CERTAIN GENERALISATIONS OF ACCEPTANCE SAMPLING PLANS BY ATTRIBUTES

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Thesis submitted to the Indian Statistical Institute for the degree of Doctor of Philosophy

August [redacted], *1984.*

A C K N O W L E D G E M E N T S

The work embodied in this thesis is the result of my research over several years. I am indebted to many persons inside and outside the Indian Statistical Institute.

Dr. B.P. Adhikari, Research Professor, Indian Statistical Institute has been a continuing source of inspiration for the past little over two decades, right from my days at the University of Lucknow. I am very grateful to him for the pains he has taken in supervising this work. His encouragements and guidance through useful discussions have greatly helped in the completion of the thesis.

Dr. K.G. Ramamurthy, Indian Statistical Institute has been very kind to help me in checking the mathematical details of this work. I am really grateful to him.

During personal correspondence, some general comments and specific suggestions we received on the contents of several of my papers which constitute the bulk of this work, from several foreign scholars from USA, Denmark and Holland. I thank them all.

I thank Prof. Ashok Maitra, Director, Indian Statistical Institute for the facilities made available to me in respect of this research work at the SQC Training and Promotional Unit of the Institute.

My sincere thanks are due to Dr. J. Roy, Research Professor and Head, Computer Science Unit (CSU) of the Indian Statistical Institute for making the necessary computer time available for running my programmes to work out the optimal plans discussed in the Chapter 5.

I thank my senior colleagues Messers S.M. Sundara Raju, C.R. Prasad, K.G. Ramamurthy and C.A. Setty who have been Head of the SQC and OR Division over the years, for their constant encouragement and good wishes. My thanks are due to all my colleagues and secretarial staff at the SQC T and P Unit and all other colleagues in the outlying units of the SQC and OR Division for their good wishes and constant cooperation in every possible way.

Mr. Jyotirmay Sarma, SQC T and P Unit has done an excellent job in typing the thesis. I thank him for assistance and cooperation. My thanks are due to Mr. Harish Chandra for excellent help in duplicating work.

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CHAPTER ONE

GENERAL INTRODUCTION AND SUMMARY

1.1 Origin and types of acceptance sampling inspection

The use of inspection gained importance when people started getting concious of maintaining the quality of manufactured product. According to Anscombe (1958) inspection for quality in the form of acceptance sampling plans originated around 1920. The classical work of Dodge and Romig (1929) put acceptance sampling on a firm footing. In general, sampling inspection can be broadly classified into (i) lot-by-lot inspection (ii) bulk sampling inspection and (iii) 'continuous' sampling inspection.

The lot-by-lot sampling inspection is applied where a series of lots or batches of discrete items are available for inspection. On the other hand, a batch of cotton, a wagon of coal or a ladle of molten metal are some examples where a lot or batch is comprised of a bulk or continuous product.

When the batches or lots consist of continuous nondiscrete material, method of bulk sampling inspection is used.

There are situations where, though the production is in terms of discrete items, continuous flow of products is available for inspection. A continuous flow of potteries on a conveyor belt in a ceramic industry is an example. Formation of lots for the purpose of lot-by-lot inspection is not possible here. Dodge (1943) designed a special type of sampling plan, called 'continuous' sampling plan, consisting of alternating hundred percent inspection and sampling inspection, in case inspection is not destructive of the item.

The present work is related to only lot-by-lot sampling inspection.

Further, a lot-by-lot sampling inspection may involve inspection either by attributes or by variables. In attribute inspection each unit in a sample is inspected visually on a go-no-go gauge basis for one or more characteristics, and classified as defective or non-defective. In variable inspection, each unit in the sample is measured for a single characteristic, such as weight or strength. The published plans by Dodge and Romig (1959), Mil-Std. 105D (1963) and IS-2500 (1973) are examples of sampling inspection plans by attributes. Examples of sampling inspection plans by variables are the plans by Bowker and Goode (1952), Mil-Std. 414 (1957) and IS-2500 (1965).

In this work we consider only sampling inspection by attributes.

