political will, objective difficulties seem to account for this immature aspect of the institutional structure in an otherwise largely planned economy. Particularly acute is the problem of identifying and reaching the proposed beneficiaries of any prospective income support scheme. The most dramatic consequence of the failure of the fiscal machinery to soften the wild income disparities generated by an essentially capitalistic economy is the permanent vulnerability of a large section of the population to extreme and persistent destitution following events such as a poor harvest, a natural disaster leading to a short-term disruption of the rural economy, a temporary slack in the labour market, sickness, old age, physical or mental handicap, gambling misfortune, marriage expenses, etc. Moreover, the damage caused to social welfare by the existence of extreme destitution should

be obvious to anyone who has witnessed the unbeatable efficiency of low-income Indians as producers of utility: health, two fair meals a day, a few buckets of water and a single pair of clothes are a very short step indeed from misery to contentment.^{20/}

Yet the resources for a viable social insurance scheme against low-end poverty are not necessarily absent. Casual evidence in support of such a contention is provided by the events following the exceptional drought of 1979, which precipitated a fall in agricultural output of more than 15%;^{21/} yet the calamity went almost unnoticed to international philanthropists, thanks perhaps to an energetic step-up of the already massive Food-for-Work programme, officially providing in that year as many as 1 billion man-days^{22/} of subsidized employment on public worksites, exclusively to the rural poor. For further supportive indication of the potential resources available for improved distribution policies (without even going into a debate about the opportunity of military spending), it suffices to contemplate the massive amount of public funds poured into such a regressive scheme as the Public Distribution System (further discussed below).

The last two illustrations bring us naturally to the next step of our argument. If few policy tools are designed to pursue distributional objectives as such, many others involve a substantial distributional element, and in some respects function very much like lump-sum transfers. In India, the most widespread of this kind of policy tools is perhaps preferential trading between the government and a particular class of consumers or producers. Preferential trading typically takes place on what are now widely

known in India as dual markets. Under this arrangement, a specific commodity is exchanged both on a "free" market at a competitive price, and on a regulated market at an administered price. The regulated market is organized by the government and exclusively accessible to a target group. Moreover, the maximum quantity exchanged by any eligible customer on the regulated market is usually limited, if only to avoid unrestrained speculative trading.

It is easy to demonstrate that such an arrangement is normally equivalent to an implicit lump-sum transfer benefiting the privileged customer, and amounting to the difference between the respective values at market price and at preferential price of the "ration" allowed on the regulated market, as soon as the customer participates in both markets. This will be the case either if he (or she) wishes to consume more than the ration, or if he (or she) is allowed to resell on the free market any excess of the ration over his (or her) preferred consumption level.

To see this formally, let q , q^i , Q^i stand respectively for the vectors of market prices, preferential prices, and rations faced by a consumer i , and r^i his (her) lump-sum income exclusive of implicit transfers via preferential trading. The equivalence of the two following budgets sets is immediate (see also Figure 1):

$$(111) \quad B^i = \left\{ x^i : \bar{q}^i Q^i + q(x^i - Q^i) = r^i \right\}$$

$$(112) \quad B_*^i = \left\{ x^i : qx^i = r_*^i \right\}$$

where $r_*^i = r^i + (q - \bar{q}^i)Q^i$ is i 's "full" lump-sum income, i.e. inclusive of implicit transfers.

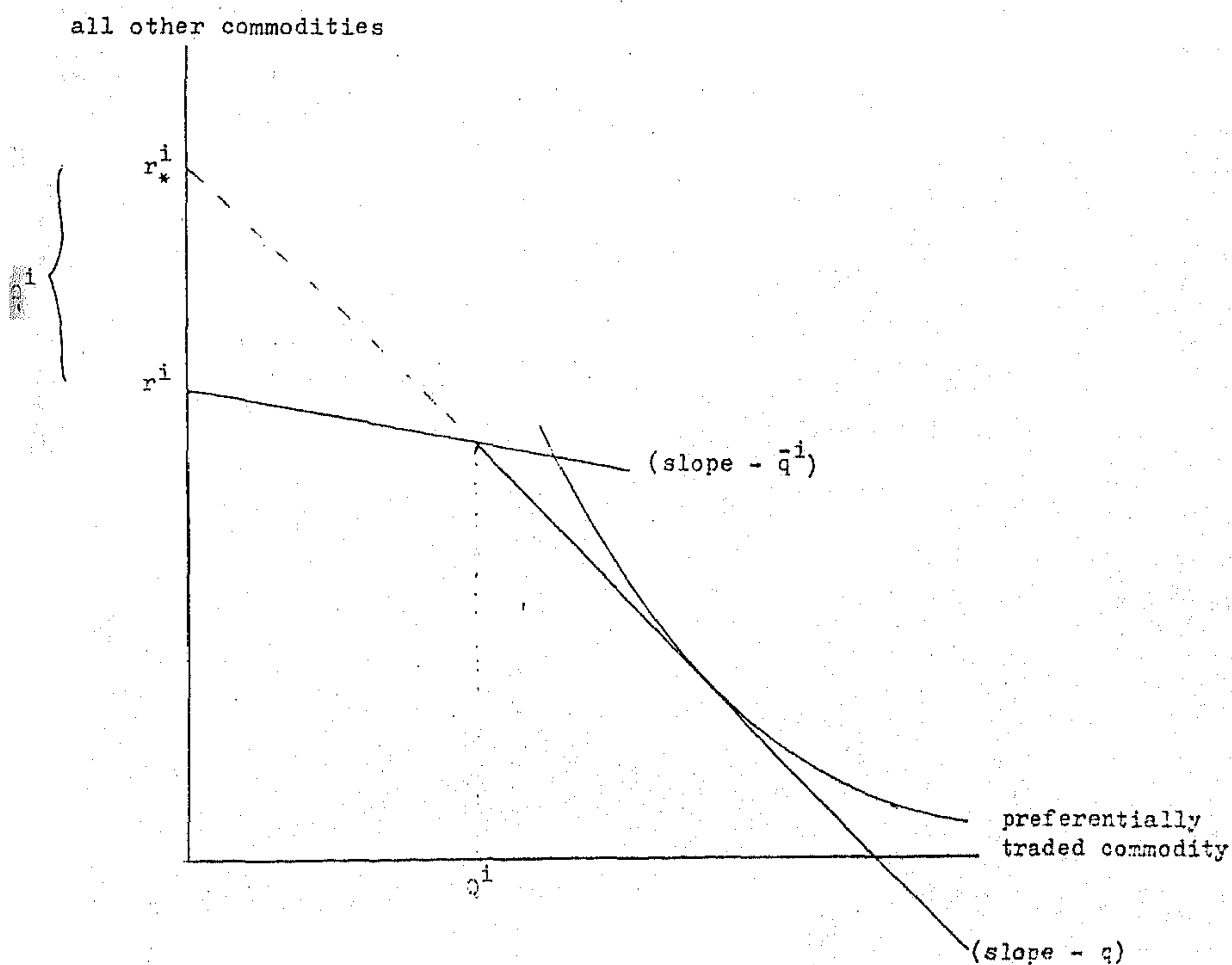


FIGURE 1

It should make no difference, therefore, whether the government engages in preferential trading with this consumer, or sells the same quantity on the free market while compensating him (or her) with a suitable direct transfer. Accordingly, when direct transfers are not feasible, the parameter $\bar{q}^i$ should be set, as far as possible, so as to bring i 's full income to an optimal level.

Of course, this result often needs qualification, e.g. when quality differences are significant or the unregulated market is not competitive.^{24/} But its intuitive appeal is undisputable. Simple as it is, however, the rough equivalence of many dual market arrangements with implicit lump-sum transfers seems to be still^{25/} insufficiently recognized by Indian policy-makers; by and large, such schemes are popularly thought to influence the allocation of resources directly (in the same fashion as indirect taxes and subsidies, or quantity constraints), whereas in fact they may only do so qua implicit transfers.

In India, a wide range of commodities are at times exchanged on dual markets, e.g. sugar, cereals, kerosene, train fares, entrance to universities, foreign exchange, cement, credit, fertilizers, medical treatment, paper products, and many others. More important quantitatively, and especially interesting for our purposes are:

- (i) The procurement system for cereals, affecting many surplus farmers.

(ii) The Public Distribution System (PDS), mostly benefiting urban consumers.

(iii) The rural public works programme, directed at rural labourers.

Such, then, are the typical institutional vehicles of income distribution policies in India. Of course, each of these schemes pursues multiple distributional as well as non-distributional objectives : some form of cereal procurement is essential to build up the buffer stocks necessary to support an effective food security system; the management of the PDS is a powerful weapon against speculative trading of essential commodities; and rural public works aim at least partly at consolidating rural capital accumulation. However, most of their non-distributional objectives should be achievable through non-preferential public participation in the market; the choice of the preferential price, therefore (in these cases, the procurement price of cereals, issue prices in "Fair Price Shops", and the wage on public works), essentially involves distributional considerations. As an illustrative discussion of feasible transfer policies in a dual economy, we shall consider in greater detail the most puzzling of the schemes mentioned above, namely the Public Distribution System. Without much loss of generality, we shall from now on restrict our interpretation of this term to preferential cereals sales.

§III.3.2. : Case study : the Public Distribution System

The Public Distribution System for cereals in India, consisting of subsidized sales of cereals in urban areas under the dual market arrangement, swallows a huge government subsidy.^{26/} As an implicit transfer, it has an essentially regressive character, since it overwhelmingly benefits the urban middle classes. The latter feature of the scheme may not be exactly intentional, and it is usually recognized only with reluctance; but it seems to be reliably established by casual observation or reasoning, besides^{27/} being acknowledged in some of the more objective official documents. The regressive nature of the scheme makes its rationale rather confusing a priori.

As we have already suggested, what prompts enquiry is not the desire of the government to participate in the cereals market. Such a decision seems justified by the concern to foil speculative activities in foodgrains trade, consolidate the food security system and soften price fluctuations. The achievements of Indian planners in these respects are indeed very impressive. But again none of these legitimate preoccupations explain why government participation on the cereals market should take the form of subsidized sales carrying regressive distributional implications.

A simple explanation for the alleged anomaly runs in terms of an arbitrary urban bias permeating most public policies: the decision-makers live in big cities, and are naturally more sensitive to their problems and more diligent at improving

28/

the urban environment. The partly arbitrary character of the obvious urban bias affecting government decision-making in India (e.g. in the provision of most public goods) is difficult to deny, and perhaps no further explanation is needed of the regressive character of the PDS as it currently functions. But such a suspicion should only make it all the more instructive to investigate the plausibility of alternative and perhaps more defensible rationalizations of the scheme. With this aim in mind let us come back to our simple dual economy model.

According to our previous theory, the fundamental features of a dualistic economy generally imply the desirability of subsidizing industrial wages. "Compensated" wage subsidies are attractive as soon as the social cost of labour falls short of its social marginal productivity. Uncompensated wage subsidies may be less uncontroversial, but provided that dualism and/or the wage elasticity of industrial employment are large enough, they are still likely to deserve approval.

Unfortunately, wage subsidies often lie, for institutional and administrative reasons, outside the range of feasible policies available to the planner. Quite interestingly, however, when urban wages follow a "perfect" indexation formula which preserves the (absolute or relative) utility of industrial workers, direct transfers to the urban working class become a perfect substitute for a wage subsidy, simply because they induce exactly compensating wage adjustments. Although this intuitively plausible assertion is amenable to formal proof in a very general

context, it is sufficient here to establish it for our particular model. Its validity is implied by the following proposition.

Propn.5. Let $\frac{\partial \xi^i}{\partial t_m} = 0$ ($i = R, U$)

Then

$$(113) \quad MSV_{t_m} = L^U \Theta^U$$

$$\begin{aligned} \text{Proof:} \quad MSV_{t_m} &\equiv \frac{\partial \mathcal{L}}{\partial t_m} \equiv - \Theta^U x_m^U - \sigma^U \frac{\partial \xi^U}{\partial t_m} - \sigma^R \frac{\partial \xi^R}{\partial t_m} \\ &= L^U \Theta^U \quad \text{if } \frac{\partial \xi^i}{\partial t_m} = 0 \quad (i = R, U) \end{aligned}$$

#

The hypotheses of the theorem are satisfied in particular if either of H_U^3 or H_U^4 hold.

The desire to hold down wages then, may in some circumstances justify the existence of an urban bias in the transfer system. This is confirmed by the empirical observations of §III.2.4.

Still, the PDS does not, of course, allow selective transfers to urban workers. For it functions only subject to well-defined constraints, one of which is the difficulty of discriminating, within the urban sector, between the poor and the rich, or according to any other categories. Hence the scheme is generally anonymous: subject to a mild eligibility condition (such as the possession of a fixed address), any urban household is

entitled to participate in the scheme, under uniform terms. Thus, as a first approximation intended at capturing the salient features of the PDS, a small revision of the issue price can be regarded as a pair (dr^U, dr^C) satisfying

$$(114) \quad dr^C = \frac{L^C}{L^U} dr^U$$

The evaluation of such a policy move, implicitly involving an inflation of the public budget, would however bear the stamp of a general bias in favour of or against inflationary policies. As a more robust and instructive exercise, we shall conclude the discussion by envisaging a marginal diversion of public funds from the PDS towards a scheme benefiting the rural poor, e.g. a public works programme. Formally, we consider a triple (dr^R, dr^U, dr^C) satisfying

$$(115) \quad dr^R + dr^U + dr^C = 0$$

$$(116) \quad dr^C = \frac{L^C}{L^U} dr^U$$

$$(117) \quad dr^U = -L^U$$

The last equality is simply a normalization rule. The marginal social value of this policy move, say Π^* is easily found to be

$$(118) \quad \Pi^* = (L^U + L^C)e^R - L^U e^U - L^C e^C$$

Since the contemplated transfer benefits the rural poor at the expense of the urban population as a whole, it is progressive, and therefore likely to be desirable, as we have seen, when

money wages are fixed or price-indexed. When one of our utility wage hypotheses is valid, however, a dilemma of the efficiency-equity type may appear: the transfer will improve distribution, but adversely affect aggregate output by precipitating urban wage increases. The evaluation may then hinge crucially upon inequality aversion, and to some extent also upon other parameters such as the wage elasticity of industrial employment.

This reasoning is amply confirmed by the numerical calculations of §III.2.4. Figure 2, drawn in (D, μ^C) - space, shows the frontier of "critical values" corresponding to the equation

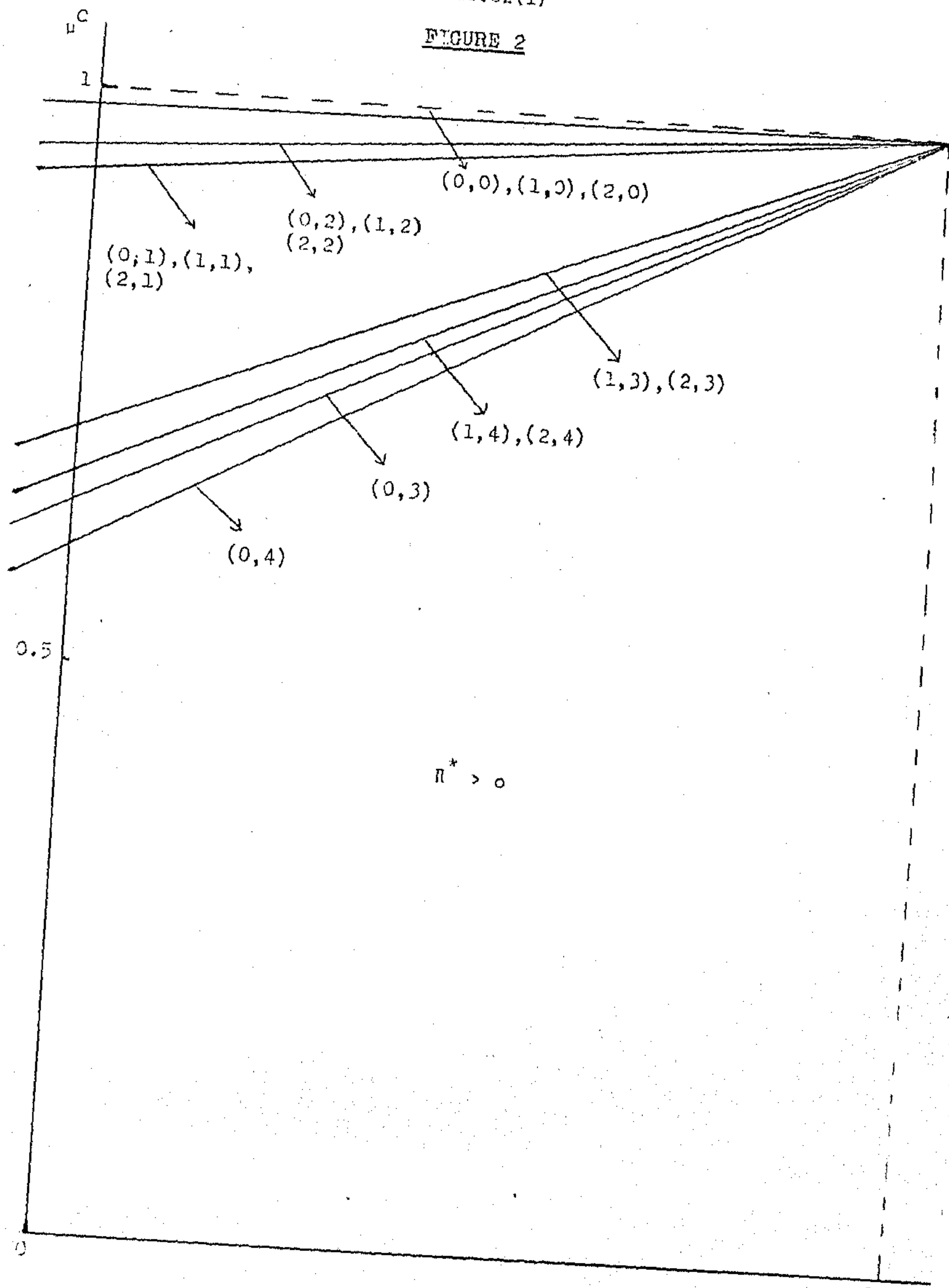
$$(119) \quad \Pi^* = 0$$

for a sample of wage hypotheses. Of course, all of them go through the point (1,1), as required by theorem 1. Elsewhere, however, the frontiers corresponding to the utility wage hypotheses in the urban sector stand markedly apart from other frontiers, and much further inside the parameter space. This confirms our earlier contention: when direct transfers are not compensated by wage adjustments, progressive transfers are unequivocally desirable as soon as a mild degree of inequality aversion exists; when urban wages are perfectly indexed, ambiguity may creep in: the PDS now appears superior to, say, rural public works when dualism is strong while inequality aversion is relatively mild.

The source of the contrast is also perfectly apparent

III.62(1)

FIGURE 2



Cont'd.....

Explanatory notes:

- (1) Each frontier corresponds to a pair or set of pairs of wage hypotheses; the first (second) element of each pair of indices refers to the code of a rural (urban) wage hypothesis (see p.III.23).
- (2) The drawing is somewhat approximate; the true frontiers are not perfectly linear, and never exactly coincide.

from Table 5, which displays, for illustrative purposes, the total effect of the contemplated policy on each endogenous variable, for the preferred values of the parameters, and under two contrasting pairs of wage assumptions (the reference "fixed-wage" assumption and the migration equilibrium hypothesis with fixed rural wages).

With fixed money wages, the impact of the reform is solely distributional. Production is hardly affected; rural classes benefit at the expense of urban classes (farmers gain from large cereal price increases); moreover, the working class as a whole gains and so with inequality aversion the policy deserves approval.

Under the migration equilibrium hypothesis, by contrast, there is a sharp drop of industrial production; urban workers now gain to the same extent as their rural counterparts, while farmers are hit by the adverse movement in their terms of trade on account of the drop in industrial production. If welfare weights are uniform, the suggested reform should be rejected.

In spite of its apparent arbitrariness, then, the Public Distribution System turns out to be consistent with a simple blend of:

- (i) a simple version of a very popular macroeconomic model for development policy (the dual economy model with migration equilibrium)
- (ii) an observed and sometimes explicit value-judgement of Indian planners (unit welfare weights, and the objective of

29/
maximum deflated GNP).

Although the PDS can be constructively criticized, then,
its rationale cannot be summarily dismissed.

TABLE 5 : Impact and evaluation of a transfer to the rural poor
financed by a cut in the PDS

a) Total (general equilibrium) impact of the policy on each
endogenous variable (with $f' = 0.4$, i.e. $D = 0.6$).

	Fixed money wages (H_U^0, H_R^0)	Migration equilibrium (H_U^4, H_R^0)
dp_1	0.147	0.140
dp_2	-0.016	0.283
$d\bar{x}_m^R$	0.001	0.099
$d\bar{y}_l^R$	-0.001	-0.099
du^R	0.166	0.117
du^U	-0.093	0.027
du^C	-0.084	-0.136
du^F	0.012	-0.037
dp_m	0	1.479
dp_l	0	0
dW ($\mu^C = 1$)	-0.001	-0.029
dW ($\mu^C = 0$)	0.073	0.144

cont'd...

b) Marginal social value under alternative wage assumptions

D=0

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	-0.00(0.07)	-0.00(0.10)	-0.00(0.09)	-0.00(0.10)
H_U^1	-0.01(0.08)	-0.01(0.10)	-0.01(0.09)	-0.00(0.09)
H_U^2	-0.00(0.08)	-0.01(0.10)	-0.01(0.09)	-0.01(0.09)
H_U^3	-0.06(0.10)	-0.06(0.13)	-0.06(0.13)	-0.05(0.05)
H_U^4	-0.07(0.10)	-0.07(0.14)	-0.07(0.14)	-0.06(0.05)

D=1

	H_R^0	H_R^1	H_R^2	H_R^4 b)
H_U^0	0 (0.07)	0 (0.10)	0 (0.09)	0 (0.10)
H_U^1	0 (0.00)	0 (0.11)	0 (0.10)	0 (0.11)
H_U^2	0 (0.00)	0 (0.11)	0 (0.10)	0 (0.10)
H_U^3	0 (0.10)	0 (0.17)	0 (0.18)	0 (0.17)
H_U^4	0 (0.18)	0 (0.20)	0 (0.21)	0 (0.20)

cont'd...

Explanatory notes

(1) The policy move analysed here is :

$$\begin{cases} dr^U = -L^U = -0.108 \\ dr^C = -L^C = -0.122 \\ dr^R = -dr^U - dr^C = 0.23 \end{cases}$$

Its effect on any endogenous variable, say α_k , is thus:

$$d\alpha_k^* = (0.23) \frac{d\alpha_k^*}{dr^R} - (0.108) \frac{d\alpha_k^*}{dr^U} - (0.122) \frac{d\alpha_k^*}{dr^C}$$

The values of $\frac{d\alpha_k^*}{dr^R}$, $\frac{d\alpha_k^*}{dr^U}$, $\frac{d\alpha_k^*}{dr^C}$ can be read immediately

from the matrix G_α^{-1} (see Table 2 above). The change in social welfare is similarly given by:

$$dW = (0.23) \frac{dW}{dr^R} - (0.108) \frac{dW}{dr^U} - (0.122) \frac{dW}{dr^C} \equiv (0.23)\theta^R - (0.108)\theta^U - (0.122)\theta^C$$

(2) The format of part b) of this table is as in Table 4.

Footnotes : Chapter III

- 1/ More recent work, concentrating more carefully on inter-temporal questions, has brought out strongly the importance of a correct modelling of savings and investment behaviour, as well as the poverty of their treatment in the present model. In the simple models of the dual economy literature, commodity prices are implicitly taken as fixed (this may be no more than a choice of numeraire), while macroeconomic equilibrium is ensured by assuming that savings are identically invested. In our model, by contrast, investment is implicitly exogenous, while commodity markets clear by price adjustment (the price level is endogenous because the model is not homogenous in prices, e.g. lump-sum transfers are fixed in money terms). While they can lead to very different welfare prescriptions, neither of these two approaches seem completely satisfactory, and they are perhaps best considered as two useful polar cases. A more synthetic model is in progress, under the pleasurable guidance of Professor M. Marchand.
- 2/ A minor discrepancy occurs due to a rather elementary treatment of savings throughout this chapter. A more elaborate and explicitly intertemporal model is in progress (see previous footnote).
- 3/ For some background material on dual economies, see, e.g. Dixit (2, 3), or Lluch (12).
- 4/ The analytical difficulty is connected with a proper identification of the socio-economic characteristics of a migrant (urban or rural?); the quantitative insignificance of the problem is due to the fair homogeneity of these characteristics in the numerical simulations. Admittedly, however, matters might be different if the model allowed for urban unemployment and probabilistic migration equilibria, as in the well-known Harris-Todaro (9) model. See also Stiglitz (14).
- 5/ This is consistent with utility-maximizing behaviour; it may imply non-differentiabilities in the utility functions, but not necessarily in the expenditure and compensated demand functions, which are used here.
- 6/ Except insofar as either
 - (i) entrepreneurial classes provide labour services directly to the government; in this case their quantity and reward has to be regarded as exogenous to the model.
 - (ii) $a^{oh} \neq 0$ ($h = C, F$); this qualification is neglected, as a first approximation.
- 7/ This cardinalization is permissible if V^i is strictly increasing in e^i for each i . But of course, it has to be carefully remembered when thinking about the form of $\psi(\cdot)$.

- 8/ This may seem to contrast with property (R2) of Chapter II. The point is that ϕ_l^1 may not be defined here, as the assumption of fixed labour supply implies a non-differentiability in the utility function.
- 9/ The independent treatment of producers' and consumers' behaviour, in spite of the possible existence of indirect taxes, is plausible for the industrial sector where dividend payments presumably accrue in cash rather than in kind; it is also permissible for rural labourers, since their behaviour as producers is confined to absorbing surplus labour. Finally, it should be acceptable for capitalist farmers unless a significant indirect tax on food exists, as is rarely the case in the kind of economies we have in mind.
- 10/ Notice that because the model is not homogeneous, it will solve for money prices rather than relative prices.
- 11/ Private savings are available to the government for spending. An explicit treatment of money and credit is beyond the scope of this paper, but presently in progress in a refined version of the model.
- 12/ Formally, it is possible to consider subsistence farms as simple self-managed enterprises, which maximize value-added per head rather than profits (see Ward (15)).
- 13/ Distributional indifference is easily shown to reconcile the planner with the objective of GNP maximization. Indeed, if

$$e \equiv \sum_i e^i \quad e^* \equiv \frac{e}{Q^*(q)}, \quad Q^*(q) \equiv \frac{\tilde{q}x^0}{e^0} \text{ (a Consumer Price Index),}$$

then

$$de^* = \frac{de}{Q^*} - \frac{e}{(Q^*)^2} \frac{\partial Q^*}{\partial q} \quad d\tilde{q} > 0 \Leftrightarrow Q^* de > e \frac{\partial Q^*}{\partial \tilde{q}} d\tilde{q} \Leftrightarrow de > \tilde{x} d\tilde{q}$$

Hence, using (55) with $\mu^i = \mu \forall i$:

$$dW > 0 \Leftrightarrow de > \tilde{x} d\tilde{q} \quad \text{or} \quad de^* > 0.$$

- 14/ Again, minor changes in the formulae occur due to the allowance for savings.
- 15/ See previous footnote.

- 16/ The results presented below are of an "intermediate" nature; they appeal to conditions which are not merely immediately verifiable statements about the "primary" parameters of the model, but also sometimes "simple" properties of the shadow price vector.
- 17/ If $d_1 = d_2 = 0$, one easily obtains $D = 1$, $\theta^R = 0$ and $v_j/v_k = p_j/p_k \quad \forall j, k \in J$. This situation is "unlikely" unless the conditions of theorem 1 are satisfied; in any case it does not refute the conjectures since these exploit the notion of dualism, which is absent in this case.
- 18/ Or, equivalently here, by an adequate tax on industrial profit.
- 19/ Dalton, for instance, long ago put forward the value of 1 for the elasticity of the marginal utility of expenditure, arguing for its "simplicity and plausibility" (see H. Dalton, Public Finance, 1939, pp. 98-99).
- 20/ One could argue, of course, that the origins of such contentment precisely lies in "escaping" from misery, so that the poverty of a few contribute to the general welfare. Believing in altruism, I would think that externalities in utility actually play in the reverse direction; in our formal model, such externalities are ruled out, and it seems futile to consider the complications they introduce.
- 21/ According to the Government of India (5) Economic Survey 1981-82, the output of foodgrains fell from 132 million tonnes in 1978-79 to 110 million tonnes in 1979-80; closing stocks amounted to 16 m.t. in April 1979-80, and 10 m.t. in April 1981-82. Imports were negligible throughout. The net availability of cereals and pulses (after allowing for net production, net imports and changes in stocks) fell from 480.3 grams per head per day in 1979 to 416.9 grams per head per day in 1980! (See Government of India (5), pp. 5, 34, and 84-85).
- 22/ See India 1980, Government of India (8), p. 255.
- 23/ This terminology obviously bears no relation with the phenomenon of "dualism".
- 24/ Notice, however, that even if, on account of these qualifications, the quantity parameter $\bar{q}^1$ may no longer be manipulated in the fashion of a lump-sum transfers, the price parameter q^1 may retain this property.
- 25/ Such is the impression one may be tempted to retain, for instance, from a glance at the very interesting Dagli (1) Committee Report on Controls and Subsidies.

- 26/ The annual quantity of foodgrains released through the PDS during the years 1977-1981 varied between 10 and 15 million tonnes (total production in 1979-79 was of the order of 132 m.t.). In 1978-79, the total "food subsidy" (consisting of all subsidies on foodgrains transactions to the Food Corporation of India) amounted to 5.7 billion rupees; the total expenditure of the Central Government was of the order of 180 billion rupees (see Government of India (5), p.85, p.5. and p.103, and Dagli (1), Annex No.11.1, p.90). For a detailed account of the PDS, see Dagli (1).
- 27/ e.g. the Dagli (1) Committee Report: "It is claimed that the public distribution system today covers nearly 95 per cent of the population of the country through fair price shops. This is highly misleading. (...) The offtake of foodgrains in urban areas was 6.5 million tonnes whereas in rural areas it was less than half, namely 3.1 million tonnes in 1971, for a total of 9.6 million tonnes distributed through fair-price shops for which break-up of urban and rural consumption is available". (Dagli (1), p.132).
- 28/ For an enthusiastic defence of the urban bias thesis, see Lipton (11).
- 29/ See footnote 13/.

Appendix A.3.1. Some formal proofs

In the following we shall assume that $\min(v_1, v_2) > 0$ and $\mu^R > 0$.

$$(R1) \quad \frac{v_m}{v_1 p_m} = D - \frac{\theta^R}{v_1} (1-D) < 1 \Leftrightarrow \frac{\theta^R}{v_1} > -1 \text{ and } D < 1.$$

Proof: (R1) follows immediately from (88), the definition of D and the equality $g' = p_m$. #

$$(R2) \quad \frac{v_m}{v_2 p_m} = \frac{v_1}{v_2} D - \frac{\theta^R}{v_2} (1-D) < 1 \Leftrightarrow \frac{\theta^R}{v_2} > \frac{\frac{v_1}{v_2} D - 1}{1-D} \text{ or } D = 1 \text{ and } v_1 < v_2$$

Proof: As in (R1). #

$$(R3) \quad \left. \begin{array}{l} v_1 > v_2 \Rightarrow \theta^R > -v_1 \\ v_2 > v_1 \Rightarrow \theta^R > -v_2 \end{array} \right\} \quad \text{In particular } \theta^R > \min(-v_1, -v_2)$$

Proof: Let β_j^i denote i 's marginal propensity to spend on good j (out of expenditure).

If $v_1 > v_2$, then

$$\theta^R = (1-\gamma^R)(\mu^R - \beta_1^R v_1 - \beta_2^R v_2)$$

$$\geq (1-\gamma^R)(\mu^R - v_1) \quad \text{since } \beta_1^R + \beta_2^R = 1 \text{ and } v_1 > v_2$$

$$\geq -v_1 \quad \text{since } \mu^R > 0 \text{ and } \gamma^R \in [0, 1]$$

A symmetric argument applies when $v_2 \geq v_1$ and establishes the result. #

$$(R4) \quad \text{Let } \begin{cases} \sigma_1 \equiv -d_1 x_1 + \frac{\partial y_1}{\partial p_1} (v_1 - \frac{v_\ell}{p_\ell}) \\ \sigma_2 \equiv -d_2 x_2 + \frac{\partial y_2}{\partial p_2} (v_2 - \frac{v_m}{p_m}) \end{cases}$$

If A*4 holds, then

- (i) $\max. (d_1, d_2) > 0$ unless $v_1 = v_2$. If $v_1 = v_2$, $d_1 = 0 \leq d_2$.
- (ii) $\sigma_1 + \sigma_2 = 0$; in particular, σ_1 and σ_2 have opposite signs.
- (iii) $v_1 > v_2 \Leftrightarrow \sigma_1 < 0 \Leftrightarrow \frac{\sigma_1}{x_1} < \frac{\sigma_2}{x_2}$

Proof:

(a) (84) implies, using (18) and (31):

$$\begin{bmatrix} v_1 & v_2 \end{bmatrix} x = \begin{bmatrix} \sigma_1 & \sigma_2 \end{bmatrix}$$

$$\text{where } x_{jk} = \frac{\hat{x}_j}{q_k} \quad (j, k = 1, 2).$$

$$\text{Since } x \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0 \text{ and } q_1 = q_2 = 1, \sigma_1 + \sigma_2 = 0,$$

establishing (ii).

(b) Further, since $\begin{bmatrix} q_1 & q_2 \end{bmatrix} x = 0$ and $x_{11} < 0$:

$$\sigma_1 = v_1 x_{11} + v_2 x_{21} = (v_1 - v_2) x_{11}$$

implying

$$v_1 > v_2 \Leftrightarrow \sigma_1 < 0 \Leftrightarrow \frac{\sigma_1}{x_1} < \frac{\sigma_2}{x_2}$$

since σ_1 and σ_2 have opposite signs; this proves (iii).

(c) If $v_1 < v_2$, then $\theta^R > -v_2$ from (R3), and so

$$\theta^R > -v_2 \geq \frac{v_1 D - v_2}{1-D} \quad (\text{since } v_1 < v_2)$$

implying $v_m < v_2 p_m$ from (R2). Moreover

$$\sigma_1 = (v_1 - v_2)x_{11} > 0. \quad \text{Then}$$

$$0 < \sigma_1 = -\sigma_2 = d_2 x_2 - \frac{\partial y_2}{\partial p_2} \left(v_2 - \frac{v_m}{p_m} \right) < d_2 x_2 \Rightarrow d_2 > 0.$$

(d) On the other hand, if $v_1 > v_2$, then, from (iii), A*4 and the definition of σ_1 : $d_1 > 0$.

So we may conclude : $v_1 \neq v_2 \Rightarrow \max. (d_1, d_2) > 0$.

(e) Now let $v_1 = v_2$. Then $\sigma_1 = (v_1 - v_2)x_{11} = 0$, and so $\sigma_2 = 0$, $d_1 = 0$ (using A*4). Reasoning as in (c) above, we find $v_m \leq v_2 p_m$, and hence:

$$0 = \sigma_2 \geq -d_2 x_2 \Rightarrow d_2 \geq 0$$

From (d) and (e), we obtain (i)

#

(R6) Under fairly general conditions (which may be expected to be sensibly weaker when dualism is strong), we may assert the existence of a weighted average of the θ^i 's which is positive:

$$\exists \left\{ w^i \right\}_{i \in I} : \sum_{i \in I} w^i \theta^i > 0, \quad \sum_{i \in I} w^i = 1$$

Moreover, each weight is positive when the marginal propensity to save in the capitalist classes and/or profit taxes are high enough.

In such case:

$$\max_i \theta^i > 0.$$

Proof:

$$\text{Let } z_j^0 \equiv - \sum_{i \in I} z_j^i \quad (j = 1, 2).$$

(a) As soon as marginal savings rates exceed average ones:

$$(i) \quad z_1^0 + z_2^0 = - \sum_{i \in I} (z_1^i + z_2^i)$$

$$(ii) \quad = \sum_i \frac{e^i}{1-\gamma^i} - \sum_i (1-a^{oi}) y_*^i \quad \text{where } y_*^i \equiv y_1^i + y_2^i$$

$$\geq \sum_i M^i - \sum_i (1-a^{oi}) y_*^i$$

$$= \left[\sum_i (y_*^i - a^{oi} r^i + r^i) - p_\ell z_\ell - p_m z_m \right] - \sum_i (1-a^{oi}) y_*^i$$

$$= \sum_i (y_*^i - r^i) a^{oi} - p_\ell z_\ell - p_m z_m + \sum_i r^i$$

$$= \sum_i a^{oi} (-p_\ell y_\ell^i - p_m y_m^i) - p_\ell z_\ell - p_m z_m + \sum_i r^i$$

$$(iii) \quad > 0 \quad \text{unless } \sum_i r^i \text{ is "large", since } y_\ell^i, y_m^i, z_m, z_\ell \leq 0.$$

Several sets of sufficient conditions exist to guarantee that the sum $z_1^0 + z_2^0$ is positive; e.g. using (ii), $z_1^0 + z_2^0 > 0$ as soon as

$$(iv) \quad e^i > (1-a^{oi})(1-\gamma^i) y_*^i \quad \forall i$$

(iv) always holds for the working classes. It will also hold for the capitalist classes if their marginal savings rates (or, alternatively, profit taxes), are high enough. A comparative static argument, building on the assumption that higher public spending should result in higher money prices, would possibly lead to the same conclusion. We shall assume, then, that $z_1^0 + z_2^0 > 0$.

(b) Let $v_1 \neq v_2$. Since $\max(d_1, d_2) > 0$, there exists a non-empty interval in $[0, 1]$, say T_1 such that

$$(v) \quad T \in T_1 \Rightarrow T d_1 x_1 + (1-T) d_2 x_2 > 0$$

Similarly, since $z_1^0 + z_2^0 > 0$, there exists a non-empty interval T_2 in $[0,1]$ such that

$$\begin{cases} \tau \in T_2 \Rightarrow \tau z_1^0 + (1-\tau) z_2^0 > 0 \\ \left\{ \frac{1}{2} \right\} \subset T_2 \end{cases}$$

We shall assume that $T_1 \cap T_2 \neq \emptyset$. Several sets of sufficient conditions for this to hold exist, one of which is:

$$v_m < v_2^{p_m}$$

since then $d_1 x_1 + d_2 x_2 = \frac{\partial y}{\partial p_2} (v_2 - \frac{v_m}{p_m}) > 0$, i.e. $\left\{ \frac{1}{2} \right\} \subset T_1 \cap T_2$.

Let then $T^* \in T_1 \cap T_2$. Then

$$\begin{aligned} 0 &< [T^* d_1 x_1 + (1-T^*) d_2 x_2] [T^* z_1^0 + (1-T^*) z_2^0]^{-1} \\ &= [-\sum_i \theta^i (T^* z_1^i + (1-T^*) z_2^i)] [T^* z_1^0 + (1-T^*) z_2^0]^{-1} \\ &= \sum_i \theta^i w^i \end{aligned}$$

where $\{w^i\}$ is a set of weights satisfying $\sum_i w^i = 1$.

(c) A sufficient set of conditions for each weight w^i to be positive and the sum $z_1^0 + z_2^0$ to be positive is

$$(vi) \quad z_j^i < 0 \quad \forall i \in I, \quad j = 1, 2$$

i.e.

$$\begin{aligned} (vii) \quad & \begin{cases} x_1^R > (1-\gamma^R) y_1^R \\ (viii) \quad x_1^F > (1-\gamma^F) (1-a^{oF}) y_1^F \\ (ix) \quad x_2^C > (1-\gamma^C) (1-a^{oC}) y_2^C \end{cases} \end{aligned}$$

(vii) is satisfied if peasants are net buyers of food;

(viii) and (ix) hold provided that the marginal propensity to save of capitalist classes and/or profits taxes are high enough. Much weaker assumptions exist conditionally upon $v_m < w_m^p$. When all the weights are positive we may conclude immediately:

$$\max_i \theta^i > 0.$$

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Appendix A.3.2. The data

In this appendix we sketch the construction of the data shown in Table 1 of this chapter.

The nature of the calculations reported in this chapter is more that of simulation than empirical estimation. Hence, even though we have taken the data set of Annex 1 relating to the Indian economy in 1979 as a basis for the numbers, the figures of Table 1 aim more at consistency than realism.

