INDIAN STATISTICAL INSTITUTE

Backpaper Examination: 2023–2024

B. Stat. (Hons.) 2nd Year. 2nd Semester

Statistical Methods IV

Date: August 25, 2025 | Total Marks: 100 | Time: 3 hours

ANSWER ALL THE QUESTIONS.

1. Let $\boldsymbol{X} = (X_1, X_2, X_3)$ be a trivariate random vector. Suppose that $\text{Corr}(X_1, X_2) = 0.25$, $\text{Corr}(X_1, X_3) = 0.50$, $\text{Corr}(X_2, X_3) = 0.25$. Compute the multiple correlation between X_1 and (X_2, X_3). [10]

2. Suppose that we have n i.i.d. observations on the random vector $(X_1, \ldots, X_d)$ with mean vector $\boldsymbol{\mu}$ and p.d. dispersion matrix $\boldsymbol{\Sigma}$. Assume $n > d \geq 3$. We wish to test the null hypothesis $H_0 : \boldsymbol{\mu} = \mathbf{0}$. Find the likelihood ratio test for this problem. What will be the rejection region? [12+3=15]

3. Suppose that for $i = 1, 2, 3$, $X_{ij}, j = 1, \ldots, n_i$ are iid observations from a $N(\mu_i, \sigma^2)$ population. Suppose, in addition, that all the X_{ij}'s are independent. We wish to test the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$. Derive the likelihood ratio test statistic for this problem. Find its null distribution. [10+15=25]

4. Consider two regression models

$$y_{i1} = \alpha_1 + \beta_1 x_{i1} + \epsilon_{i1},\ i = 1, \ldots, m,$$

and

$$y_{j2} = \alpha_2 + \beta_2 x_{j2} + \epsilon_{j2},\ j = 1, \ldots, n.$$

The quantities x_{i1}'s and x_{j2}'s are assumed to be non-stochastic. Also, the ϵ_{i1}'s and ϵ_{j2}'s are i.i.d. $\text{N}(0, \sigma^2)$ errors. Suppose we wish to test the hypothesis that $H_0 : \beta_1 = \beta_2$. Develop a suitable test for H_0, making necessary assumptions. [15]

5. The following table gives the results of 100 pupils in a class in the half-yearly and annual examinations.

		Half-yearly exam	
		Passed	Failed
Annual exam	Passed	65	6
	Falied	8	21

We wish to test if the annual exam results have any association with the results of the half-yearly exam. Describe a suitable test along with its large-sample distribution. Find an expression for the p-value in terms of Φ, the CDF of $\text{N}(0, 1)$. [10+5=15]

[P.T.O.]

6. Suppose we have n i.i.d. observations from a bivariate normal distribution with unknown parameters. Denote by ρ and ρ_n, the population and sample correlation coefficients, respectively.

   (a) Find a function h such that the expression for variance in the asymptotic normal distribution of $h(\rho_n)$ does not depend on ρ. [10]

   (b) Suppose now that $n = 100$ and $\rho_n = 0.30$. Obtain a 95% confidence interval for ρ. [Note. $\Phi(1.96) = 0.975$] [10]