1.2 Purpose of sampling inspection

In brief, inspection has two purposes: (i) discrimination of good items from bad, and (ii) collection of information on the quality of the items inspected. It is well known that 100% inspection is not necessarily efficient due to inspection fatigue and considerations of cost and time. When inspection is destructive, 100% inspection is obviously ruled out. In view of these points sampling inspection has almost replaced regular hundred percent inspection in industries. However, hundred percent inspection on a non-regular basis i.e., in case of only some lots, is the common feature of some acceptance sampling plans (e.g., Dodge and Romig's plans).

In the present work it is assumed that the inspection of an item is not destructive. The sampling plans discussed include the possibility of 100% inspection of some lots.

1.3 Three-decision plans and protection provided by them

The lot-by-lot acceptance sampling plans by the method of attributes, in which each unit in a sample is inspected on a go-no-go gauge basis for one or more characteristics, and the lot-by-lot acceptance sampling plans by the method of variables, in which each unit in a sample is measured for a single characteristic, such as weight or strength, are either acceptance-rectification or acceptance-rejection plans.

The classical work of Dodge and Romig (1929) introduced new concept into acceptance sampling. The situation considered by them was the inspection of a long series of batches such as would occur if components are passed from one stage of production to another, or from one department to another, or from one factory to another, in the course of manufacture of some assembly. Dodge and Romig's plans are two-decision plans with acceptance or rectification of the lot as the two terminal actions. In other words, the batches are ordinarily not rejected but, if they are not accepted, they are to be rectified by being inspected completely and defectives either removed and replaced with good items or corrected.

Thus, under a two-decision plan: (i) a lot is either accepted as a good one or classified as bad one and screened for the purpose of acceptance or (ii) a lot is either accepted as a good one or classified as bad one and rejected.

In practice, situations exist where the above type of two-terminal actions may not be found adequate in the economic sense. The following points provide the basis for such consideration:

(a) 100% inspection of very bad lots and replacing or rectifying large number of defective items may not always prove to be an economically valid proposition.

(b) Out right rejection of a lot, in particular, a moderately good lot, may not be always agreeable to the producer. However, producer may agree to an out right rejection of a very bad lot under certain conditions.

(c) Three terminal-action procedure may be a natural course in certain cases and three-decision plans may make it more objective.

In such cases it may be economical to operate a three-decision plan instead of a two-decision plan. For example, if large number of bad lots are submitted for inspection, three-terminal actions - accept a good lot, screen a bad lot and reject a very bad lot - may be more appropriate.

Three-decision plans with various types of three-terminal actions as a generalisation of two-decision plans, are developed in this work and constitute the main theme of the discussions. Four full chapters : Chapter 2 [Pandey (1972a)], Chapter 3 [Pandey (1977)] , Chapter 4 [Pandey (1977)] and Chapter 5 [Pandey (1974a)] are devoted to the discussion of three-decision plans.

Some of the typical three terminal actions under a three-decision plan are listed below:

(i) Classify the lots in three quality grades A, B and C. Accept grade A as good lot, screen grade B lot and accept, and reject a grade C lot outright. Plans with such three decisions are called ASR plans in this work and are discussed in the Chapter 2 (section 2.3), Chapter 3 (section 3.4) and Chapter 5. They have different optimal properties.

(ii) Classify the lots in three quality grades A, B and C as before and accept grade A as good lot, accept grade B as moderately good lot or salvagable lot and, screen and accept a grade C lot. Plans with such three decisions are called AMS plans in this work and are discussed in the Chapter 2 (section 2.4).

(iii) Classify the lots in three quality grades A, B and C as before and accept grade A as good lot, screen and accept a grade B lot and accept a grade C lot with a penalty imposed on the supplier. Plans with such three decisions are called ASP plans in this work and are discussed in Chapter 3 (section 3.5).

A single sampling ASR plan is defined by the parameters n, c_1 and c_2 and is to be operated as follows:

Take a random sample of size n from a lot of size N and let x be the number of defectives in the sample then

$$\begin{aligned} &\text{accept the lot if} \quad 0 \leq x \leq c_1 \;; \\ &\text{screen the lot if} \quad c_1 < x \leq c_2 \;; \\ &\text{reject the lot if} \quad c_2 < x \leq n \end{aligned} \qquad (1.3.1)$$

(1.3.1) generalises two-decision plans. For example, some of the two-decision plans are special cases of (1.3.1) viz., accept-reject plan ($c_1 = c_2$), accept-screen plan ($c_2 = n$) and screen-reject plan ($c_1 = -1$).