For convenience, the exposition will simply follow through the entries of Table 1. It is sufficient to concentrate on these parameters because the data needed to carry out all the calculations reported in this chapter can be derived from Table 1 and the accounting identities of the model. The data of Table 1 assumes $z_j = 0 \forall j$; this involves no loss of generality since the public sector can always be formally lumped into the "private" sector by a suitable specification of production sets and profit shares.

(I) All the "consumer parameters" appearing in Table 1(a) (namely L^i , e^i , α^i , β^i , γ^i , APS^i) have been obtained directly from the "core data set" and the aggregation formulae in Annex 1. The aggregation keys are as follows.

The 89 commodities of the core data set (which, in this respect, follows the classification scheme of the Technical Note to the VIth five-year plan of India) have been grouped into two broad categories. The first category consists of commodities 1-15 of the core classification (namely "paddy", "wheat", "jowar",

"bajra", "other cereals", "pulses", "sugarcane", "jute", "cotton", "plantation", "other crops", "milk and milk products", "other animal husbandry", "forestry" and "fishing"). This category corresponds precisely to the sum of the broad commodity groups "agriculture", "forestry and logging", and "fishing" appearing in the 14 x 14 aggregated input-output table of the Technical Note; as in the theoretical model, then, we did not distinguish between "rural" and "agricultural". The second category comprises all other commodities (see Table B.1 in Annex 1).

Our point of departure for dividing the population of each sector into the two required groups was the estimate of the "poverty line" given in the VIth five-year plan (this seems permissible, since our model implicitly allows the "capitalist" classes to supply labour services as well; see the remarks on p.III.10). It turned out to be computationally convenient, however, to include in each population group a finite number of the 28 basic income classes included in the core data set. Allowing for this, we were led to include 66% of the rural population in class R, and 47% of the population in class U, giving the figures of the first column of Table 1(a) (the rural population in our core data set consists of 77% of the total population, which is itself normalized to unity).

(II) The figures appearing in Table 1(b) were obtained as follows:

(i) The entries of the first two columns are obtained by letting $z_j = 0$ ($j=1,2$), and calculating y_j from Table 1(a):

$$(1) \quad y_j = x_j = \varepsilon_j \cdot L^j \cdot \frac{e^j}{L^j} \cdot \alpha_j^j \quad (j = 1, 2)$$

(ii) Similarly, the next two columns follow from letting

$$z_m = z_q = 0, \text{ implying}$$

$$(2) \quad y_m = -L^U, \quad y_q = -L^R$$

(iii) Since the rural sector is notoriously known to escape most of the direct tax net in India, we have naturally chosen

$$a^{oR} = a^{oF} = 0.$$

On the other hand, if we include the wages of class c in industrial "profits" π^C , we may then calculate a^{oC} from:

$$(3) \quad \frac{1-a^{oC}}{a^{oC}} = \frac{\text{share of class C in industrial value-added}}{\text{share of govt. in industrial value-added}}$$

which gives $a^{oC} = 0.2$ using the estimates of the distribution of value-added in the core data set. This estimation procedure is rather rough, but since the parameter a^{oC} is probably not very significant greater caution did not seem crucial here.

(III) Turning to Table 1(c), the core data set, suitably aggregated, gives the following 2 x 2 uncompensated price elasticity matrix for aggregate consumer demands:

$$(4) \quad \varepsilon = \begin{bmatrix} -0.805 & -0.217 \\ -0.309 & -0.657 \end{bmatrix}$$

Since we wish our estimated matrix of compensated price elasticities $\hat{\epsilon}$ to satisfy the a priori restrictions

$$(5) \quad \hat{\epsilon}_{11} + \hat{\epsilon}_{12} = 0, \quad \hat{\epsilon}_{21} + \hat{\epsilon}_{22} = 0, \quad \alpha_1 \hat{\epsilon}_{11} + \alpha_2 \hat{\epsilon}_{21} = 0$$

it is sufficient to know one of its elements in order to compute the others. Furthermore, one of its elements can be obtained from any entry of the ϵ matrix, together with the Slutsky equation and the data of Table 1(a). If the matrix ϵ satisfied the required theoretical properties, it would not matter which of its entries we chose for this purpose. Unfortunately, although ϵ satisfies adding-up, it does not satisfy homogeneity (this is discussed in detail in §A.IV of Annex 1), and therefore different results would be obtained by choosing the required element from the first or the second column of the matrix ϵ . Since elementary sensitivity exercises did not seem to confer a great deal of importance to this choice, we have worked for definiteness with the own-price elasticity of demand for food ϵ_{11} , and therefore computed $\hat{\epsilon}$ from (5) and the formula

$$(6) \quad \hat{\epsilon}_{11} = \epsilon_{11} + \sum_i \frac{x_1^i}{x_1} \beta_1^i$$

The second column of Table 1(c) reflects our assumption of price-inelastic aggregate food supply $A^*.4$.

In the third column, we find the normalization rule $p_1 = p_2 = 1$, and also $p_m = 0.7$. The latter figure is obtained as follows. If class U derives all its income from wages, we

should have

$$(7) \quad p_m = \frac{M^U}{L^U} = \frac{e^U}{(1-APS^U)L^U} = 0.76 \text{ from Table 1(a)}$$

This figure was slightly modified, however, because of our desire to be able to capture the "first-best" solution of Propn.1. More precisely, consider the following inequalities, holding in a first-best environment :

- (i) $p_1 f' = p_2 g' = p_m$ (no dualism)
- (ii) $p_1 f' \leq p_\ell$ (no forced labour)
- (iii) $p_1 f' \leq \frac{p_1 y_1^R}{R}$ (concavity of f)

where ℓ^R denotes self-employment on peasant farms. If $z_\ell = r_\ell^R = 0$, these inequalities together with the identity

$$(9) \quad \frac{M^R}{L^R} = \frac{\ell^R}{L^R} \frac{p_1 y_1^R}{\ell^R} + \left(1 - \frac{\ell^R}{L^R}\right) p_\ell$$

imply

$$(10) \quad p_m \leq \frac{M^R}{L^R} = \frac{e^R}{(1-APS^R)L^R} = 0.7 \text{ from Table 1(a).}$$

We have therefore chosen to let $p_m = 0.7$; to reconcile this with the last equality of (7), we had to implicitly assume a transfer $r^U = 0.06$.

(4) The unobservable parameters of Tables 1(d) have been the focus of our sensitivity exercises, which are briefly mentioned in the text. Their "reference values" have been chosen from simple rules of thumb. The reference value of η_{mm} (the wage elasticity

of industrial employment) is purely arbitrary. So is that of f' (the marginal productivity of labour on peasant farms; remember also that, with $p_m = 0.7$, $D = f'/0.7$), though it has seldom been used.

To think about the value of y_1^R/y_1 , consider the ratio, say x^R , of cash expenditure to total consumption expenditure in a typical rural peasant household. If this household is deficit in food, the ratio x^R should satisfy

$$(11) \quad x^R = \frac{p_2 x_2^R + p_1 (x_1^R - y_1^R)}{e^R} = 1 - \frac{p_1 y_1^R}{e^R}$$

According to the 28th Round of the National Sample Survey, the ratio x^R takes the following average value for different classes of rural households:

Monthly per capita expenditure group(Rs.)	x^R
0-15	0.64
15-28	0.63
28-43	0.61
43-55	0.60
55-75	0.59
75-100	0.61
100 and above	0.72
All households	0.62

Source: R.J.Chelliah and R.M.Lal, Incidence of indirect taxation in India 1973-74, New Delhi, 1977, p.101.

The value of x^R seems remarkably stable, at any rate among the lower income groups. Taking $x^R = 0.62$ for simplicity,

we obtain

$$(12) \quad y_1^R = e^R(1-x^R) = (0.326)(0.38) = 0.124$$

whence

$$\frac{y_1^R}{y_1} = \frac{0.124}{0.612} \approx 0.2 \quad \text{as in Table 1(d).}$$

Finally, to think about the value of p_ℓ , we may rewrite

(9) as

$$(14) \quad p_\ell = \frac{M^R}{L^R} - \left(1 - \frac{p_\ell}{\bar{f}^R}\right) \frac{p_1 y_1^R}{L^R}$$

$$(15) \quad = 0.7 - \left(1 - \frac{p_\ell}{\bar{f}^R}\right) (0.245)$$

where $\bar{f}^R$ is the average product on peasant plots. When the rural wage takes its "reference" value, then, both urban and rural wages are equal to the average product on subsistence farms; this fits with the simple rationalization of dualism alluded to in § III.1.3 (more recent, and more realistic, models of dual economies allow for migration costs, and uncertain prospects of finding urban employment; such refinements can only be regarded here as useful directions of extension for the present model).

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CHAPTER IV : Shadow prices and tax reform
in a two-factor economy

Section IV.1 : Preliminaries : further remarks on shadow prices in a reform context.

In this preliminary section, we mention a few simple features of shadow prices in situations of reform, which will prove useful later in the chapter. The results presented below apply to the planner's model without side constraints ($S^* \equiv R^L$). They easily generalize, but this will not be necessary for the purposes of this chapter.

R.1. Consider the planner's model without side constraints:

$$(i) \quad W = W(s)$$

$$(ii) \quad E(s) - z = 0$$

$$(iii) \quad z \in Z^*$$

A fictitious price vector is simply a vector $\phi \in R^M$ of the form:

$$(iv) \quad \phi = W_s E^- \text{ where } E^- \text{ is an } L \times M \text{ matrix s.t. } E_s^- E^- = I$$

i.e. E^- is a right-inverse of E_s (see Section I.6, lemma 1 and its corollaries).

R.2. Let E^- be a right-inverse of E_s and $\phi \equiv W_s E^-$ the corresponding FP vector. Now each column of E^- can be interpreted as a (potentially feasible) reform:

$$(v) \quad E^- \text{ not. } (\dot{s}^{(1)}, \dots, \dot{s}^{(M)})$$

In a search for a shadow price vector λ , suppose we require shadow prices to be such that each of these reforms breaks even, i.e.

$$(vi) \quad W_s \dot{s}^{(k)} - \lambda E_s \dot{s}^{(k)} = 0, \quad k = 1, \dots, M$$

When (vi) holds:

$$(vii) \quad W_s E^{-} - \lambda E_s E^{-} = 0 \quad \text{or} \quad \lambda = \phi$$

R.3. Conversely, let $A \equiv (\dot{s}^{(1)}, \dots, \dot{s}^{(M)})$ be a set of M potential reforms which are linearly independent in the sense that the matrix $E_s A$ has full rank. If we require a shadow price vector λ to be s.t. all these reforms break even, then:

$$(viii) \quad 0 = W_s A - \lambda E_s A \Rightarrow \lambda = W_s E^{-} \text{ with } E^{-} \equiv A(E_s A)^{-1}$$

R.4. Hence, a FP vector can always be regarded to be defined implicitly by the requirement that M linearly independent reforms should break even.

R.5. By a suitable reparametrization of the space of signals around s^0 necessary, a vector ϕ of fictitious prices can always be written

$$(ix) \quad \phi = W_s E^{-1} = W_s \begin{bmatrix} E^{-1} \\ \alpha \\ 0 \end{bmatrix} \quad \text{not. } W_s r^\alpha$$

for some vector $\alpha = (s_{\alpha_j})_{j=1, \dots, M} \in R^M$ of "endogenous variables".

Proof: Let $\phi = W_s E^{-}$ be a FP vector, where E^{-} is a right-inverse of E_s . Let C be an $L \times L$ matrix of the form $C = [A \ B]$, where A has the same column span as E^{-} , while the columns of B span the linear subspace orthogonal to the column span of E^{-} , so that $B'A = 0$ and C has full rank. Define $\tilde{s} \equiv [\alpha \ \beta]$ implicitly through the identity:

$$s \equiv s^0 + C(\tilde{s} - s^0)$$

and write

$$\tilde{E}(\tilde{s}) \equiv E(s^0 + C(\tilde{s} - s^0))$$

$$\tilde{W}(\tilde{s}) \equiv W(s^0 + C(\tilde{s} - s^0))$$

Then

$$\tilde{E}_\alpha = E_s A$$

$$\tilde{W}_\alpha = W_s A$$

and $\phi = W_s E^{-1} = W_s A (E_s A)^{-1}$ since A and E^{-1} have the same column span

$$= \tilde{W}_\alpha (\tilde{E}_\alpha)^{-1}$$

#

R.6. If $\dot{s}^{(k)}$ is the k^{th} column of E^{-1} interpreted as a potential reform, and $\phi = W_s E^{-1}$, then $E_s \dot{s}^{(k)} = e_k$ (the k^{th} unit vector in R^M) and:

$$(x) \quad W_s \dot{s}^{(k)} = \phi_k$$

i.e. the k^{th} shadow price may simply be interpreted as a trade-off between the k^{th} resource and welfare inherent in the specific reform $\dot{s}^{(k)}$.

R.7. Now let $\bar{s}$ be any reform s.t. $E_{\bar{s}} = e_k$. The marginal social value of $\bar{s}$ is:

$$\Pi^*(\bar{s}) = W_{\bar{s}} - \phi_k \quad \text{not.} \quad \hat{\phi}_k - \phi_k$$

where $\hat{\phi}_k$ is a "shadow price" obtained by looking at the trade-off between the k^{th} resource and welfare implicit in $\bar{s}$. In this chapter we shall call $\hat{\phi}_k$ a "suggestive shadow price".

Section IV.2 : Tax reform with two scarce factors: theoretical analysis

As an illustration of the methodological issues arising for CBA in a context of reform, we shall now consider a simple production and tax reform problem for a mixed economy. The model of this chapter is both a special case of the abstract models of Chapters I and II, and an extension of the Ahmad-Stern (1) model of tax reform for India, to which the reader is referred for greater detail.

§ IV.2.1 : The model

The problem is set in a medium-run perspective, say that of a five-year plan. There are M goods, consisting of $(M-2)$ produced commodities (or "commodities" for short), and two scarce factors (or "resources" for short), which we shall interpret as foreign exchange and domestic value added (or sometimes "labour" for short). Thus $J = \{1, \dots, M-2, f, \ell\}$ not. $\tilde{J} \cup \{f, \ell\}$. Henceforth, the symbol " $\tilde{\cdot}$ " will indicate the truncation of a vector to its first $(M-2)$ components, e.g. $z = (\tilde{z}, z_f, z_\ell)$ is the public production plan. Private producers operate with a fixed-coefficients technology; their gross output vector is denoted by $g = (g_j)_{j \in \tilde{J}}$.

Let A^m be a matrix of technical coefficients for non-competitive imported intermediates, and m^o denote all other (net) imports. The exchange rate is fixed throughout at unity. If p^m is the vector of world prices, the balance of payment constraint reads:

$$(1) \quad p^m (A^m g + m^o) - z_f = 0$$

where z_f denotes the net depletion of foreign exchange reserves.

Similarly, if $-z_l$ denotes value added in the public sector and l a vector of domestic value added coefficients for the private sector, aggregate value added (or "employment"), say l^* , is given by:

$$(2) \quad l^* - lg + z_l = 0$$

If A^D is the matrix of domestic input-output coefficients for the private sector, then the private production plan y and the aggregate net domestic supply vector Y become

$$(3) \quad y \equiv \begin{bmatrix} (I - A^D)g \\ -p^m A^m g \\ -lg \end{bmatrix}, \quad Y \equiv y + z$$

Further, the producer price vector p satisfies ^{1/}

$$(4) \quad p = pA^D + p^m A^m + l$$

and, given our assumption of fixed coefficients, p is fixed throughout:

$$(5) \quad p = p^m A^m (I - A^D)^{-1} + l (I - A^D)^{-1} \text{ not } \bar{p}$$

By a suitable choice of units we may take $\bar{p}_j = 1 \forall j \in \tilde{J}$.

Private income payments $(\ell^i)_{i \in I}$ are assumed to depend in a deterministic way on aggregate value added:

$$(6) \quad \ell^i = \ell^i(\ell^*) \quad \text{with} \quad \sum_i \ell^i(\ell^*) \equiv \ell^*$$

We can therefore write private consumption demands directly as:

$$(7) \quad x = x(p + t, \ell^*, r) \equiv \sum_i x^i(p + t, \ell^*, r^i)$$

where r is still a vector of lump-sum transfers and $x^i(\cdot)$ solves

$$(X) \quad \text{Max.}_{x^i} U^i(x^i; \ell^*) \text{ s.t. } (p + t)x^i = e^i(\ell^i(\ell^*) + r^i)$$

(As in Chapter III we take i 's consumption expenditure e^i to be a simple function of income). The maximum value function of (X) is i 's indirect utility function, $V^i(p + t; \ell^*, r^i)$. As before $q \equiv p + t$.

As a specification of the abstract model of Chapter I, then, the present model reads:

$$(8) \quad s \equiv (t, g, \ell^*, m^0, r)$$

$$(9) \quad W(s) \equiv \psi(\dots V^i(\bar{p} + t, \ell^*, r^i) \dots)$$

$$(10) \quad E(s) \equiv \begin{bmatrix} \sum_i x^i(\bar{p}+t, \ell^*, r^i) - (I-A^D)g - m^0 \\ p^m(A^m g + m^0) \\ -\ell^* + \ell g \end{bmatrix}$$

and there are no side constraints. Walras' law is again easily verified:

$$(11) \quad B \equiv t x + p z + z_f + z_\ell + (p-p^m)m^0 - \sum_i r^i + \sum_i (r^i + \ell^i - e^i)$$

= 0 when $E(s) = z$ and (5), (6) hold. (The last term in (11) is total private savings).

Finally, the Lagrangian of the model reduces to the function

$$(12) \quad \mathcal{L}(s, z; \phi) \equiv W(s) - \phi [E(s) - z]$$

where $\phi \equiv (\phi, \phi_f, \phi_\ell)$ is a shadow price vector.

The absence of specific restrictions on the choice of g deserves some attention. It will be justified under two distinct sets of circumstances. First, if private firms produce according to demand. Alternatively, the planner may be powerful enough to effectively manipulate the private production plan in the medium run, e.g. through fiscal incentives, licensing procedures, quota restrictions etc.

Although the first line of argumentation is not very convincing in view of the importance of supply constraints in the Indian

economy, the second viewpoint is certainly in tune with the fundamental premises of Indian planning, as the following quotation from the VIth five-year Plan makes clear:

"As for the private sector, the relevant macro plans are formulated by the Planning Commission, in complete tune with the general development strategy of the country and of the public sector. This part of the plan is regarded as indicative and subsequently appropriate measures are undertaken by different Ministries through fiscal, monetary and income policies, to ensure their fulfilment". 2/

These assumptions have been the target of perhaps pertinent criticism (see e.g. Rudra (4)). However, our search for a tax reform model coherent with the framework of the Plan seems to justify their endorsement here.

§IV.2.2 : Tax reform and shadow prices

Consider a tax reform proposal $\dot{t} = (\dot{t}_j)_{j \in J}$. In the terminology of Chapter I, $\dot{t}$ is a "potentially feasible reform". Its resource requirement is:

$$(13) \quad \frac{\partial x}{\partial q} \dot{t}$$

and its marginal social value

$$(14) \quad \Pi^* = \sum_i \frac{\partial \Psi}{\partial v^i} \frac{\partial v^i}{\partial q} \dot{t} - \tilde{\phi} \frac{\partial x}{\partial q} \dot{t}$$

$$(15) \quad = \left[-\sum_i \mu^i x^i - \tilde{\phi} \frac{\partial x}{\partial q} \right] \dot{t}$$

where $\{\mu^i\}_{i \in I}$ is a set of welfare weights and $\tilde{\phi}$ is a vector of shadow prices for produced commodities.

As we have discussed at some length in Chapter I, the choice of a shadow price vector in a reform context is a subtle question, often not amenable to a simple and compelling solution concept. In search of an appealing choice criterion, we shall here concentrate on aggregate production efficiency considerations. More precisely, cost-benefit tests in this economy may conceivably be required to possess the following property:

(P1) If a project $\dot{z}$ allows the same aggregate supply vector of commodities Y to be produced with less resources, it should be accepted.

As the following proposition makes clear, this requirement solidly restricts the choice of shadow prices by imposing a definite relationship between the shadow prices of commodities and those of scarce factors.

Propn.1: In the above model, let $\phi \equiv (\bar{\phi}, \phi_f, \phi_\ell)$ be a vector of shadow prices for project evaluation. Then the CB test defined in Section I.3 possesses property (Pl) iff

$$(16) \quad \phi_f, \phi_\ell > 0 \text{ and } \bar{\phi} = \phi_\ell \ell (I-A^D)^{-1} + \phi_f p^m A^m (I-A^D)^{-1}$$

$$\text{not. } \phi_\ell L + \phi_f F$$

Proof: Let $\dot{Y}$ satisfy $\dot{Y} = 0$, $\dot{Y}_f \geq 0$, $\dot{Y}_\ell \geq 0$, $\dot{Y}_f \dot{Y}_\ell > 0$. Then $\dot{G} = - (I-A^D)^{-1} \dot{Z}$ and

$$\dot{z}_f \geq - \dot{Y}_f = - p^m A^m (I-A^D)^{-1} \dot{Z} \text{ not. } = - F \dot{Z}.$$

$$\dot{z}_\ell \geq - \dot{Y}_\ell = - \ell (I-A^D)^{-1} \dot{Z} \text{ not. } = - L \dot{Z}.$$

with at least one strict inequality. Therefore a necessary and sufficient condition for a project $\dot{z}$ to be acceptable according to the production efficiency criterion is

$$(17) \quad \dot{z}_f \geq - F \dot{z}, \quad \dot{z}_\ell \geq - L \dot{z}, \quad (\dot{z}_f + F \dot{z})(\dot{z}_\ell + L \dot{z}) > 0$$

(i) Let ϕ satisfy (16). Then for every project satisfying (17):

$$\phi \dot{z} = \bar{\phi} \dot{z} + \phi_f \dot{z}_f + \phi_\ell \dot{z}_\ell = \phi_f (\dot{z}_f + F \dot{z}) + \phi_\ell (\dot{z}_\ell + L \dot{z}) > 0$$

and so the CB test associated with ϕ satisfies (Pl).

(ii) Conversely, if ϕ is such that (P1) holds, then,

$$\forall \dot{z} \in R^{M-2}:$$

$$\left. \begin{aligned} \dot{z}_f &\geq -F\dot{z}, \quad \dot{z}_\ell \geq -L\dot{z} \\ (\dot{z}_f + F\dot{z})(\dot{z}_\ell + L\dot{z}) &> 0 \end{aligned} \right\} \Rightarrow \phi \dot{z} > 0$$

Equivalently, letting $\epsilon_f \equiv \dot{z}_f + F\dot{z}$, $\epsilon_\ell \equiv \dot{z}_\ell + L\dot{z}$:

$$(18) \quad \left. \begin{aligned} \epsilon_f &\geq 0, \quad \epsilon_\ell \geq 0 \\ \epsilon_f \epsilon_\ell &> 0 \end{aligned} \right\} \Rightarrow \forall \dot{z} \in R^{M-2} : \tilde{\phi} \dot{z} + \phi_f(\epsilon_f - F\dot{z}) + \phi_\ell(\epsilon_\ell - L\dot{z}) > 0$$

(18) implies $\phi_f > 0$, $\phi_\ell > 0$, $\tilde{\phi} = \phi_f F + \phi_\ell L$ and completes the proof.

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Notice that (16) is just the familiar marginal social cost rule for shadow pricing; it is reminiscent of rule 6 of Chapter II. It is also usefully contrasted with the market pricing rule (5). In particular

$$(19) \quad \tilde{\phi} \div p \Leftrightarrow \phi_f = \phi_\ell \quad (\text{unless } F \text{ and } L \text{ in (16) are colinear}).$$

Finally, we note for future reference the obvious interpretation of F and L as vectors of total direct and indirect foreign exchange and labour coefficients, respectively.

The values of ϕ_f and ϕ_ℓ (the "social value of foreign exchange", and the "shadow wage rate", respectively) of course depend upon the alternative possible sources of foreign exchange and labour, e.g. whether unemployment exists or whether an appropriate

set of tax and transfer policies is proposed to maintain full employment. Pending a complete solution of the problem, we shall simply regard ϕ_f and ϕ_g as parameters, and later mention some of their plausible values under alternative "closing rules".

In effect, then, we are investigating an a priori specified set of potential reforms $(\dot{g}, \dot{t})$ satisfying

$$(20) \quad (I - A^D) \dot{g} = \frac{\partial x}{\partial q} \dot{t}$$

Indeed, the marginal social value of such a reform, irrespective of the shadow prices of commodities, is

$$(21) \quad \Pi^* = \left[-\sum_i \mu_i^i x^i - \phi_f p^m A^m (I - A^D)^{-1} \frac{\partial x}{\partial q} - \phi_g \ell (I - A^D)^{-1} \frac{\partial x}{\partial q} \right] \dot{t}$$

as in (15) when (16) holds.

Notational convention: For clarity, the pair (ϕ_f, ϕ_g) will henceforth be denoted by $v \equiv (v_f, v_g)$ and loosely called "the shadow price vector". Further we let $\tilde{\phi} \stackrel{\text{not.}}{=} \eta = (\eta_j)_{j \in J}$

§IV.2.3 : A simple characterization of desirable tax reforms

Let $\dot{t}$ be a tax reform proposal. As we have just recalled, its marginal social value in the present framework is

$$(22) \quad \Pi^*(\dot{t}) = (-\sum_i \mu_i^i x_i^i \dot{t}) - v_f(p^m A^m (I-A^D)^{-1} \frac{\partial x}{\partial q} \dot{t}) - v_l(l(I-A^D)^{-1} \frac{\partial x}{\partial q} \dot{t})$$

$$\stackrel{\text{not.}}{=} \psi^o(\dot{t}) - v_f \psi^f(\dot{t}) - v_l \psi^l(\dot{t})$$

where ψ^o , ψ^f , ψ^l are, respectively, the "gross" welfare gain, foreign exchange cost, and labour cost of the contemplated reform.

As a convenient indexation scheme we shall write $\dot{t} \stackrel{\text{not.}}{=} \dot{t}_\alpha$ when the j^{th} element of $\dot{t}$ is

$$(23) \quad \frac{-\alpha_j}{\sum_i \mu_i^i x_j^i} \stackrel{\text{not.}}{=} \frac{-\alpha_j}{x_j^*} \quad \begin{array}{l} (\alpha \in \mathbb{R}^{M-2}) \\ (j \in \tilde{J}) \end{array}$$

Under this convention:

$$(24) \quad \psi^o(\dot{t}_\alpha) = \sum_j \alpha_j$$

For simplicity, and at little loss of insight, we shall confine most of the analysis to tax reforms with a unit welfare gain. Hence

$$(25) \quad \sum_j \alpha_j = 1$$

unless explicit statement is made to the contrary. Any tax reform may then to be identified with an element α of the unit simplex in $\mathbb{R}^{M-2}$.

Since marginal changes are additive, it will also prove convenient to concentrate much of our attention on a set of "basic" tax reforms $(t^{(j)})_{j \in \tilde{J}}$ defined simply by:

$$(26) \quad t^{(j)} = t_{e_j} \quad \text{where } e_j \text{ is the } j^{\text{th}} \text{ unit vector } (j \in \tilde{J}).$$

The j^{th} basic tax reform therefore consists of generating a unit welfare gain by a small manipulation of the j^{th} tax in isolation. Its foreign exchange cost ψ_j^f , labour cost ψ_j^l , and marginal social value π_j^* are readily found:

$$(27) \quad \psi_j^f = -F \frac{\partial x}{\partial q_j} / x_j^*$$

$$(28) \quad \psi_j^l = -L \frac{\partial x}{\partial q_j} / x_j^*$$

$$(29) \quad \pi_j^* = 1 - v_f \psi_j^f - v_l \psi_j^l \quad \text{not.} \quad 1 - v \psi_j$$

ψ_j^f is naturally interpreted as the total foreign exchange cost, per unit welfare gain, of reducing the j^{th} tax rate; and correspondingly for ψ_j^l .

Letting $\psi^f = (\psi_j^f)_{j \in \tilde{J}}$, $\psi^l = (\psi_j^l)_{j \in \tilde{J}}$ and $\pi^* = (\pi_j^*)_{j \in \tilde{J}}$,

(27)-(29) generalize immediately to

$$(30) \quad \psi_\alpha^f = \psi^f(t_\alpha) = \alpha \psi^f$$

$$(31) \quad \psi_\alpha^l = \psi^l(t_\alpha) = \alpha \psi^l$$

$$(32) \quad \pi_{\alpha}^* \equiv \pi^*(t_{\alpha}) = 1 - v_f \psi_{\alpha}^f - v_{\ell} \psi_{\alpha}^{\ell} \quad \text{not.} \quad 1 - v \psi_{\alpha} = \alpha \pi^*$$

Thus a tax reform t_{α} will be deemed desirable if

$$(33) \quad \boxed{v \psi_{\alpha} < 1}$$

The structure of this simple tax reform model permits a wide range of variations around two main themes. The first theme suggests investigating how policy recommendations depend upon the location of the shadow price vector v .

By contrast, the second theme consists of examining how "suggestive" values for the shadow price pair (v_f, v_{ℓ}) can be inferred from a careful study of the policies available to the planner.

The duality between these two important themes should already be clear from the introductory remarks of Section IV.1. Particularly obvious here is the duality between the search for the set of tax reforms which are desirable given a certain location of the SP vector, and the identification of the set of SP pairs which sanction or "support" a given set of tax reform proposals. Formally, let us write, for any vector $a \in R^2$:

$$(34) \quad P^+(a) = \left\{ \lambda \in R^2 : a \cdot \lambda < 1 \right\}$$

and

$$(35) \quad P^*(a) = \left\{ \lambda \in R^2 : a \lambda = 1 \right\} = \text{bdy } (P^+(a)).$$

Obviously $\lambda \in P^+(a) \Leftrightarrow a \in P^*(\lambda)$

With any tax reform α we may associate a set Q_α^+ of shadow price vectors which support it:

$$(36) \quad Q_\alpha^+ \equiv \left\{ v : \Pi_\alpha^* > 0 \right\} = P^+(\Psi_\alpha) \quad \text{using (33).}$$

Following straightforward notational convention we write

$Q_j^+ \equiv Q_{e_j}^+$ and $Q_j^* = \text{bdy}(Q_j^+)$. Similarly, each SP vector v defines a set of desirable tax reforms T_v^+ :

$$(37) \quad T_v^+ \equiv \left\{ \alpha : \Pi_\alpha^* > 0 \right\}$$

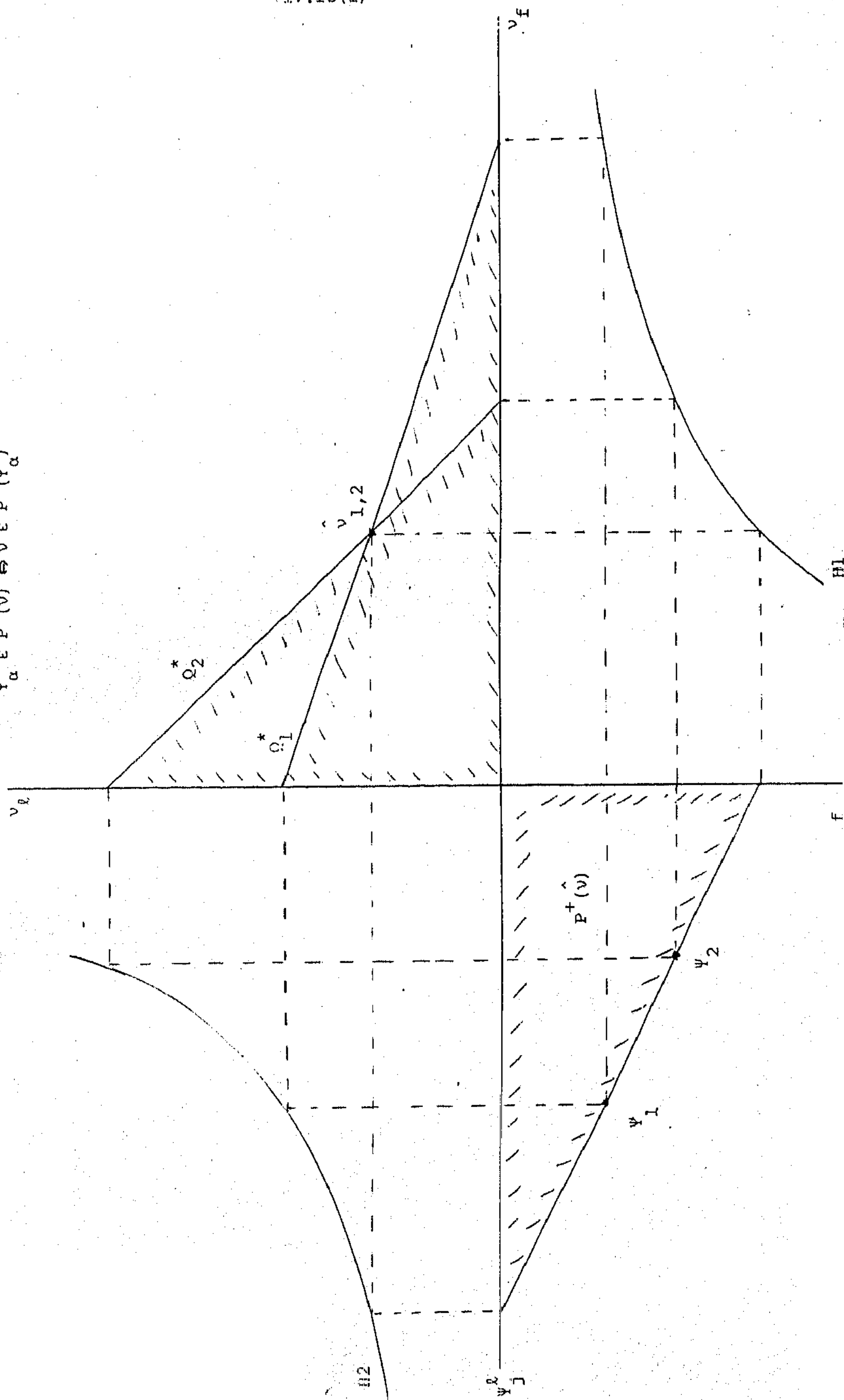
Clearly then

$$(38) \quad \alpha \in T_v^+ \Leftrightarrow v \in Q_\alpha^+ \Leftrightarrow v \in P^+(\Psi_\alpha) \Leftrightarrow \Psi_\alpha \in P^+(v)$$

These relationships are illustrated in Figure A for a hypothetical example involving only two taxes. To see how it is constructed, consider first its north-eastern quadrant, drawn in v -space. A frontier Q_j^* ($j = 1, 2$) corresponds to the equation $v\Psi_j = 1$ and can therefore readily be drawn. It intersects with the axes at the points $(\frac{1}{\Psi_j^1}, 0)$ and $(0, \frac{1}{\Psi_j^2})$, which then yield

FIGURE A

$$\psi_\alpha \in P^+(\nu) \Leftrightarrow \nu \in P^+(\psi_\alpha)$$



the actual coordinates of ψ_j by simple "reflection" against the rectangular hyperbolae H_1, H_2 .

Now two frontiers generally have an intersection, called $\hat{v}_{1,2}$ in this example. Generally, as we have briefly explored in the introductory remarks, any pair (α_1, α_2) of tax reforms (satisfying a mild linear independence condition) define a "suggestive" SP pair, denoted $\hat{v}(\alpha_1, \alpha_2)$ and such that

$$(39) \quad \Pi_{\alpha_1}^* = \Pi_{\alpha_2}^* = 0 \quad \text{when} \quad v = \hat{v}(\alpha_1, \alpha_2)$$

Thus

$$(40) \quad Q_{\alpha_1}^* \cap Q_{\alpha_2}^* = \left\{ \hat{v}(\alpha_1, \alpha_2) \right\} \quad (\text{unless } Q_{\alpha_1}^*, Q_{\alpha_2}^* \text{ are parallel}).$$

For any $j, k \in \tilde{J}$ we write $\hat{v}_{j,k} = \hat{v}(e_j, e_k)$; this justifies the notation of the diagram. As one expects, the vector $\hat{v}_{j,k}$ is the SP vector which emerges when the balance of payments and full employment constraints (1) and (2) are met by suitable manipulations of the taxes t_j and t_k (or, equivalently, when the model is "closed" by "endogenizing" t_j and t_k). For then, according to R.4 of Section IV.1, (39) should hold, and solve uniquely for $\hat{v}$ provided ψ_j and ψ_k are linearly independent. Furthermore, (39) implies that the vectors ψ_j, ψ_k both lie on the line $P^*(\hat{v})$, as a glance at Figure A indeed confirms.

The relations (38) and the construction of Figure A can prove useful in exercising our intuition about tax reform and shadow pricing in this simple model. An exhaustive investigation of the insights they offer would be tedious and we shall confine ourselves, in the next section, to a largely informal discussion drawing upon an empirical example.

The present section concludes with a glimpse into the closely related issue of optimal taxation.

§IV.2.4 : Optimal Taxation

The theory of optimality, regarded as the study of optimality conditions, and the theory of reform, viewed as an attempt to describe the set of available reforms and suggest directions of change, are strongly complementary. A necessary condition for optimality is that the set of desirable reforms should be empty; on the other hand a knowledge of the features of an optimum will often influence our views about appealing directions of change. The anatomy of the tax reform problem should therefore gain some clarity from a short digression into the optimal taxation problem.

The optimal taxation problem can be written here as

$$(T) \quad \text{Max.} \quad \Psi(\dots v^i(\bar{p} + t, \ell^*, r^i) \dots) \\ t, \dots$$

$$\text{s.t. (1)} \quad F[\bar{x}(\bar{p} + t, \ell^*, r) - m^0 - \tilde{z}] + p^m m^0 - z_f = 0 \quad (v_f) \\ (2) \quad L[\bar{x}(\bar{p} + t, \ell^*, r) - m^0 - \tilde{z}] - \ell^* - z_\ell = 0 \quad (v_\ell)$$

where, as before, $F \equiv p^m A^m (I - A^D)^{-1}$, $L \equiv \ell (I - A^D)^{-1}$, and $F + L = (1, \dots, 1) \stackrel{\text{not}}{=} 1$. The problem may involve other policy instruments besides t ; their choice will be determinant for the actual value of v at the optimum. For our limited viewpoint on the problem, v will again be regarded as a parameter.

The necessary conditions for a maximum are:

$$(50) \quad \pi_j^* = 0 \quad \forall j \in J$$

or

$$(51) \quad \mathbf{1} = v_f \psi^f + v_g \psi^g$$

where $\mathbf{1}$ is a vector of ones. Their solution can be clarified as follows.

