Indian Statistical Institute

Periodical Examination of Second Semester (2024-2025)
M. TECH. (CrS) First Year
Subject: Information and Coding Theory
Date: ~~03~~/~~03~~/2025 27.02 Time: 2 hours Maximum Marks: 40

Note: **Notations used are as explained in the class.**

1. For random variables (X, Y, Z), prove that $H(X, Y|Z) = H(X|Z) + H(Y|X, Z)$. [5]

2. Prove that $H(Y|X) = 0$ if and only if Y is a function of X. [10]

3. Prove that any sequence of $(2^{nR}, n)$ codes with $\lambda^{(n)} \to 0$ must have $R \leq C$. [10]

4. Let $\mathcal{X} = \{1, 2, 3\}$ and $X_1, X_2, X_3 \ldots$ be a time invariant Markov chain where X_is take values from the set $\mathcal{X}$. The transition matrix is

$$P = \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}.$$

   Find the stationary distribution of the Markov chain. [5]

5. Let (X, Y) have the following joint distribution:

Y \ X	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

   Find $H(X), H(Y), H(X|Y), H(Y|X)$ and $I(X; Y)$. [10]