The average amount of inspection of an ASR plan is

$$\bar{I} = n + (N-n) \sum_{r=c_1+1}^{r=c_2} e^{-n\bar{p}} (n\bar{p})^r / r! \quad \ldots (1.3.2)$$

where $\bar{p}$ denotes the process average quality. The minimum average amount of inspection at process average quality, $\bar{I}$, has been chosen as a criterion by Dodge and Romig (1959) to obtain their optimal plans satisfying certain other conditions and risk.

It is well known that when a sampling plan is operated the problem of conflicting interest of producer and consumer arises. A producer should not grudge if a very bad lot is rejected out right by the consumer. The consumer is supposed in this work to operate a plan for receiving inspection to safeguard his interest against accepting a lot which is very bad.

At the same time 'consumer' should not be notional in the sense of Tippett (1958, pp. 137). The consumer should agree to allow a chance of 100% inspection for lots which are not accepted ordinarily but are not very bad. Every such lot should be accepted after screening. The lots which are very bad (grade C) may involve high cost of screening and neither the consumer who bears the cost of inspection would allow screening of such lots nor the producer would find the cost of replacing or rectifying too many defective items as attractive. In fact, whenever such a lot is rejected the consumer's loss may be very low.

We have assumed that the consumer bears the cost of inspection. It may be a departure from the common cases in which producer bears the cost of inspection. This assumption may be justified under the argument and the arrangement described below:

The out right rejection of any lot may be deemed by the producer as a drastic action on the part of a consumer if screening be the general practice under two-decision plan. However, the producer may agree to such a proposal provided:

(i) The consumer agrees to accept the moderately good (grade B) lot after either imposing a penalty on the producer (supplier) or screening.

(ii) Only very bad (grade C) lots are rejected out right.

(iii) The consumer agrees to bear the cost of inspection (or at least 100% inspection).

The consumer's and producer's risks are most widely used for determining systems of sampling plans. The consumer's risk is defined as the probability of acceptance of a lot or process with the deteriorated quality level or lot tolerance percent defective (LTPD) quality under a given plan. The producer's risk, a concept opposite to consumer's risk is defined as the probability of rejecting a lot of good quality level or acceptable quality level (AQL) by the consumer under a given plan. The Dodge and Romig's (1959) LTPD systems of plans provide lot quality protection in terms of consumer's risk.

The notion of two consumer's risks and two producer's risks, as introduced in this work, is new in the field of acceptance sampling and is explained next.

When a three-decision plan is operated for receiving inspection any misclassification of an inferior quality lot or process as a superior quality lot or process entails a risk to the consumer. To provide a lot quality protection in case of the three-decision plans on the line of Dodge and Romig's LTPD systems of plans, it may be therefore, logical to specify the consumer's risk in terms of the probabilities of misclassification of the above type.

When a three-decision ASR plan is operated the misclassification entailing a risk to the consumer would arise in two ways: (a) when a fairly good (grade B) lot is classified as grade A or (b) when a fairly bad (grade C) lot is classified as grade B or grade A lot. The consumer's risk, therefore, need be specified at two quality levels - one of grade B and the other of grade C. For this purpose the two quality levels p_1 and p_2 ($p_1 < p_2$) are chosen depending on consumer's specification as follows:

The quality level p_1 is specified in such a way that the probability of misclassifying a lot of this quality under the plan as grade A is quite low β_1 (say 0.01 to 0.10) and the probability of classifying it correctly as grade B is fairly high. Similarly, the quality level p_2 is specified by the consumer in such a way that the probability of misclassifying a lot of this quality as grade B or grade A is quite low β_2 (say 0.01 to 0.10) and the probability of classifying it correctly as grade C is fairly high (Fig.6).

Thus, the risk β_1 is the probability of misclassifying a lot of quality p_1 as grade A. The risk β_2 is the probability of misclassifying a lot of quality p_2 either as grade B or A. These risks (β_1 and β_2) are mathematically written as

$$\beta_1 = B\,(c_1;\ n,\ p_1) \quad \ldots \quad (1.3.3)$$

and

$$\beta_2 = B\,(c_2;\ n,\ p_2) \quad \ldots \quad (1.3.4)$$

using the sample criteria (1.3.1) where the above two conditions are to be satisfied as closely as possible.