For any vector $\mathbf{a} = (a_j)$ let $\mathbf{a}$ be the diagonal matrix with $\mathbf{a}$ on its main diagonal, and $\mathbf{a}^{-1}$ be the vector with components (a_j^{-1}) . Further let

$$(52) \quad T_j \equiv \frac{q_j - p_j}{q_j} \quad (j^{\text{th}} \text{ nominal tax rate as a proportion of consumer price})$$

$$(53) \quad T_j \equiv \frac{q_j - \eta_j}{q_j} \quad (j^{\text{th}} \text{ fictitious } G_j \text{ tax as a proportion of consumer price})$$

Since $\bar{p} = \mathbf{1}$, the vectors $\mathbf{T}$ and $\mathbf{T}$ are related by the identity

$$(54) \quad \mathbf{1} - \mathbf{T} \equiv (\mathbf{1} - \mathbf{T}) \mathbf{\eta}$$

If

$$(55) \quad D_j \equiv \sum_i \mu_i^j x_j^i / x_j \quad (j \in \tilde{J})$$

denotes the "gross" distributional characteristic of the j^{th} commodity, we may write ψ^f as

$$(56) \quad \psi^f = -F \frac{\partial x}{\partial q} \underline{x}^{-1} \underline{D}^{-1} \quad \text{where} \quad \left[\frac{\partial x}{\partial q} \right]_{jk} = \frac{\partial x_j}{\partial q_k} \quad (j, k \in \tilde{J})$$

$$(57) \quad = F \underline{q}^{-1} \underline{\Lambda} \underline{D}^{-1} \quad \text{where} \quad \underline{\Lambda} \equiv -\underline{q} \frac{\partial x}{\partial q} \underline{x}^{-1}$$

$$(58) \quad = \underline{q}^{-1} \underline{F} \underline{\Lambda} \underline{D}^{-1} \quad \text{since} \quad \underline{a} \underline{b} = \underline{b} \underline{a} \quad \text{and} \quad \underline{a}^{-1} = (\underline{a})^{-1}$$

$$(59) \quad = (1-T) \underline{\eta}^{-1} \underline{F} \underline{\Lambda} \underline{D}^{-1} \quad \text{since} \quad 1-T = \underline{q}^{-1} \underline{\eta}$$

Similarly

$$(60) \quad \psi^l = (1-T) \underline{\eta}^{-1} \underline{L} \underline{\Lambda} \underline{D}^{-1}$$

Each column of $\underline{\Lambda}$ can be interpreted as a vector of averaging weights since

$$(61) \quad 1 \underline{\Lambda} = -\underline{q} \frac{\partial x}{\partial q} \underline{x}^{-1} = \underline{x} \underline{x}^{-1} = 1 \quad \text{using the adding-up property}$$

$$\frac{\partial x}{\partial q} = -x.$$

The matrix $(I-\underline{\Lambda})$ has zero column sums; its j^{th} column precisely indicates how the expenditure on each commodity is affected by a small change of the j^{th} price:

$$(62) \quad [\underline{I}-\underline{\Lambda}]_{jk} = \frac{\partial (q_j x_j)}{\partial q_k} \underline{x}_k^{-1}$$

Thus, for instance, with the Linear Expenditure System,

$$\underline{\Lambda} = \underline{I}.$$

Substituting (59) and (60) in (51) we obtain the optimality conditions

$$\begin{aligned}
 (63) \quad 1 &= v_f (1-T^*) \eta^{-1} F \wedge D^{-1} + v_\ell (1-T^*) \eta^{-1} L \wedge D^{-1} \\
 &= (1-T^*) \eta^{-1} (v_f F + v_\ell L) \wedge D^{-1} \\
 &= (1-T^*) \wedge D^{-1} \quad \text{since } \eta \equiv v_f F + v_\ell L \text{ (see Propn.1)}
 \end{aligned}$$

whence:

$$(64) \quad T^* = (1-D) \wedge^{-1}$$

Finally, recalling (54):

$$(65) \quad T_j^* = \frac{T_j^* + (n_j - 1)}{n_j}$$

The j^{th} optimal tax can therefore be regarded as the sum of two elements.

The first, namely $\frac{n_j - 1}{n_j}$, reflects the divergence between the market and shadow prices of scarce resources; its contribution precisely brings consumer prices in line with shadow prices since

$$(66) \quad \text{if } T_j^* = \frac{\eta_j^{-1}}{\eta_j}, \text{ then } q_j = \eta_j$$

When v_f and v_ℓ are known, this first element is independent of consumer tastes, as well as of any value judgement; in fact, it can be computed as a weighted average of an economy-wide "labour tax" and "foreign exchange tax", the weights being the shares of each input in the (average or marginal) social cost of production of commodity j :

$$(67) \quad \frac{\eta_j^{-1}}{\eta_j} = \frac{v_f F_j}{v_f F_j + v_\ell L_j} \bar{T}_f + \frac{v_\ell L_j}{v_f F_j + v_\ell L_j} \bar{T}_\ell \quad \text{where}$$

$$(68) \quad \bar{T}_f \equiv \frac{v_f^{-1}}{v_f}, \quad \bar{T}_\ell \equiv \frac{v_\ell^{-1}}{v_\ell} \quad (\text{recall that } F + L = 1)$$

The second component of the j^{th} optimal tax, T_j^*/η_j , corrects the first by allowing for distributional considerations, as (64) indicates. It is the same for all commodities if and only if all distributional characteristics are equal; this seems plausible in general only when $\mu^i = \mu \quad \forall i \in I$, i.e. "distributional indifference" holds.

We conclude this section with a brief examination of the simple case where $\mu^i = \mu \quad \forall i \in I$, for some $\mu > 0$. When this holds, $T^* = (1-\mu)1\Lambda^{-1} = (1-\mu)1$ and so

$$(69) \quad T_j^* = \frac{\eta_j^{-1} - \mu}{\eta_j}$$

Hence

$$(70) \quad t_j^* = \frac{T_j^*}{1-T_j^*} = \frac{\eta_j - \mu}{\mu} = \bar{t}_f F_j + \bar{t}_\ell L_j \quad \text{where}$$

$$(71) \quad \bar{t}_f = \frac{v_f - \mu}{\mu}, \quad \bar{t}_\ell = \frac{v_\ell - \mu}{\mu}$$

Although the absolute level of taxes will depend upon the aggregate tax requirement (see the government budget constraint (11)), the ranking of tax rates is determinate. Indeed

$$(72) \quad t_j^* - t_k^* = \frac{v_f - v_\ell}{\mu} (F_j - F_k) = \frac{v_\ell - v_f}{\mu} (L_j - L_k)$$

Three cases may then occur

$$(73) \quad (i) \quad v_f = v_\ell ; \quad \text{then } t_j^* = t_k^* \quad \forall j, k \in \tilde{J}$$

$$(ii) \quad v_f < v_\ell ; \quad \text{then } t_j^* > t_k^* \Leftrightarrow L_j > L_k \quad \forall j, k \in \tilde{J}$$

$$(iii) \quad v_\ell < v_f ; \quad \text{then } t_j^* > t_k^* \Leftrightarrow F_j > F_k \quad \forall j, k \in \tilde{J}$$

The ranking of commodities by increasing order of tax rates is therefore either exactly the same or exactly the reverse as their ranking by increasing order of their total (unit) foreign exchange cost of production (F_j), according as $v_\ell \lessgtr v_f$. Such features of the optimum will prove useful in understanding the anatomy of tax reform in an economy with two scarce factors.

Section IV.3 : An illustrative application to India

§IV.3.1 : Outline of the exercise and the results

To illustrate the findings of Section IV.2. we have estimated the values of ψ_j^f , ψ_j^l , with Indian data (relating to 1979-80) for 9 broad commodity groups. The statistical material has been assembled from various sources all obtained from or available to the Planning Commission and is fully described in Annex 1.

The estimation of the welfare weights $(\mu^i)_{i \in I}$ of course involves ethical judgement. The nature of this judgement is however fairly clear. Unless consumers are regarded to differ significantly in their tastes (V^i) or social importance ($\partial \Psi / \partial V^i$), the welfare weights $\{\mu^i\}$ should differ across consumers solely on account of differences in their respective levels of expenditure.

It seems reasonable, then, as a first approximation, and to allow for an intuitive and operational representation of our value judgements, to constrain the weights $\{\mu^i\}$ to depend in an iso-elastic way on expenditure levels:

$$(74) \quad \mu^i = K(e^i)^{-\delta}$$

The elasticity parameter (δ) may then be interpreted as a measure of inequality aversion, presumably ranging from 0 (for a Benthamite) to $+\infty$ (for a Rawlsian). This procedure has become a popular element of the standard CBA kit (see e.g. Squire-Van Der Tak (5)), though it is usually defended by a slightly different

argument. In implementing it, consumers have been divided into 10 implicitly homogeneous groups, each roughly corresponding to two contiguous deciles of the expenditure scale in either the rural or the urban sector. The normalization rule

$$(75) \quad u^1 = 1$$

has been adopted, the first consumer group consisting of the poorest rural class.

The full data set, as well as the welfare weights, are presented in Appendix A.4.2 to this chapter. The results are shown in Table 1 and Figures B, C (the empirical equivalents of Figure A) for two extreme degrees of inequality aversion : $\delta = 0$ (distributional indifference) and $\delta = 5$ (approximately maximin SWF).^{4/}

To ease the interpretation of the results, we also present in Table 1 our estimates of the characteristics of the various commodity groups which appear most significant for the problem at hand. Finally, before embarking upon a detailed interpretation of the results, it is useful to explore a little further the mechanics of Figures B and C. This is the purpose of the next section.

TABLE 1 Commodity groups and their characteristics

a) General characteristics

	D_j			t_j	$ \varepsilon_j $	$q_j x_j$
	$\delta = 0$	$\delta = 1$	$\delta = 5$			
(1) CL	1	0.53 (1)	0.19 (1)	-0.026 (9)	0.53	0.399 (1)
(2) ML	1	0.39 (7)	0.06 (7)	0.001 (7)	1.13	0.079 (4)
(3) EO	1	0.46 (3)	0.12 (4)	0.100 (3)	0.48	0.028 (8)
(4) MFE	1	0.44 (6)	0.11 (5)	0.000 (8)	0.85	0.028 (9)
(5) SG	1	0.45 (5)	0.11 (6)	0.060 (4)	0.59	0.031 (7)
(6) OF	1	0.45 (4)	0.12 (3)	0.133 (2)	0.84	0.159 (2)
(7) CT	1	0.36 (9)	0.05 (9)	0.042 (5)	0.74	0.062 (5)
(8) FL	1	0.50 (2)	0.16 (2)	0.166 (1)	0.49	0.041 (6)
(9) ONF	1	0.36 (8)	0.06 (8)	0.033 (6)	0.94	0.124 (3)

cont'd.....

b) Labour content

	ℓ_j	L_j	$\frac{L_j}{1+\tau_j}$	$\delta = 0$	ψ_j^{ℓ} $\delta = 1$	$\delta = 5$
(1) CL	0.78 (1)	0.964 (4)	0.990 (1)	0.95 (3)	1.78 (8)	5.0 (9)
(2) MM	0.55 (3)	0.984 (1)	0.983 (2)	0.99 (1)	2.56 (1)	16.7 (2)
(3) EO	0.17 (9)	0.917 (8)	0.834 (8)	0.91 (5)	2.00 (6)	7.4 (6)
(4) MFE	0.43 (6)	0.978 (2)	0.978 (3)	0.98 (2)	2.22 (4)	8.9 (4)
(5) SG	0.38 (7)	0.951 (5)	0.897 (6)	0.90 (7)	2.00 (5)	8.5 (5)
(6) OF	0.68 (2)	0.976 (3)	0.861 (7)	0.87 (8)	1.92 (7)	7.0 (7)
(7) CT	0.34 (8)	0.940 (6)	0.902 (5)	0.91 (4)	2.55 (2)	19.0 (1)
(8) FL	0.47 (5)	0.701 (9)	0.601 (9)	0.76 (9)	1.52 (9)	4.6 (8)
(9) ONF	0.54 (4)	0.933 (7)	0.903 (4)	0.90 (6)	2.53 (3)	15.8 (3)

cont'd....

c) Foreign exchange content

	f_j	F_j	$\frac{F_j}{1+t_j}$	$\delta = 0$	ψ_j^F $\delta = 1$	$\delta = 5$
(1) CL	0.022 (4)	0.036 (6)	0.037 (6)	0.042 (6)	0.079 (7)	0.223 (9)
(2) MM	0.000 (9)	0.016 (9)	0.016 (9)	0.025 (9)	0.064 (9)	0.414 (6)
(3) EO	0.039 (2)	0.083 (2)	0.075 (2)	0.059 (3)	0.128 (4)	0.484 (4)
(4) MFE	0.000 (8)	0.022 (8)	0.022 (7)	0.039 (7)	0.089 (6)	0.358 (7)
(5) SG	0.009 (6)	0.049 (5)	0.046 (5)	0.048 (5)	0.107 (5)	0.454 (5)
(6) OF	0.005 (7)	0.024 (7)	0.021 (8)	0.030 (8)	0.067 (8)	0.245 (8)
(7) CT	0.010 (5)	0.060 (4)	0.058 (4)	0.055 (4)	0.154 (3)	1.147 (1)
(8) FL	0.241 (1)	0.299 (1)	0.256 (1)	0.147 (1)	0.296 (1)	0.899 (3)
(9) ONF	0.029 (3)	0.067 (3)	0.065 (3)	0.064 (2)	0.179 (2)	1.119 (2)

cont'd.....

Notational conventions

D_j : gross distributional characteristic of good j (see 55)).

t_j : nominal tax on good j ($t_j = q_j - p_j$)

ε_j : own-price elasticity of aggregate consumer demand for good j .

$q_j x_j$: aggregate consumer expenditure on good j (excluding imports).

l_j : direct labour content of good j (i.e. value added generated per unit of gross output).

L_j : direct and indirect labour content of good j :
 $L = l(I - A^D)^{-1}$

ψ_j^l : labour cost per unit welfare gain when lowering t_j

f_j : direct foreign exchange cost of good j (i.e. foreign exchange cost of imported inputs per unit of gross output).

F_j : direct and indirect foreign exchange cost of good j :
 $F = f(I - A^D)^{-1}$

ψ_j^f : foreign exchange cost per unit welfare gain when lowering t_j .

δ : inequality aversion parameter

CL = cereals

MM = milk and milk products

EO = edible oils

MFE = meat, fish and eggs

SG = sugar and gur

OF = other food

CT = clothing

FL = fuel and light

ONF = other non-food

cont'd....

Explanatory notes:

- (1) The numbers in brackets indicate the rank-order of commodities according to the specified characteristic.
- (2) The normalization rules are
 - (i) $p_j = 1 \quad \forall j$
 - (ii) the exchange rate and the wage are equal to unity
 - (iii) $\mu^1 = 1$, where μ^1 is the welfare weight of the poorest rural class
 - (iv) aggregate consumer expenditure is unity. The last column of the first table only adds up to 0.951 however, because it excludes imports of consumer goods (see Appendix A.4.2. for further detail).

FIGURE B

Critical shadow price values ($\delta = 0$)

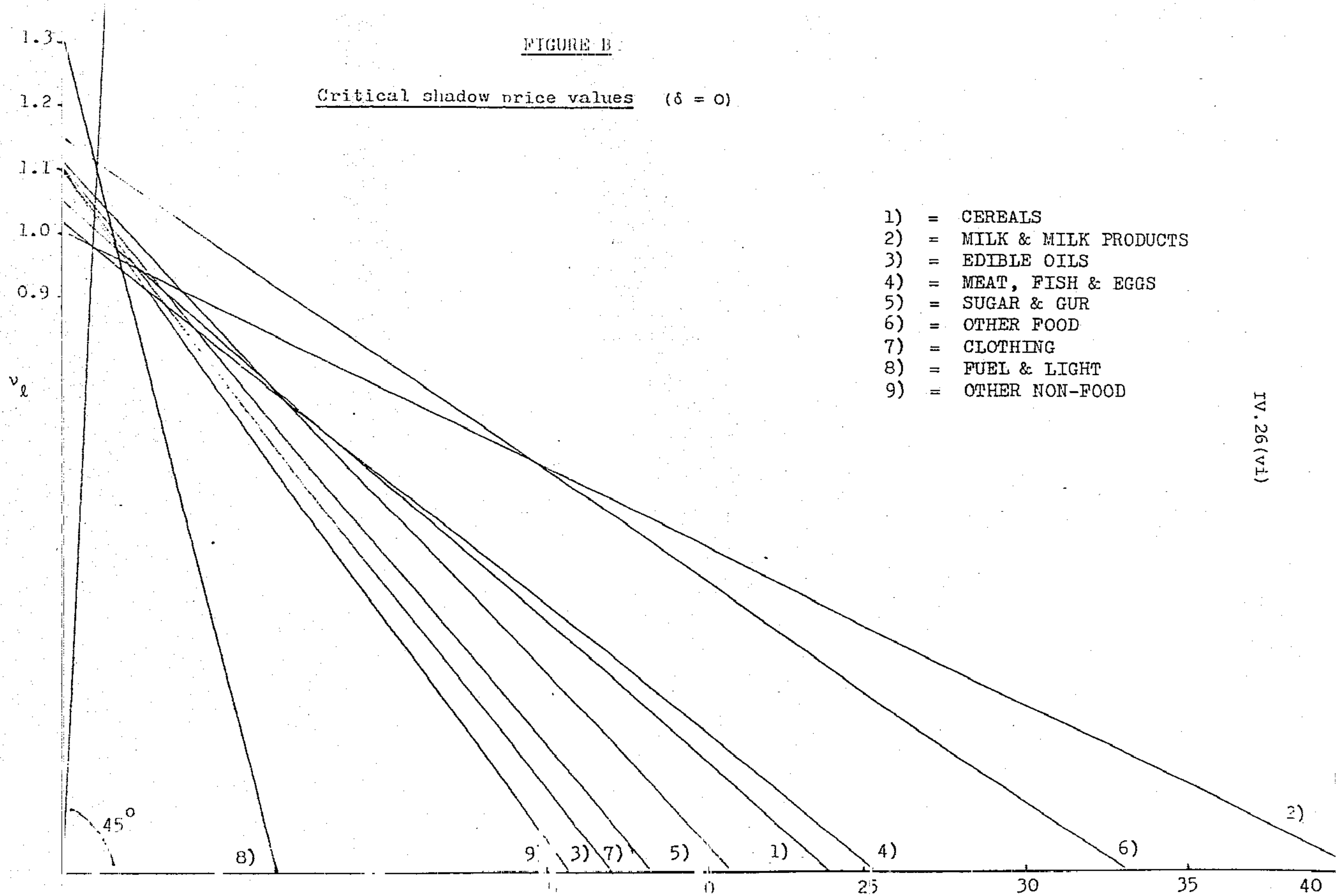
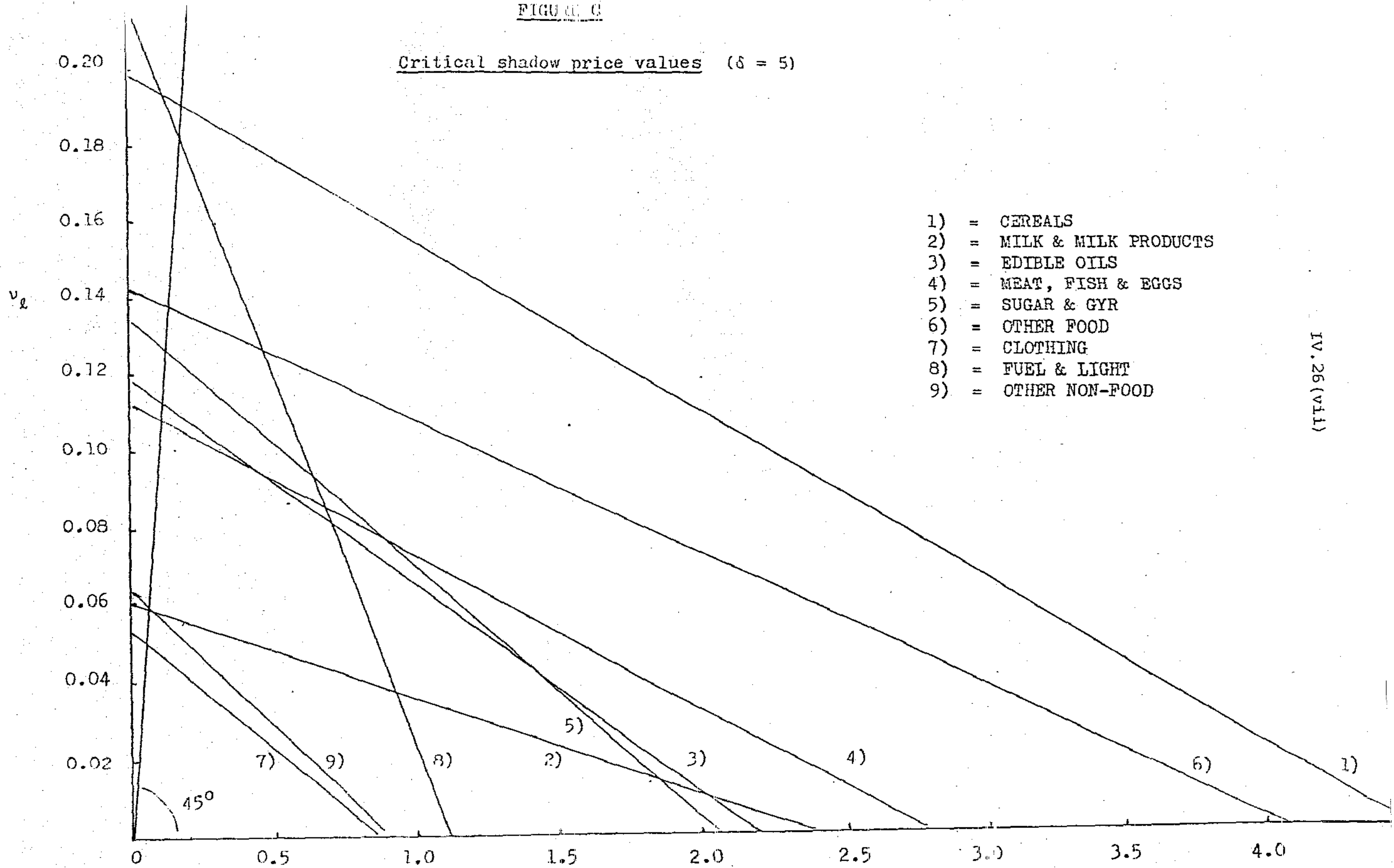


FIGURE C

Critical shadow price values ($\delta = 5$)



IV.3.2 : Diagrammatic interpretation

Preliminary definitions and notational conventions

- 1) The symbol σ denotes the SP ratio : $\sigma \equiv v_\ell / v_f$
- 2) The "true" SP pair is denoted by v^* . We assume $v^* \geq 0$.
- 3) For any $(j,k) \in J \times J$, the solution for v of the equations

$$\Pi_j^* = \Pi_k^* = 0$$

is noted by $\hat{v}(j,k) = (\hat{v}_f(j,k), \hat{v}_\ell(j,k))$. This is just the "suggestive" SP pair associated with the tax instruments (t_j, t_k) (see R7, Section IV.1). Moreover

$$(76) \quad \hat{\sigma}(j,k) \text{ not. } \frac{\hat{v}_\ell(j,k)}{\hat{v}_f(j,k)}$$

It may be easily verified that:

$$(77) \quad \begin{cases} \hat{\sigma}(j,k) \equiv \frac{\psi_k^f - \psi_j^f}{\psi_j^\ell - \psi_k^\ell} \\ \hat{v}_f(j,k) \equiv (\psi_j^f + \hat{\sigma}(j,k) \psi_j^\ell)^{-1} = (\psi_k^f + \hat{\sigma}(j,k) \psi_k^\ell)^{-1} \end{cases}$$

- 4) For any σ , we denote by $\hat{v}_f^{(j)}(\sigma)$, the value of v_f for which t_j breaks even when σ is the correct SP ratio

$$\hat{v}_f^{(j)}(\sigma) \text{ solves } 1 - v_f \psi_j^f - \sigma v_f \psi_j^\ell = 0$$

Thus

$$(78) \quad \hat{v}_f^{(j)}(\sigma) \equiv (\psi_j^f + \sigma \psi_j^l)^{-1}$$

- 5) If a tax reform modifies only two taxes, it is called a pairwise reform; if it involves three taxes, a triple-wise reform.
- 6) A tax reform α is
- a) "foreign-exchange preserving" if $\psi_\alpha^f = 0$
 - b) "labour preserving" if $\psi_\alpha^l = 0$
 - c) "resource preserving" for σ if $\psi_\alpha^f + \sigma \psi_\alpha^l = 0$
 - d) "robust" if $\Pi_\alpha^* > 0 \quad \forall v \geq 0$
- 7) A tax t_j dominates a tax t_k if $\psi_j \leq \psi_k$

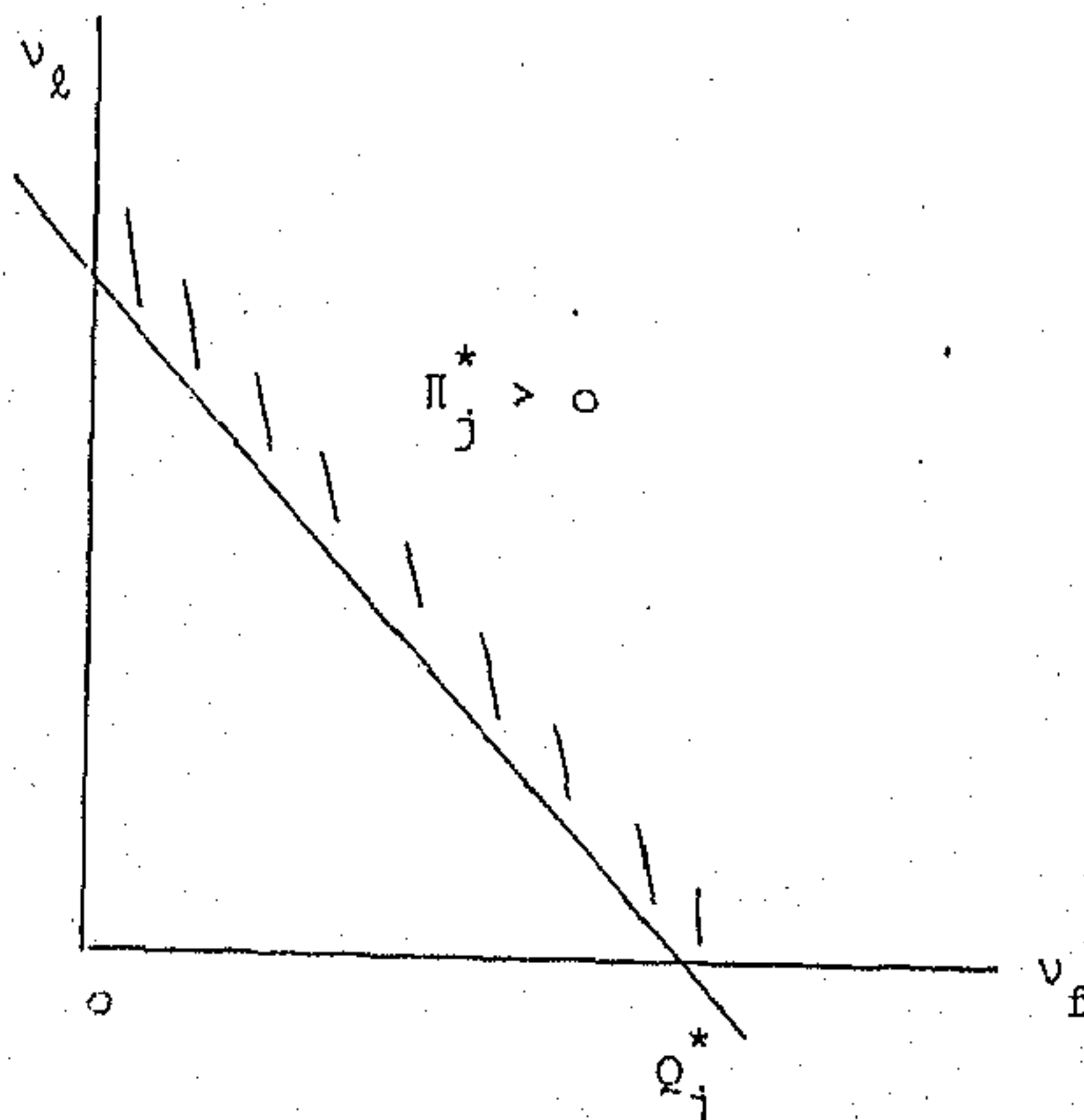
We shall now point out a few instructive inferences that can be drawn from an inspection of Figures A, B or C. They are proved in Appendix A.4.1. to this chapter under the following unnecessary but simplifying assumptions:

- A.1. a) $v^* \geq 0$
- b) $\psi_j > 0 \quad \forall j \in \tilde{J}$
- c) $\psi_j^l \neq \psi_k^l, \quad \psi_j^f \neq \psi_k^f \quad \forall j, k \in \tilde{J}, \quad j \neq k$
- d) $\psi_j^l / \psi_j^f \neq \psi_k^l / \psi_k^f \quad \forall j, k \in \tilde{J}, \quad j \neq k.$

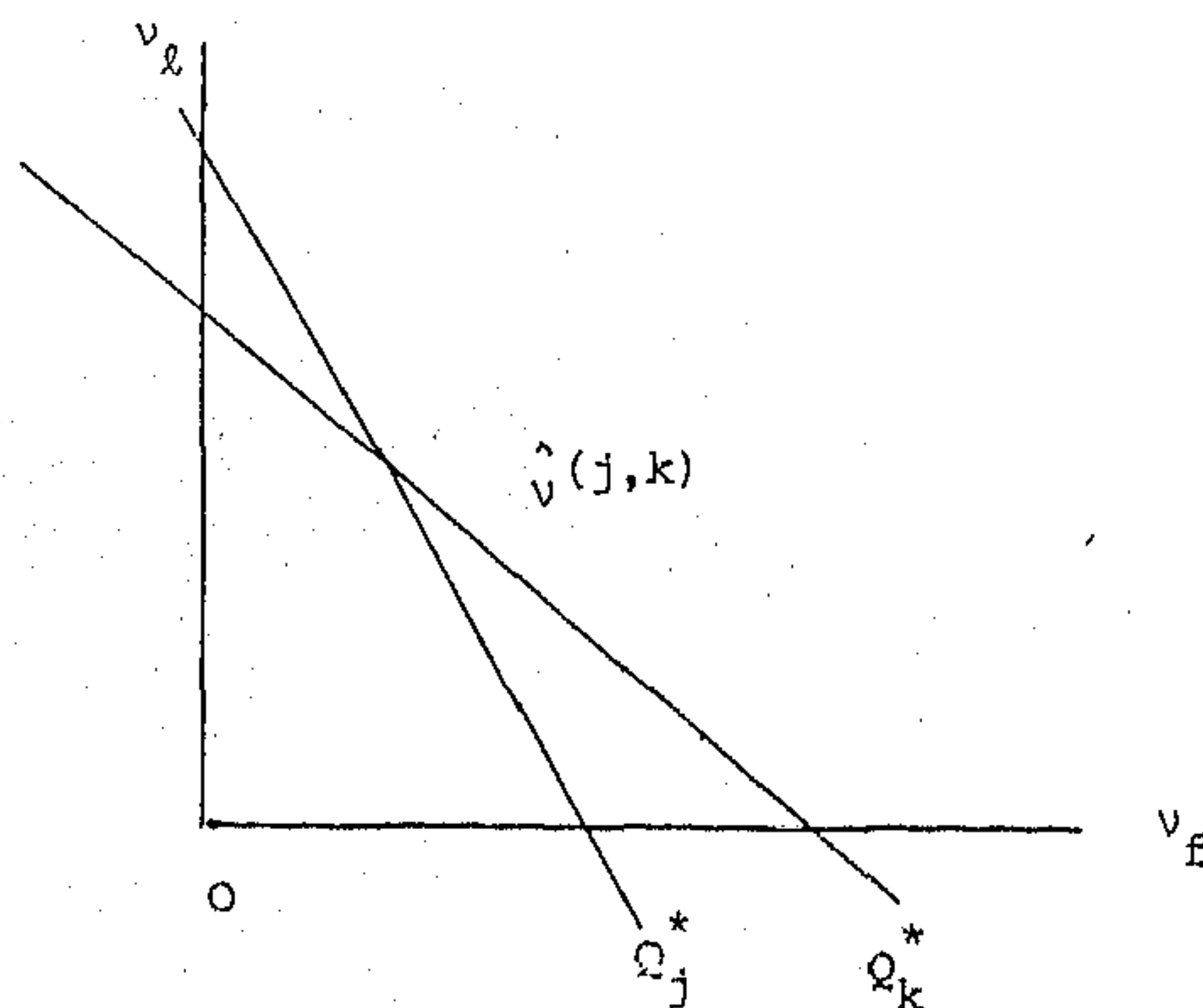
The last three hypotheses are of course verified in our empirical example.

- (II) Each frontier Q_j^* demarcates the sets of SP pairs according to which the j^{th} tax ought to be revised upwards or downwards.

(79) $\pi_j^* > 0 \Leftrightarrow v \in Q_j^+ \quad (\text{see (36)})$



- (I2) Any pair of tax instruments (t_j, t_k) yields a "suggestive" SP pair $\hat{v}^{(j,k)}$, lying at the intersection of the frontiers Q_j^* , Q_k^* . The associated full SP vector $(\tilde{\phi}, \hat{v}^{(j,k)})$ detects welfare-improving projects when public sector projects "crowd-out" the private sector and the resource constraints are met by suitable manipulation of these taxes. $\hat{v}^{(j,k)}$ lies outside the non-negative orthant iff a dominance relation exists between t_j and t_k .



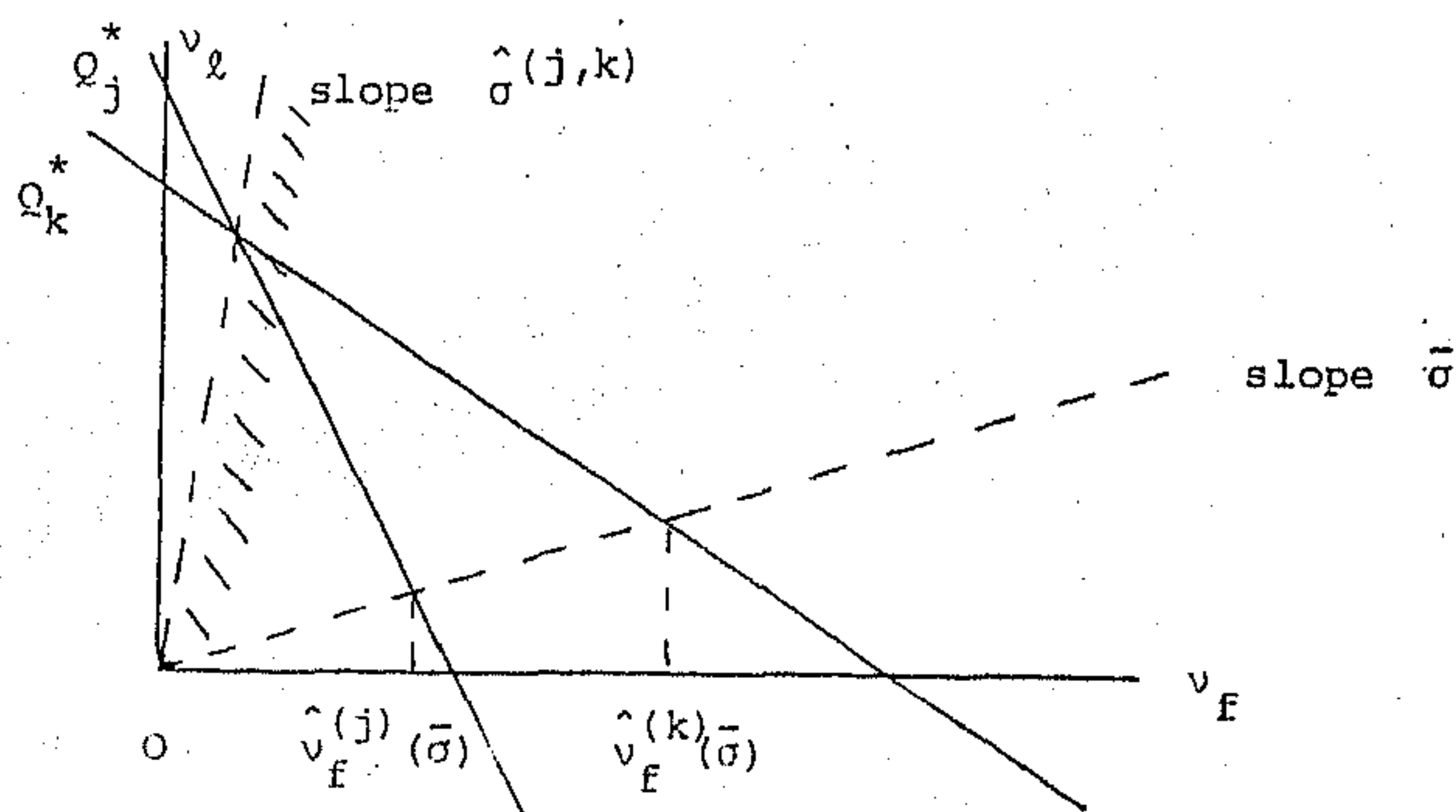
- (I3) Let $\sigma = \bar{\sigma} \geq 0$ be given, and consider a pairwise tax reform $\bar{\alpha}$ which involves t_j and t_k only and is resource-preserving for $\bar{\sigma}$. Such a reform will be desirable for every vector v along the ray indicated by $\bar{\sigma}$. Moreover

$$(80) \quad \bar{\alpha}_j > 0 \Leftrightarrow \hat{v}_F^{(j)}(\bar{\sigma}) > \hat{v}_F^{(k)}(\bar{\sigma}) \Leftrightarrow \bar{\alpha}_k < 0$$

$$(81) \quad \Leftrightarrow \text{either } \psi_j^l > \psi_k^l \text{ and } \bar{\sigma} < \hat{\sigma}^{(j,k)}$$

$$\text{or } \psi_j^l < \psi_k^l \text{ and } \bar{\sigma} > \hat{\sigma}^{(j,k)}$$

Thus, for a given SP ratio σ , the ranking of commodities along the corresponding ray and on their respective frontiers indicates how to construct pairwise tax reforms which are desirable for every SP pair on that ray. For instance, by observing the ranking of commodities along the horizontal axis, we may infer pairwise tax revisions which are unequivocally desirable when labour is not considered to be scarce (i.e. $v_\ell = 0$); and similarly on the vertical axis.



