

INDIAN STATISTICAL INSTITUTE

ENDTERM EXAMINATION M.TECH(CS/CrS) & M. MATH II YEAR

AUTOMATA THEORY, LANGUAGES AND COMPUTATION

Date: 24.11.2025 Maximum marks: 100 Duration: 3.0 hours.

The paper contains 110 marks. Answer as much as you can, the maximum you can score is 100.

1. Consider the following class of languages numbered **Class 1** to **Class 7**.

Class 1: Empty

Class 2: Finite

Class 3: Regular

Class 4: Context Free

Class 5: Recursive

Class 6: Recursively Enumerable

Class 7: All Languages

For the listed languages below, what is the lowest numbered class to which the language surely belongs. For example, the correct classification for a context free language which is not regular would be class 4 (though it also belongs to all the classes with numbers 4 and above. Similarly, suppose a language L is known to be recursively enumerable and it could possibly be recursive but the available information do not guarantee that it is recursive, then L should be classified into **Class 6** .

You just need to write the class number against the following languages, no justification is required.

- (a) A language in **coNP**.
- (b) The complement of a nonrecursive language.
- (c) The complement of a context free language.
- (d) The intersection of a recursive language and a recursively enumerable language.
- (e) The intersection of a recursive language and a language which is not recursively enumerable.

[5 × 2 = 10]

2. State if the following statements are TRUE or FALSE. Give a short argument in favor of your answer.

- (a) Every subset of a regular language is regular.
- (b) The class of non-regular languages is closed under complementation.
- (c) Let \equiv be any right congruence of finite index on Σ^* . Any equivalence class of \equiv is a regular language.
- (d) $L(M)$ is recursive if and only if the Turing machine M is total.
- (e) It is known that there is no polynomial time algorithm to decide $\overline{\text{SAT}}$.
- (f) If B is recursively enumerable and $A \leq_p B$ then A is recursively enumerable.
- (g) If A is recursive and $A \leq_p B$ then B must be recursive.
- (h) If A is NP-complete and $A \leq_p B$ then B must be NP-complete.
- (i) Suppose $L_1 \in \mathbf{P}$ and L_2 is NP-complete, then it is known that there cannot be a polynomial time reduction from L_2 to L_1 .
- (j) If $A \leq_p B$, then it is possible that A is in \mathbf{P} and B is not in \mathbf{NP} .

[10 × 2 = 20]

3. (a) You are given two DFAs A and B . Describe the construction of a DFA M , such that

$$L(M) = (L(A) - L(B)) \cup (L(B) - L(A))$$

(b) Prove that given a DFA M which accepts $L \subseteq \Sigma^*$ there exists a Myhill Nerode relation \equiv_M on Σ^* for L .

[10+10 = 20]

4. (a) Show that the language $L_1 = \{(0^k 1^k)^k : k \geq 1\}$ is not a CFL.

(b) Is the language $L_2 = \{0^m 1^n : m \neq n\}$ a CFL? If L_2 is a CFL, give a CFG for it.

[10+10 = 20]

5. (a) Consider the following languages A and B :

$$A = \{(M, w) : w \in L(M)\}$$

$$B = \{M : L(M) \text{ is finite}\}$$

Prove that $A \leq_m B$.

(b) Let $L = \{(M_1, M_2) : L(M_1) = L(M_2)\}$, i.e., L contains encodings of two Turing Machines M_1, M_2 , such that the language accepted by M_1 and M_2 are the same. Prove that L is not recursive by using a reduction from any known non-recursive language.

[10+10 = 20]

6. (a) Let P be a property of recursively enumerable sets. When is P said to be monotone?
- (b) Let P be a non monotone property of recursively enumerable sets and let $T_P = \{M : P(L(M)) = \mathbb{T}\}$. Prove that T_P is not recursively enumerable. You can assume \overline{HP} to be not recursively enumerable.

[5+15 = 20]