These equations can be also written using form of Dodge and Romig (1959) page 19 e.g., (1.3.4) can be written as

$$\beta_2 = \sum_{x=0}^{c_2} \binom{M}{x} \left(1 - \frac{a}{M}\right)^{M-x} \left(\frac{a}{M}\right)^{x} \quad \ldots \quad (1.3.4a)$$

where $M = Np_2$, $a = np_2$

The concept of producer's risks say a_1 and a_2 at quality levels p_1' and p_2' (say) respectively can be similarly defined (Fig.6). Mathematically

$$a_1 = \sum_{x=c_1+1}^{n} b\,(x;\ n,\ p_1') \qquad \ldots \quad (1.3.5)$$

and

$$a_2 = \sum_{x=c_2+1}^{n} b\,(x;\ n,\ p_2') \qquad \ldots \quad (1.3.6)$$

where p_1' denotes the quality level such that when a three-decision plan is operated the probability of misclassification of a lot of quality p_1' (grade A) as either grade B or C is quite low say a_1 whereas the probability of classifying it correctly as grade A is fairly high. Similarly, p_2' denotes the quality level such that the probability of misclassifying a lot of quality p_2' (grade B) as grade C is quite low say a_2 whereas the probability of classifying it correctly as grade B is fairly high. The values of a_1 and a_2 may range from 0.01 to 0.10.

Since, the concept of two risks is quite new in the field of acceptance sampling it was felt necessary to talk about both consumer's and producer's risk. However, the determinations of three-decision plans discussed in this thesis, do not involve producer's risk, hence the notion of this risk is not relevant to the present work.

The plans discussed in the Chapters 2 and 5 (section 5.4) are on the line of Dodge and Romig's LTPD systems of plans (consumer's risk plans), and are based on the notion of two consumer's risks β_1 and β_2.

Two types of protection (consumer's protection) are provided under Dodge and Romig's plans: lot quality protection (LTPD plans) and average quality protection (AOQL plans). In each case the value of either LTPD or AOQL is specified, and the plans in their tables provide minimum average amount of inspection for specified process average quality.

In Chapter 2 [Pandey (1972a)] a new type of three-decision plans (n, c_1, c_2) is discussed. These plans satisfy two consumer's risks β_1 and β_2 defined in terms of probability of misclassification, and give minimum average amount of inspection at process average quality $\bar{p}$. These plans are to be operated as (1.3.1). It is shown that the optimal three-decision plan gives a smaller value of $\bar{I}$, defined by (1.3.2), than the 'corresponding' Dodge and Romig's plan [Pandey (1974b)]. A set of Dodge and Romig's plans and the 'corresponding' three-decision plans are tabulated along with their values of $\bar{I}$. A comparison shows that the values of $\bar{I}$ for the three-decision plans are smaller in each case. A set of illustrative optimal three-decision plans are tabulated and given in the table 1 in the end. Numerical comparison is provided in the table 2.

In Chapter 2 [Pandey (1972a)] in section 2-4, further, a second type of new three-decision plans with the following sample criteria, different from the above, are discussed:

$$\begin{aligned} &\text{accept as good if } 0 \leq x \leq c_1 \,; \\ &\text{accept as moderately good if } c_1 < x \leq c_2 \quad \ldots(1.3.7) \\ &\text{accept after screening if } x > c_2 \end{aligned}$$

A set of illustrative 450 three-decision plans of (1.3.7) type are provided in Pandey (1972a) in the tabular form.

In Chapter 3 [Pandey (1977)] single sampling three-decision plans providing average quality protection in terms of AOQL (instead of lot quality protection as in Chapter 2), and minimising average amount of inspection at process average quality $\bar{p}$ are discussed, and a set of illustrative optimal plans are given in the tables 5 and 8 in the end.

In an unpublished thesis, Soundararajan (1972) has obtained some results related to the inflexion point on the operating characteristic function of the two-decision single sampling plan. In Chapter 4 [Pandey (1977)] some results on the characterisation of the single sampling three-decision plan with an original and rigorous proof are discussed.