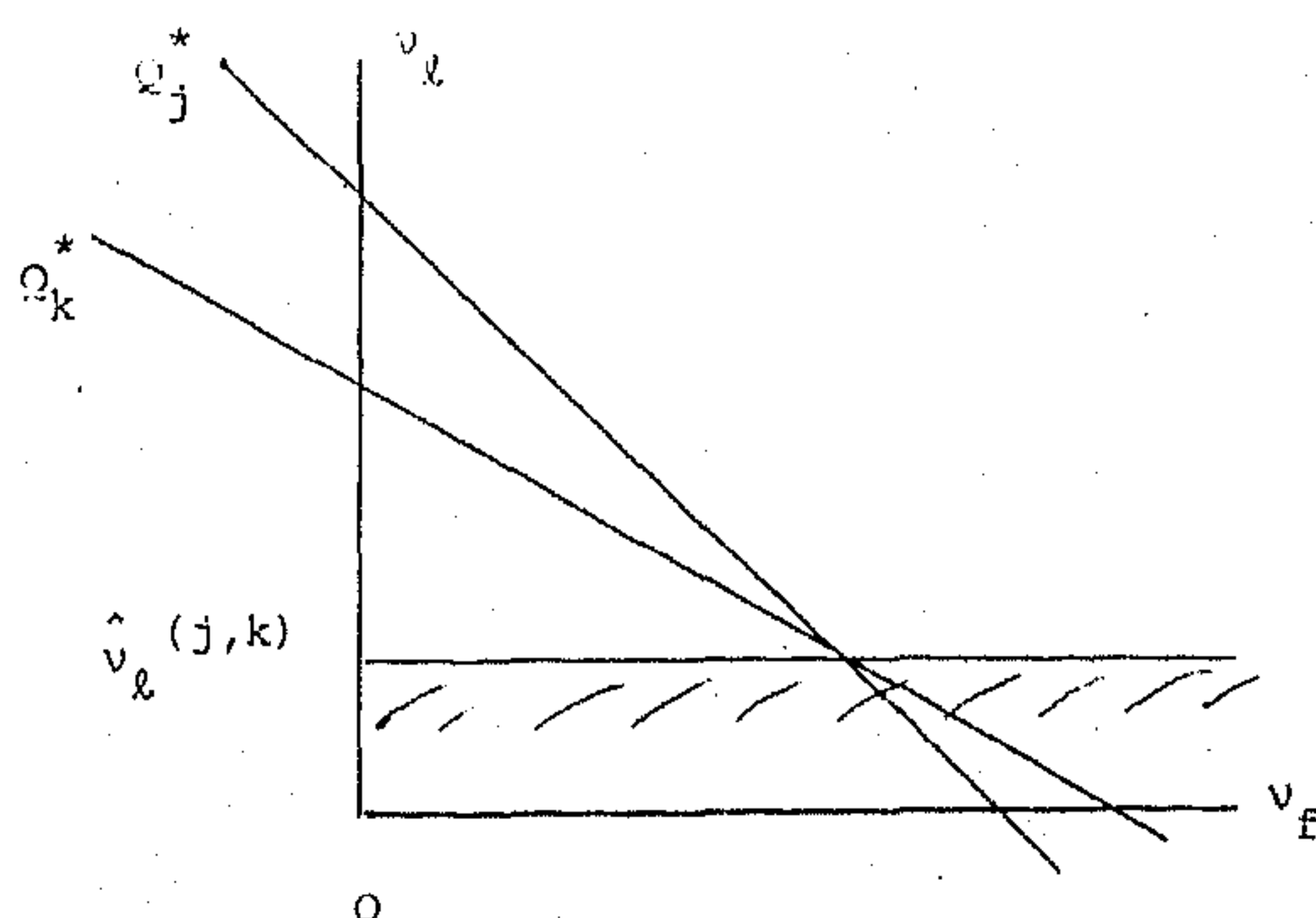
(I4) Consider a foreign-exchange preserving pairwise reform $\bar{\alpha}$ involving t_j and t_k . It is desirable iff

$$\text{either } \hat{v}_\ell^{(j,k)} < 0 \text{ or } v_\ell^* < \hat{v}_\ell^{(j,k)}$$

Moreover

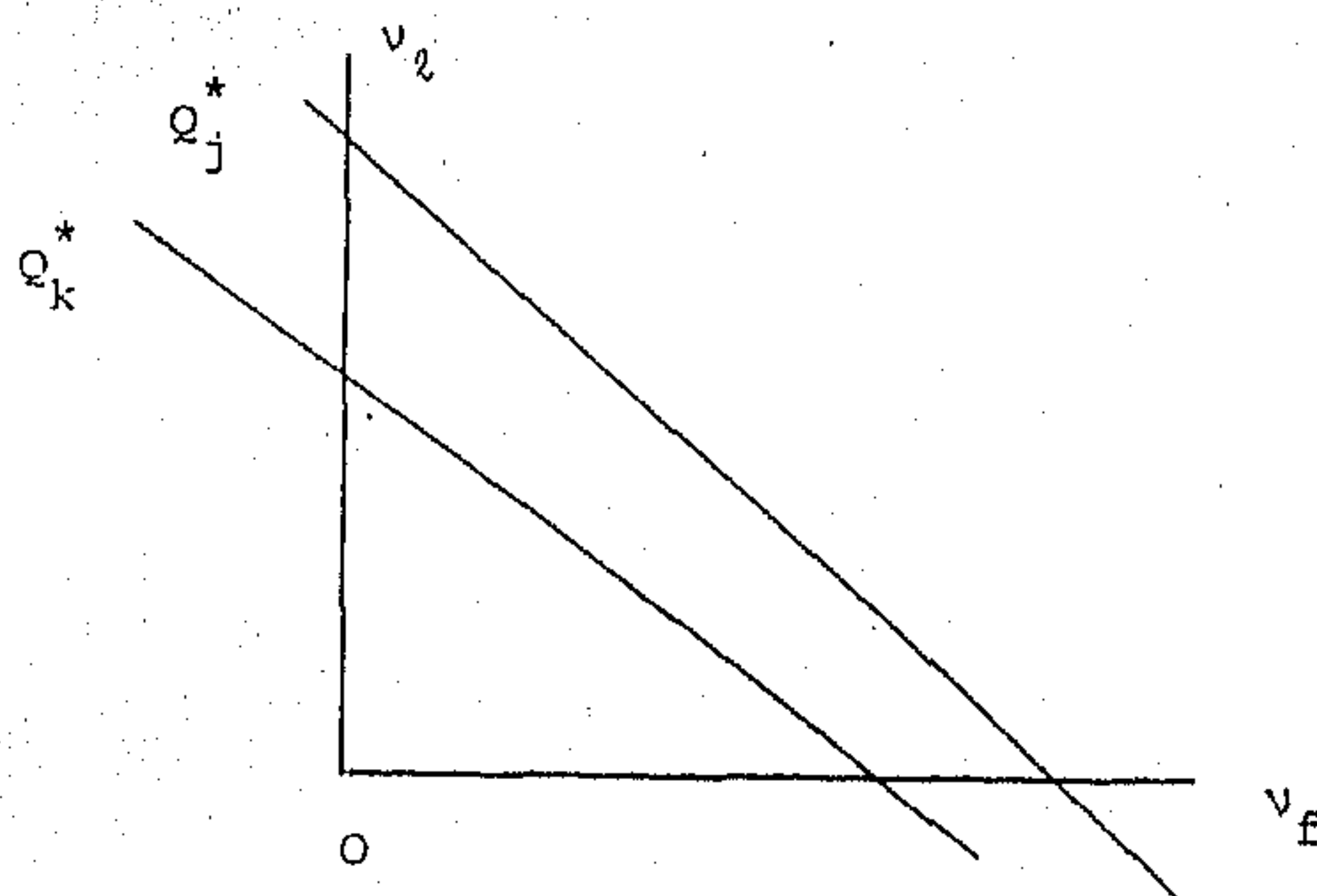
$$\bar{\alpha}_j > 0 \Leftrightarrow \psi_j^f < \psi_k^f \Leftrightarrow \bar{\alpha}_k < 0$$

Symmetric statements obviously apply for labour-preserving reforms.



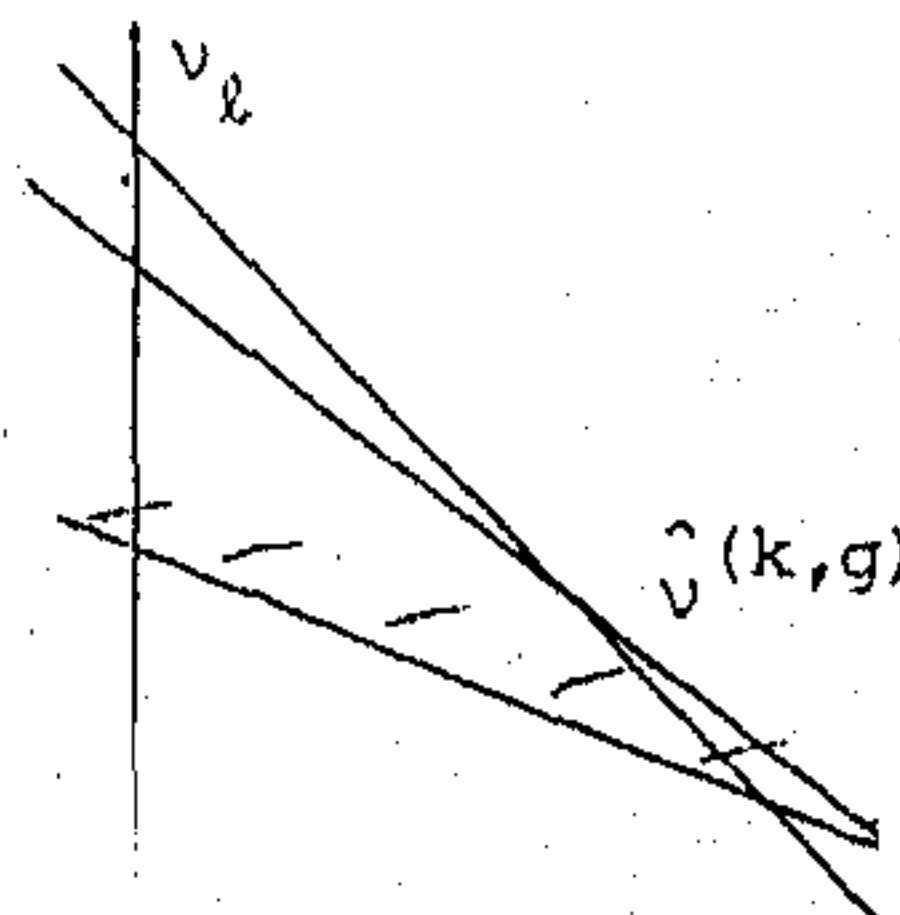
- (I5) A necessary and sufficient condition for the existence of a robust pairwise reform $\bar{\alpha}$ involving t_j and t_k is that a dominance relation holds between t_j and t_k . Moreover for such a reform

$$(84) \quad \bar{\alpha}_j > 0 \Leftrightarrow \psi_j \leq \psi_k \Leftrightarrow \bar{\alpha}_k < 0$$



- (I6) Any triple of tax instruments, say (t_j, t_k, t_g) , allows the construction of robust triple-wise tax reforms. Moreover for such a reform, say $\bar{\alpha}$: if $\hat{v}(k,g) > 0$, then

$$(85) \quad \bar{\alpha}_j > 0 \Leftrightarrow \hat{v}(k,g) \in Q_j^+$$



§IV.3.3 : Discussion of the results

To a large extent, the set of desirable directions of tax reform in this model is conditioned by the shape of the existing tax structure, relatively to the optimal one. As we have learned in §IV.2.4, the latter essentially reflects both the presence of factor price distortions ($v_f \neq v_l$) and the influence of equity considerations. Loosely speaking, in Figures B and C, the slope of Q_j^* depends primarily upon the relative total foreign exchange and labour costs of a small change in the consumer price of the j^{th} commodity, while its distance from the origin hinges on j 's distributional characteristic D_j and the tax it initially bears (since, ceteris paribus, the higher the existing tax, the smaller the set of shadow prices which support an upward revision). The comments to follow first concentrate on each of the three influences just mentioned in turn.

1) As a convenient starting point for inspection, consider the 45° line^{5/} of Figure B, along which $v_f = v_l$ and $\delta = 0$. When the true SP ratio σ^* is known, the whole analysis can be shown to reduce to the single scarce factor case, with any pair of factor inputs (y_f, y_l) being now equivalent to an amount $y^* = y_f + \sigma^* y_l$ of a single homogeneous input. A particularly interesting case occurs here with $\sigma = 1$. For then, letting $v_f = v_l = \lambda > 0$:

$$(86) \quad \eta = v_f F + v_l L = \lambda p$$

and therefore the marginal social value of the j^{th} tax becomes:

$$(87) \quad \text{MSV}_{t_j} = \frac{\partial \mathcal{L}}{\partial t_j} = - \sum_i \mu^i x_j^i - \lambda p \frac{\partial x}{\partial q_j}$$

$$(88) \quad \begin{aligned} &= - \sum_i \mu^i x_j^i + \lambda \frac{\partial (tx)}{\partial t_j} \quad \text{since} \quad p \frac{\partial x}{\partial q_j} = (q-t) \frac{\partial x}{\partial q_j} \\ &= -x_j - t \frac{\partial x}{\partial q_j} = \frac{\partial (tx)}{\partial q_j} \end{aligned}$$

Now (88) is precisely the formula one would obtain for the marginal social value of the j^{th} tax according to a partial equilibrium model involving only consumer welfare and tax revenue, as in Ahmad-Stern (1)'s early versions of the tax reform problem in India, where

$$(89) \quad \mathcal{L} = \Psi(\dots v^i(\dots) \dots) + \lambda(tx)$$

This analogy is sustained by Walras' law, which allows us to reinterpret our single scarce factor as government revenue.

Now, from (88) (and when $\frac{\partial (tx)}{\partial t_j} > 0$ as one should expect):

$$(90) \quad \text{MSV}_{t_j} > 0 \Leftrightarrow \lambda > \frac{\sum_i \mu^i x_j^i}{\partial (tx) / \partial t_j} \quad \text{not.} \quad \hat{\lambda}_j = \hat{v}_f^{(j)}(\sigma) \Big|_{\sigma=1}$$

As pointed out in (I3) above, a pairwise reform (dt_j, dt_k) which improves welfare while leaving tax revenue (or

the use of the scarce resource) unchanged satisfies

$$(91) \quad dt_j < 0 \quad \text{iff} \quad \hat{\lambda}_j > \hat{\lambda}_k$$

The ranking of the λ_j 's can be observed along the 45° line of Figures B and C. Their exact values are given in Table 2^{6/} along with the corresponding Ahmad-Stern estimates.

When $\delta = 0$, a strong correlation emerges between $\hat{\lambda}$ and t , the vector of actual tax rates. This is expected, since, according to (73) (i) in this situation ($\delta = 0$ and $v_f = v_g$) the optimal tax rates are all equal. Alternatively, we may notice that, when $\delta = 0$ and $\sigma = 1$:

$$(92) \quad \psi^f + \psi^g = \mathbf{1} \mathbf{q}^{-1} \mathbf{\Lambda} \mathbf{D}^{-1} \quad (\text{see (57)})$$

$$= \mathbf{D}_*^{-1} \mathbf{q}^{-1} \mathbf{\Lambda} \quad \text{where } \mathbf{D}_* = \mathbf{D}_j \quad \forall j \in J$$

$$= \mathbf{D}_*^{-1} (\mathbf{1} - \mathbf{T}) \mathbf{\Lambda}$$

$$= \mathbf{D}_*^{-1} (\mathbf{1} - \bar{\mathbf{T}})$$

where $\bar{\mathbf{T}}_j$ is a tax index obtained by averaging $\mathbf{T}$ according to the weights contained in the j^{th} column of $\mathbf{\Lambda}$; in particular, if $\Lambda_{jj} = 1$ and $\Lambda_{jk} = 0 \quad \forall j \neq k$, then $\bar{\mathbf{T}}_j = \mathbf{T}_j$. Finally, recalling (90) and

(79)

$$(93) \quad \frac{\hat{\lambda}_j}{\hat{\lambda}_k} = \frac{1 - \bar{\mathbf{T}}_k}{1 - \bar{\mathbf{T}}_j}$$

TABLE 2 Break-even values of λ

$\hat{\lambda}_j$

	$\delta = 0$		$\delta = 5$	
	JD	AS	JD	AS
CL	1.008 (7)	1.034 (8)	0.190 (1)	0.00465 (2)
MM	0.980 (8)	1.004 (9)	0.059 (8)	0.00079 (9)
EO	1.036 (5)	1.067 (6)	0.126 (4)	0.00268 (5)
MFE	0.979 (9)	1.053 (7)	0.108 (6)	0.00282 (4)
SG	1.058 (3)	1.089 (5)	0.112 (5)	0.00207 (6)
OF	1.113 (1)	1.136 (4)	0.137 (3)	0.00367 (3)
CT	1.040 (4)	1.245 (1)	0.050 (9)	0.00096 (8)
FL	1.107 (2)	1.162 (2)	0.181 (2)	0.00537 (1)
ONF	1.035 (6)	1.145 (3)	0.059 (7)	0.00142 (7)

Explanatory notes:

- (1) For commodity codes and normalization rules see the previous table .
- (2) Numbers in brackets indicate rank-orders.
- (3) The columns labelled "JD" indicate our own estimates; those labelled "AS" report the estimates of Ahmad and Stern (see Ahmad and Stern (1), p.74). Only the respective rank-orders should be compared, since Ahmad and Stern use different normalization rules.
- (4) The formula for computing $\hat{\lambda}_j$ is : $\hat{\lambda}_j = (\psi_j^f + \psi_j^l)^{-1}$

again suggesting the likelihood of a strong positive correlation between $\hat{\lambda}$ and T (or t).

More generally, careful observation will reveal a fair degree of harmony between tax recommendations based on the location of the optimum and those aiming at local improvements.

Unfortunately, this intuition seems difficult to formalize further or test.

2) Consider next the ranking of commodities along the horizontal axis in Figure B. As argued in (I3), this ranking indicates how to construct desirable pairwise reforms when labour is not considered to be scarce ($v_L = 0$).

Now the intercept of Q_k^* along the horizontal axis is $(\psi_k^f)^{-1}$, where ψ_k^f is a measure of the total foreign exchange cost (per unit welfare gain) of reducing the k^{th} tax rate. Further, from (58) with $D_j = 1 \ \forall j$ (since $\delta = 0$):

$$(94) \quad \psi_k^f = \sum_j \Lambda_{jk} \frac{F_j}{1+t_j}$$

where $(\Lambda_{jk})_{j \in J}$ is a vector of averaging weights reflecting consumers' demand response. In principle, therefore, ψ_k^f depends upon all tax rates, price elasticities, and foreign exchange costs of production. However, it seems to be predominantly determined by the tax rate and the (direct and indirect) foreign exchange cost applying to the k^{th} commodity itself. Indeed, the rankings of commodities in terms of ψ_j^f or of $F_j/1+t_j$ are almost identical (theoretically,

they coincide when all cross-price elasticities are zero, i.e.

$$\Lambda = I).$$

The consequent robustness of tax recommendations with respect to errors in the estimation of the price elasticity matrix in this region of the shadow price space is exemplified by the following exercise. Suppose foreign exchange is deemed to be the only scarce factor (as may happen if there is involuntary unemployment and domestic excess capacity), and it is proposed to search for pairwise tax manipulations which improve welfare but leave the balance of payments unaffected. When a pair of tax rates, say (t_j, t_k) is quoted, the CB analyst is asked to state which of the two should be reduced. The proper criterion, of course, is

$$(95) \quad \text{increase } t_j \text{ iff } \psi_j^f > \psi_k^f \text{ (see (I3)).}$$

and we may conclude from a careful comparison of the rank-order of $\{\psi_j^f\}$ and $\{F_j/1+t_j\}$ that an answer based on the assumption of zero cross-price elasticities (and hence unit own-price elasticities) would be incorrect in only one case out of 36 (involving the pair (t_3, t_9)).

Much of this robustness disappears, however, when labour is the only scarce factor (compare ψ_j^l with $(L_j/1+t_j)_{j \in J}$). In the same type of exercise (with now $v_f = 0$), 6 recommendations out of 36 would be incorrect when cross-price elasticities are ignored. This asymmetry occurs because commodities are much more sharply differentiated with respect to their relative foreign

exchange costs than to their relative labour costs.

To summarize, in the example studied here, the importance of obtaining precise estimates of consumer response parameters seems to fade away to a large extent when foreign exchange is at a high premium relative to labour in the SP system. This observation, while falling far short of a rigorous formalization of robustness problems, points to the potential usefulness of their study.

As far as the actual results are concerned, the horizontal axis in Figure B points to the relatively high foreign exchange cost of industrial products ((7), (8), (9)) compared to agricultural products ((1), (2), (4), (6)). The "fuel and light" group particularly stands out in this respect. The ranking tends to get reversed along the vertical axis, as one should expect since, when $s = 0$ (implying $D_j = 1 \forall_j$) and t is small:

$$(96) \quad \psi_j^F + \psi_j^L = 1 \quad \forall_j \in \tilde{J}$$

A conspicuous exception to this reversal rule occurs for the "other food" group (6), whose frontier lies outwards of most others on account of the very high tax it presently bears.

3) The effect of allowing for equity considerations appears from a comparison of Figures B and C. The frontiers Q_j^* of Figure C are parallel inward shifts of those of Figure B, the depth of each shift being precisely proportional to the (gross)

distributional characteristic of the corresponding commodity:

$$(97) \quad \frac{[\hat{v}_f^{(j)}(\sigma)]_{\delta=\bar{\delta}}}{[\hat{v}_f^{(j)}(\sigma)]_{\delta=0}} = [D_j]_{\delta=\bar{\delta}} \quad \forall \sigma > 0$$

The gross distributional characteristic of a commodity, in turn, depends upon its relative importance in the consumption basket of the poor and the rich; for high degrees of inequality aversion, the ranking of commodities by distributional characteristics is very close to their ranking by expenditure elasticity of demand.

The most conspicuous shifts are those of "cereals" (1) (outwards relatively to other commodities), and, at the other end, "clothing" (7) and "other non-food" (9) (inwards); this is natural since these categories lie at the extremes of the ranking by distributional characteristics. Another noteworthy item is "milk and milk products" (2), which becomes increasingly attractive to tax at higher levels of inequality aversion: however "essential" this commodity may seem to compassionate politicians and economists, it unfortunately remains by and large a luxury good in India (with an expenditure elasticity of demand for exceeding unity according to our data).

An immediate consequence, then, of the wide disparity of distributional characteristics, is a much reduced amount of "bunching" of the frontiers for high levels of inequality aversion, as well as a much wider spread of their intersection points and an increasing frequency of dominance relationships. Dominance

relationships are shown in Figure D for $\delta = 0$, $\delta = 1$ and $\delta = 5$. Strong inequality aversion implies a definite dominance of cereals over all other commodities, and a general dominance of food items over industrial products (with the noteworthy exception of milk).

The tendencies we have just mentioned significantly improve the scope for robust policy recommendations at high levels of inequality aversion (see (I5) above). To see this point, consider for instance the vector

$$(98) \quad \hat{v}^\sigma \stackrel{\text{def.}}{=} (\hat{v}_f^{(j)}(\sigma))_{j \in \tilde{J}}$$

locating the intersections of the frontiers with a particular ray (e.g. for $\sigma = 1$, $\hat{v}^\sigma = \hat{\lambda}$).

Since

$$(99) \quad \begin{cases} \psi^f = q^- F \Lambda D^{-1} & (\text{see (58)}) \\ \psi^l = q^- L \Lambda D^{-1} & (\text{by analogy}) \\ \hat{v}^\sigma = [\psi^f + \sigma \psi^l]^- & (\text{see (78)}) \end{cases}$$

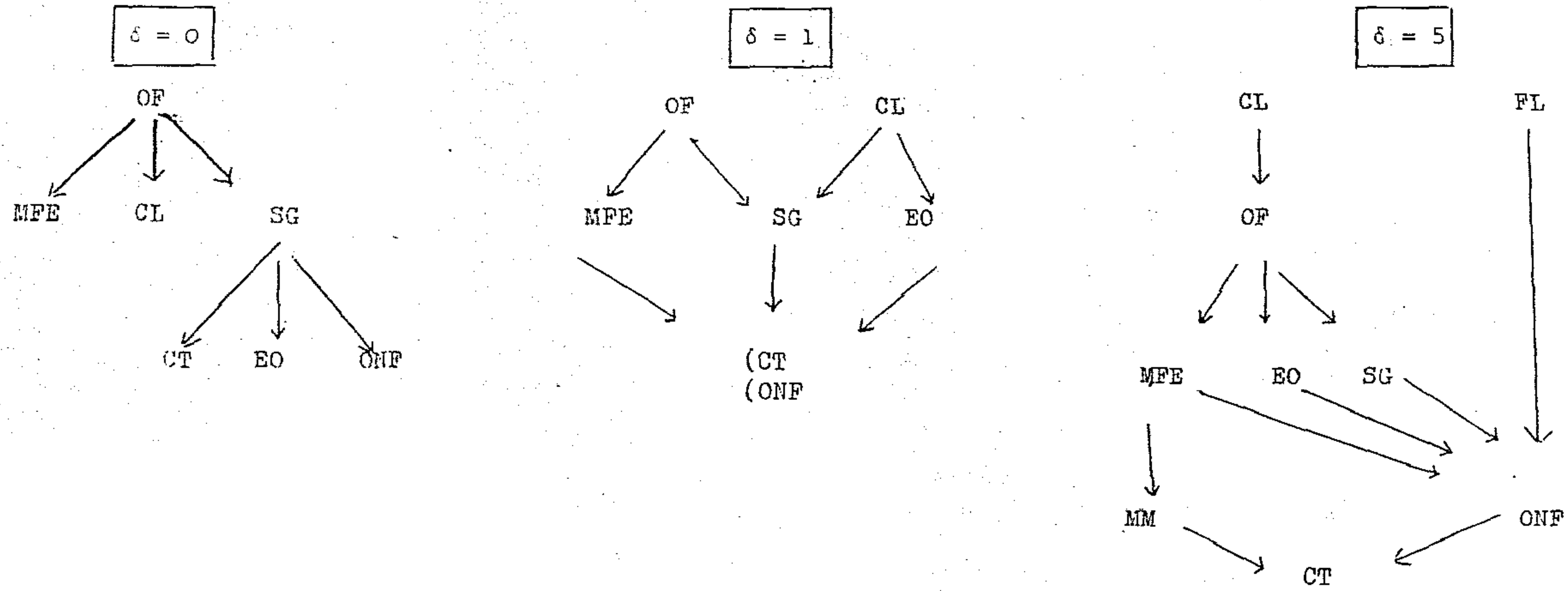
we may write

$$(100) \quad \hat{v}^\sigma = \bar{v}^\sigma D$$

where

FIGURE D

Dominance relationships



IV.40(1)

Explanatory notes:

- (1) Good $j \rightarrow$ good k if t_j "dominates" t_k , i.e. if $\psi_j^f < \psi_k^f$ and $\psi_j^l < \psi_k^l$
- (2) δ is the inequality aversion parameter.
- (3) For commodity codes, see previous figures and tables.

$$(101) \quad \bar{v}^\sigma \equiv [g^-(F + \sigma L) \Lambda]^- = \hat{v}^\sigma \quad \text{if } \delta = 0$$

Thus

$$(102) \quad \frac{\hat{v}_j^\sigma}{\hat{v}_k^\sigma} = \left(\frac{\bar{v}_j^\sigma}{\bar{v}_k^\sigma} \right) \left(\frac{D_j}{D_k} \right) > 1 \quad \text{iff}$$

$$(103) \quad \frac{D_j}{D_k} > \frac{\bar{v}_k^\sigma}{\bar{v}_j^\sigma}$$

7/

The left-hand side of (103) is bounded. However, when distributional characteristics are highly differentiated at high levels of inequality aversion, one may reasonably expect that, for large values of δ , (103) will hold, for any given pair (j,k) , for a wide range of values of σ . Furthermore, if the vector $\bar{v}^\sigma$ is not too strongly differentiated, the rank-order of commodities in the vectors $\hat{v}^\sigma$ and D should be fairly similar.

These conjectures are amply confirmed by the calculations. When $\delta = 5$, a striking correlation holds, for most "reasonable" values of σ , between the vectors $\hat{v}^\sigma$ and D ; e.g. when $\sigma = 1$ the ranking of commodities in each of these two vectors is as follows :

<u>Distributional characteristic (D_j)</u>	<u>"Switching value" of v_F ($\hat{v}_{\cdot j}^\sigma$)</u>
1. Clothing (7)	1. Clothing (7)
2. Other non-food (9)	2. Other non-food (9)
3. Milk and milk products (2)	3. Milk and milk products (2)
4. Sugar and gur. (5)	4. Meat, fish and eggs (4)
5. Meat, fish and eggs (4)	5. Sugar and gur (5)
6. Edible oils (3)	6. Edible oils (3)
7. Other food (6)	7. Other food (6)
8. Fuel and light (8)	8. Fuel and light (8)
9. Cereals (1)	9. Cereals (1)

Yet more strikingly, the right-hand side ranking is invariant for all values of σ in the range, say, $\sigma > 0.1$, except possibly for the exact position of "fuel and light" within the last three cells. To put it another way, and conclude, when inequality aversion is strong, frontiers tend to cross in a rather extreme range of (v_f, v_l) -space, and therefore the region where controversy may arise over suitably constructed pairwise reforms considerably shrinks.

4) Let us now pay some attention to a few individual commodities.

We have already noted the rather poor performance of the "milk and milk products" (2) group on the distributional side, reflected by the strongly inwards position of its frontier in Figure C. Planners have been keen, however, to subsidize this apparently "essential" commodity, and indeed, according to our data, its overall (and negligible) tax rate is composed both of a sales tax and of a subsidy.^{8/} The desirability of milk subsidy schemes^{9/} appears questionable if their regressiveness is

a cause of concern. Matters would be different, of course, if these schemes specifically benefited the poor; as they are administered, however, they cater predominantly to the urban middle-class. If such policies are politically motivated, it is wise to nevertheless recognize their social cost.

By contrast, a case seems to exist for reducing taxes on the "other food" category (6). The strongly outwards position of its frontier in Figure B is due to the very high existing tax it bears (combined with a rather average mix of labour and foreign exchange in its production costs). This relative position is preserved or strengthened in Figure C, except against "cereals" (1), thanks to a favourable distributional characteristic. However, a glance at the composition of tax revenue in the "other food" category ^{10/} reveals a major share originating from tobacco manufactures, and to a lesser extent beverages. Our earlier tentative conclusion may then have to be qualified in two ways. Firstly, the distributional characteristic for the heavily taxed subgroup may be at variance with that of the group as a whole; presumably depending upon the distribution of the tax burden ^{11/} between cigarettes and "bidies". Secondly, there clearly are special considerations behind the high existing tax rates applying to these items, perhaps connected with their "merit" qualities. In these cases, therefore, the discussion may have to take place at a more disaggregated level before meaningful recommendations are possible.

On both figures, "fuel and light" stands out by its

extremely steeply sloped frontier, reflecting a very high foreign exchange cost of production. The latter is easily traced to the contribution of the "petroleum products" item, which accounts for slightly over 25% of the gross output of this group; its (direct) foreign exchange cost of production (at CIF prices) per rupee of gross output is around Rs.0.6, all on account of intermediate imports of "Petroleum and Natural Gas". For the "fuel and light" category, then, the relative price of labour and foreign exchange in the SP system is a particularly crucial parameter for tax policy decisions; in Figure B, the relative position of its frontier along a ray cuts right through the whole commodity vector as one moves across the positive orthant. As a corollary, the tax policy issue for this item has a fairly simple structure, and can largely dispense with sophisticated estimates of its distributional characteristic and demand parameters. This is fortunate, because the highly heterogeneous composition of the "fuel and light" category (exhibiting such contrasting items as firewood, electricity and petroleum products) makes the interpretation of its distributional characteristic delicate and the estimation of its demand parameters rather unreliable.

Perhaps the most intuitively appealing result of the exercise is the complete "dominance" of cereals over all other commodities at high levels of inequality aversion, where a cereal tax (or subsidy) exhibits both the lowest foreign exchange cost and the lowest labour cost per unit welfare gain (except compared to "fuel and light", in a very narrow range of the SP-space). Hence, as shown in (I5) above, the introduction or encouragement

of a cereal subsidy, suitably compensated by an increase in any other commodity tax (or combination of them), will be welfare-improving irrespective of the actual value of ψ . This result only confirms and generalizes the observations of Ahmad-Stern (1); but it should be qualified in at least two ways.

First, the simple exercise performed here can at best provide a first step towards a convincing evaluation of a cereals subsidy; the latter would appeal to a more careful modelling of the foodgrains market, involving producers as well as consumers' response and distributional characteristics.

Second, the kind of taxes and subsidies envisaged in our analysis may not be feasible at all in practice for commodities such as cereals, as when the phenomenon of subsistence production prohibits the taxation of production as opposed to sales. The evaluation of a feasible subsidy may be very different from that of an unconstrained subsidy of the type considered here. In this respect, it is worth remembering that, as argued in Chapter III, the huge "foodgrains subsidy" poured into the Public Distribution System does not presently function as an indirect tax or subsidy, but rather as an implicit transfer to the urban classes, and as such seems regressive rather than progressive.

At the other end of the dominance tree (see Figure D) we find "other non-food" and, at the rock bottom, "clothing". The last observation may seem surprising at first sight. An examination of the consumer expenditure matrix, however, confirms that

the average propensity to consume on clothing steadily increases with income, ranging from less than 3% for most poorer groups to 16% for the highest rural expenditure class. On the other hand, the distribution across finer commodity groups of the existing tax on "clothing" seems distinctly progressive, ranging from a small nominal subsidy on "Handloom and Khadi cotton textiles" to a tax rate of 16% on "art silk fabrics". There seems to be some scope, therefore, to increase the tax on clothing according to the existing schedule (but not by evening it out!).

The three remaining commodity groups, namely "edible oils", "meat, fish and eggs" and "sugar and gur" tend to bunch in the middle of our tables and figures and may be considered as a fairly homogeneous "reference" subgroup.

5) Let us conclude with a brief discussion of "suggestive" shadow prices. It should be emphasized at the outset that a proper choice of the shadow wage and social value of foreign exchange cannot dispense with a careful analysis of trade as well as direct transfer policies, which lies beyond the scope of this study. The suggestive shadow price pairs examined at this stage are relevant only when the planner is constrained to resort to indirect taxation in order to maintain full employment and balance-of-payments equilibrium, a rather implausible situation.

To recall our earlier theory, any pair (t_j, t_k) of taxes provides a potential "closure" of the model and induces a shadow price pair for scarce resources (lying at the intersection of Q_j^* and

Q_k^*), which is non-negative iff no dominance relation exists between t_j and t_k (see (I2)). Matrices of suggestive shadow prices are shown in Table 3 for $\delta = 0$, $\delta = 1$ and $\delta = 5$.

Even for $\delta = 0$, many of these shadow price configurations are intuitively unappealing, suggesting that the choice of a closing rule must involve more than the verification of a simple non-singularity condition. Intuitively, the best candidate for maintaining balance-of-payments equilibrium is the tax on "fuel and light" (the most efficient foreign-exchange saver). Considering the domestic resource next, and looking through column 8 of the first three matrices, many combinations of taxes seem possible. Most imply a premium on foreign exchange, presumably because more foreign exchange allows a reduction of the heavy existing tax on fuel and light, and hence a greater harmony between consumer tastes and technology. This intuition is confirmed by the exception to the rule: the pair consisting of "fuel and light" and "other food" implies a premium on labour, because "other food" is also heavily overtaxed ((see 4) above) if $\delta = 0$.

Quantitatively, however, a tax on, say, milk is not a very powerful instrument to maintain full domestic capacity! The best candidates to complete the closing rule therefore seem to be "cereals", "other food" and "other non-food", yielding the following suggestive shadow prices :

TABLE 3 Suggestive shadow prices

(i)

$\delta = 0$

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
Suggestive shadow price of foreign exchange $(\hat{v}_f(j,k))$									
CL	0.000	2.493	2.416	8.435	6.585	-10.510	3.088	1.787	2.070
MM	2.493	0.000	2.453	0.910	3.832	14.822	2.764	1.867	2.249
EO	2.416	2.453	0.000	3.398	-1.003	-1.600	0.105	1.686	0.803
MFE	8.435	0.910	3.398	0.000	7.143	-24.945	4.163	1.961	2.916
SG	6.585	3.832	-1.003	7.143	0.000	-1.964	-1.774	1.468	-0.404
OF	-10.510	14.822	-1.600	-24.945	-1.964	0.000	-1.914	1.065	-1.220
CT	3.088	2.764	0.105	4.163	-1.774	-1.914	0.000	1.631	0.487
FL	1.787	1.867	1.686	1.961	1.468	1.065	1.631	0.000	1.726
ONF	2.070	2.249	0.803	2.916	-0.404	-1.220	0.487	1.726	0.000
Suggestive shadow wage $(\hat{v}_e(j,k))$									
CL	0.000	0.943	0.946	0.680	0.762	1.517	0.917	0.974	0.961
MM	0.943	0.000	0.944	0.982	0.910	0.636	0.936	0.958	0.949
EO	0.946	0.944	0.000	0.882	1.168	1.207	1.096	0.994	1.051
MFE	0.680	0.982	0.882	0.000	0.732	2.019	0.852	0.940	0.902
SG	0.762	0.910	1.168	0.732	0.000	1.220	1.210	1.036	1.136
OF	1.517	0.636	1.207	2.019	1.220	0.000	1.218	1.115	1.194
CT	0.917	0.936	1.096	0.852	1.210	1.218	0.000	1.004	1.073
FL	0.974	0.958	0.994	0.940	1.036	1.115	1.004	0.000	0.986
ONF	0.961	0.949	1.051	0.902	1.136	1.194	1.073	0.986	0.000
Suggestive SP ratio $(\hat{\sigma}(j,k))$									
CL	0.000	0.378	0.392	0.081	0.116	-0.144	0.297	0.545	0.464
MM	0.378	0.000	0.385	1.080	0.237	0.043	0.339	0.513	0.422
EO	0.392	0.385	0.000	0.260	-1.165	-0.754	10.397	0.590	1.309
MFE	0.081	1.080	0.260	0.000	0.103	-0.081	0.205	0.479	0.309
SG	0.116	0.237	-1.165	0.103	0.000	-0.621	-0.682	0.706	-2.810
OF	-0.144	0.043	-0.754	-0.081	-0.621	0.000	-0.636	1.046	-0.978
CT	0.297	0.339	10.397	0.205	-0.682	-0.636	0.000	0.616	2.203
FL	0.545	0.513	0.590	0.479	0.706	1.046	0.616	0.000	0.571
ONF	0.464	0.422	1.309	0.309	-2.810	-0.978	2.203	0.571	0.000

cont'd....

(ii) $\delta = 1$

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
Suggestive shadow price of foreign exchange $(\hat{v}_f(j,k))$									
CL	0.000	8.793	-2.487	26.990	-6.565	4.148	-10.426	0.656	-6.235
MM	8.793	0.000	2.937	3.942	3.767	13.197	0.039	1.580	0.096
EO	-2.487	2.937	0.000	2.321	0.857	-0.426	24.819	1.161	-19.986
MFE	26.990	3.942	2.321	0.000	3.567	-13.256	-2.904	1.347	-1.821
SG	-6.565	3.767	0.857	3.567	0.000	-1.208	-15.793	1.136	-6.054
OF	4.148	13.197	-0.426	-13.256	-1.208	0.000	-5.065	0.861	-3.519
CT	-10.426	0.039	24.819	-2.904	-15.793	-5.065	0.000	1.986	0.288
FL	0.656	1.580	1.161	1.347	1.136	0.861	1.986	0.000	2.127
ONF	-6.235	0.096	-19.986	-1.821	-6.054	-3.519	0.288	2.127	0.000
Suggestive shadow wage $(\hat{v}_l(j,k))$									
CL	0.000	0.172	0.670	-0.631	0.850	0.377	1.021	0.531	0.836
MM	0.172	0.000	0.318	0.293	0.297	0.063	0.390	0.351	0.388
EO	0.670	0.318	0.000	0.358	0.453	0.536	-1.104	0.433	1.808
MFE	-0.631	0.293	0.358	0.000	0.308	0.982	0.567	0.397	0.524
SG	0.850	0.297	0.453	0.308	0.000	0.563	1.344	0.438	0.823
OF	0.377	0.063	0.536	0.982	0.563	0.000	0.698	0.492	0.644
CT	1.021	0.390	-1.104	0.567	1.344	0.698	0.000	0.272	0.375
FL	0.531	0.351	0.433	0.397	0.438	0.492	0.272	0.000	0.245
ONF	0.836	0.388	1.808	0.524	0.823	0.644	0.375	0.245	0.000
Suggestive SP ratio $(\hat{\sigma}(j,k))$									
CL	0.000	0.020	-0.269	-0.023	-0.130	0.091	-0.098	0.810	-0.134
MM	0.020	0.000	0.108	0.074	0.079	0.005	9.945	0.222	4.063
EO	-0.269	0.108	0.000	0.154	0.528	-1.260	-0.044	0.373	-0.090
MFE	-0.023	0.074	0.154	0.000	0.086	-0.074	-0.195	0.295	-0.288
SG	-0.130	0.079	0.528	0.086	0.000	-0.467	-0.085	0.386	-0.136
OF	0.091	0.005	-1.260	-0.074	-0.467	0.000	-0.138	0.571	-0.183
CT	-0.098	9.945	-0.044	-0.195	-0.085	-0.138	0.000	0.137	1.300
FL	0.810	0.222	0.373	0.295	0.386	0.571	0.137	0.000	0.115
ONF	-0.134	4.063	-0.090	-0.288	-0.136	-0.183	1.300	0.115	0.000

Cont'd....

(111)

$\delta = 5$

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
Suggestive shadow price of foreign exchange $(\hat{v}_f^{(j,k)})$									
CL	0.000	7.168	-3.074	21.262	-8.561	5.983	-9.027	0.122	-5.097
MM	7.168	0.000	1.851	3.411	2.020	8.291	-0.208	0.922	0.070
EO	-3.074	1.851	0.000	0.896	1.426	-0.260	17.403	0.635	-12.519
MFE	21.262	3.411	0.896	0.000	0.445	-5.658	-2.938	0.677	-1.596
SG	-8.561	2.020	1.426	0.445	0.000	-1.288	-9.675	0.699	-3.198
OF	5.983	8.291	-0.260	-5.658	-1.288	0.000	-3.506	0.466	-2.199
CT	-9.027	-0.208	17.403	-2.938	-9.675	-3.506	0.000	1.219	1.020
FL	0.122	0.922	0.635	0.677	0.699	0.466	1.219	0.000	1.236
ONF	-5.097	0.070	-12.519	-1.596	-3.198	-2.199	1.020	1.236	0.000
Suggestive shadow wage $(\hat{v}_l^{(j,k)})$									
CL	0.000	-0.118	0.334	-0.741	0.576	-0.066	0.597	0.193	0.423
MM	-0.118	0.000	0.014	-0.025	0.010	-0.146	0.065	0.037	0.058
EO	0.334	0.014	0.000	0.076	0.042	0.151	-0.997	0.093	0.948
MFE	-0.741	-0.025	0.076	0.000	0.094	0.339	0.230	0.085	0.176
SG	0.576	0.010	0.042	0.094	0.000	0.187	0.636	0.081	0.289
OF	-0.066	-0.146	0.151	0.339	0.187	0.000	0.264	0.126	0.219
CT	0.597	0.065	-0.997	0.230	0.636	0.264	0.000	-0.021	-0.009
FL	0.193	0.037	0.093	0.085	0.081	0.126	-0.021	0.000	-0.024
ONF	0.423	0.058	0.948	0.176	0.289	0.219	-0.009	-0.024	0.000
Suggestive SP ratio $(\hat{\sigma}^{(j,k)})$									
CL	0.000	-0.016	-0.109	-0.035	-0.067	-0.011	-0.066	1.580	-0.083
MM	-0.016	0.000	0.008	-0.007	0.005	-0.018	-0.313	0.040	0.835
EO	-0.109	0.008	0.000	0.085	0.029	-0.582	-0.057	0.147	-0.076
MFE	-0.035	-0.007	0.085	0.000	0.212	-0.060	-0.078	0.125	-0.110
SG	-0.067	0.005	0.029	0.212	0.000	-0.145	-0.066	0.115	-0.090
OF	-0.011	-0.018	-0.582	-0.060	-0.145	0.000	-0.075	0.270	-0.099
CT	-0.066	-0.313	-0.057	-0.078	-0.066	-0.075	0.000	-0.017	-0.009
FL	1.580	0.040	0.147	0.125	0.115	0.270	-0.017	0.000	-0.020
ONF	-0.083	0.835	-0.076	-0.110	-0.090	-0.099	-0.009	-0.020	0.000

Cont'd.....