1.3.1 Other three-decision or 'three-class' plans

The work discussed in the Chapter 2 was presented at the Fifth All India Conference on Quality Control, Indian Statistical Institute 17-19 March 1971, New Delhi and subsequently published in 1972 in Sankhyā [Pandey (1972a)]. The contents of Chapters 2-5 excepting the sections 5.6 and 5.8 are published as indicated in the reference. A brief summary of the contents of Chapters 2, 3 and 5 entitled "Sampling Plans with Three Decisions" is included in the book "STATISTICAL THEORY OF SAMPLING INSPECTION BY ATTRIBUTES" by Anders Hald, 1981, pages 427-428, Academic Press.

Some work carried out independently in the field of acceptance sampling, which has an apparent similarity with the three-decision plans developed by us are due to Golub (1953), Umarov (1970), Bray, Lyon and Burr (1973) and Schmidt, Case and Bennet (1980). The 'three-class' plans of Bray, Lyon and Burr (1973), however, are essentially two-decision plans.

1.3.1(a) Golub (1953) considered the determination of c_1 and c_2 for specified value of n such that the sum of the probabilities of acceptance, screening and rejection at the specified incoming quality is maximised.

1.3.1(b) Umarov (1970) has discussed a special case of sequential acceptance sampling plans with three terminal actions, viz., accept without inspection, reject without inspection and carryout 100% inspection.

1.3.1(c) Bray et.al. (1973) have defined a 'three-class' plan by n, c_1 and c_2. The probability that a sample of size n includes d_o good items, d_1 marginal items and d_2 bad items is given by

$$\frac{n!}{d_o!\, d_1!\, d_2!}\; p_o^{d_o}\, p_1^{d_1}\, p_2^{d_2} \quad \ldots \quad (1.3.8)$$

where p_o is the fraction of items of good quality in the lot, p_1 the fraction of items of marginal quality and $p_2 = 1 - p_o - p_1$ is the fraction of items of bad quality. The probability of acceptance of the lot on the basis of the sample is given by

$$\sum_{j=o}^{c_2} \sum_{i=o}^{c_1-j} \frac{n!}{i!\,(n-i-j)!}\; p_o^{n-i-j}\, p_1^{i}\, p_2^{j} \quad \ldots \quad (1.3.9)$$

Under this 'three-class' plan an item in the sample is classified either as good or marginal or bad and depending on the sample results two decisions are taken i.e., a lot is either (i) accepted or (ii) rejected.

In their work the authors consider only the case with $c_2 = o$, thus simplifying the expression (1.3.9) to

$$\sum_{i=o}^{c_1} \binom{n}{i} p_o^{n-i} \; p_1^{i} \qquad \ldots \qquad (1.3.10)$$

They have constructed a set of plans giving values of n and c where c denotes c_1, and c_2 is always equal to zero. Some applications in the area of hazardous substances in food and amount of active ingredient in dosage forms of drugs are pointed out.

1.3.1(d) Schmidt Jr, Case and Bennet (1980) have considered three-action viz., accept-screen-reject, cost model for plan by variables for two-sided specification case and have taken a straightforward cost function. They have attempted an approximate solution by simplifying the cost function. Their work is on the lines similar to Pandey (1972b).

It may be noticed that the plans summarised in 1.3.1(a),(b),(c) and (d) above give very restricted sets for in (a) n is fixed in advance and a criterion of maximum sum of probabilities is used; in (b) sequential plans are discussed, with one of the terminal actions being 100% inspection; in (c) c_2 is taken as 0, thus reducing the plans in fact to a very restricted two-decision plans; and in (d) three-action cost model for plans by variable similar to Pandey (1972b) are attempted.

In our work in the following Chapters (2,3,4 and 5) we have tried to use practical criteria of choice and develop a much wider variety of proper three-decision plans.

It is recognised that, inspite of theoretically demonstrated advantages, a set of plans is useful only to the extent that these are employed in practice. For this, two essential requirements are practioners' familiarity with the plans and the availability of readily usuable tables. We are in the process of compiling such tables and if they are published along with a summary of the contents of the present work, it is hoped that these three-decision plans may gain currency of use.

.2 Bayesian three-decision plans based on cost model

It is well recognised in industry that the loss or regret function when expressed in terms of costs, is more tangible than the average amount of inspection. The wealth of information gathered over time about a production or inspection process may be used to make the decisions more realistic.

Robbins (1950) has discussed an empirical approach to identify the form of the prior distribution based on the past information. Several authors like Barnard (1954), Guthrie and Johns (1959), Pfanzagl (1963), Wetherill (1960), Hald (1960b, 1965, 1967a, 1967b, 1968a, 1968b, 1973, 1981), Johansen (1970), Guenther (1971), Hald and Thyregod (1971), Hald and Keiding (1969, 1972), Chiu (1974), Thyregod (1974, 1975), etc., assume a prior distribution for incoming lot quality, and, generally under a linear cost model, work out optimal two-decision plans.

In Chapter 5 [Pandey (1974a)] three-decision plans, on a cost structure basis of the model, have been introduced. Bayesian single sampling three-decision plans by attributes for discrete and continuous prior distributions are discussed. The plans minimise the sum of average total cost of inspection and the cost of decision per item.

In the discrete prior case:one, two and three point prior distributions for the lot quality are used. Some illustrative set of optimal plans are also given. A numerical comparison is made of average total cost of inspection plus cost of decision per item between two-decision plans and the 'corresponding' three-decision plans in the case of two-point prior. It is found that three-decision plans are more advantageous than the two-decision plans. The uniqueness of the solution is attempted to be shown and it is observed that the expected decision loss is a non-increasing function of the sample size with a falling rate of decrease.

For the case of continuous prior [Pandey (1973)], beta prior is chosen as the distribution of the incoming lot quality in Chapter 5 (section (5.8)) and three-decision ASR plans are discussed. A few illustrative plans are also worked out and given.

Finally, for completeness and the benefit of practitioners in the application of statistical quality control in industry, sampling inspection tables for Bayesian single sampling three-decision plans by attributes, based on the work of Chapter 5, are compiled separately for various values of the parameters λ_{ij}, i, j = 1, 2 and μ_{ij}, i, j = 1,2,3.

In the present work both, non-Bayesian approach (Chapter 2, 3 and 4) and Bayesian approach (Chapter 5) are adopted to discuss three-decision sampling plans. The role of Bayesian approach, though quite important in obtaining optimal sampling inspection plans, does not qualify ''Bayesianism'' to be overemphasised [as remarked by De Finetti (1974)] at least in practice, so as to pose a threat to the statistical profession in view of Hamaker (1977a).

The work presented here is based on the consideration of one quality characteristic only whereas in practice it may be desirable to consider several types of defects simultaneously. However, inspite of some commendable attempts to consider multiple characteristics by defining a composite index or criterion [Harrington (1965), Mukherjee (1970) and Schmidt et. al. (1972)] , work in this field has remained rare and more academic than practical. For the present, as is done in Mil-Std. 105D (1963), for example, the plans presented here may be used by choosing a single characteristic, out of many, on the basis of seriousness or other practical considerations.

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CHAPTER TWO

SINGLE SAMPLING THREE-DECISION PLAN BY ATTRIBUTES PROVIDING LOT QUALITY PROTECTION

2.1 Introduction

In Pandey (1975) some limitations of percentage sampling inspection such as dependence of its sample size only on lot size and undesirable increase in the average quality of the accepted products due to random pairing and doubling of batches under certain condition [Nath (1948)] were pointed out and established respectively. The system of sampling plans by attributes developed by Dodge and Romig (1929) is, to a large extent, free from such limitations. Among other plans Dodge and Romig consider the single sampling plan (n, c) providing lot quality protection.

For specified values of lot size (N), process average ($\bar{p}$), LTPD (p_t) and consumer's risk (P_c = 0.10) the plan (n,c) satisfies

$$\sum_{x=0}^{c} \binom{M}{x} \left(\frac{n}{N}\right)^{x} \left(1 - \frac{n}{N}\right)^{M-x} = P_c \quad \ldots \quad (2.1)$$

where $M = Np_t$, and minimises $\bar{I}$ given by

$$\bar{I} = n + (N-n) \sum_{x=c+1}^{n} e^{-n\bar{p}} (n\bar{p})^{x} / x! \quad \ldots \quad (2.2)$$

[Dodge and Romig (1959) pages 33-34]. Under the plan (n, c) decision on a lot is taken as follows:

$$\begin{array}{ll} \text{Accept the lot if} & 0 \leq x \leq c \\ \text{Screen the lot if} & x > c \end{array} \qquad \ldots \quad (2.3)$$

where ''screening'' of a lot amounts to an 100% inspection and accepting the lot after either rectifying or replacing with good items all the defectives found. To cover the practical situations where ''screening'' of a very bad lot is quite costly and it may be economically preferable to reject such lots out right, we propose in this Chapter a three-decision single sampling plan.