Explanatory notes:

- (1) For each pair (j,k) , the pair $(\hat{v}_f(j,k), \hat{v}_\ell(j,k))$ solves the equations

$$\pi_j^* = 0 \text{ and } \pi_k^* = 0$$

$$(2) \quad \hat{\sigma}(j,k) \equiv \hat{v}_\ell(j,k) / \hat{v}_f(j,k)$$

- (3) For commodity codes, see previous tables.

		v_F	v_L	σ
(104)	(FL, CL) :	1.79	0.97	0.55
	(FL, OF) :	1.07	1.12	1.05
	(FL, ONF) :	1.73	0.99	0.57

Looking at column 8 of matrices 1 and 2, one easily surmises that any of these SP pairs imply negative marginal social values of income $\{\theta^i\}$ where as usual

$$(105) \quad \theta^i \equiv \frac{\partial \mathcal{L}}{\partial r^i} = \frac{\partial \Psi}{\partial v^i} \cdot \frac{\partial v^i}{\partial r^i} - \sum_{j \in J} \eta_j \frac{\partial x_j^i}{\partial r^i}$$

More careful calculations confirm this intuition. In other words, a switch from indirect to direct taxation is desirable (when $\delta = 0$).

When inequality aversion is introduced, suggestive shadow prices become increasingly unstable and more frequently negative, as one should expect since dominance relationships become more likely. When $\delta = 5$, very few appealing shadow price pairs are left. The tax on "fuel and light" seems to be a compelling element of any closing rule (the pair (MM, ONF) seems too versatile; it gives negative SP's for close but lower values of δ). For the three tax pairs introduced above, we now obtain:

		v_f	v_l	σ
(106)	(FL,CL) :	0.12	0.19	1.58
	(FL,OF) :	0.47	0.13	0.27
	(FL,ONF) :	1.24	-0.02	-0.02

Notice that the pair (FL,CL) no longer implies a premium on foreign exchange: greater domestic resources now permit lower cereal prices, with large distributional gains.

Section IV.4 : Two simple extensions

In this section we shall outline, without attempting an in-depth analysis, two simple extensions of the model studied in the previous sections. The first gives a straightforward treatment of lump-sum transfers. The second brings the estimation of the shadow wage within the scope of the model by giving an explicit treatment of the labour market, and shows how unemployment affects shadow prices as well as policy recommendations.

§IV.4.1 : Direct taxation

In the model of Section IV.2, the usual formula for the marginal social value of lump-sum transfers applies:

$$(107) \quad \theta^i = \frac{\partial \mathcal{L}}{\partial r^i} = \frac{\partial \Psi}{\partial v^i} \frac{\partial v^i}{\partial r^i} - \eta \frac{\partial x^i}{\partial r^i}$$

Given the shadow price formula of Propn.1, (107) here specializes to

$$(108) \quad \theta^i = (1-\gamma^i) \left(\mu^i - v_f^i F \frac{\partial x^i}{\partial e^i} - v_\ell^i L \frac{\partial x^i}{\partial e^i} \right)$$

$$= \text{not. } (1-\gamma^i) (\mu^i - v_f^i b_f^i - v_\ell^i b_\ell^i)$$

where b_f^i and b_ℓ^i are, respectively, the marginal (direct and indirect) foreign exchange content and marginal (direct and

indirect) labour content of i 's expenditure, while γ^i is i 's marginal savings rate.

It is a straightforward exercise, then, to adapt the methodology developed above in order to obtain an integrated picture of the related problems of shadow pricing, direct and indirect taxation.

To illustrate, Figures E and F reproduce Figures B and C and complete them with a selection of frontiers Q_*^i , where Q_*^i has equation

$$(109) \quad \theta^i = 0$$

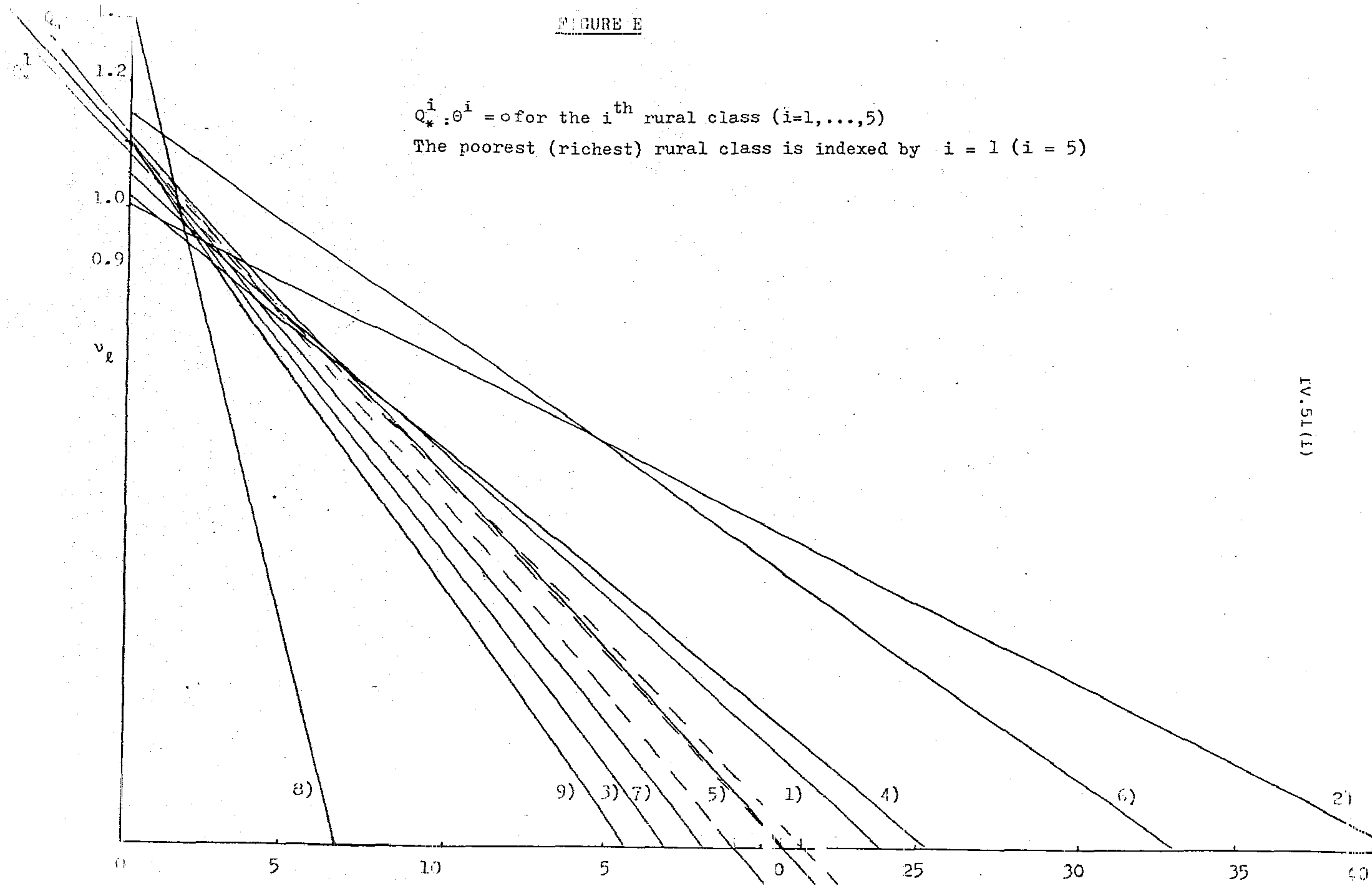
Not surprisingly, these frontiers are very close when $\delta = 0$, since then their distance hinges upon differences in marginal expenditure patterns, which tend to be rather small. Transfers from the rich to the poor seem appropriate when foreign exchange is particularly scarce, because the rich have a more foreign-exchange intensive marginal consumption basket. The reverse applies when labour is at a high social premium.

If the Ahmad-Sternone-factor model is applicable (i.e. $\sigma^* = 1$), the "suggestive" shadow price of the scarce resource emerging from an analysis of direct taxation lies between 1.02 and 1.06, according to the identity of the tax-payers. This is always slightly above unity, as one should expect when initial (indirect) taxes are positive on average; when $\mu^i = 1$ and $v_F = v_G = \lambda$,

FIGURE E

$Q_*^i, \theta^i = 0$ for the i^{th} rural class ($i=1, \dots, 5$)

The poorest (richest) rural class is indexed by $i = 1$ ($i = 5$)



then

$$(110) \quad \theta^i = 0 \quad \text{only if} \quad 0 = 1 - \lambda p \frac{\partial x^i}{\partial e^i} = 1 - \lambda + \lambda t \frac{\partial x^i}{\partial e^i}$$

$$(111) \quad \text{i.e. only if} \quad \lambda = \hat{\lambda}^i \equiv \frac{1}{1 - t \frac{\partial x^i}{\partial e^i}} > 1 \quad \text{as soon as} \quad t \frac{\partial x^i}{\partial e^i} > 0$$

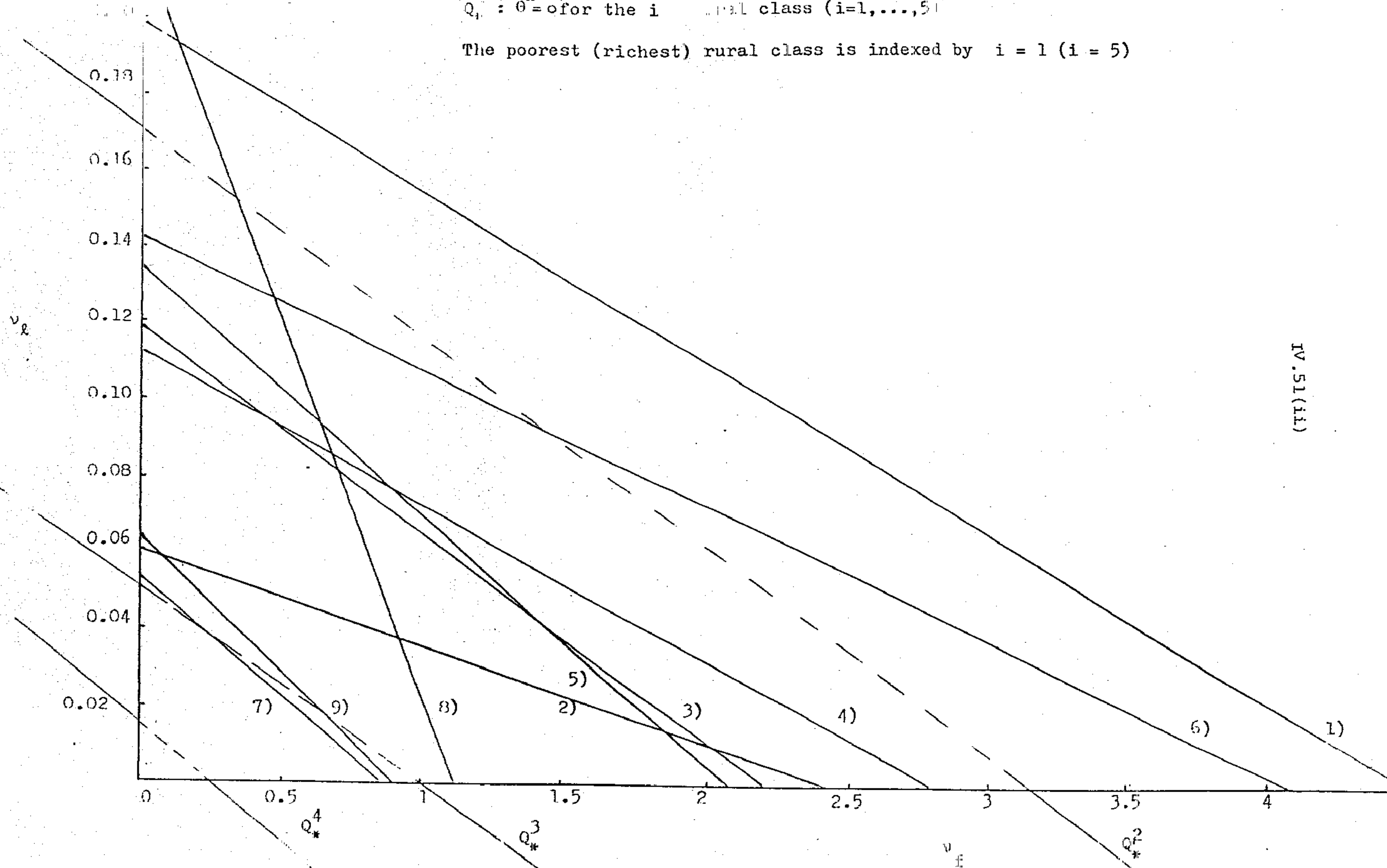
Again when $\sigma = 1$ and $\delta = 0$, the rich are slightly more "income-deserving" than the poor because their marginal consumption basket is more tax-intensive (so that their utility is less costly in terms of real resources).

When $\delta = 5$, the picture becomes obviously dominated by the overwhelming desirability of lifting up the poor. Progressive transfers are the only acceptable ones and are "robust" reforms. Any shadow price pair can emerge from a suitable combination of direct and indirect taxation.

It is a straightforward exercise to compute suggestive SP pairs by combining reforms of the direct as well as indirect tax structure; but we shall not enter into this issue here.

$Q_i : \theta = 0$ for the i rural class ($i=1, \dots, 5$)

The poorest (richest) rural class is indexed by $i = 1$ ($i = 5$)



IV.51(11)

then

$$(110) \quad \theta^i = 0 \quad \text{only if} \quad 0 = 1 - \lambda p \frac{\partial x^i}{\partial e^i} = 1 - \lambda + \lambda t \frac{\partial x^i}{\partial e^i}$$

$$(111) \quad \text{i.e. only if} \quad \lambda = \lambda^i \equiv \frac{1}{1 - t \frac{\partial x^i}{\partial e^i}} > 1 \quad \text{as soon as} \quad t \frac{\partial x^i}{\partial e^i} > 0$$

Again when $\sigma = 1$ and $\delta = 0$, the rich are slightly more "income-deserving" than the poor because their marginal consumption basket is more tax-intensive (so that their utility is less costly in terms of real resources).

When $\delta = 5$, the picture becomes obviously dominated by the overwhelming desirability of lifting up the poor. Progressive transfers are the only acceptable ones and are "robust" reforms. Any shadow price pair can emerge from a suitable combination of direct and indirect taxation.

It is a straightforward exercise to compute suggestive SP pairs by combining reforms of the direct as well as indirect tax structure; but we shall not enter into this issue here.

IV.4.2 : Involuntary unemployment

To conclude, let us consider a situation of involuntary unemployment and excess capacity, where the level of employment becomes endogenous to the model. We then obtain the additional "shadow price equation"

$$(112) \quad \frac{\partial \mathcal{L}}{\partial \ell^*} = 0$$

When there is no disutility of labour, (112) easily reduces to:

$$(113) \quad v_\ell = - \sum_i \theta^i \frac{\partial \ell^i}{\partial \ell^*}$$

confirming the shadow wage formula under involuntary unemployment found in Chapter II (see (47*) on p.II.18).

Combining (108) and (113), then:

$$(114) \quad v_\ell = - \sum_i (1-\gamma^i) (\mu^i - v_f b_f^i - v_\ell b_\ell^i) \frac{\partial \ell^i}{\partial \ell^*}$$

$\Rightarrow$

$$(115) \quad v_\ell = - \frac{\sum_i (1-\gamma^i) (\mu^i - v_f b_f^i) \frac{\partial \ell^i}{\partial \ell^*}}{1 - \sum_i (1-\gamma^i) b_\ell^i \frac{\partial \ell^i}{\partial \ell^*}}$$

Foreign exchange is now the only effectively scarce factor and by substituting (115) into our earlier expressions we may find, say, the critical value $\hat{v}^{(j)}$ of v_f which solves

$$(116) \quad \pi_j^* = 0 \quad \text{for } v_f$$

These critical values are reproduced in Table 4, for
^{12/}
 $\delta = 0, 1$ and 5 ; they are readily interpreted as trade-offs between foreign exchange and welfare implicit in the corresponding tax instruments. The outstanding position of "fuel and light" is not surprising. The high values obtained reflect the large multiplier effects associated with greater foreign exchange availability when domestic excess capacity exists. Quite significantly, the ranking of these critical values is rather insensitive to the inequality aversion parameter : saving foreign exchange has become an overriding "intermediate" objective, eventually furthering the welfare of the poor as well as the rich. Notice finally how these trade-offs look significantly higher than those which would emerge from an analysis of international trade (recall R4, section IV.1, and Chapter II, (50)). Indeed the situation envisaged here is plausible only if all import-substitution and export-promotion opportunities have been exhausted.

As a concluding exercise, it is interesting to compare the tax recommendations one arrives at in two alternative single-scarce-factor models. The first is the simplest version of the Ahmad-Stern full employment model, discussed in § IV.3.3, pp.33-35. The second is the present model with involuntary unemployment. Table 5 gives the ranking of commodities in terms of the trade-off between welfare and the scarce resource implicit in the corresponding tax instrument, for both models and two extreme distributional judgments. Not surprisingly, few pairwise tax recommendations resist such dramatic sensitivity exercises. These are useful, however, in bringing out the considerations of particular significance in different problems, e.g. the importance

TABLE 4 Break-even values of the shadow price of foreign exchange,
with excess domestic capacity.

	$\hat{v}_f^{(j)}$		
	$\delta = 0$	$\delta = 1$	$\delta = 5$
CL	22.61 (4)	11.82 (3)	4.36 (1)
MM	29.44 (2)	12.22 (2)	2.37 (5)
EO	17.57 (7)	8.17 (6)	2.27 (6)
MFE	24.34 (3)	11.25 (4)	3.57 (3)
SG	20.77 (5)	9.42 (5)	2.44 (4)
OF	32.16 (1)	14.67 (1)	4.17 (2)
CT	18.41 (6)	6.73 (7)	1.06 (8)
FL	6.34 (9)	3.43 (9)	1.11 (7)
ONF	15.71 (8)	5.62 (8)	0.90 (9)

Explanatory notes:

(1) The numbers in brackets indicate rank-orders

(2) The derivation of $\hat{v}_f^{(j)}$ is explained in the text (p.53-54). The marginal value-added shares $\left\{ \frac{\partial \ell^i}{\partial \ell^*} \right\}$ have been inferred by assuming constant value-added shares in each industry, and using the estimates of the distribution of value-added from the core data set along with the usual aggregation formulae (see Annex 1, § A.IV and § A.V).

TABLE 5 Two alternative single-scarce-resource models

Rank	$\delta = 0$		$\delta = 5$	
	Full Employment	Unemployment	Full Employment	Unemployment
1	MFE	FL	CT	ONF
2	MM	ONF	MM	CT
3	CL	EO	ONF	FL
4	ONF	CT	MFE	EO
5	EO	SG	SG	MM
6	CT	CL	EO	SG
7	SG	MFE	OF	MFE
8	FL	MM	FL	OF
9	OF	OF	CL	CL

Explanatory notes:

- (1) Each column gives the ranking of commodities in increasing order of the break-even value of the shadow price of the resource, in a particular model and for a particular value of the inequality aversion parameter.
- (2) Columns 2 and 4 correspond to the "full employment" model, with $\sigma = 1$. They therefore match Table 2.
- (3) Columns 3 and 5 correspond to the "excess capacity" model, and match Table 4.
- (4) CL = cereals
MM = milk and milk products
EO = edible oils
MFE = meat, fish and eggs
SG = sugar and gur
OF = other food
CT = clothing
FL = fuel and light
ONF = other non-food

of a correct choice of model for fuel tax policy, or of inequality aversion for cereal tax policy, or again the strength of the prima facie case for reducing the tax on "other food". The only feature which stands out in both models is the desirability of subsidizing cereals when value-judgments are strongly egalitarian: as we should expect, analysts overwhelmingly sympathetic to the poor may agree on the opportunity of such a reform, even when they hold very different views about the way the economy functions.

Section IV.5 : Concluding comments

The results of our investigation are too varied and special to lend themselves to a useful summary. The exercise, however, is not devoid of wider methodological lessons. Two of them perhaps deserve special mention.

First, the discussion of tax reform seems to benefit from a careful blend of intuition and theory, going beyond either sophisticated mathematical algorithms or journalistic discussion. Economic analysis and econometric observation are invaluable in exercising, sharpening or refuting our intuition about tax policies. But equally well, intuition and judgment should remind us of the limitations of our models, as when they leave out "the merit qualities" of some commodities or suffer from aggregation biases. The analysis of tax policy seems to be a particularly challenging meeting ground for informed common sense and theoretical analysis, and in this chapter we have, as a rule, found a strong degree of harmony between the two.

Hence a second lesson: an empirical approach to shadow pricing and project evaluation problems seems to qualify to some extent the negative theoretical results of Chapter I, stating the impossibility, in principle, of unambiguously evaluating "potentially feasible reforms". Consider for instance the issue of whether a marginal shift of the tax burden away from cereals should be implemented. As we have noticed, and leaving aside here all the limitations of the model, the marginal social value of this reform is positive in a very wide range of the shadow price space, and very differing views about the functioning of the economy are consistent with its approval. Of course, as the theory of Chapter I tells us, it is always possible to devise a suitable "closing rule" under which the proposed policy will appear unfavourable. However, the perversion of such constructs will normally be apparent (e.g. in the form of negative shadow prices). As long as our theoretical tools are not powerful enough to rule them out, we should be able to dispose of them by careful introspection or informed discussion.

Footnotes : Chapter IV

- 1/ The more sophisticated work of Ahmad-Stern (1) stresses the fact that the Indian indirect tax system drives a wedge between buying and selling prices rather than between producer and consumer prices, so that private producers pay taxes on their purchases of intermediates. For clarity of exposition, we have adopted the simple and more conventional assumption of production efficiency in the private sector; however, the empirical calculations have been performed both using the simple formula (4) and the more plausible formula:

$$p = (p+t)A^D + (p^m+t^m)A^m + \pi$$

where π is a vector of profit rates. It turned out that both specifications gave close results, so that no further account of these complications seems to be necessary for our limited purposes.

- 2/ Government of India (3), p.4.
- 3/ See Chapter II, Section II.2, (24).
- 4/ The scale of the axes on Figures B and C should not confuse the reader: the foreign exchange cost of a unit of welfare is generally much lower than its labour cost simply because non-competitive intermediates contribute only a small fraction of average costs. The deflation in the shadow price system when inequality aversion increases reflects the normalization rule $\mu^1 = 1$.
- 5/ By "45° line" we mean the line passing through the origin and the point (1,1). Because of an asymmetric choice of units, it does not have the conventional slope in Figures B and C.
- 6/ Although a broad agreement seems to exist between the findings of Ahmad-Stern and ours, some discrepancies remain due to differences in models, estimation procedures, aggregation keys, etc. Admittedly, a more careful comparison of the respective approaches and results should be carried out at some stage.
- 7/ With our normalization rule:

$$D_j \in \left[\frac{x_j^1}{x_j}, 1 \right]$$

and so

$$\frac{D_j}{D_k} \in \left[\frac{x_j^1}{x_j}, \frac{x_k^1}{x_k} \right]$$

- 8/ The nominal sales tax rate for this category is 0.1%. The nominal subsidy rate is 0.03%. See Ahmad-Stern (1), p.37 and 34, commodity category No.12.
- 9/ Such as the well-known Operation Flood of the Indian Dairy Corporation (see Dagli (2)).
- 10/ See Annex 1 and the aggregation key reproduced in Appendix A.4.2.to this chapter.
- 11/ A bidi is a cheap tobacco product, consisting of a single dried and rolled tobacco leaf.
- 12/ The coefficients $\frac{\partial \ell^i}{\partial \ell}$ have been computed by assuming fixed value added shares across consumer groups within each production sector.

Appendix A.4.1. Some formal proofs

We shall here prove statements (I1)-(I6) of § IV.3.2 under assumption A.1 of p.IV.29.

(I1) Immediate (see (36) on p.IV.16). #

(I2) See the argument on p.IV.17. Further

$$\hat{\sigma}^{(j,k)} \equiv \frac{\psi_k^f - \psi_j^f}{\psi_j^l - \psi_k^l} < 0 \Leftrightarrow \text{either } \psi_k < \psi_j \text{ or } \psi_j < \psi_k$$

establishing the last statement of (I2) (since $\hat{\sigma}^{(j,k)}$ must have at least one positive component). #

(I3) Let $\alpha_k = 1 - \alpha_j$ and $\alpha_g = 0$ ($g \neq j, k$). α is then resource-preserving for $\bar{\sigma}$ iff

$$\alpha_j \psi_j^f + (1 - \alpha_j) \psi_k^f + \bar{\sigma} \alpha_j \psi_j^l + \bar{\sigma} (1 - \alpha_j) \psi_k^l = 0$$

$$\Leftrightarrow \alpha_j [\hat{v}_f^{(j)}(\bar{\sigma})]^{-1} + (1 - \alpha_j) [\hat{v}_f^{(k)}(\bar{\sigma})]^{-1} = 0$$

$$\Leftrightarrow \alpha_j = \frac{\hat{v}_f^{(j)}(\bar{\sigma})}{\hat{v}_f^{(j)}(\bar{\sigma}) - \hat{v}_f^{(k)}(\bar{\sigma})}$$

$$\text{So } \alpha_j > 0 \Leftrightarrow \hat{v}_f^{(j)}(\bar{\sigma}) > \hat{v}_f^{(k)}(\bar{\sigma})$$

$$\Leftrightarrow \psi_j^f + \bar{\sigma} \psi_j^l < \psi_k^f + \bar{\sigma} \psi_k^l$$

$$\Leftrightarrow \text{either } \psi_j^\ell > \psi_k^\ell \text{ and } \bar{\sigma} < \frac{\psi_k^f - \psi_j^f}{\psi_j^\ell - \psi_k^\ell} \equiv \hat{\sigma}(j,k)$$

$$\text{or } \psi_j^\ell < \psi_k^\ell \text{ and } \bar{\sigma} > \hat{\sigma}(j,k)$$

Finally $\alpha_j > 0 \Leftrightarrow \alpha_k < 0$ since $\alpha_k = 1 - \alpha_j$ and $\alpha_j \in [0,1]$

#

(I4) Let $\alpha_k = 1 - \alpha_j$, $\alpha_g = 0$ ($g \neq j, k$), and $\psi_\alpha^f = 0$. Then

$$\alpha_j \psi_j^f + (1 - \alpha_j) \psi_k^f = 0 \Rightarrow \alpha_j = \frac{\psi_k^f}{\psi_k^f - \psi_j^f} \in [0,1]$$

So $\alpha_j > 0 \Leftrightarrow \psi_j^f < \psi_k^f \Leftrightarrow \alpha_k < 0$, as required.

Moreover

$$\begin{aligned} \pi_\alpha^* &= 1 - v^* \psi_\alpha \\ &= (\hat{v}(j,k) - v^*) \psi_\alpha \text{ since } \hat{v}(j,k) \psi_j = \hat{v}(j,k) \psi_k = 1 \\ &= (\hat{v}_\ell(j,k) - v_\ell^*) \psi_\alpha^\ell \text{ since } \psi_\alpha^f = 0 \end{aligned}$$

So $\pi_\alpha^* > 0 \Leftrightarrow$ either $v_\ell^* < \hat{v}_\ell(j,k)$ and $\psi_\alpha^\ell > 0$
or $v_\ell^* > \hat{v}_\ell(j,k)$ and $\psi_\alpha^\ell < 0$

But

$$1 = \hat{v}(j,k) \psi_\alpha = \hat{v}_\ell(j,k) \psi_\alpha^\ell \Rightarrow \psi_\alpha^\ell = \frac{1}{\hat{v}_\ell(j,k)}$$

so

$$\hat{v}_\ell(j,k) < 0 \Leftrightarrow (\psi_\alpha^\ell < 0 \text{ and } v_\ell^* > \hat{v}_\ell(j,k))$$

which completes the proof. #

(I5) Let $\alpha_k = 1 - \alpha_j$ and $\alpha_g = 0$ ($g \neq j, k$). α will be a robust reform iff

$$v\psi_\alpha < 1 \quad \forall v \geq 0$$

$$\Leftrightarrow \psi_\alpha \leq 0$$

$$\Leftrightarrow \alpha_j \psi_j + (1 - \alpha_j) \psi_k \leq 0$$

$$\Leftrightarrow \alpha_j (\psi_j - \psi_k) \leq -\psi_k < 0$$

$$\Leftrightarrow \text{either } \psi_j < \psi_k \text{ and } \alpha_j \geq \frac{-\psi_k^\ell}{\psi_j^\ell - \psi_k^\ell} > 1, \alpha_k = 1 - \alpha_j < 0$$

$$\text{or } \psi_j > \psi_k \text{ and } \alpha_j < 0, \alpha_k > 1 > 0$$

establishing (I5). #

(I6) Let $\alpha_j = 1 - \alpha_k - \alpha_g$ and $\alpha_h = 0$ ($h \neq j, k, g$). The reform α is robust iff

$$\psi_\alpha \leq 0$$

This can always be achieved by a suitable choice of α . Further,

when $\psi_\alpha \leq 0$ and $\hat{v}^{(k,g)} > 0$:

$$(1-\alpha_k-\alpha_g)\psi_j + \alpha_k\psi_k + \alpha_g\psi_g \leq 0$$

$$\Rightarrow \alpha_j(\hat{v}^{(k,g)}\psi_j) \leq -\alpha_k - \alpha_g \text{ since } \hat{v}^{(k,g)}\psi_k = \hat{v}^{(k,g)}\psi_g = 1$$

$$\Rightarrow \text{either } \alpha_j > 0 \text{ and } \hat{v}^{(k,g)}\psi_j \leq \frac{\alpha_j^{-1}}{\alpha_j} < 1$$

$$\text{or } \alpha_j < 0 \text{ and } \hat{v}^{(k,g)}\psi_j \geq \frac{\alpha_j^{-1}}{\alpha_j} > 1$$

(notice that $\alpha_j \neq 0$ since otherwise $\alpha_k + \alpha_g = 1$ and $\alpha_k\psi_k + \alpha_g\psi_g \leq 0$, implying a dominance relationship between t_k and t_g and violating $\hat{v}^{(k,g)} > 0$).

Thus, when $\psi_\alpha \leq 0$ and $\hat{v}^{(k,g)} > 0$:

$$\alpha_j > 0 \Leftrightarrow \hat{v}^{(k,g)} \in Q_j^+$$

as required.

#

Appendix A.4.2 The data

The calculations reported in this chapter rely on estimates of the following:

- (i) The matrix $((x_j^i))$ of consumer demands for commodities.
- (ii) The matrix $\frac{\partial x}{\partial q}$ of price derivatives of consumer demands
- (iii) The vector γ of marginal savings rates ($\gamma^i = 1 - \frac{\partial e^i}{\partial r^i}$).
- (iv) The matrix β of marginal budget shares ($\beta_j^i = q_j \frac{\partial x_j^i}{\partial e^i}$).
- (v) The vector t of indirect taxes.
- (vi) The matrices A^m , A^D of input-output coefficients.
- (vi) The vectors f , g of foreign exchange and labour coefficients.
- (vii) The welfare weights $\{u^i\}$.

The source of each of these estimates is explained in this Appendix. In a nutshell, all of them are simply taken from the core data set described in detail in Annex 1, following the aggregation scheme outlined in § A.V (Annex 1). The aggregation keys are shown in Table A.1 below. The only point which needs some formal caution (although its quantitative significance is very small) is the treatment of consumer imports, about which we shall here be more explicit.

- (i) Consumer demands: As in Annex 1, we shall consider consumer imports to be non-substitutable with domestic production. Their

quantities $\{\bar{x}_j^m\}$ and prices $\{\bar{q}_j^m\}$ are fixed and exogenous.

This is fairly realistic in view of the very strict quotas applying to consumer imports in India (these constitute, in the core data set, less than 5% of total consumer expenditure; according to some official statistical classification schemes, they even do not exist at all!).

If c_j is the aggregate consumer expenditure on good j , we now write

$$\begin{aligned} (1) \quad c_j &= q_j x_j + \bar{q}_j^m \bar{x}_j^m \\ &= (1 + t_j) x_j + \bar{q}_j^m \bar{x}_j^m \text{ by choice of units} \end{aligned}$$

The consumer demands $\{x_j^i\}$ in this chapter should therefore be interpreted to exclude consumer imports. Accordingly, the tax changes envisaged must be regarded as changes in those indirect taxes which apply to domestic production only (such as excise taxes, but not sales taxes or tariffs), though of course it is easy to carry out a parallel analysis for other kinds of taxes.

Let $1 - \omega_j$ be the percentage of consumer expenditure on the j^{th} good which consists of imports. For simplicity, we shall assume this ratio to be constant across income classes. Since the core data set of Annex 1 contains estimates of $\{c_j^i\}$, $\{\omega_j\}$ and $\{t_j\}$, it is then straightforward to estimate x_j^i as

$$(2) \quad x_j^i = \frac{\omega_j c_j^i}{1 + t_j} \quad \forall i \in I, \quad \forall j \in J$$

Table A.2 shows the matrix $((c_j^i))$ (implicitly, since the displayed matrix is in per capita terms; class sizes can be read from Table A.1) and the vectors (ω_j) and (t_j) for the 9 commodities, 10 income groups classification adopted in this exercise.

(ii) Price derivatives : Let Q_j be a price-index for the j^{th} commodity, of the form:

$$(3) \quad Q_j = \omega_j^0 q_j + (1 - \omega_j^0) \bar{q}_j^m$$

where $1 - \omega_j^0$ is the share of imports in consumer expenditure on good j in some base year. This form of the price index is in line with the methodology of the consumer demand studies upon which the price elasticity matrix of the core data set is based (see § A.IV.11, Annex 1). In these studies, no distinction is drawn between expenditure on imports and expenditure on domestic production; it seems natural therefore, to choose units for consumer imports so that, in the base year:

$$(4) \quad q_j = \bar{q}_j^m$$

This is permissible, and need not conflict with any of our other normalization rules, since consumer imports constitute in effect a separate set of commodities.

With $\bar{q}^m$ and $\bar{x}^m$ exogenous, let us write

$$(5) \quad x_j(q; \cdot) = \frac{c_j(Q(q); \cdot) - \bar{q}_j^m \bar{x}_j^m}{q_j}$$

with $Q_j(q)$ as in (3). From (5) we easily derive:

$$(6) \quad \frac{\partial x_j}{\partial q_k} = \frac{\partial c_j}{\partial Q_k} \frac{\omega_k^0}{q_j} - \delta_{jk} \frac{x_j}{q_j}$$

where δ_{jk} is Kronecker's delta.

The consumer demand studies used in building the core data set of Annex 1 attempt to estimate aggregate price elasticities $\{\epsilon_{jk}\}$ of the form

$$(7) \quad \epsilon_{jk} = \frac{\partial c_j}{\partial Q_k} \frac{Q_k}{c_j} - \delta_{jk}$$

Substituting (7) into (6), we obtain

$$(8) \quad \frac{\partial x_j}{\partial q_k} = \frac{\omega_k^0}{q_j} (\epsilon_{jk} + \delta_{jk}) \frac{c_j}{Q_k} - \delta_{jk} \frac{x_j}{q_j}$$

$$(9) \quad \approx \epsilon_{jk} \frac{c_j \omega_k^0}{q_j q_k} \quad \text{if } \omega_k \approx \omega_k^0 \text{ and } q_k \approx q_k^m$$

The last formula is exact in the base year, and otherwise approximate. It forms the basis of our estimate of the matrix of price derivatives. The matrix ϵ of price elasticities appears in Table A.3; it is discussed in detail in § A.IV.11 of Annex 1.

(iii) Marginal savings rates: These are taken without alteration from the core data set, suitably aggregated. They appear in Table A.4 below.

(iv) Marginal budget shares: When imports are rationed, the marginal budget shares applying to expenditure on domestic production are identical to the marginal budget shares for total consumption expenditure; formally, using the equivalent of (5) for the i^{th} consumer:

$$(10) \quad \frac{\partial(q_j x_j^i)}{\partial e^i} = \frac{\partial c_j^i}{\partial e^i} \quad \text{not.} \quad \beta_j^i$$

The matrix β can therefore also be obtained by straightforward aggregation from the core data set. The results are shown in Table A.4.

(v) Indirect taxes: The vector t of indirect taxes appearing in Table A.2 is again a product of the core data set and the aggregation formulae of Annex 1.

After completion of the exercise, a slight error was detected in the construction of the index weights used for the aggregation of tax rates. The vector t of Table A.2 is therefore not perfectly correct. However, the discrepancy is quantitatively insignificant and did not seem to warrant a new set of calculations.

(vi) Input-output matrices: By contrast with the model of this chapter, the core data set allows for joint production (see Annex 1, §A.IV.2). At the 9 commodity level of aggregation this refinement hardly makes any difference, since the corresponding "MAKE" matrix is very nearly an identity matrix (only one of its entries on the main diagonal falls short of 0.99). Although in the actual

calculations we have used formulae which allow for joint production, therefore, there is no need to go into these complications here.

When we ignore joint production, then, the matrices A^D and A^m can be identified with the matrices B^D and B^m of the core data set. The matrix B^D is reproduced in Table A.5, as well as the vector $p^m B^m$ (which is sufficient for our purposes). As in Annex 1, we have adopted the normalization rules

$$(11) \quad p_j = p_j^m = 1 \quad \forall j$$

which is permissible since imported intermediates are considered as separate commodities.

(vii) Foreign exchange and labour coefficients: The vector

$$(12) \quad f \equiv p^m A^m = p^m B^m$$

already appears in Table A.5. The vector g is then computed from the identity

$$(13) \quad g \equiv p - p^m A^m - p A^D$$

The latter vector is slightly different from what we would obtain by aggregation from the core data set; the discrepancy arising because in Annex 1 value-added coefficients are calculated under the (realistic) assumption that private producers pay indirect taxes on intermediates (this refinement of the model is discussed in footnote 1 to this chapter). Since internal consistency is

crucial for the calculations, (13) is the proper formula to use here.

(viii) Welfare weights: The estimation of the welfare weights has been discussed in the text. Although the calculations have been performed for a wide range of values of the inequality aversion parameter δ , the welfare weights reproduced in Table A.6 refer only to the values of 0, 1 and 5 for δ relevant for the results reported throughout this chapter.

TABLE A.1 Aggregation keys

(a) Commodities

Commodity group	Commodities covered	Consumer expenditure
Cereals (CL)	1,2,3,4,5	0.389
Milk and milk products (MM)	12	0.080
Edible oils (EO)	23,24	0.041
Meat, fish, eggs (MFE)	13,15	0.028
Sugar and gur (SG)	7,21,22	0.032
Other food (OF)	6,11,20,25,26,27	0.180
Clothing (CT)	28,29,30,31,32,33,34,35	0.065
Fuel and light (FL)	14,16,43,44,80	0.056
Other non-food (ONF)	8-10,17-19,36-42,45-79,81-89	0.129
Total	1-89	1.00

cont'd....

(b) Consumer groups

Consumer group	RURAL					URBAN					
	1	2	3	4	5	1	2	3	4	5	Total
Classes covered	1-7	8	9	10	11-14	15-21	22	23	24	25-28	1-28
Group size	0.196	0.151	0.158	0.146	0.119	0.029	0.035	0.045	0.052	0.070	1.00
Average per capita consumption expenditure	0.46	0.67	0.85	1.11	1.90	0.49	0.67	0.85	1.12	2.16	1.00

cont'd....

Explanatory notes:

- (1) In part (a) of the table, the commodities covered by each of our 9 broad commodity groups are indicated by their code number in the 89-commodity classification of Annex 1. The full list of these commodities and their code numbers can be found in Table B.1 of Annex 1.
- (2) In the last column of part (a) of the table, consumer expenditure is inclusive of consumer imports.
- (3) In part (b), the income classes covered by each of our 10 broad consumer groups is indicated by their code number in the 28-class breakdown of the population used in Annex 1 (see Table B.1 of Annex 1). The group sizes indicate their share of the total population.
- (4) For the application of the aggregation formulae of Section A.V. (Annex 1), the aggregation key for "industries" is the same as that for "commodities".
- (5) The normalization rules are:
aggregate consumer expenditure = 1 = total population.

TABLE A.2 Consumer expenditure and tax rates

	Matrix of per capita expenditures											
	Rural Classes					Urban Classes						
	1	2	3	4	5	1	2	3	4	5	w_j	t_j
CL	0.272	0.360	0.419	0.482	0.575	0.229	0.285	0.319	0.357	0.372	1.00	-0.027
MM	0.012	0.032	0.056	0.093	0.194	0.021	0.042	0.067	0.107	0.224	1.00	0.001
EO	0.016	0.026	0.033	0.043	0.063	0.023	0.036	0.049	0.065	0.101	0.74	0.100
MFE	0.010	0.016	0.022	0.031	0.050	0.014	0.020	0.028	0.039	0.075	0.98	0.000
SG	0.010	0.020	0.027	0.039	0.065	0.016	0.024	0.031	0.042	0.062	1.00	0.060
OF	0.075	0.115	0.144	0.183	0.300	0.098	0.135	0.175	0.231	0.505	1.00	0.163
CT	0.009	0.020	0.038	0.080	0.232	0.006	0.011	0.021	0.037	0.150	0.99	0.043
FL	0.033	0.042	0.049	0.059	0.081	0.036	0.048	0.057	0.073	0.110	0.85	0.166
ONF	0.021	0.037	0.058	0.099	0.336	0.041	0.068	0.105	0.165	0.552	0.99	0.033
Total	0.46	0.67	0.85	1.11	1.90	0.49	0.67	0.86	1.12	2.16		

Notational conventions:

- $1 - w_j$ = share of imports in consumer expenditure on good j
 t_j = tax rate on good j as a proportion of the producer price.

Explanatory notes:

- (1) For codes and normalization rules, see previous table.
(2) Column totals involve rounding.

TABLE A.3 Consumer price elasticity matrix

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
CL	-0.530	-0.018	-0.018	-0.038	-0.013	-0.038	-0.012	0.003	0.003
MM	-0.553	-1.130	-0.153	0.147	0.018	0.113	-0.033	-0.050	-0.035
EO	-0.287	-0.187	-0.478	0.050	-0.056	0.134	-0.016	-0.026	-0.024
MFE	-0.514	0.379	0.056	-0.846	0.011	0.087	-0.016	-0.030	-0.017
SG	-0.410	0.063	-0.084	0.003	-0.593	-0.165	-0.023	-0.400	-0.030
OF	-0.187	0.108	0.029	0.011	-0.023	-0.843	-0.016	-0.019	-0.009
CT	-0.297	-0.037	-0.028	-0.021	-0.017	-0.107	-0.739	-0.039	-0.025
FL	-0.131	0.002	-0.010	-0.008	-0.003	-0.039	-0.012	-0.486	-0.011
ONF	-0.298	-0.038	-0.030	-0.029	-0.018	-0.125	-0.032	-0.039	-0.939

Explanatory notes:

- (1) The $(j,k)^{th}$ entry of the table is

$$\epsilon_{jk} = \frac{\partial c_j}{\partial Q_k} \frac{Q_k}{c_j} - \delta_{jk} \quad (\text{see (7)})$$

- (2) For a detailed discussion of the construction of this matrix see § A.IV.11. of Annex 1.
- (3) For commodity codes see Table A.1 above.

TABLE A.5 Input-output coefficients

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
	Matrix of technical coefficients for domestic intermediates (A^D)								
CL	0.066	0.056	0.001	0.054	0.003	0.012	0.001	0.000	0.004
MM	0.000	0.000	0.000	0.000	0.000	0.020	0.000	0.000	0.001
EO	0.000	0.003	0.068	0.003	0.045	0.003	0.000	0.000	0.001
MFE	0.014	0.000	0.000	0.000	0.004	0.018	0.004	0.000	0.004
SG	0.000	0.000	0.000	0.000	0.316	0.005	0.000	0.000	0.000
OF	0.001	0.279	0.248	0.346	0.021	0.081	0.008	0.000	0.004
CT	0.000	0.018	0.007	0.016	0.007	0.001	0.193	0.001	0.005
FL	0.013	0.000	0.011	0.003	0.023	0.015	0.036	0.175	0.041
ONF	0.102	0.090	0.458	0.146	0.189	0.160	0.405	0.113	0.372
	Vector of direct foreign exchange costs (f)								
	0.022	0.000	0.039	0.000	0.009	0.005	0.010	0.241	0.029

Cont'd....

Explanatory notes:

- (1) The $(j,k)^{th}$ element of the matrix A^D indicates the expenditure on intermediate purchases of good k (domestically produced) per rupee of output of good j .
- (2) The j^{th} element of the vector f indicates the total foreign exchange cost (at c.i.f. prices, and with the exchange rate normalized to unity) of the imported intermediates required per rupee of gross output of good j .

TABLE A.6 Welfare weights

	RURAL CLASSES					URBAN CLASSES				
	1	2	3	4	5	1	2	3	4	5
$\delta = 0$	1	1	1	1	1	1	1	1	1	1
$\delta = 1$	1	0.69	0.54	0.42	0.24	0.94	0.69	0.54	0.41	0.21
$\delta = 5$	1	0.15	0.05	0.01	0.00	0.75	0.15	0.05	0.01	0.00

Explanatory note:

(1) The formula used is

$$w_i = \left(\frac{e^{-1}}{e^{-i}} \right)^\delta$$

where e^{-i} is average per capita expenditure in class i and $i = 1$ corresponds to the poorest rural class.

References : Chapter IV

- (1) Ahmad, S.E. and N.H.Stern (1983), "Effective taxes and tax reform in India", Discussion Paper No.25, Development Economics Research Centre, University of Warwick, January.
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- (3) Government of India (1981), A technical note on the sixth plan of India (1980-1985), Planning Commission, New Delhi.
- (4) Rudra, A. (1982), paper presented at the ISI Golden Jubilee Conference at the Indian Statistical Institute, Calcutta, January.
- (5) Squire, L. and H.Van Der Tak (1975), Economic Analysis of projects : IBRD, Washington.

CONCLUDING COMMENTS

The topics which we have touched upon in the four preceding chapters have a common inspiration. The evaluation of such schemes as a Food-for-Work programme should benefit from a better understanding of the general principles of shadow pricing (Chapter I), a reminder of some of the best known shadow pricing rules (Chapter II), a reflection on distributional issues in a dual economy (Chapter III), and an example of the potential usefulness and shortcomings of the "reform viewpoint" (Chapter IV). At the same time, each of these topics has been treated very much in isolation; without necessarily attempting at a grand synthesis, we should conclude with a short step backwards, aiming both at reconsidering our original enquiry, and identifying useful lines of further research. In doing so I shall draw to some extent upon the lessons I learnt from extensive exercises on shadow pricing with Indian data although, as stated in the Introduction, the result of these exercises cannot be presented here.

The problem of choosing shadow prices to evaluate such projects as the construction of an anicut through a rural public works scheme still appears to me to be a very intricate one. Even when conceptual issues are clarified, formidable problems of empirical application remain and different methodologies, none of which seems totally unattractive, can lead to very different results.

Much of the problem stems from the fact that in an economy lying away from an optimum, each resource seems to have a very different social opportunity cost in a variety of alternative uses. Our

estimate of the shadow price of a unit of cement affected to a FFW programme, for instance, can surely vary a great deal according as to whether we consider its provision to impose higher imports, promote domestic production, or reduce the size of the last stadium to be built in Delhi for the Asian Games.

In principle, economic theory suggests an unequivocal way of dealing with such situations. The "planner", whoever he is, should identify his opportunity set, and jointly solve the problems of choosing shadow prices and designing (constrained) optimal policies. In the above example, cement would then have the same social value in each alternative use which the planner can control, other alternative uses being irrelevant for the problem because effectively unavailable. This is the essential teaching of second-best theory for project evaluation.

Why, then, do we find it so hard to put these simple principles into practice? Our short experience of empirical estimation with Indian data consistently points to two major difficulties in applying the theory.

The first is a modelling problem. Much of the literature on "public economics" is founded on a gross dichotomy between a large number of rational "private agents" and a mythical "planner" entrusted with all public decisions. Such models can hardly explain why cement in India should have such apparently contrasting social opportunity costs in a variety of alternative uses, most of which are under the control of some public agency or the other. In reality, of course, governments are not monolithic; even worse, government

agencies need not behave according to systematic or observable rules. In practice, then, the choice of shadow prices may be very sensitive to the precise identity of the "planner", the opportunities available to him, and the behaviour of other government agencies. Realistic modelling will then require a great deal of practical acquaintance with the institutional features of the economy under study (rather than, or in addition to, an understanding of the various general "guidelines" explored in the literature on project evaluation). Our lack of such detailed empirical knowledge, in the case of the Indian economy, is the first reason for our persistent inability to give a confident answer as to how one should evaluate such public projects as a FFW programme.

Secondly, empirical applications obviously lead to acute estimation problems. The informational requirements involved in the design of constrained optimal policies and choice of shadow prices are often formidable. How project evaluation should take place when information is incomplete or inaccurate is still an insufficiently explored issue. Perhaps useful lessons could be drawn from placing project evaluation explicitly in the context of statistical decision theory (beyond the comparably comfortable confines of sensitivity analysis).

These difficulties point to challenging areas for further research. Hopefully their patient study will one day rescue the peasant of Manohar Thanna from the caprices of the rain gods and the comprehensible ad hockery of Indian planning.

ANNEX 1 : Statistical material

Annex I.

The data construction described in this Annex was undertaken with the aim of carrying out extensive numerical simulations of some versions of the model of Chapter II. The statistical material relates to the Indian economy in 1979-80; in assembling it we followed as closely as possible the framework of the VIth five-year plan of India (9) and endeavoured to resort to data sources and techniques that are available to a government agency such as, say, the Project Appraisal Division in the Planning Commission.

The purpose of this exercise was threefold.

First, numerical simulations carried out with consistent (though not necessarily realistic) data are potentially useful in improving our understanding of the shadow pricing, optimal policy, and policy reform rules derived in Chapter II; in particular, sensitivity analysis is amenable to reveal features of these rules which would be extremely difficult to guess a priori or analyse theoretically (as well as to orient our theoretical preoccupations).

Second, such exercises can provide a useful preliminary step towards further empirical research (e.g. by pointing to the most valuable pieces of empirical evidence, and underlying areas where econometric accuracy is particularly important).

Finally, we did not exclude the possibility of arriving at a better understanding of some important areas of policy reform

in India, in spite of the difficulty of gathering consistent, wide-ranging and reliable data. In this respect, we attempt to follow the pioneering steps of Ahmad and Stern (3,4), who concentrated on the issue of tax reform.

Since our emphasis was primarily on the first two of these aims, the data construction stresses consistency more than strict conformity to the contemporary Indian economy.

As stated in the introduction, the empirical exercises reported in this thesis make use of only a thin segment of the data presented here. However, the full data construction will be outlined, both for the record, and in the hope of completing a fruitful groundwork for further research.

The content of this Annex is as follows:

Section A.I.	Main data sources
Section A.II.	Notational conventions
Section A.III.	The model
Section A.IV.	The core data set
§ A.IV.1.	Normalization rules
§ A.IV.2.	The input-output table
§ A.IV.3.	Prices and taxes
§ A.IV.4.	Population
§ A.IV.5.	The consumer expenditure matrix
§ A.IV.6.	Imports of consumer goods
§ A.IV.7.	Income distribution
§ A.IV.8.	Average savings rates
§ A.IV.9.	Marginal propensities to save and marginal expenditure shares
§ A.IV.10.	Income and expenditure growth factors

§ A.IV.11.

The consumer price elasticity matrix

§ A.IV.12.

Value-added coefficients

§ A.IV.13.

The distribution of value-added

Section A.V.

Aggregation scheme

Data tables

Footnotes

References

Section A.I : Main data sources

Our major data sources are listed below, preceded by their respective codes:

- (TN) The VIth five-year plan of India (1980-85) and in particular its Technical Note (9, 10).
- (NSS) The 28th round of the National Sample Survey of India (29) relating to 1973-74.
- (NCAER) The 1975-1976 survey on "Household income and its disposition", conducted by the National Council of Applied Economic Research, New Delhi (28).
- (A-S) The recent published and unpublished work of E.Ahmad and N.Stern (1,2,3,4) on tax reform in India.
- (SINHA) The study of R.Sinha et al (36) on "Income distribution, growth and basic needs in India".
- (MR) Various Discussion Papers of K.N.Murty and R.Radhakrishna (24,25) reporting their econometric work on consumer demand in India.
- (PUB) Miscellaneous Publications such as the Indian Government's annual Economic Survey , statistical abstracts of the Central Statistical Organization , articles from the Economic and Political Weekly , etc.

Section A.II: Notational conventions

This Annex follows a completely autonomous notation.

The indices $h \in H$, $j \in J$, $k \in K$ will always refer respectively to industries, commodities (excluding labour services) and classes of consumers. The indices f, g will be used freely as auxiliaries. The indexes R, U refer to the rural and the urban sector, respectively. We write $k \in K_R$ ($k \in K_U$) if the k^{th} class belongs to the rural (urban) sector. The size of the k^{th} class is denoted by N_k . Finally, the superscripts "D" and "M" should be read as "domestic" and "import", their more precise meaning being usually clear from the context.

Matrices are represented by capital letters, as well as their elements. If $Y = ((Y_{fg}))_{f \in F, g \in G}$ is a matrix, the following notational conventions hold:

(i) Aggregation :

$$\begin{aligned} Y_{f\cdot} &= \sum_g Y_{fg} \\ Y_{\cdot g} &= \sum_f Y_{fg} \\ Y_{\cdot\cdot} &= \sum_f \sum_g Y_{fg} \end{aligned}$$

(ii) Partial aggregation: $\bar{F} \subseteq F \Rightarrow Y_{\bar{F}g} = \sum_{f \in \bar{F}} Y_{fg}$, $Y_{f\bar{F}} = \sum_{g \in \bar{F}} Y_{fg}$
(similarly for $\bar{G} \subseteq G$)

(iii) Vector expansion :

$$\begin{aligned} Y_{f*} &= (Y_{fg})_{g \in G} \\ Y_{*g} &= (Y_{fg})_{f \in F} \end{aligned}$$

(iv) Transposition
$$Y = ((Y_{gf}))_{\substack{g \in G \\ f \in F}}$$

Furthermore, if in the above g indexes consumers or sets of consumers and Y records a specific consumer characteristic (e.g. expenditure on a specific commodity), the following notation automatically applies:

(i) Y_{fk} = corresponding characteristic of class k

Y_{fR} = " " " for the rural sector

Y_{fU} = " " " for the urban sector

(ii) $\bar{Y}_{fk} = Y_{fk}/N_k$, $\bar{Y}_{fR} = Y_{fR}/N_R$, $\bar{Y}_{fU} = Y_{fU}/N_U$ (per capita figures)

(iii) $\tilde{Y}_{fk} = Y_{fk}/Y_{fR}$ if $k \in K_R$ (share of sector total)
 $= Y_{fk}/Y_{fU}$ if $k \in K_U$

(iv) $\tilde{\tilde{Y}}_{fk} = Y_{fk}/(\sum_k Y_{fk})$ (share of overall total)

(v) $\tilde{\tilde{Y}}_{fk}^+ = (\sum_{\substack{g \leq k \\ g \in K_R}} Y_{fg})/Y_{fR}$ if $k \in K_R$
 $= (\sum_{\substack{g \leq k \\ g \in K_U}} Y_{fg})/Y_{fU}$ if $k \in K_U$

(cumulative share of sector total)

Similarly for $Y_{\cdot k}$, $Y_{\cdot R}$, $Y_{\cdot U}$, $\bar{Y}_{\cdot k}$, $\bar{Y}_{\cdot R}$, $\bar{Y}_{\cdot U}$, $\tilde{Y}_{\cdot k}$, $\tilde{Y}_{\cdot R}$, $\tilde{Y}_{\cdot U}$, $\tilde{\tilde{Y}}_{\cdot k}^+$, $\tilde{\tilde{Y}}_{\cdot R}^+$, $\tilde{\tilde{Y}}_{\cdot U}^+$ etc.

Vectors are considered as single-column matrices and the same notation holds for them, except that in their case the column index (or row index when they are transposed) can be omitted without confusion.

The lower-case letters a, b, c, d will be used freely as auxiliaries.

We conclude with a catalogue of the matrices subsequently used (in alphabetical order):

- B^D : matrix of input-output coefficients for domestically produced intermediates ($|J|X|H|$).
- B^M : matrix of input-output coefficients for imported intermediates ($|J|X|H|$).
- D : "MAKE" matrix ($|H|X|J|$).
- G : vector of industry gross outputs ($|H|X1$).
- L : matrix of value-added payments per unit of gross output ($|H|X|K|$).
- N : vector of class sizes ($|K|X1$).
- P : vector of sellers' prices ($|J|X1$).
- P^M : vector of import prices ($|J|X1$).
- Q^D : vector of commodity gross domestic outputs ($|J|X1$).
- Q^M : vector of gross imports ($|J|X1$).
- R : vector of "full incomes" (inclusive of wages) ($|K|X1$).

- S : vector of private savings ($|K|X1$).
 T : vector of indirect taxes on domestic production ($|J|X1$).
 T^M : vector of indirect taxes on imports ($|J|X1$).
 W : matrix of value-added payments ($|H|X|K|$).
 X : matrix of private consumer expenditure (at consumer prices) ($|J|X|K|$).
 α : matrix of average propensities to consume (out of expenditure) ($|J|X|K|$).
 β : matrix of marginal propensities to consume (out of expenditure) ($|J|X|K|$).
 γ : vector of marginal propensities to save (out of income) ($|K|X1$).
 δ : vector of average propensities to save (out of income) ($|K|X1$).
 ε : aggregate consumer price elasticity matrix ($|J|X|J|$).
 Π : vector of "profit" rates ($|H|X1$).
 ω : vector of domestic production - proportions for consumer expenditure ($|J|X1$).
 λ : see text.

Section A.III : The model

A wide range of empirical exercises have been undertaken based on a simple variant of the model analysed in Chapter II. The main departures from our previous model are explained below.

(i) Prices: Following Ahmad and Stern (2), we assume indirect taxes to bear on intermediate purchases as well as on final consumption. The following identity therefore applies:

$$(1) \quad \Pi + L_{**} = D^{-1}P - B^D \cdot (P+T) - B^M (P^M + T^M)$$

The left-hand side of this identity is a vector of value-added coefficients. The h^{th} component of the vector on the right-hand side is the difference between the average revenue and the average cost (labour excluded) of a unit of gross output in the h^{th} industry.

(ii) Savings and investment: As in the applications of Chapters III and IV, private savings are taken to be simple functions of private incomes :

$$(2) \quad X_{.k} = X_{.k}(R_k) \quad \forall k$$

As far as investment is concerned, two approaches have so far been considered. In the first, investment is exogenous. In the second, private investment follows the accelerator-type model built in the VIth five-year plan. This second approach will be

ignored here.

(iii) Imports: Imports of consumer goods are considered as rationed and exogenous:

$$(3) \quad x_{jk}(\cdot) \equiv (P_j + T_j) x_{jk}^D(\cdot) + (P_j^M + T_j^M) \bar{x}_{jk}^M$$

In most applications so far, imports of intermediates have been treated as non-substitutable with domestic production, and required in fixed proportion to gross outputs (see Ahmad-Stern (2) for a detailed discussion of this assumption). In principle, however, our model is flexible enough to allow relaxation of this assumption if necessary.

(iv) Labour: The typology of labour services (e.g. differentiated according to the individuals who supply them, or the industry which employs them) is application-specific. Utilities are separable between consumption and leisure. In the applications so far, labour supplies are fixed.

(v) Side constraints: Any kind of side constraints can be introduced in the applied version of the model, provided that the required data is available to take them into account. We have concentrated particularly on alternative price and wage fixing hypotheses.

In all the applications so far, we have adopted the linear technology framework of the VIth five-year plan (though an appropriate set of side constraints would allow, in principle, the introduction of flexible technical coefficients). The model can then be written

as follows:

$$(M1) \quad W = \Psi(\dots, V^k(\dots), \dots)$$

$$(M2) \quad \sum_k X_{*k}^D (P + T, X_k) - (D^{-1} - B^D) G - \Delta^D = 0$$

$$(M3) \quad P^M [B^M G - \Delta^M] = 0$$

$$(M4) \quad X_k - X_k(R_k) = 0 \quad \forall k$$

$$(M5) \quad R_k - \sum_h L_{hk} G_h - \Delta^k = 0 \quad \forall k$$

$$(M6) \quad \Pi + L_* = D^{-1} P - B^D (P + T) - B^M (P^M + T^M) = 0$$

$$(M7) \quad (\text{Other side constraints})$$

where Δ^D , Δ^M , Δ^k are exogenous components of the model (e.g. imports of consumer goods, lump-sum transfers, public production plan, etc.) which can in principle become endogenous by a suitable specification of further side constraints.

The empirical exercises performed so far all rely on a "core data set" consisting of the following 19 matrices and vectors (in alphabetical order):

$$\left\{ \begin{array}{l} B^D, B^M, D, G, L, N, Q^D, Q^M, R, T, T^M, X \\ \beta, \gamma, \delta, \varepsilon, \lambda, \pi, \omega \end{array} \right.$$

where

$$(i) \quad \beta_{jk} \equiv \frac{\partial X_{jk}}{\partial X_{.k}} \quad (j \in J, k \in K)$$

$$(ii) \quad \gamma_k \equiv 1 - \frac{\partial X_{.k}}{\partial R_k} \quad (k \in K)$$

$$(iii) \quad \delta_k \equiv 1 - \frac{X_{.k}}{R_k} \quad (k \in K)$$

$$(iv) \quad \varepsilon_{jq} \equiv \frac{\partial X_{j.}}{\partial q_q} \frac{q_q}{X_{j.}} \quad \text{if } q \neq j \quad (j, q \in J)$$

$$\equiv \frac{\partial X_{j.}}{\partial q_q} \frac{q_q}{X_{j.}} - 1 \quad \text{if } q = j \quad (j, q \in J)$$

$$(v) \quad \omega_j \equiv \frac{(P_j + T_j) X_j^D}{X_j} \quad (j \in J)$$

(λ will be introduced in the text).

Ideally, the estimation of these statistics should be integrated in a full set of complete and consistent national accounts. However, since it was doubtful whether the exacting efforts involved

in such a construction would be worthwhile for our purposes, we did not push self-discipline further than to insist upon internal consistency (a difficult enough task when data from very heterogeneous sources has to be assembled!). In the next section, therefore, we shall outline separately each step of the statistical construction.

In the classification scheme of the core data set, there are 89 commodities, 89 industries, and 28 classes of consumers. The choice of a classification scheme is a difficult problem. When different data sources deal with different levels of aggregation, this choice often involves a dilemma between loss of information through aggregation, and unreliability of data at a very disaggregated level. We have chosen the strategy of constructing the core data set at a highly disaggregated level (even though at some points disaggregation involves rough estimation procedures), since it is generally easy to reaggregate according to convenience in particular exercises. The aggregation formulae are given in Section A.V.

A display of the full core data set would be cumbersome and difficult to interpret. The data tables B.2-B.8 therefore refer to a classification scheme involving 26 commodity groups and 10 income classes which we have found empirically very useful. The aggregation keys are given in Table B.1.

Section A.IV. The core data set

§ A.IV.1 : Normalization rules

We have adopted the following normalization rules:

(i) The total population size is unity.

$$(5) \quad \sum_k N_k = 1$$

(ii) Aggregate consumer expenditure (and hence, per capita consumption expenditure) amounts to unity:

$$(6) \quad X_{..} = 1$$

(iii) Sellers' price are equal to unity:

$$(7) \quad P_j = 1 \quad \forall_j \in J$$

(iv) Since we shall consider imported commodities as non-substitutable with domestic production (as in the VIth five-year plan), their c.i.f. prices $\{P_j^M\}$ can also be normalized to unity:

$$(8) \quad P_j^M = 1 \quad \forall_j \in J.$$

§ A.IV.2 : The input-output table

The Technical Note to the VIth five-year plan of India (10) (hereafter TN) contains extensive data on the source and destination of commodities (classified into 89 groups) in various industries (89 of

them, roughly matching the commodity classification) as well as in final use, for the years 1979-80 and 1984-85. Three of the published tables are of interest to us:

(i) (TN-UFLOW): The core input-output table (for 1979-80). Its structure is displayed schematically in Table A.1. below.

All the accounts are at producer prices; e.g. the $(j,h)^{th}$ entry of the upper left-hand block in the table indicates the expenditure on commodity j in industry h , at producer prices; a separate entry in the same column (corresponding to row (91) of the table) records the sum of all indirect tax payments made by industry h in its purchases of inputs. However, we shall ignore the indirect tax row, since the indirect tax estimates of A-S seem much more reliable. By implication, we shall also ignore the value-added row (row (92)), since it is obtained in TN as a residual, thus using row (91); rather, we shall compute our own residual value-added row on the basis of the A-S indirect tax estimates.

In the TN accounts, the private consumption expenditure column (91) is also a residual, and does not seem very reliable. NSS consumer expenditure data, based on large-scale surveys which have a long history, should prove far superior. Unfortunately, its published report for the 28th round (29) deals with a much higher level of aggregation than TN. We have therefore decided to give priority to the NSS consumer expenditure data, relying however on the TN estimates whenever evidence about the distribution of expenditure within the NSS commodity groups was needed.

(ii) (TN-MAKE): Although the TN industry classification closely matches the commodity classification, it does not altogether exclude joint production. The $(h, j)^{th}$ entry of the (89×89) "make" matrix D indicates the percentage of domestic gross output of the j^{th} commodity originating from the h^{th} industry. The columns of the "make" matrix thus add up to unity. From inspection, D is not very far from an identity matrix, and for convenience the reader may wish to ignore the complications arising from the existence of joint production.^{1/}

(iii) (TN-UIMPORT) : This matrix has 90 rows and 94 columns. Columns (1)-(93) precisely match the North-Western (90×93) block of the TN-UFLOW matrix, giving, for each entry of that block, the "import" element in the recorded transaction. Column (94) is simply a "total" column, which replicates column (96) of TN-UFLOW. All the entries of this matrix are at c.i.f. prices.

Since they are published in TN, it would be wasteful to reproduce these tables here. However, some of their basic features are apparent from Tables B.2, B.3.

The following items of our core data set have been computed directly from the above 3 tables:

(i) G: By simply scaling the (industry) gross output row of TN-UFLOW (row (93), entries 1 to 89). The scaling factor assumes a total consumer expenditure figure of unity, and a ratio of aggregate consumer expenditure to total gross domestic output as per the TN-UFLOW data (implying $G. = 2.51$).

(ii) $\underline{Q}^D$: Obtained by similarly scaling the (commodity) gross domestic output column of TN-UFLOW (column (98), entries 1 to 89).

Notice that $G = DQ^D$.

(iii) $\underline{Q}^M$: Taken from column (96) (entries 1 to 89) of TN-UFLOW, scaled as above.

(iv) $\underline{B}^D, \underline{B}^M$: By dividing each entry of the first 89 columns of the TN-UFLOW table, one obtains a set of input-output coefficients appropriate for the "industry technology" assumption (see previous footnote). These have been split between domestic I-O coefficients and I-O coefficients for (non-substitutable) imports according to the formulae

$$(9) \quad (B^M)_{jh} = \frac{(TN-UIMPORT)_{jh}}{(TN-UFLOW)_{93,h}}$$

$$(10) \quad (B^D)_{jh} = \frac{(TN-UFLOW)_{jh} - (TN-UIMPORT)_{jh}}{(TN-UFLOW)_{93,h}}$$

§A.V.3: Prices and taxes

Following A-S, in our empirical work we let indirect taxes drive a wedge between buying and selling prices, rather than between consumer and producer prices (e.g. private producers pay indirect taxes on intermediates).^{2/} If q_j is the buyers' price, and P_j the sellers' price, of the j^{th} commodity (domestically produced):

$$(11) \quad q_j = P_j + T_j = 1 + T_j \quad \text{by choice of units } (j \in J)$$

The vector T is taken from the work of A-S; this is permissible since their methodology is broadly consistent with ours. When necessary the vector T can be further broken down as

$$(12) \quad T = T^e + T^a + T^r + T^s$$

where T^e , T^a , T^r , T^s are vectors of excise taxes, sales taxes, remaining indirect taxes, and subsidies, respectively.

When imported commodities are considered as non-substitutable with domestic production, their c.i.f. prices can be normalized to unity (see § A.IV.1. above). The buyers' price of an imported commodity, q_j^M then becomes

$$(13) \quad q_j^M = P_j^M + T_j^M = 1 + T_j^M \quad (j \in J)$$

In (13), T^M is a vector of indirect taxes which has also been taken from the A-S study, and consists of the components:

$$(14) \quad T^M = T^a + T^f$$

where T^f is a vector of tariffs.

Perhaps a word of clarification is in order here. The A-S tax estimates have been computed from tax revenue data on the assumption that:

- (i) Excise tax rates apply to the value of transactions at sellers' prices.
- (ii) Tariff rates apply to the c.i.f. value of imports.

- (iii) Sales tax rates apply to the value of transactions at sellers' prices in the case of domestically produced goods, and to the c.i.f. value of transactions in the case of imports.

It may seem more realistic to assume that sales tax rates apply to the value of transactions inclusive of excises and/or tariffs; however, the discrepancy between the two estimation procedures is quantitatively insignificant and we have preferred to accept the A-S figures.

The vectors T and T^M appear in A-S^{3/} for the full 89 commodity classification, and in Table B.2. below for our 26 commodity classification.

If we wish to allow for substitution between domestically produced and imported commodities, a very rough approach could be as follows. Let $1/\phi_j$ be the number of units of the j^{th} imported commodity which are equivalent to one unit of the same commodity domestically produced. With our normalization rules, we should find, when buyers are not strongly rationed:

$$(15) \quad q_j = q_j^M \frac{1}{\phi_j}$$

or

$$(16) \quad \phi_j = \frac{q_j^M}{q_j} = \frac{1 + T_j^M}{1 + T_j}$$

These conversion factors are important, for instance, if one wishes to use the influential semi input-output model (see e.g. Kuyvenhoven (18)), or the closely related Little-Mirrlees (19) methodology for project appraisal. Unfortunately, the assumption of only mildly rationed buyers does not seem very realistic in the case of many imported commodities. More accurate estimation could be attempted from specific data on world prices.

§ ATV.4 : Population

Consumers are divided into 28 "classes" of supposedly identical individuals belonging to two broad "sectors" : rural (14 groups) and urban (14 groups). It will be convenient to interpret each class as a specific fractile of the intra-sectoral per capita expenditure scale (e.g. the first rural class may consist of the poorest 5% in the rural sector in terms of per capita expenditure, etc.) We shall assume that per capita expenditure increases monotonically with per capita income, and that the intra-sectoral distributions of p.c. expenditure and p.c. income are stable over the time period covered by our different data sources (1968 to 1979). Each class may therefore equally be interpreted as a specific fractile of an intra-sectoral per capita income scale. Furthermore the shares $\tilde{N}_k$ of each class in sectoral population are constant over time.

These have been computed from the NSS 28th round Report (29) according to the formula:

$$(17) \quad \tilde{N}_k = \frac{\sum_s [a_{ks} \times b_{ks}] \times c_s}{\sum_{f \in K_R} \left\{ \sum_s [a_{fs} \times b_{fs}] \times c_s \right\}} \quad \text{for } k \in K_R \text{ (and correspondingly for } k \in K_U)$$

where a_{ks} is the average household size, in the NSS sample, in class k and state s , b_{ks} is the number of sample households in class k and state s , and c_s is a population multiplier giving the ratio of sample population to total population in each state. The share of class k in total population at time t is then:

$$(18) \quad \begin{cases} \tilde{N}_k^t = \tilde{N}_k \tilde{N}_R^t & \text{if } k \in K_R \\ \tilde{N}_k^t = \tilde{N}_k \tilde{N}_U^t & \text{if } k \in K_U \end{cases}$$

Furthermore

$$(19) \quad \begin{cases} \tilde{N}_U^t = \tilde{N}_U^0 \left[\frac{1+d_U}{1+d} \right]^t \\ \tilde{N}_R^t = 1 - \tilde{N}_U^t \end{cases}$$

where $\tilde{N}_U^0$ is the urban population share in some base year (here 1971) and d_U, d are respectively the rates of growth of urban and total population.

According to the Census data reported by R.Mohan and C.Pant (20):

$$(20) \quad \begin{cases} \tilde{N}_U^0 = 0.202 \\ d_U = 0.0386 \text{ over the period } 1971-81 \\ d_r = 0.0175 \quad " \quad " \quad " \quad " \end{cases}$$

Hence $d = (0.8) (0.0175) + (0.2) (0.0386) = 0.0217$. Therefore:

$$(21) \begin{cases} \text{in 1974: } \tilde{N}_U = (0.202) (1.0165)^3 = 0.212 \\ \text{in 1979: } \tilde{N}_U = (0.202) (1.0165)^8 = 0.230 \end{cases}$$

These estimates are consistent with the reported figure of

$\tilde{N}_u = 0.237$ in 1981, from the same source.

Combining (5), (17), (18) and (21) we obtain the 1979 population vector displayed in Table B.4.

§A.IV.5 : The consumer expenditure matrix

The matrix $\bar{X}$ of per capita expenditure on each of 89 commodities by each of our 28 expenditure classes for 1979-80 is obtained by updating and expanding an initial expenditure matrix, say $\bar{X}^0$, published in the NSS Report (29) on the 28th Round (1973-74) which covers 27 broad commodity groups. The adjustment procedure is as follows:

Step 1: The matrix $\bar{X}^0$ relates to the year 1973-74. The ratio of total per capita expenditure between 1973-74 and 1979-80 is assumed to be the same for each expenditure group:

$$(22) \quad \frac{\bar{X}_{.k}^0}{\bar{X}_{.k}^1} = \text{not. } \rho \quad \forall k, \quad \text{with } \rho = 0.96 \text{ (see §A.IV.10)}$$

where the matrix $\bar{X}^1$ now relates to 1979.

The value of 0.96 for ρ is higher than one would infer from the Technical Note figures; the reason is presumably that 1979-80 was a very depressed year, a fact not fully reflected in the quick estimates of the Technical Note. However, it should be stressed that the only effective use we shall make of this ratio is in updating the average propensity to consume matrix; since the shift of this matrix between the two periods is very small in any case (see below), no great change would result from revising the value of ρ .

In order to adjust the per capita expenditure matrix for the growth in average p.c. consumer expenditure between 1974 and 1979 (implying, for instance, a lower share for foodgrains in total expenditure in the latter period), we shall consider that, within each sector, individuals differ only in respect of their employment (and hence income) opportunities. Since leisure and consumption are assumed to be weakly separable in each utility function, the 1973-74 cross-section data then provides us with 14 points on each sectoral Engel curve for each good, from which we can infer the 1979 expenditure pattern by linear interpolation:

$$(23) \quad \bar{x}_{jk}^1 = \bar{x}_{jk}^0 + \beta_{jk}^+ [\bar{x}_{.k}^1 - \bar{x}_{.k}^0]$$

where β_{jk}^+ is the "right-hand" marginal propensity to spend on good j for an individual in class k :
7/

$$(24) \quad \beta_{jk}^+ = \frac{\bar{x}_{j(k+1)} - \bar{x}_{jk}}{\bar{x}_{.(k+1)} - \bar{x}_{.k}}$$

Dividing by $\bar{X}_{.k}^1$ and juxtaposing across k and j :

$$(25) \quad \alpha^1 = \alpha^0 \rho + \beta^+ (1-\rho)$$

where α^0, α^1 are matrices of average propensities to consume.

Since ρ is very close to unity, the average propensity to consume matrix is rather insensitive to this adjustment.

The matrix $\bar{X}^1$, further normalized so that $\bar{X}_{..} = 1$, can finally be estimated from:

$$(26) \quad \left\{ \begin{array}{l} \bar{X}_{jk}^1 = \alpha_{jk}^1 \bar{X}_{.k}^1 \\ \bar{X}_{.k}^1 = \frac{\bar{X}_{.k}^0}{\bar{X}_{..}^0} \end{array} \right.$$

Step 2: In order to match the 89 commodity classification of TN, the matrix $\bar{X}^1$ was expanded by distributing the expenditure on each of the NSS commodity groups (for each class) according to the intra-group patterns revealed by the TN aggregate consumer expenditure vector (the latter is otherwise ignored in view of its unreliable nature). This involved no great difficulty since, by broadly following the Planning Commission's (B) own classification scheme, each commodity of the TN classification seemed to be sensibly allocated to a single NSS commodity group; the only exceptions being the following:

a) The "other cereals" commodity (TN) was identified with the sum of "Maize", "Ragi", "Barley", "Small millets", "Gram" and "Cereal substitutes" (NSS), since the latter were not covered elsewhere.

b) The "Miscellaneous food products" commodity (TN) was assumed to consist of "Vegetables", "Fruits and Nuts", "Salt", and "Spices" (NSS), each of these contributing to the "Miscellaneous food products" in proportion to their share in total expenditure as per the NSS data.

c) The "Other crops" item of TN was similarly divided into "Vegetables", "Fruit and Nuts" and "Spices".

Generally, then, if a_{jg} is the share of commodity j in group g ($j = 1, \dots, 89$; $g = 1, \dots, 27$), a matrix $\bar{X}$ of size (89×28) is computed according to the formula:

$$(27) \quad \bar{X}_{jk} = \sum_g a_{jg} \bar{X}_{gk}^1$$

The per capita expenditure matrix so obtained (and reaggregated to the size 26×10) is displayed in Table B.5.

3A.IV.6 : Imports of consumer goods

In some exercises it is necessary to identify the proportion of consumer expenditure on each commodity met from imports, here denoted by $1 - \omega_j$;

$$(28) \quad 1 - \omega_j = \frac{(P_j^M + T_j^M) \bar{X}_j^M}{X_j}$$

There is no evidence on this from the NSS data. On the other hand, the TN input-output matrix splits total consumer expenditure between domestic purchases and imports. We have thus computed the ω_j 's on the basis of the latter figures; this conforms with our strategy of preferring NSS consumer expenditure data to TN data across the 27 commodity groups of the published NSS Report, while accepting the TN intra-group figures when finer classification is required. It should be stressed that the intra-group refinements generally play a minor role. In this case, the values of ω_j are very near unity for most commodities; only for 7 commodities out of 89 does ω_j fall below 0.9. The vector ω for the 26 commodity classification is contained in Table B.5.

Whenever an explicit consideration of imported consumer goods was necessary, we have treated them as rationed commodities, consumed in exogenous quantities. Under this assumption, all the formulae derived in this section without distinguishing between domestically produced and imported consumption can be shown to remain valid.

§ A.IV.7 : Income distribution

The NCAER (28) 1975-76 survey on "Household income and its disposition" gives extensive information on the size distribution of income, expenditure and savings, separately for urban and rural areas.

The consistency between the NSS data and the NCAER data on expenditure distribution is a question of some importance. It is

a tricky one, too, since both sets of data deal with different fractiles of the per capita expenditure scale, and therefore the consistency issue partly boils down to the assumptions one is willing to make about the intra-class distribution of expenditure (at this point the assumption of identical individuals within each class is of course not tenable for both sets of data simultaneously). A minimal consistency test simply requires the existence of some pattern of intra-class distribution which reconciles the NCAER and NSS inter-class distribution patterns. This requirement will be satisfied provided that all the Lorenz-curve points revealed by both sets of data can be fitted on a single, convex Lorenz curve.