Under a three-decision plan lots are classified in three grades A, B and C. Two cases of three-decisions are considered in this Chapter:

Case (a) : Accept grade A, screen grade B and reject out right a grade C lot.

Case (b) : Accept grade A, Accept grade B as moderately good and screen grade C lot.

Three-decision single sampling plans (n, c_1, c_2) ensuring specified values of two consumer's risks (β_1 and β_2) as defined in (1.3.3) and (1.3.4) and minimising average amount of inspection (AOI) at given $\bar{p}$ are developed.

2.2 Choice of grades

To define three quality grades, two quality levels p_1 and p_2 $(p_1 < p_2)$ are predetermined. A lot of incoming quality p is of grade either

$$\begin{aligned} &A \quad \text{if} \quad p < p_1 \quad \text{or} \\ &B \quad \text{if} \quad p_1 \leq p < p_2 \quad \text{or} \\ &C \quad \text{if} \quad p \geq p_2 . \end{aligned}$$

Consumer's specification guides the choice of p_1, p_2 and the associated risks β_1 and β_2. The risks β_1 and β_2 are taken as probability of misclassification as follows:

β_1 : Probability of misclassifying a lot of quality p_1 as grade A.

β_2 : Probability of misclassifying a lot of quality p_2 either as grade B or grade A.

Defined as such the risks β_1 and β_2 have the mathematical expression as (1.3.3) and (1.3.4) respectively. Dodge and Romig (1959 page 19) have used f= binomial expression for consumer's risk β in (2.1) but we use ordinary binomial for the two consumer's risks. Generally, the values of β_1 and β_2 are such that in practice for a three-decision plan $c_1 = -1$ is ruled out.

The parameters (n, c_1, c_2) of a three-decision plan discussed in this Chapter are defined as follows:

A random sample of n items is drawn from the lot of size N. Decisions in case (a) and case (b) are taken as in the table next, on the basis of the number of defectives observed in the sample.

number of defectives in the sample	decision in case (a).	decision in case (b).
$\leq c_1$	accept the lot	accept the lot
$> c_1$ but $\leq c_2$	screen and accept the lot	accept the lot as moderately good
$> c_2$	reject the lot	screen and accept the lot

The significant features of these plans may be summed up as follows:

(i) <u>Consumer's protection</u> : the two consumer's risks are kept at specified low value β_1 and β_2 closely for given p_1 and p_2 $(p_1 < p_2)$

(ii) <u>Producer's interest</u> : Out right rejection takes place only if the lot is too bad and if the lot is not very bad it is invariably accepted after screening (case (a)). In the second case (case (b)) not only a very bad lot is accepted after screening but even a lot which is not very bad is accepted as moderately good lot without any screening.

(iii) Two-decision plans as special cases : Two-decision plans can be obtained as a special cases of three-decision plans e.g., from accept-screen-reject type of three-decision plan of case (a) we can get the following type of two decision plans:

Accept - Reject plan when $c_1 = c_2$

Accept - Screen plan when $c_2 = n$

Screen - Reject plan when $c_1 = -1$

Such special cases for the three-decision plans of case (b) are obvious.

In view of (i) and (ii) we may state that these plans take care of both the consumer and the producer and do not protect only imagined interest of consumer independent of producer in the sense of Tippett (1958, pp.137).

The plans with Accept-Screen-Reject type of decision (case(a)) are referred to as ASR plans and similarly those with Accept-Accept as Moderately (M) good-Screen type of decision (case(b)) as AMS plans in the sequel.

2.3 Case (a) : ASR plans

For specified lot size (N), p_1, p_2, $\bar{p}$, β_1 and β_2 the problem of determining (n, c_1, c_2) which satisfy (1.3.3) and (1.3.4) very closely and minimise $\bar{I}$ given by

$$\bar{I} = n + (N-n) \sum_{x=c_1+1}^{c_2} e^{-n\bar{p}} (n\bar{p})^x / x! \quad \ldots \quad (2.3.1)$$

is considered. In (2.3.1) Poisson approximation for $\text{Prob}(c_1 < x \leq c_2 \mid N, \bar{p}, n, c_1, c_2)$ is used as done by Dodge and Romig (1959, page 19) in (2.2). The plan (n, c_1, c_2) is to be operated as described in the section 2.2 and the three-decisions under it are to be taken as given in column 2 of the table there.