Formally, let the indexes k and g refer respectively to the NSS and the NCAER p.c. expenditure classes, and, restricting our attention for the moment to one particular sector (urban or rural) let $\tilde{N}_k^+$, $\tilde{X}_{.k}^+$ denote the cumulative shares of all classes up to and including k in sectoral population and expenditure, when classes are arranged in ascending order of p.c. expenditure (separately for each sector). If both surveys deal with the same expenditure concepts, and if the distribution of expenditure within each sector has not shifted significantly between 1973-74 and 1974-75, the consistency of the two data sets is equivalent to the following:

$$(29) \left\{ \begin{array}{l} \tilde{N}_g^+ \leq \tilde{N}_k^+ \leq \tilde{N}_{g'}^+ \Rightarrow \tilde{X}_{.g}^+ \leq \tilde{X}_{.k}^+ \leq \tilde{X}_{.g}^+ + (\tilde{X}_{.g'}^+ - \tilde{X}_{.g}^+) \frac{\tilde{N}_k^+ - \tilde{N}_g^+}{\tilde{N}_{g'}^+ - \tilde{N}_g^+} \\ \tilde{N}_k^+ \leq \tilde{N}_g^+ \leq \tilde{N}_{k'}^+ \Rightarrow \tilde{X}_{.k}^+ \leq \tilde{X}_{.g}^+ \leq \tilde{X}_{.k}^+ + (\tilde{X}_{.k'}^+ - \tilde{X}_{.k}^+) \frac{\tilde{N}_g^+ - \tilde{N}_k^+}{\tilde{N}_{k'}^+ - \tilde{N}_k^+} \end{array} \right.$$

This is in turn equivalent to

$$(30) \quad \left\{ \begin{array}{l} \tilde{x}_{.g_k}^+ \leq \tilde{x}_{.k}^+ \leq \tilde{x}_{.g_k}^+ + (\tilde{x}_{.g_{k+1}}^+ - \tilde{x}_{.g_k}^+) \frac{\tilde{N}_k^+ - \tilde{N}_{g_k}^+}{\tilde{N}_{g_{k+1}}^+ - \tilde{N}_{g_k}^+} \text{ not. } y_k \\ \tilde{x}_{.k_g}^+ \leq \tilde{x}_{.g}^+ \leq \tilde{x}_{.k_g}^+ + (\tilde{x}_{.k_{g+1}}^+ - \tilde{x}_{.k_g}^+) \frac{\tilde{N}_g^+ - \tilde{N}_k^+}{\tilde{N}_{k_{g+1}}^+ - \tilde{N}_k^+} \text{ not. } y_g \end{array} \right.$$

$$(31) \quad \left\{ \begin{array}{l} g_k \text{ is s.t. } \tilde{N}_{g_k}^+ \leq \tilde{N}_k^+ \leq \tilde{N}_{k_{g+1}}^+ \\ k_g \text{ is s.t. } \tilde{N}_{k_g}^+ \leq \tilde{N}_g^+ \leq \tilde{N}_{k_{g+1}}^+ \end{array} \right.$$

In (30), y_k can be interpreted as an estimate of $\tilde{x}_{.k}^+$ by linear interpolation between $\tilde{x}_{.g_k}^+$ and $\tilde{x}_{.g_{k+1}}^+$; and similarly for y_g .

The values of $\tilde{N}_k^+$, $\tilde{x}_{.k}^+$, y_k and $\tilde{N}_g^+$, $\tilde{x}_{.g}^+$, y_g are presented in Table A.2. below. The points at which the consistency test is violated ($\tilde{x}_{.k}^+ \geq y_k$ or $\tilde{x}_{.g}^+ \geq y_g$) are marked by an asterisk. On the whole, inspection reveals a substantial degree of agreement between the two sets of data. The NCAER data exhibits a slightly more skewed expenditure distribution in the urban sector, presumably because the NCAER survey took greater care to sample very high income groups (the NSS survey data is generally believed to be rather weak at the extreme ends of the expenditure scale).

The consistency test seems very mildly violated in the urban sector, hardly at all in the rural sector. On the other hand, the overall degree of agreement between the two data sets is rather striking at this point (but see below). The hypothesis of consistent

TABLE A.2.

a) Rural sector

k	$\tilde{N}_k^+$	$\tilde{X}_{.k}^+$	y_k	g	$\tilde{N}_g^+$	$\tilde{X}_{.g}^+$	y_g
1	0.36	0.07	0.11	1	4.37	1.34	1.46
2	0.75	0.17	0.23	2	33.29	17.56	18.34
3	1.95	0.55	0.60	3	59.48	39.53	40.06
4	4.14	1.37	1.27*	4	77.75	60.05	60.22
5	7.28	2.65	2.97	5	86.58	71.89	71.64*
6	13.28	5.59	6.34	6	91.82	80.47	80.06*
7	25.42	12.65	13.14	7	96.87	90.13	90.24
8	44.98	26.80	27.37	8	98.57	94.24	94.21*
9	65.56	45.62	46.36	9	99.45	97.28	97.05*
10	84.52	68.33	69.14	10	99.74	98.34	98.61
11	93.63	82.97	83.93	11	99.92	99.14	99.57
12	98.42	93.72	93.87	12	100.00	100.00	100.00
13	99.44	96.98	97.23				
14	100.00	100.00	100.00				

Cont'd....

b) Urban sector

k	$\tilde{N}_k^+$	$\tilde{X}_{.k}^+$	y_k	y	$\tilde{N}_g^+$	$\tilde{X}_{.g}^+$	y_g
1	0.05	0.006	0.017	1	9.96	3.43	3.83
2	0.17	0.03	0.06	2	29.50	13.21	14.50
3	0.46	0.10	0.16	3	45.80	24.28	25.76
4	0.98	0.24	0.34	4	60.75	36.72	39.06
5	2.30	0.67	0.79	5	70.12	45.78	47.70
6	5.32	1.78	1.83	6	84.72	63.97	65.72
7	12.44	4.92	4.67*	7	90.87	74.34	76.20
8	27.45	13.09	12.18*	8	94.64	82.13	82.95
9	46.84	26.48	25.14*	9	96.75	87.29	88.05
10	69.57	47.03	45.24*	10	97.82	90.22	91.16
11	84.09	64.65	63.19*	11	99.29	95.71	97.12
12	94.19	81.85	81.19	12	99.69	97.60	98.74
13	97.50	89.88	89.35	13	100.00	100.00	100.00
14	100.00	100.00	100.00				

Notational conventions:

k : income-class index for NSS income groups.

g : income-class index for NCAER income groups.

$\tilde{N}_k^+$, $\tilde{N}_g^+$: cumulative share of class k (class g) in sectoral population

$\tilde{X}_{.k}^+$, $\tilde{X}_{.g}^+$: cumulative share of class k (class g) in sectoral consumer expenditure

y_k , y_g : see text

evidence on the same population from the two data sets, while greatly simplifying our task, does not seem too far-fetched; it has therefore been accepted as far as intra-sectoral distribution is concerned.

On the other hand, the NSS and NCAER data give somewhat conflicting pictures of the inter-sectoral distribution of expenditure. According to the NSS data the share of the rural sector in aggregate consumer expenditure in 1973-74 works out at:

$$(32) \quad \bar{X}_{.R} \approx 0.736$$

On the other hand, from the NCAER Report, p.186-187, we may compute, for 1975-76:

$$(33) \quad \bar{X}_{.R} \approx \frac{1-\delta}{1-\delta} \bar{X}_{.R} = \frac{0.84}{0.82} (0.668) = 0.684$$

This is sensibly lower than the previous figure, in spite of the fact that the NCAER study assumes a value of 0.791 for $\bar{N}_R$ (p.58), which exceeds the corresponding value of 0.788 we used in dealing with the NSS data for 1974.

It is difficult to see why either survey should arrive at an accurate picture of intra-sectoral distribution while at the same time substantially misstating the inter-sectoral one, and strictly speaking the disagreement on inter-sectoral distribution should qualify the apparent harmony revealed by the intra-sector distributions. Nevertheless we shall accept both sets of data on intra-sectoral distribution. As far as inter-sectoral distribution is concerned, it seemed wise, in order to leave the NSS data integrally

intact, to ignore the NCAER evidence altogether.

The income distribution across the NSS expenditure groups within each sector has been inferred as follows. A curve showing the cumulative income distribution against the cumulative expenditure distribution is constructed by joining the points corresponding to the NCAER data by linear segments (linearity is a plausible assumption in this context; convexity is not a general feature of this curve). The cumulative share of income for each NSS class is then easily read from this curve. Formally, $\tilde{R}_k^+$ is estimated as

$$(34) \quad \tilde{R}_k^+ = \tilde{R}_{g_k}^+ + (\tilde{R}_{g_k+1}^+ - \tilde{R}_{g_k}^+) \frac{\tilde{X}_{:k}^+ - \tilde{X}_{g_k}^+}{\tilde{X}_{g_k+1}^+ - \tilde{X}_{g_k}^+}$$

where now

$$(35) \quad g_k \text{ is s.t. } \tilde{X}_{g_k}^+ \leq \tilde{X}_{:k}^+ \leq \tilde{X}_{g_k+1}^+$$

Since the expenditure groups are rather narrow, this simple linear interpolation formula should be acceptable.

The income distribution vector for 1979 (shown in Table B.4) is finally obtained by adopting the same intra-sectoral distributions as in 1973-74, and combining them with the inter-sectoral distribution implied by the NSS expenditure data together with the formula:

$$(36) \quad \tilde{R}_R = \tilde{X}_R \frac{(1-\delta_U)}{(1-\delta_R) + \tilde{X}_R(\delta_R - \delta_U)}$$

where δ_R , δ_U are sectoral average savings rates, discussed below.
Formula (36) is obtained from the identities:

$$(i) \quad \bar{R}_R = \bar{X}_R \frac{1-\delta}{1-\delta_R}$$

(37)

$$(ii) \quad \delta = \bar{R}_R \delta_R + \bar{R}_U \delta_U$$

With $\delta_R = 0.175$, $\delta_U = 0.234$ and $\bar{X}_R = 0.715$ in 1979, (36) gives:

$$(38) \quad \bar{R}_R = 0.700$$

SA.IV.8 : Average savings rates

The NCAER Report mentions the following average and
9/
marginal savings rates for 1975-76:

	Rural	Urban
Average savings rate	0.16	0.22
Marginal savings rate	0.39	0.48

The marginal savings rates may seem rather high; we shall only make a very minor use of them (to update the average savings rates for 1979; see below). The average savings rates, however, are quite consistent with national income account statistics, and we shall adopt them, assuming the same to hold for 1973-74.

Let the superscripts "1" and "0" refer to the time periods 1979-80 and 1973-74, respectively. Denoting average savings rate by δ and marginal savings rates by γ , we should have,

when $(\bar{R}_U^1 - \bar{R}_U^0)$ is small:

$$(39) \quad \bar{s}_U^{-1} = \bar{s}_U^{-0} + \gamma_U^0 (\bar{R}_U^1 - \bar{R}_U^0)$$

or

$$(40) \quad \delta_U^1 = \delta_U^0 \frac{\bar{R}_U^0}{\bar{R}_U^1} + \gamma_U^0 \frac{\bar{R}_U^1 - \bar{R}_U^0}{\bar{R}_U^1}$$

Assuming the inter-sectoral ratio of per capita income to be constant between 1973-74 and 1979-80 we have:

$$(41) \quad \frac{\bar{R}_U^0}{\bar{R}_U^1} = \frac{\bar{R}_R^0}{\bar{R}_R^1} = 0.947 \quad (\text{see } \S A.IV.10)$$

and hence

$$(42) \quad \delta_U^1 = (0.22) (0.947) + (0.48) (0.053) = 0.234$$

Using the same formula for the rural sector would give:

$$(43) \quad \delta_R^1 = (0.16) (0.947) + (0.39) (0.053) = 0.172$$

Observe, however, that our assumptions of constant inter-sectoral ratios for both per capita expenditure and per capita income imply a constant inter-sectoral ratio for the average propensity to consume:

$$(44) \quad \frac{1 - \delta_U^1}{1 - \delta_R^1} = \frac{1 - \delta_U^0}{1 - \delta_R^0} = \frac{0.78}{0.84} = 0.929$$

Combining (42) and (44) suggests:

$$(45) \quad \delta_R^1 = 1 - \frac{0.766}{0.929} = 0.175$$

This is almost identical to the earlier figure (in (43)) and has been adopted for consistency. The overall average savings

$$(46) \quad \delta^1 = \tilde{R}_R^1 \delta_R^1 + \tilde{R}_U^1 \delta_U^1 = 0.193 \text{ using (45), (42) and (37)(ii).}$$

Finally, average savings rates for each class are easily obtained from the income distribution figures and the formulae:

$$(47) \quad \begin{aligned} (i) \quad \delta_k &= \frac{S_k}{R_k} = \frac{\tilde{S}_k}{\tilde{R}_k} \delta \\ (iii) \quad \tilde{S}_k &= \frac{\tilde{R}_k - \tilde{X}_k(1-\delta)}{\delta} \end{aligned}$$

Our estimates of average savings rates appear in Table B.4. along with the marginal savings rates discussed below.

§ A.IV.9 : Marginal propensities to consume and save

In our model, individuals differ only in respect of their employment opportunities (and hence their income); since the NSS data results from a large-scale survey, we may regard the reported average expenditure on each good by each class as a point on the true and unique Engel curve of that good. The simplest estimate of the matrix of marginal expenditure shares becomes:

$$(48) \quad \beta_{jk} = \frac{\bar{X}_{j(k+1)} - \bar{X}_{j(k-1)}}{\bar{X}_{.(k+1)} - \bar{X}_{.(k-1)}} \quad (j \in J, k \in K)$$

This is just a properly weighted average of the "right-hand" marginal propensity to consume β_{jk}^+ introduced in § A.IV.5, and a similarly defined "left-hand" MPC. It results in the estimate of β shown in Table B.6.

By the same reasoning, we may estimate the vector of marginal

propensities to save as:

$$(49) \quad \gamma_k = \frac{\bar{S}_{k+1} - \bar{S}_{k-1}}{\bar{R}_{k+1} - \bar{R}_{k-1}}$$

The results of this calculation appear in Table B.4.

§A.IV.10 : Income and expenditure growth factors

We shall here compute the income and expenditure growth factors which have been used in order to update the consumer expenditure matrix and the average propensity to save vector (in §A.IV.5, §A.IV.8). As we have already pointed out, these parameters only play a very minor role in the calculations; the apparent precision of the numbers adopted is only motivated by the convenience of overall consistency.

Under our assumptions:

$$(50) \quad (i) \quad \frac{\bar{R}_R^0}{\bar{R}_R^1} = \frac{\bar{R}_U^0}{\bar{R}_U^1} \quad \text{not.} \quad \eta$$

$$(ii) \quad \frac{\bar{X}_R^0}{\bar{X}_R^1} = \frac{\bar{X}_U^0}{\bar{X}_U^1} \quad \text{not.} \quad \rho$$

where the indexes "0" and "1" refer to 1973-74 and 1979-80 respectively. Define

$$(51) \quad \bar{\eta} = \frac{\bar{R}_K^0}{\bar{R}_K^1}, \quad \bar{\rho} = \frac{\bar{X}_{\cdot K}^0}{\bar{X}_{\cdot K}^1} \quad \text{where} \quad \bar{R}_K^0 = \frac{\sum_k R_{kK}^0}{\sum_k N_k} \quad \text{etc.}$$

Then

$$(i) \quad \bar{n} = \frac{(N_R^0 \bar{R}_R^0 + N_U^0 \bar{R}_U^0)/N^0}{(N_R^1 \bar{R}_R^1 + N_U^1 \bar{R}_U^1)/N^1} = \eta \quad \frac{\tilde{N}_R^0 \bar{R}_R^0 + \tilde{N}_U^0 \bar{R}_U^0}{\tilde{N}_R^1 \bar{R}_R^0 + \tilde{N}_U^1 \bar{R}_U^0}$$

(52)

$$(ii) \quad \rho = \frac{(N_R^0 \bar{X}_R^0 + N_U^0 \bar{X}_U^0)/N^0}{(N_R^1 \bar{X}_R^1 + N_U^1 \bar{X}_U^1)/N^1} = \rho \quad \frac{\tilde{N}_R^0 \bar{X}_R^0 + \tilde{N}_U^0 \bar{X}_U^0}{\tilde{N}_R^1 \bar{X}_R^0 + \tilde{N}_U^1 \bar{X}_U^0}$$

According to our data:

$$(53) \quad \frac{\bar{X}_U^0}{\bar{X}_R^0} = 1.333, \quad \tilde{N}_R^0 = 0.788, \quad \tilde{N}_R^1 = 0.77$$

Also, using $\delta_R^0 = 0.16$ and $\delta_U^0 = 0.22$ (see §A.IV.8), we find:

$$(54) \quad \frac{\bar{R}_U^0}{\bar{R}_R^0} = 1.436$$

From (52), (53) and (54), then :

$$(55) \quad \frac{\bar{n}}{\eta} = 0.9929, \quad \frac{\bar{\rho}}{\rho} = 0.9944.$$

According to the 1982 Economic Survey (11), the figures for per capita net national product at constant 1970-71 prices in 1973-74 and 1979-80 are respectively:

$$(56) \quad \left\{ \begin{array}{ll} 1973-74 & : 621.2 \text{ Rs./Year} \\ 1979-80 & : 661.0 \text{ Rs./Year.} \end{array} \right.$$

This provides us with an estimate of $\bar{n}$:

$$(57) \quad \bar{n} = \frac{621.2}{661.0} = 0.9398$$

which, along with (55), gives

$$(58) \quad n = 0.947$$

Moreover, under our assumptions :

$$(59) \quad \rho = \frac{1-\delta_U^0}{1-\delta_U^1} \quad n = \frac{1-\delta_R^0}{1-\delta_R^1} \quad n$$

where, as in §A.IV.8, δ_U^1 is given by the formula:

$$(60) \quad \delta_U^1 = n\delta_U^0 + (1-n)\gamma_U^0 = n(0.22) + (1-n)(0.48) = 0.234$$

Hence

$$(61) \quad \rho = 0.964$$

§A.IV.11 : The consumer price elasticity matrix

There exist numerous empirical studies of consumer demand in India, most of them based on the extensive data gathered by the National Sample Survey Organization over several decades. Only a few of the more recent ones, however, go as far as attempting to estimate complete consumer demand systems (e.g. Coondoo (6,7), Murty et al (21,22, 26), Murty-Radhakrishna (24,25); Ray (32,33,34)). After an extensive survey of empirical consumer demand studies in India

we came to the conclusion that the work of K.N.Murty and R.Radhakrishna (25) seemed to have far more to offer for our purposes than any other study. Although some other very interesting research has been done in this area, the main other contributions of comparable depth seemed either preliminary (Coondoo (7)) incompletely available (GVSN Murty et al. (26)), or dealing with a very high level of aggregation (Ray (32,33)). In view of these shortcomings, and of the difficulties involved in pooling econometric results from different sources, the price elasticity estimates are derived exclusively from the work of K.N.Murty and R.Radhakrishna, as reported in (25), and hereafter referred to as MR.

The commodity classification adopted by MR has become quite popular in recent years (it is used e.g. in the work of A-S as well as by the Planning Commission of India in several studies); it consists of the following 9 broad commodity groups (the figures in brackets indicate the aggregate budget share α_j of each commodity group, for 1979, according to our data):

(CL)	Cereals	(0.389)
(MM)	Milk and milk products	(0.080)
(EO)	Edible Oils	(0.041)
(MFE)	Meat, fish and eggs	(0.028)
(SG)	Sugar and gur	(0.032)
(OF)	Other food	(0.180)
(CT)	Clothing	(0.065)
(FL)	Fuel and light	(0.056)
(ONF)	Other non-food	(0.129)
	Total	(1.00)

For this commodity classification, MR have estimated an extended version of the Linear Expenditure System due to Nasse (27), using time-series consumer expenditure data from NSS rounds 2 to 25 (covering the period 1951 to 1971). The estimation was carried out separately for 10 expenditure groups (see MR for the details of the estimation procedure). Further, by suitably averaging the price elasticities estimated for each expenditure group (computed "at their respective group means"), MR report the aggregate price elasticity matrix for 1970-71 presented in Table A.3. below.

According to the analysis of MR, this elasticity matrix (as well as other elements of their study) reveals the dominant importance of own-price elasticities, as well as of cross-price elasticities w.r.t. cereals; other cross-price elasticities being comparatively small.

The above elasticity matrix is inconsistent with our aggregate consumer expenditure vector. In particular, the adding-up property

$$(62) \quad \sum_j \alpha_j \epsilon_{jg} = -\alpha_g \quad (g = 1, \dots, 9)$$

is violated. Two reasons may account for this. First, the published work of MR does not seem to provide evidence that their estimation procedure ensures the fulfilment of this property, even with the appropriate consumer expenditure vector. Second, even if it does, (62) can of course not be expected to hold with our 1979 consumer expenditure vector.

Preliminary calculations revealed the crucial importance of working with a price elasticity matrix which satisfied the adding-up property (62). Consider, for instance, the "generalized Ramsey rule" derived in Chapter II (see (18) and (35^{*}) of Section II.2):

$$(63) \quad -v \frac{\partial x}{\partial q_j} = \sum_i \mu^i x_j^i \quad \text{if } j \text{ is a "procured good".}$$

Under some special assumptions (the details of which need not concern us here), (63) will hold for each consumer good, giving an autonomous system of shadow price equations for consumer goods: ^{12/}

$$(64) \quad -v \frac{\partial x}{\partial q} = x^* \quad (x_j^* \equiv \sum_i \mu^i x_j^i)$$

or

$$(65) \quad v = -x^* \left[\frac{\partial x}{\partial q} \right]^{-1}$$

and it is certainly an interesting exercise to carry out numerical solutions of this equation, examine its sensitivity to the welfare weights and the price elasticities, etc.

Now observe that if $\mu^i = 1 \quad \forall i$ (distributional indifference), then (65) has the unique solution $v = q$, by the adding-up property of the price elasticity matrix. If this property fails to appear in the empirical estimation of (65) the estimated shadow prices will fail to coincide with consumer prices when $\mu^i = 1 \quad \forall i$; a fortiori it will be difficult to interpret the results of sensitivity analysis. Furthermore, our calculations reveal that the solution of (65), although perhaps quite insensitive to errors of estimation in price

elasticities which nevertheless preserve the adding-up restriction (62), is definitely extremely sensitive to even minor violations of that property.

If we accept the conjecture that high accuracy in the estimation of price elasticities may not be crucial as long as appropriate theoretical restrictions are satisfied, it may seem reasonable to utilize the MR price elasticity matrix (in spite of all its possible shortcomings, e.g. being outdated), and merely adjust it in an ad hoc manner to satisfy the required restrictions. In doing so, we confined ourselves to the adding-up restriction, since we had evidence of its importance.^{13/} Several adjustment procedures were experimented with. The following was retained for its simplicity, intuitive appeal and apparently acceptable results; it simply consists of minimizing the "Euclidean distance" between the adjusted and unadjusted matrices.

Let ϵ^0 be the original MR elasticity matrix, and ϵ^1 the adjusted matrix. Consider the minimum-distance problem

$$(66) \quad \min_{\epsilon^1} \sqrt{\sum_{j,f} (\epsilon_{jf}^1 - \epsilon_{jf}^0)^2} \quad \text{nat. } ||\Delta||$$

$$\text{s.t. } \sum_j \alpha_j \epsilon_{jf}^1 = -\alpha_f \quad (\chi_f) \quad (f = 1, \dots, 9)$$

The first-order conditions for a minimum are

$$(67) \quad (\epsilon_{jf}^1) \quad \frac{\Delta_{jf}}{||\Delta||} = \chi_f \alpha_j \quad (j, f = 1, \dots, 9)$$

implying

$$(68) \quad \Delta_{jf} = \chi_f \alpha_j ||\Delta||$$

Now

$$(69) \quad \sum_j \alpha_j \epsilon_{jf}^1 = -\alpha_f \Rightarrow \sum_j \alpha_j (\epsilon_{jf}^0 + \chi_f \alpha_j ||\Delta||) = -\alpha_f \quad \forall f$$

or

$$(70) \quad \chi_f = \frac{\hat{\alpha}_f - \alpha_f}{||\alpha||^2 ||\Delta||} \quad \text{where} \quad \hat{\alpha}_f \equiv -\sum_j \alpha_j \epsilon_{jf}^0$$

Hence

$$(71) \quad \Delta_{jf}^* = \frac{\alpha_j (\hat{\alpha}_f - \alpha_f)}{||\alpha||^2}$$

The adjusted elasticity matrix ϵ^1 is given in Table A.4. below. The shift between ϵ^0 and ϵ^1 seems rather small, at least by the standards of accuracy that one would expect the matrix ϵ^0 itself to conform with.

No serious attempt has been made at further disaggregating the (9 x 9) consumer price elasticity matrix ϵ^1 , (except, trivially, by supplementing it with a set of zero entries corresponding to non-consumption goods i.e. goods for which the TN consumption vector reveals zero aggregate consumption). In all the empirical exercises, therefore, care has been taken to ensure that, whenever the specifi-

TABLE A.3 Unadjusted price elasticity matrix

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
CL	-0.504	-0.044	-0.018	-0.023	-0.009	-0.020	-0.007	-0.019	-0.010
MM	-0.548	-1.135	-0.155	0.150	0.019	-0.117	-0.032	-0.054	-0.036
EO	-0.281	-0.188	-0.478	0.051	-0.055	0.134	-0.015	-0.028	-0.020
MFE	-0.512	0.377	0.057	-0.345	0.011	0.088	-0.016	-0.032	-0.019
SG	-0.408	0.061	-0.085	0.004	-0.593	-0.164	-0.023	-0.402	-0.030
OF	-0.175	0.096	0.029	0.018	-0.021	-0.835	-0.014	-0.029	-0.015
CT	-0.293	-0.041	-0.028	-0.019	-0.016	-0.104	-0.738	-0.043	-0.027
FL	-0.127	-0.002	-0.010	-0.006	-0.002	-0.036	-0.011	-0.489	-0.013
ONF	-0.296	-0.047	-0.020	-0.024	-0.017	-0.120	-0.030	-0.046	-0.946

Source: Murty-Radhakrishna (25), p.13.

TABLE A.4 Adjusted price elasticity matrix (ϵ^1)

	CL	MM	EO	MFE	SG	OF	CT	FL	ONF
CL	-0.520	-0.018	-0.018	-0.038	-0.013	-0.038	-0.012	0.003	0.003
MM	-0.553	-1.130	-0.155	0.147	0.018	0.113	-0.033	-0.050	-0.033
EO	-0.284	-0.185	-0.478	0.049	-0.055	0.132	-0.016	-0.026	-0.019
MFE	-0.514	0.379	0.057	-0.346	0.011	-0.087	-0.016	-0.030	-0.018
SG	-0.410	0.063	-0.085	0.003	-0.593	-0.165	-0.023	-0.400	-0.029
OF	-0.187	0.108	0.029	0.011	-0.023	-0.843	-0.016	-0.019	-0.009
CT	-0.297	-0.037	-0.028	-0.021	-0.017	-0.107	-0.739	-0.039	-0.025
FL	-0.131	0.002	-0.010	-0.008	-0.003	-0.039	-0.012	-0.486	-0.011
ONF	-0.299	-0.038	-0.030	-0.029	-0.018	-0.126	-0.032	-0.039	-0.942

Source: see text (§ A.IV.11)

cation of the model is such that consumer prices are allowed to vary, the exercise is carried out at a level of aggregation at which consumer goods follow a classification no finer than the MR classification, with two exceptions. These exceptions relate to the "fuel and light" and "other non-food" categories; it was felt that a significant loss of information would be incurred by lumping into these categories some important commodities of an essentially intermediate nature (e.g. "Miscellaneous coal and petroleum products"), which, in any case, contribute only an insignificant amount to the total consumption expenditure on these two groups. It was thought preferable, then, when finer disaggregation was desired within these two groups, to exclude such items from the group, and simply consider them as rationed commodities from the point of view of consumers. We shall return to this point below.

For computational purposes, it proved convenient to build a fictitious (89 x 89) elasticity matrix, even though, as we have just explained, all the exercises allowing for consumer price changes would involve re-aggregation. This (89 x 89) matrix was simply constructed as follows. Within each MR commodity group, a single commodity was selected as the "freely bought" commodity for that group, all other commodities being considered as rationed commodities. From this assumption it is straightforward to derive appropriate elasticity formulae.

Formally, let the indexes $j \in J = \{1, \dots, 89\}$, and $g \in G = \{1, \dots, 9\}$ refer to the TN and the MR commodity classifications, respectively. Write

$$(72) \quad J = \bar{J} \cup J^*, \text{ where } j \in \bar{J} \text{ iff } j \text{ is rationed}$$

$$(73) \quad J = \bigcup_g J_g, \text{ where } j \in J_g \text{ iff } j \text{ belongs to group } g$$

$$\text{with } J^* \equiv \left\{ j_*(g) \right\}_{g \in G}, \text{ and } j_*(g) \in J_g \forall g.$$

Let Q_g be a price index for group g , which we shall take as

$$(74) \quad Q_g = Q_g^0 \frac{\sum_{j \in J_g} q_j c_j^0}{\sum_{j \in J_g} q_j^0 c_j^0}$$

where (c_j^0) is a reference commodity bundle and (q_j^0) , (Q_g^0) are base-level prices. (74) implies, for the base year:

$$(75) \quad \frac{\partial Q_g}{\partial q_j} \frac{q_j}{Q_g} = \frac{q_j c_j^0}{\sum_{j \in J_g} q_j^0 c_j^0} \stackrel{\text{not.}}{=} \lambda_j \text{ if } j \in J_g$$

$$= 0 \text{ otherwise}$$

In (75), λ_j is the share of the j^{th} commodity in the expenditure on the group to which it belongs (for the reference bundle and the base year); we have taken it as the actual shares in the 1979 consumption vector (this conforms with the MR methodology for computing price indices, though they used the base-year 1961-62 throughout).

The work of MR consists of estimating functions of the form:

$$(76) \quad X_g (\dots Q_g \dots)$$

with

$$(77) \quad \frac{\partial X_f}{\partial Q_g} \frac{Q_g}{X_f} = \epsilon_{fg} + \delta_{fg} \quad \text{where } \delta_{fg} \text{ is the Kronecker delta} \\ (f, g \in G)$$

To obtain the (89×89) elasticity matrix ϵ , we postulate

$$(78) \quad \begin{cases} X_{j*} = q_j \bar{X}_{j*} & j \in \bar{J} \\ X_{j*} = X_g(\dots Q_g(\cdot), \dots) & \text{if } j = j_*(g) \in J^* \end{cases}$$

Using (75) and (78) we easily obtain:

$$(79) \quad \begin{cases} \epsilon_{jl} = 0 & \text{if } j \in \bar{J} \quad (\forall l \in J) \\ \epsilon_{jl} = \epsilon_{gf} \frac{\lambda_l}{\lambda_j} & \text{if } j = j_*(g) \text{ and } l \in J_f \end{cases}$$

Notice that ϵ satisfies the adding-up property; moreover, our aggregation formulae are such as to retrieve the 9×9 elasticity matrix ϵ^1 when commodities are grouped according to the MR classification.

Our introductory remarks should by now be clearer: when consumer prices are allowed to vary, the commodity classification for consumer goods is required to conform to the MR aggregation scheme, except for "fuel and light" and "other non-food"; in these groups, some commodities of secondary quantitative importance in the consumption vector are allowed to appear separately, and are treated, for consumption purposes, as rationed commodities.

The nine "unrationed" commodities are the following, simply chosen according to the criterion of highest share in each of the MR groups (the numbers in brackets indicate the commodity index in the TN classification):

CL :	Paddy (1)
MM :	Milk and milk products (12)
EO :	Edible oil, excluding vanaspati (24)
MFE :	Other animal husbandry (13)
SG :	Gur and khandsari (22)
OF :	Other crops (11)
CT :	Cotton textiles (28)
FL :	Forestry and logging (14)
ONF :	Miscellaneous manufacturing industries (78)

The vector λ and the matrix ϵ appear in Table B.7. for the usual 26 commodity classification.

3A.IV.12 : Value-added coefficients

If V_h denotes value-added, per unit of gross output, in industry h , then the following identity holds:

$$(80) \quad V = D^{-1} P - B^D(P+T) - B^M(P^M + T^M) \quad . \text{ (see also (1))}$$

Since each element contributing to the right-hand side has been estimated, we already possess implicit estimates of V .

These estimates to some extent conflict, as one would

expect, with the value-added row of the TN input-output table. Rather than attempting to reconcile the two pieces of evidence, we have decided to ignore the TN value-added row altogether. This can be justified on the grounds that the value-added row of the TN input-output table is a residual row as well, calculated using the same kind of identity as above, but with much more dubious estimates of indirect taxes than those patiently gathered by A-S.

§ A.IV.13 : The distribution of value-added

Sinha et al (36) have attempted to estimate the distribution of value-added across income classes in different industries for the year 1967-68. Admittedly their work is somewhat outdated, but not exceptionally so by the standards of TN, and for want of more recent studies on the subject at a comparable level of detail, their distribution matrix was adopted and adapted.

The Sinha et al (hereafter SINHA) matrix deals with a 77-industries breakdown of economic activity; this is a slightly revised version of the Saluja 1970-71 input-output table classification. Saluja (35) provides detailed information on the composition of most of these industries as well as on the relative contribution of each sub-sector to an industry's gross output. This provided the basis for a match between the SINHA classification and the TN classification, from which a first estimate of the distribution of value-added for each of our 89 sectors was inferred.

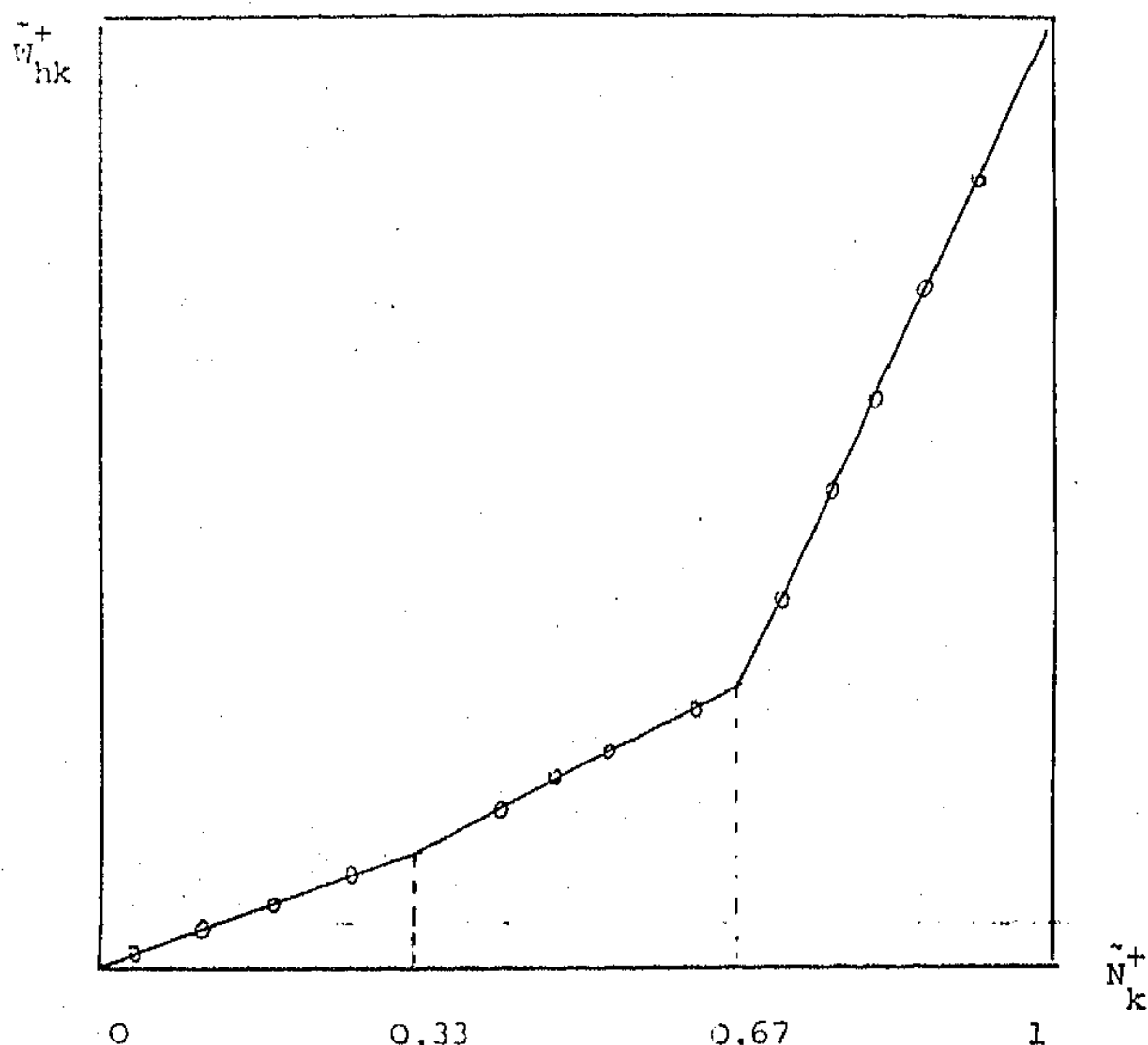
The recipients of value-added in the work of SINHA are, on the one hand, six income classes representing private households (three

classes in each sector, corresponding to equal-sized fractiles of the per capita expenditure scale) and, on the other hand, a residual "other income" group, which receives "non-personal (corporate and government) income", which in their model model is assumed to be saved. The other income category will here simply be called "profits". Our estimate of "profits" in industry h is simply obtained from the combination of the SINHA estimates with the match between the SINHA classification and the TN classification, without further adjustment.

By contrast, class-wise private shares in the distributed component of value-added have been obtained after two further adjustments to the preliminary estimates obtained above. First, for each industry we had to infer the distribution of private value-added (hereafter simply "wages") across our 28 income classes from the preliminary estimates pertaining to 6 broader income groups. Second, the resulting (89 x 28) matrix of wage shares was roughly updated so as to bring the implied overall distribution of wage income in agreement with our separate estimates of income distribution in 1979. These two steps will now be described in greater detail.

Step 1: Let W^0 be the 89 x 6 matrix of wage payments obtained by combining the above preliminary estimates of the distribution of value-added with the value-added vector derived in the previous paragraph. Each row of this matrix has been converted into a 28-component vector of wage payments (matching our own typology of income classes) by linear interpolation along the industry-wise Lorenz curves for wage income; thus, for each industry and each sector, we compute, by linear interpolation using four initial points of the Lorenz curve

for wages, the cumulative wage share of each of our 14 classes, and finally ordinary wage shares:



Formally, if $k = 1, \dots, 14$ and $g = 1, \dots, 3$ are rural income-class indices for our own classification and for the SINHA classification respectively, and if $\tilde{w}^1$ is the (89×28) matrix of wage payments here constructed, the following formulae are used:

$$(81) \quad \tilde{w}_{hk}^{1+} = \tilde{w}_{h,g_k}^{0+} + (\tilde{w}_{h,g_k+1}^{0+} - \tilde{w}_{h,g_k}^{0+}) \frac{\tilde{N}_k^+ - \tilde{N}_{g_k}^+}{\tilde{N}_{g_k+1}^+ - \tilde{N}_{g_k}^+}$$

where g_k is s.t. $\tilde{N}_{g_k}^+ \leq \tilde{N}_k^+ \leq \tilde{N}_{g_k+1}^+$.

$$(82) \quad \tilde{w}_{hk}^1 = (\tilde{w}_{hk}^{1+} - \tilde{w}_{h,k-1}^{1+}) \cdot \tilde{w}_{hR}^0$$

$$(83) \quad W_{hk}^1 = \tilde{W}_{hk}^1 (V_h - \Pi_h) G_h$$

The urban sector is dealt with using analogous formulae.

Step 2: The column sums of the matrix W^1 imply the following distribution of wage income across the six private income groups of the SINHA study:

$$(84) \quad (\tilde{W}_{\cdot g}^1)_{g=1, \dots, 6} = \begin{bmatrix} 0.093 & 0.176 & 0.370 & 0.042 & 0.084 & 0.235 \end{bmatrix}$$

By contrast, our estimate of the distribution of income across these groups, obtained by aggregation of the vector R , is
17/
the following:

$$(85) \quad (\tilde{R}_g)_{g=1, \dots, 6} = \begin{bmatrix} 0.112 & 0.183 & 0.405 & 0.043 & 0.072 & 0.185 \end{bmatrix}$$

The difference between (84) and (85) could be argued to arise from the transfer system. But one would expect the quantitative impact of the transfer system in India to be rather small; and besides there is no evidence that it would work in the direction apparent in (84) and (85). We shall prefer to assume that the transfer system does not have a significant impact on income distribution. As a result, the distribution of income should coincide with the distribution of wage receipts (broadly defined, as we have done here), and the difference between (84) and (85) appears as a statistical discrepancy. More precisely, relative to our estimates of the distribution of

income, the unadjusted figures derived from the work of SINHA seem to underestimate the share of the poor.

This was not unexpected. In fact, Sinha et al., who did not have reliable direct evidence on savings rates and income distribution, could only reconcile their own $\bar{W}$ vector (for 1967-68) with the NSS evidence on expenditure distribution by accepting the strongly negative implicit savings rates for the poorer classes (-77% and -59% for the bottom rural and urban class, respectively!). In our case, a different conclusion is in order: since our estimates of the distribution of income are probably reasonably accurate, while those implicit in the work of SINHA are hardly defensible, the matrix W^0 (or W^1) should be corrected.

Since we do not have the adequate data to trace the source of the discrepancies and make more economically meaningful adjustments, ^{18/} we resorted to the numerical RAS adjustment routine, applied to the matrix W^1 , taking as a row control our value-added vector, and as a column control the wage income vector corresponding to our estimate of the distribution of income:

$$(86) \quad \begin{cases} \bar{W}_{h.}^1 = (V_h - \pi_h) G_h & \text{(row controls)} \\ \bar{W}_{.k}^1 = R_k W_{..}^1 & \text{(column controls)} \end{cases}$$

Finally, the elements of the matrix L are derived from the adjusted matrix W as :

$$(87) \quad L_{hk} = \frac{W_{hk}}{G_h}$$

Table B.8. summarizes the distribution of value-added.

Section A.V : Aggregation scheme

Very few of the exercises we set out to perform can sensibly take place at the level of disaggregation (89 commodities, 28 income groups) present in the core data (one of the few exceptions consists of estimating shadow prices at the 89-commodity level under the assumptions of the non-substitution theorem). A simple computer program was therefore written to carry out the task of re-aggregation, taking as input (besides the core data) a set of three aggregation keys (for commodities, industries, and consumer groups, respectively).

The aggregation formulae are given in Table A.5. below, where J , H , K are index sets identifying specific groups of commodities, industries and consumers, respectively. The superscripts " ^{0} " and " ^{1} " indicate the core data, and the aggregated data, respectively. The chosen formulae preserve consistency in estimation.

TABLE A.5

Aggregation formulae

$$N_K^1 = \sum_{k \in K} N_k^0$$

$$G_H^1 = \sum_{h \in H} G_h^0$$

$$Q_J^{D1} = \sum_{j \in J} Q_j^{D0}$$

$$Q_J^{M1} = \sum_{j \in J} Q_j^{M0}$$

$$B_{JH}^{D1} = \sum_{j \in J} \sum_{h \in H} B_{jh}^{D0} \frac{G_h^0}{G_H^1}$$

$$B_{JH}^{M1} = \sum_{j \in J} \sum_{h \in H} B_{jh}^{M0} \frac{G_h^0}{G_H^1}$$

$$D_{HJ}^1 = \sum_{j \in J} \sum_{h \in H} D_{hj}^{D0} \frac{Q_j^{D0}}{Q_J^{D1}}$$

$$\Pi_H^1 = \sum_{h \in H} \Pi_h^0 \frac{G_h^0}{G_H^1}$$

$$L_{HK}^1 = \sum_{h \in H} \sum_{k \in K} L_{hk}^0 \frac{G_h^0}{G_H^1}$$

$$\epsilon_{JG}^1 = \sum_{j \in J} \sum_{g \in G} \epsilon_{jg}^0 \frac{x_{j.}^0}{x_{J.}^1}$$

$$T_J^1 = \sum_{j \in J} T_j^0 \frac{Q_j^{D0}}{Q_J^{D1}}$$

$$T_J^{M1} = \sum_{j \in J} T_j^{M0} \frac{Q_j^{M0}}{Q_J^{M1}}$$

$$R_K^1 = \sum_{k \in K} R_k^0$$

$$\gamma_k^1 = \sum_{k \in K} \gamma_k^0 \frac{R_k^0}{R_K^1}$$

$$\delta_K^1 = \sum_{k \in K} \delta_k^0 \frac{R_k^0}{R_K^1}$$

$$x_{JK}^{-1} = \sum_{j \in J} \sum_{k \in K} x_{jk}^{-0} \frac{N_k^0}{N_K^1}$$

$$x_{.k}^1 = \sum_{k \in K} \sum_{j \in J} N_k^0 x_{jk}^0$$

$$x_{J.}^1 = \sum_{j \in J} \sum_{k \in K} N_k^0 x_{jk}^0$$

$$\lambda_J^1 = \sum_{j \in J} \lambda_j^0$$

$$\omega_J^1 = \sum_{j \in J} \omega_j^0 \frac{x_{j.}^0}{x_{J.}^1}$$

$$\beta_{JK}^1 = \sum_{j \in J} \sum_{k \in K} \beta_{jk}^0 \frac{x_{.k}^0}{x_{.K}^1}$$

TABLE B.1. Classification schemes(I) Commodity classifications

In the following, the numbers in brackets indicate the codes of the commodities from the TN 89-commodity classification which are covered by the commodity group of interest. An asterisk indicates partial coverage of the concerned commodity, when the latter is split across several commodity groups (e.g. in the NSS 27-commodity classification, the group "Maize and products" consists of only a fraction of the TN "Other cereals" commodity).

(i) The 89-commodity classification (TN)

Index	Commodity
1	Paddy (1)
2	Wheat (2)
3	Jowar (3)
4	Bajra (4)
5	Other Cereals (5)
6	Pulses (6)
7	Sugarcane (7)
8	Jute (8)
9	Cotton (9)
10	Plantation (10)
11	Other Crops (11)
12	Milk and Milk Products (12)
13	Other Animal Husbandry (13)
14	Forestry and Logging (14)
15	Fishing (15)
16	Coal and Lignite (16)
17	Petroleum and Natural Gas (17)
18	Iron Ore (18)
19	Other Minerals (19)
20	Miscellaneous Food Products (20)
21	Sugar (21)
22	Gur and Khandsari (22)
23	Hydrogenated Oil Vanaspati (23)
24	Edible Oil excluding Vanaspati (24)
25	Tea and Coffee (25)
26	Other Beverages (26)
27	Tobacco Manufacture (27)
28	Cotton Textiles (excluding Handloom and Khadi) (28)
29	Cotton Textiles-Handloom and Khadi (29)
30	Woollen and Silk Fabrics (30)
31	Art Silk Fabrics (31)
32	Jute Textiles (32)
33	Readymade Garment, Textile (33)
34	Miscellaneous Textile Products (34)
35	Carpet Weaving (35)
36	Wood Products (36)
37	Paper, Paper products, Newsprint (37)
38	Printing and Publishing (38)
39	Leather and Leather Products (39)
40	Leather Footwear (40)

Cont'd....

41	Rubber Products (41)
42	Plastic and Synthetic Rubber (42)
43	Petroleum Products (43)
44	Miscellaneous Coal & Petroleum Products (44)
45	Inorganic Heavy Chemicals (45)
46	Organic Heavy Chemicals (46)
47	Chemical Fertilisers (47)
48	Insecticide, Fungicide etc. (48)
49	Drugs and Pharmaceuticals (49)
50	Soaps and Glycerine (50)
51	Cosmetics (51)
52	Man Made Fibres (52)
53	Other Chemicals (53)
54	Refractories (54)
55	Cement (55)
56	Other Nonmetallic Products (56)
57	Iron and Steel, Ferroalloys (57)
58	Iron and Steel Castings and Forgings (58)
59	Iron and Steel Structure (59)
60	Nonferrous Metals including Alloys (60)
61	Metal Products (61)
62	Tractors and Other Agricultural Implements (62)
63	Machine Tools (63)
64	Office, Domestic and Commercial Equipment (64)
65	Other Nonelectrical Machinery (65)
66	Electrical Motors (66)
67	Electrical Cables, Wires (67)
68	Batteries (68)
69	Electrical Household Goods (69)
70	Communications and Electronic Equipment (70)
71	Other Electrical Machinery (71)
72	Ships and Boats (72)
73	Rail Equipment (73)
74	Motor Vehicles (74)
75	Motor Cycles and Bicycles (75)
76	Other Transport Equipment (76)
77	Watches and Clocks (77)
78	Miscellaneous Manufacturing Industries (78)
79	Construction (79)
80	Gas, Electricity, Water Supply (80)
81	Railways (81)
82	Other Transport (82)
83	Communication (83)
84	Trade, Storage and Warehousing (84)
85	Banking and Insurance (85)
86	Real Estate and Ownership of Dwellings (86)
87	Education (87)
88	Medical Health (88)
89	Other Services (89)

Cont'd.....

(ii) The 27-commodity classification (NSS)

Index	Commodity
1	Rice and Products (1)
2	Wheat and Products (2)
3	Jowar and Products (3)
4	Bajra and Products (4)
5	Maize and Products (5*)
6	Ragi and Products (5*)
7	Barley and Products (5*)
8	Small millets and Products (5*)
9	Gram and Products (5*)
10	Cereal substitutes (5*)
11	Pulses and Products (6)
12	Milk and milk Products (12)
13	Edible oil (10, 23, 24)
14	Meat, fish and eggs (13, 15)
15	Vegetables (11*, 20*)
16	Fruits and nuts (11*, 20*)
17	Sugar and gur (7, 21, 22)
18	Salt (20*)
19	Spices (11*, 20*)
20	Beverages and refreshments (25, 26)
21	Pan, tobacco and intoxicants (27)
22	Fuel and light (14, 16, 43, 44, 80)
23	Clothing (28-35)
24	Footwear (40)
25	Miscellaneous goods and services (37, 38, 42, 48, 50, 51, 53, 77, 81-85, 87-89)
26	Rents (86)
27	Durable goods (36, 39, 41, 56, 61, 64, 65, 67-71, 74, 75, 78)

(iii) The 9-commodity classification (MR)

Index	Commodity
1	Cereals (1, 2, 3, 4, 5)
2	Milk and Milk Products (12)
3	Edible Oils (23, 24)
4	Meat, Fish, Eggs (13, 15)
5	Sugar and Gur (7, 21, 22)
6	Other Food (6, 11, 20, 25, 26, 27)
7	Clothing (28, 29, 30, 31, 32, 33, 34, 35)
8	Fuel and Light (14, 16, 43, 44, 80)
9	Other Non-food (8-10, 17-19, 36-42, 45-79, 81-89)

Cont'd....

(iv) The 26-commodity classification

The following 26-commodity classification has proved very useful in many empirical exercises and is used in this Annex for displaying the data (Tables B.2-B.8).

Index	Commodity
1	Cereals (1-5)
2	Milk and Milk Products (12)
3	Edible Oils (23-24)
4	Meat, Fish and Eggs, Animal Husbandry (13,15)
5	Sugar and Gur (7,21,22)
6	Other food (and tobacco) (6,11,20,25-27)
7	Clothing (28-35)
8	Fuel and Light (14,16,43,44)
9	Miscellaneous Consumer Goods (38-41,49-51,61,68,69,75,77,78)
10	Miscellaneous Services (85-89)
11	Plantations, Raw Jute and Cotton (8,9,10)
12	Iron Ore (18)
13	Other Minerals (19)
14	Paper and Paper Products (37)
15	Plastics and Synthetic Rubber (42)
16	Heavy Chemicals, Manmade Fibres and Other Chemicals (45,46,52,53)
17	Chemicals, Fertilisers, Insecticides etc. (47,48)
18	Iron and Steel (57,58,59)
19	Non-ferrous Metals (60)
20	Petroleum and Natural Gas (17)
21	Gas, Electricity and Water Supply (80)
22	Construction Materials (36,54-56)
23	Construction (79)
24	Railways, Transport and Communication (81-83)
25	Trade, Storage and Warehousing (84)
26	Miscellaneous Investment Goods (62-67,70-74,76)

(II) Industry classification

For our purposes, the "industry" classification is always identical to the "commodity" classification.

(III) The consumer classification(i) With 28 income-classes

In the core data set, there are 28 income (or expenditure) classes, each of which should be interpreted as a specific fractile of a sectoral (i.e. rural or urban) per capita expenditure scale (see § A.IV.4). The table below gives the share of each class in sectoral population, as well as the corresponding per capita expenditure ranges in 1973-74.

Cont'd.....

Index.	Share of sectoral population	Monthly per capita expenditure range in 1973-74. (Rs.)
RURAL	1	0.0036
	2	0.0039
	3	0.0120
	4	0.0219
	5	0.0314
	6	0.0601
	7	0.1213
	8	0.1957
	9	0.2058
	10	0.1897
	11	0.0911
	12	0.0479
	13	0.0102
	14	0.0056
URBAN	15	0.0005
	16	0.0012
	17	0.0030
	18	0.0051
	19	0.0132
	20	0.0302
	21	0.0713
	22	0.1501
	23	0.1939
	24	0.2273
	25	0.1453
	26	0.1009
	27	0.0332
	28	0.0250

(ii) With 10 income classes

In many empirical exercises, as well as for the purposes of displaying the data (Tables B.2-B.8. below), the population has been divided into 10 income groups, consisting of the following sets of classes from the previous classification:

Cont'd....

Index		Classes covered
RURAL	1	1-7
	2	8
	3	9
	4	10
	5	11-14
URBAN	6	15-21
	7	22
	8	23
	9	24
	10	25-28

TABLE B.2

j	G	Q^D	Q^M	T	T^M
1	0.2244	0.2235	0.0002	-0.0265	0.0136
2	0.0784	0.0784	0.0002	0.0007	0.0010
3	0.0307	0.0306	0.0081	0.1000	0.1086
4	0.0472	0.0472	0.0004	0.0000	0.4278
5	0.0739	0.0732	0.0000	0.0600	0.0000
6	0.2550	0.2560	0.0016	0.1326	0.4751
7	0.1413	0.1421	0.0005	0.0425	0.0817
8	0.0784	0.0794	0.0146	0.2653	0.2892
9	0.1733	0.1739	0.0053	0.1428	0.5355
10	0.3635	0.3634	0.0000	0.0015	0.0000
11	0.0447	0.0447	0.0003	-0.0030	0.0050
12	0.0015	0.0015	0.0000	0.0985	0.0000
13	0.0042	0.0042	0.0074	-0.0159	0.0402
14	0.0116	0.0118	0.0025	0.1690	0.2637
15	0.0103	0.0097	0.0014	0.2322	1.0320
16	0.0285	0.0314	0.0059	0.3711	1.0643
17	0.0361	0.0341	0.0090	-0.2387	0.1221
18	0.0423	0.0409	0.0090	0.0745	0.4660
19	0.0191	0.0199	0.0048	0.0703	0.6098
20	0.0061	0.0061	0.0306	0.1652	0.0038
21	0.0510	0.0510	0.0000	0.0394	0.0000
22	0.0651	0.0654	0.0013	0.0652	0.2172
23	0.2058	0.2058	0.0000	-0.0005	0.0000
24	0.1439	0.1439	0.0000	0.0259	0.0000
25	0.2707	0.2707	0.0000	-0.0077	0.0000
26	0.1044	0.1027	0.0143	0.1130	0.8147

Explanatory notes:

- (1) j = commodity (or industry) index (see Table B.1)
 G = vector of industry gross domestic outputs
 Q^D = vector of commodity gross domestic outputs
 Q^M = vector of commodity gross imports
 T = vector of indirect taxes for domestically produced commodity
 T^M = vector of indirect taxes for imports
- (2) For normalization rules, see §A.IV.1.
- (3) For further detail, see text (§A.IV.2, A.IV.3).

TABLE B.3. The domestic input-output matrix (B^D)

COL	1	2	3	4	5	6	7	8	9	10
ROW										
1	0.0659	0.0563	0.0008	0.0539	0.0031	0.0115	0.0014	0.0000	0.0029	0.0145
2	0.0000	0.0000	0.0000	0.0000	0.0001	0.0201	0.0000	0.0000	0.0001	0.0028
3	0.0000	0.0030	0.0682	0.0035	0.0450	0.0034	0.0000	0.0000	0.0120	0.0001
4	0.0144	0.0000	0.0000	0.0000	0.0037	0.0185	0.0044	0.0000	0.0136	0.0026
5	0.0000	0.0000	0.0000	0.0000	0.3157	0.0049	0.0000	0.0000	0.0012	0.0006
6	0.0007	0.2787	0.2478	0.3460	0.0211	0.0810	0.0081	0.0000	0.0188	0.0048
7	0.0004	0.0179	0.0074	0.0159	0.0068	0.0011	0.1932	0.0022	0.0109	0.0016
8	0.0103	0.0000	0.0067	0.0031	0.0102	0.0105	0.0158	0.0315	0.0161	0.0025
9	0.0001	0.0091	0.0490	0.0146	0.0072	0.0098	0.0067	0.0017	0.1049	0.1155
10	0.0117	0.0105	0.0093	0.0359	0.0177	0.0152	0.0501	0.0421	0.0236	0.0552
11	0.0000	0.0000	0.2386	0.0000	0.0000	0.0426	0.1422	0.0000	0.0164	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000	0.0006	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0005	0.0026	0.0020	0.0000	0.0120	0.0080
15	0.0000	0.0000	0.0001	0.0000	0.0000	0.0005	0.0031	0.0005	0.0141	0.0018
16	0.0000	0.0000	0.0071	0.0000	0.0012	0.0028	0.0429	0.0025	0.0417	0.0001
17	0.0656	0.0000	0.0000	0.0000	0.0260	0.0125	0.0000	0.0000	0.0003	0.0000
18	0.0000	0.0000	0.0063	0.0000	0.0000	0.0001	0.0013	0.0000	0.0136	0.0042
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0211	0.0001
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0020	0.0038	0.0188	0.0087	0.0000
21	0.0031	0.0000	0.0042	0.0000	0.0130	0.0044	0.0199	0.0130	0.0104	0.0199
22	0.0000	0.0000	0.0007	0.0000	0.0003	0.0023	0.0022	0.0000	0.0071	0.0007
23	0.0130	0.0008	0.0039	0.0023	0.0198	0.0167	0.0144	0.0089	0.0163	0.0419
24	0.0015	0.0076	0.0310	0.0095	0.0191	0.0088	0.0307	0.0239	0.0344	0.0321
25	0.0096	0.0625	0.1073	0.0818	0.0923	0.0409	0.0850	0.0066	0.1144	0.0374
26	0.0010	0.0000	0.0048	0.0020	0.0042	0.0029	0.0210	0.0017	0.0060	0.0051

Cont'd....

COL	11	12	13	14	15	16	17	18	19	20
ROW										
1	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0026	0.0000	0.0000	0.0000	0.0000
4	0.0152	0.0000	0.0000	0.0057	0.0000	0.0005	0.0004	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0007	0.0000	0.0031	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0087	0.0012	0.0061	0.0009	0.0000	0.0000	0.0000
7	0.0025	0.0000	0.0000	0.0242	0.0036	0.0106	0.0163	0.0028	0.0009	0.0000
8	0.0095	0.1591	0.1016	0.0787	0.0140	0.0442	0.0404	0.0729	0.0418	0.0319
9	0.0000	0.1352	0.0650	0.0171	0.0241	0.0366	0.0188	0.0130	0.0120	0.0325
10	0.0511	0.0564	0.0129	0.0351	0.0398	0.0320	0.0254	0.0377	0.0126	0.0981
11	0.0003	0.0000	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0085	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0018	0.0005	0.0032	0.0140	0.0037	0.0247	0.0000
14	0.0000	0.0000	0.0000	0.0657	0.0076	0.0249	0.0003	0.0005	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0064	0.1458	0.0036	0.0144	0.0011	0.0004	0.0000
16	0.0000	0.0255	0.0081	0.1057	0.0903	0.0964	0.0583	0.0024	0.0351	0.0160
17	0.1788	0.0000	0.0000	0.0000	0.0059	0.0009	0.1280	0.0000	0.0000	0.0000
18	0.0000	0.0000	0.0000	0.0003	0.0003	0.0045	0.0011	0.0527	0.0050	0.0000
19	0.0000	0.0000	0.0000	0.0000	0.0005	0.0160	0.0001	0.0454	0.1545	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0056	0.0002	0.0000	0.0001	0.0039	0.0000
21	0.0022	0.0466	0.0383	0.0470	0.0157	0.0239	0.0702	0.0308	0.0618	0.0161
22	0.0000	0.0000	0.0000	0.0137	0.0019	0.0053	0.0007	0.0043	0.0004	0.0010
23	0.0209	0.0052	0.0065	0.0791	0.0174	0.0375	0.0290	0.1167	0.0260	0.0035
24	0.0139	0.0282	0.0139	0.0387	0.0142	0.0370	0.0366	0.0582	0.0229	0.0050
25	0.0322	0.0451	0.0218	0.1372	0.0477	0.1095	0.1157	0.1170	0.0957	0.0114
26	0.0007	0.0437	0.0210	0.0040	0.0023	0.0105	0.0094	0.0028	0.0022	0.0105

Cont'd.....

ROW	COL	21	22	23	24	25	26
1		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4		0.0000	0.0007	0.0000	0.0132	0.0000	0.0000
5		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6		0.0000	0.0042	0.0000	0.0020	0.0000	0.0000
7		0.0001	0.0168	0.0000	0.0074	0.0000	0.0093
8		0.1437	0.1016	0.0266	0.0729	0.0004	0.0130
9		0.0007	0.0169	0.0020	0.0535	0.0081	0.0512
10		0.0434	0.0261	0.0082	0.0560	0.1119	0.0484
11		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13		0.0000	0.0128	0.0053	0.0000	0.0000	0.0001
14		0.0000	0.0024	0.0000	0.0012	0.0053	0.0012
15		0.0000	0.0053	0.0000	0.0000	0.0000	0.0070
16		0.0006	0.0174	0.0089	0.0000	0.0001	0.0115
17		0.0000	0.0012	0.0000	0.0000	0.0000	0.0000
18		0.0000	0.0198	0.0993	0.0023	0.0000	0.0532
19		0.0000	0.0000	0.0003	0.0000	0.0000	0.0923
20		0.0000	0.0238	0.0000	0.0000	0.0000	0.0016
21		0.2309	0.0310	0.0070	0.0069	0.0168	0.0119
22		0.0000	0.0422	0.1935	0.0000	0.0050	0.0045
23		0.0025	0.0199	0.0000	0.0204	0.0001	0.0188
24		0.0440	0.0529	0.0571	0.0211	0.0766	0.0249
25		0.0177	0.1459	0.2339	0.0585	0.0223	0.0811
26		0.0141	0.0030	0.0001	0.0552	0.0001	0.1044

Explanatory notes: (1) The $(j,h)^{th}$ element of the matrix, B_{jh}^D , indicates the expenditure on intermediate domestic purchases of commodity j per rupee of gross output in industry h (see text, §A.IV.2 for details).

(2) For the commodity classification, see Table B.1.

TABLE B.4

k	N	$\tilde{R}$	δ	γ
1	0.1957	0.0760	0.0388	0.1252
2	0.1507	0.0887	0.0790	0.1934
3	0.1584	0.1215	0.1055	0.2440
4	0.1460	0.1547	0.1524	0.3257
5	0.1192	0.2589	0.2940	0.4586
6	0.0286	0.0119	0.0478	0.1335
7	0.0345	0.0204	0.0791	0.1662
8	0.0446	0.0342	0.0989	0.2402
9	0.0523	0.0556	0.1508	0.2921
10	0.0700	0.1781	0.3160	0.4834

Explanatory notes:

(1) k = income class index (see Table B.1)

N = vector of class sizes ($\sum_k N_k = 1$).

$\tilde{R}$ = vector of shares in total income ($\sum_k \tilde{R}_k = 1$)

δ = vector of average savings rates

γ = vector of marginal savings rates.

(2) For further detail, see text (§ A.IV.4., A.IV.7, A.IV.8., A.IV.9).

TABLE B.5

Matrix of per capita consumer expenditures ($\bar{X}$)

j	RURAL					URBAN					w
	1	2	3	4	5	6	7	8	9	10	
1	0.2727	0.3601	0.4194	0.4824	0.5759	0.2294	0.2855	0.3190	0.3573	0.3725	0.9989
2	0.0126	0.0326	0.0566	0.0935	0.1940	0.0216	0.0424	0.0675	0.1070	0.2247	0.9979
3	0.0162	0.0265	0.0331	0.0430	0.0635	0.0239	0.0368	0.0491	0.0653	0.1014	0.7398
4	0.0101	0.0167	0.0222	0.0311	0.0504	0.0149	0.0207	0.0280	0.0394	0.0759	0.9820
5	0.0107	0.0202	0.0274	0.0393	0.0658	0.0168	0.0244	0.0317	0.0427	0.0624	1.0000
6	0.0755	0.1152	0.1443	0.1832	0.3009	0.0989	0.1351	0.1757	0.2311	0.5055	0.9952
7	0.0096	0.0200	0.0386	0.0806	0.2327	0.0062	0.0117	0.0210	0.0379	0.1504	0.9945
8	0.0271	0.0346	0.0408	0.0487	0.0661	0.0300	0.0397	0.0473	0.0601	0.0902	0.8153
9	0.0015	0.0032	0.0062	0.0133	0.0725	0.0021	0.0036	0.0062	0.0108	0.0623	0.9443
10	0.0098	0.0167	0.0250	0.0414	0.1142	0.0225	0.0377	0.0586	0.0909	0.2794	1.0000
11	0.0002	0.0003	0.0004	0.0005	0.0007	0.0003	0.0004	0.0006	0.0008	0.0012	1.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
14	0.0001	0.0002	0.0002	0.0004	0.0011	0.0002	0.0003	0.0004	0.0006	0.0017	0.7164
15	0.0001	0.0001	0.0002	0.0003	0.0009	0.0001	0.0002	0.0003	0.0005	0.0015	1.0000
16	0.0000	0.0001	0.0001	0.0002	0.0004	0.0001	0.0001	0.0001	0.0002	0.0007	0.9428
17	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0002	1.0000
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
21	0.0061	0.0078	0.0091	0.0109	0.0148	0.0067	0.0089	0.0106	0.0135	0.0202	1.0000
22	0.0001	0.0003	0.0008	0.0021	0.0215	0.0001	0.0002	0.0005	0.0011	0.0160	0.9910
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
24	0.0042	0.0071	0.0105	0.0172	0.0472	0.0069	0.0111	0.0165	0.0256	0.0765	1.0000
25	0.0056	0.0095	0.0141	0.0230	0.0631	0.0092	0.0149	0.0220	0.0342	0.1023	1.0000
26	0.0000	0.0002	0.0005	0.0014	0.0143	0.0001	0.0001	0.0004	0.0007	0.0106	0.8896

Cont'd.....

Explanatory notes:

(1) $\bar{x}_{jk}$ = per capita expenditure on commodity j
in class k

$1-\omega_j$ = import proportion in consumer expenditure on
commodity j

(2) For commodity and class indices, see Table B.1.

(3) For further details, see text (§A.IV.5, A.IV.6).

TABLE B.6.

Matrix of marginal expenditure shares (β)

j \ k	RURAL					URBAN				
	1	2	3	4	5	6	7	8	9	10
1	0.4526	0.3535	0.2773	0.2094	0.0880	0.3347	0.2240	0.1610	0.0892	0.0046
2	0.0829	0.1241	0.1382	0.1398	0.1181	0.1099	0.1285	0.1449	0.1387	0.1019
3	0.0464	0.0434	0.0373	0.0321	0.0254	0.0673	0.0697	0.0639	0.0582	0.0256
4	0.0295	0.0318	0.0326	0.0289	0.0250	0.0296	0.0372	0.0419	0.0405	0.0340
5	0.0412	0.0449	0.0433	0.0388	0.0325	0.0429	0.0394	0.0410	0.0351	0.0142
6	0.1856	0.1754	0.1542	0.1525	0.1477	0.1951	0.2130	0.2153	0.2406	0.2574
7	0.0439	0.0842	0.1375	0.1705	0.1957	0.0235	0.0454	0.0588	0.0754	0.1183
8	0.0373	0.0347	0.0319	0.0273	0.0208	0.0518	0.0468	0.0458	0.0404	0.0261
9	0.0074	0.0134	0.0228	0.0363	0.0926	0.0061	0.0128	0.0161	0.0258	0.0629
10	0.0310	0.0417	0.0562	0.0740	0.0970	0.0769	0.1034	0.1194	0.1465	0.1893
11	0.0005	0.0005	0.0004	0.0004	0.0003	0.0008	0.0008	0.0007	0.0007	0.0003
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0003	0.0004	0.0005	0.0007	0.0009	0.0005	0.0006	0.0007	0.0009	0.0012
15	0.0003	0.0003	0.0004	0.0006	0.0008	0.0004	0.0005	0.0006	0.0007	0.0010
16	0.0001	0.0002	0.0002	0.0003	0.0004	0.0002	0.0003	0.0003	0.0003	0.0005
17	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0084	0.0078	0.0072	0.0061	0.0047	0.0116	0.0105	0.0103	0.0091	0.0059
22	0.0010	0.0022	0.0040	0.0068	0.0338	0.0001	0.0015	0.0020	0.0050	0.0202
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0133	0.0171	0.0228	0.0304	0.0401	0.0207	0.0276	0.0324	0.0383	0.0527
25	0.0178	0.0229	0.0305	0.0406	0.0536	0.0278	0.0369	0.0434	0.0512	0.0705
26	0.0004	0.0014	0.0027	0.0045	0.0225	0.0000	0.0010	0.0013	0.0033	0.0134

Explanatory notes: (1) β_{jk} = marginal share of commodity j in k's expenditure ($\sum_j \beta_{jk} = 1$)

(2) For commodity and class indices, see Table B.1.

(3) For further details see text (§ A.IV.5, A.IV.9)

TABLE B.7. The consumer price elasticity matrix (e)

COL	1	2	3	4	5	6	7	8	9	10
ROW										
1	-0.5305	-0.0181	-0.0180	-0.0377	-0.0128	-0.0376	-0.0120	0.0021	0.0004	0.0014
2	-0.5534	-1.1297	-0.1533	0.1470	0.0182	0.1134	-0.0330	-0.0405	-0.0046	-0.0147
3	-0.2871	-0.1874	-0.4780	0.0500	-0.0560	0.1337	-0.0157	-0.0212	-0.0026	-0.0083
4	-0.5139	0.3789	0.0563	-0.8461	0.0107	0.0867	-0.0164	-0.0249	-0.0025	-0.0080
5	-0.4102	0.0632	-0.0840	0.0028	-0.5933	-0.1655	-0.0234	-0.3269	-0.0040	-0.0128
6	-0.1873	0.1080	0.0286	0.0112	-0.0227	-0.8432	-0.0163	-0.0155	-0.0012	-0.0039
7	-0.2974	-0.0367	-0.0277	-0.0214	-0.0166	-0.1069	-0.7388	-0.0322	-0.0034	-0.0110
8	-0.1602	0.0021	-0.0121	-0.0099	-0.0031	-0.0472	-0.0144	-0.4859	-0.0019	-0.0060
9	-2.1697	-0.2790	-0.2159	-0.2097	-0.1325	-0.9137	-0.2300	-0.2304	-0.9417	-3.0203
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Cont'd.....

COL	11	12	13	14	15	16	17	18	19	20
ROW										
1	-0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	-0.0018	0.0000	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000	0.0000	0.0000
3	-0.0055	0.0000	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000	0.0000	0.0000
4	0.0007	0.0000	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000	0.0000	0.0000
5	-0.0010	0.0000	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000	0.0000	0.0000
6	0.0003	0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000
7	-0.0003	0.0000	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000	0.0000	0.0000
8	-0.0001	0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000
9	-0.0025	0.0000	0.0000	-0.0236	-0.0200	-0.0095	-0.0026	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Cont'd.....

COL	21	22	23	24	25	26	λ
ROW							
1	0.0005	0.0001	0.0000	0.0005	0.0006	0.0001	1.0000
2	-0.0091	-0.0011	0.0000	-0.0051	-0.0068	-0.0007	1.0000
3	-0.0048	-0.0006	0.0000	-0.0029	-0.0038	-0.0004	0.9886
4	-0.0056	-0.0006	0.0000	-0.0028	-0.0037	-0.0004	1.0000
5	-0.0733	-0.0010	0.0000	-0.0044	-0.0059	-0.0006	1.0000
6	-0.0035	-0.0003	0.0000	-0.0014	-0.0018	-0.0002	1.0000
7	-0.0072	-0.0008	0.0000	-0.0038	-0.0051	-0.0005	1.0000
8	-0.1090	-0.0005	0.0000	-0.0021	-0.0028	-0.0003	0.8168
9	-0.0517	-0.2264	0.0000	-1.0455	-1.3984	-0.1503	0.1377
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4417
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0114
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0035
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0029
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0014
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1832
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0331
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1529
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2045
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0220

Cont'd.....

Explanatory notes:

- (1) For details on the construction of this matrix, and on the meaning of λ , see text (§A.IV.11).
- (2) Strictly speaking, the 26-commodity classification scheme adopted here does not exactly satisfy the requirements referred to in §A.IV.11 : these would imply $\lambda_j = 1$ for $j = 1, \dots, 7$, which is slightly violated for $j = 3$ ($\lambda_3 = 0.9886$). This quantitatively insignificant discrepancy is due to the omission of "Plantations" from the third commodity group in the 26-commodity classification (see Table B.1.). Nevertheless, consistency in estimation is maintained by the application of formula (79) (§A.IV.11).

TABLE B.8.

Matrix of value-added coefficients (L)

h	RURAL					URBAN					II	V
	1	2	3	4	5	6	7	8	9	10		
1	0.1203	0.0638	0.0603	0.1980	0.3401	0.0006	0.0011	0.0010	0.0015	0.0060	0.0000	0.7927
2	0.0159	0.0591	0.0954	0.1388	0.2335	0.0001	0.0002	0.0009	0.0015	0.0047	-0.0000	0.5502
3	0.0081	0.0137	0.0203	0.0125	0.0200	0.0013	0.0022	0.0035	0.0079	0.0429	0.0069	0.1391
4	0.0123	0.0457	0.0738	0.1073	0.1805	0.0001	0.0002	0.0007	0.0012	0.0036	-0.0000	0.4253
5	0.0208	0.0220	0.0294	0.1023	0.1758	0.0005	0.0008	0.0017	0.0028	0.0088	0.0071	0.3721
6	0.0360	0.0443	0.0616	0.1806	0.3096	0.0014	0.0024	0.0032	0.0054	0.0205	0.0013	0.6662
7	0.0097	0.0162	0.0240	0.0138	0.0219	0.0071	0.0121	0.0189	0.0321	0.1162	0.0095	0.2816
8	0.0168	0.0339	0.0516	0.0600	0.1000	0.0069	0.0119	0.0124	0.0157	0.0333	0.0972	0.4396
9	0.0039	0.0070	0.0105	0.0123	0.0205	0.0068	0.0117	0.0211	0.0388	0.1553	0.0566	0.3446
10	0.0522	0.0731	0.1044	0.0560	0.0881	0.0075	0.0128	0.0261	0.0434	0.1361	0.0154	0.6150
11	0.0540	0.0506	0.0650	0.1677	0.2868	0.0004	0.0006	0.0011	0.0023	0.0111	0.0046	0.6441
12	0.0192	0.0408	0.0625	0.0435	0.0703	0.0148	0.0254	0.0205	0.0196	0.0074	0.0364	0.3605
13	0.0336	0.0715	0.1094	0.0762	0.1230	0.0260	0.0446	0.0359	0.0342	0.0130	0.0878	0.6552
14	0.0019	0.0037	0.0055	0.0090	0.0153	0.0041	0.0071	0.0178	0.0314	0.1045	0.0390	0.2392
15	0.0019	0.0036	0.0055	0.0088	0.0149	0.0061	0.0105	0.0166	0.0283	0.1022	0.0475	0.2460
16	0.0005	0.0008	0.0012	0.0020	0.0034	0.0074	0.0127	0.0203	0.0381	0.1634	0.0781	0.3279
17	0.0046	0.0090	0.0135	0.0220	0.0372	0.0046	0.0078	0.0127	0.0226	0.0880	0.1432	0.3653
18	0.0017	0.0035	0.0053	0.0088	0.0148	0.0036	0.0062	0.0278	0.0466	0.1182	0.0154	0.2518
19	0.0018	0.0038	0.0058	0.0095	0.0161	0.0009	0.0016	0.0036	0.0211	0.1825	0.0898	0.3365
20	0.0319	0.0673	0.1029	0.0717	0.1156	0.0246	0.0421	0.0338	0.0322	0.0121	0.2172	0.7514
21	0.0054	0.0103	0.0155	0.0255	0.0431	0.0033	0.0057	0.0098	0.0202	0.0967	0.2200	0.4555
22	0.0163	0.0277	0.0411	0.0269	0.0432	0.0069	0.0119	0.0209	0.0339	0.1058	0.0323	0.3671
23	0.0223	0.0278	0.0388	0.0445	0.0742	0.0038	0.0065	0.0114	0.0189	0.0626	0.0045	0.3153
24	0.0223	0.0317	0.0455	0.0385	0.0630	0.0215	0.0369	0.0422	0.0511	0.0732	0.0816	0.5074
25	0.0696	0.0926	0.1309	0.0664	0.1038	0.0090	0.0155	0.0304	0.0508	0.1612	0.0187	0.7489
26	0.0022	0.0044	0.0067	0.0108	0.0182	0.0066	0.0113	0.0218	0.0400	0.1568	0.0505	0.3293

Cont'd.....

Explanatory notes:

(1) k = industry index (see Table B.1.)

L_{hk} = wages of class k per unit of gross output in industry h

Π_h = "profits" in industry h per unit of gross output

V_h = value-added in " " " " " " "

$$(V_h = \sum_k L_{hk} + \Pi_h)$$

(2) For further details see text (§ A.IV.12, A.IV.13).

Footnotes:

- 1/ One of these complications arises from the distinction between a linear "commodity technology" and a linear "industry technology" (Saluja (35), pp.42-45); the TN methodology follows the "industry technology" assumption. For greater detail see A-S (2).
- 2/ So far, however, no allowance could be made for household production, though the A-S data set should in principle make it possible.
- 3/ See Ahmad and Stern (3), p.44-45, Table 3.6, column 3, for the vector $P+T$ of consumer prices; p.31-32, Table 3.2(b), column 2, for T^f ; and p.37-38, Table 3.4, column 2 for T^a .
- 4/ NSS (29), pp.7-19 (rural), and pp.70-83 (urban).
- 5/ Idem.
- 6/ These multipliers do not figure in the published NSS Report; they have been taken from the A-S data set. Strictly speaking, these multipliers apply to households rather than individuals. The formula is therefore valid only if, for each state and each class, the average household size in the sample is representative of the average household size in the state; this assumption is plausible from the sampling frame.
- 7/ For the highest expenditure class in each sector, β_{jk}^+ is taken to be identical to $\beta_{j(k-1)}^+$.
- 8/ Some differences of concepts are apparent from the NSS and NCAER reports, particularly relating to the treatment of indirect taxes and durable goods; no serious attempt has been made to take these discrepancies into account. Their quantitative importance is not likely to be very great for our purposes.
- 9/ NCAER (28), p.187.
- 10/ For the expenditure classes at the top of each intra-sectoral distribution, a zero weight was given to the right-hand MPC; and correspondingly for the bottom classes.
- 11/ Government of India (11), p.71.
- 12/ We are implicitly assuming labour supplies to be fixed or rationed here, so that the price derivatives of labour supplies disappear.
- 13/ More recent work, however, revealed that in different contexts other restrictions, e.g. the homogeneity restriction, may play an important role as well.

- 14/ Saluja (35), p.87-104.
- 15/ Sinha et al (36), p.53.
- 16/ Strictly speaking, the name "Lorenz curve" is justified only when, in the industry and sector of interest, the three SINHA classes rank in the same order in the distribution of wage income in that industry and sector as in the overall income distribution for that sector. This is usually the case; nevertheless our estimation procedure remains permissible when it is not.
- 17/ The aggregation formulae used here are approximate; but further refinements would not significantly modify the estimates in (84) and (85), and since we only wish to illustrate the argument such refinements would be useless.
- 18/ See e.g. Slater (37). The RAS technique is frequently used in TN for similar purposes.

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