Let S be the set of plans (n, c_1, c_2) satisfying (1.3.3) and (1.3.4). When β_1 and β_2 are specified for given p_1 and p_2, for any plan (n, c_1, c_2) in S, if any one of the numbers n, c_1 and c_2 is fixed the other two are uniquely determined. Thus the plans in the set S can be uniquely defined according to any one of the numbers n, c_1 and c_2. We have chosen c_1 for this purpose.

Let $S(c_1)$ denote a plan (n, c_1, c_2) from S and $\bar{I}(c_1)$ be the corresponding value of $\bar{I}$ given by (2.3.1) at process average $\bar{p}$. A plan $S(c_1^*)$ will be locally optimal if

$$\Delta I\ (c_1^* - 1) \leq 0 < \Delta I\ (c_1^*) \quad \ldots \quad (2.3.2)$$

For two plans $S(c_1)$ and $S(c_1+1)$ in S for some specified $\bar{p}$ we have

$$\Delta \bar{I}(c_1) = k(1-f_2) + (N-n)\ (f_2-f_1) \quad \ldots \quad (2.3.3)$$

where $k > 0$ is an integer and f_1 and f_2 are given by

$$f_1 = \sum_{x=c_1+1}^{c_2} e^{-n\bar{p}}\ (n\bar{p})^x / x\,!$$

$$f_2 = \sum_{x=c_1+2}^{c_2+\delta} e^{-(n+k)\bar{p}}\ (\overline{n+k}\ \bar{p})^x / x\,!,\ \delta > 0, \text{ an integer.}$$

During the investigation of the nature of $\bar{I}(c_1)$ for increasing value of c_1 it is found that the expression (2.3.3) becomes quite cumbersome for analytical treatment and it is not possible to make an exact analytical statement

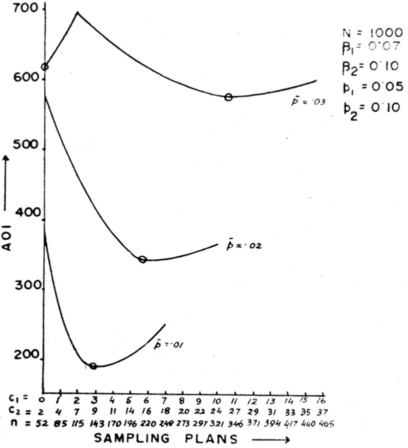

Fig.1 THE CURVE OF AOI AGAINST INCREASING C_1 FOR VARYING PROCESS AVERAGE ($\bar{p}$)

about the form of the function $\bar{I}(c_1)$ for different values of c_1. However, extensive tabulation of $\bar{I}(c_1)$ for increasing values of c_1 shows that there exist at the most two local minima for $\bar{I}(c_1)$ when N and $\bar{p}$ are fixed. This fact has been utilised in obtaining optimal ASR plans in the next section.

The nature of $\bar{I}(c_1)$ for increasing values of c_1 as a result of extensive numerical investigation is illustrated in figures 1 and 2. In figure 1, for the fixed lot size $N = 1000$ the general nature of the curve obtained by plotting AOI values at fixed three different values of $\bar{p} = .01, .02$ and $.03$ against a set of plans (satisfying $p_1 = 0.05$, $p_2 = 0.10$, $\beta_1 = 0.07$ and $\beta_2 = 0.10$) arranged in ascending order of c_1 is shown.

In figure 2, for the fixed three different values of lot size $N = 400$, 800 and 1200 the general nature of the curve obtained by plotting AOI values at fixed value of $\bar{p} = 0.03$, against a set of plans (satisfying $p_1 = 0.05$, $p_2 = 0.10$, $\beta_1 = 0.07$ and $\beta_2 = 0.10$) arranged in ascending order of c_1 is shown.

2.3.1 Determination of optimal ASR plans

One may write (1.3.3) and (1.3.4) in terms of f-binomial on the line of Dodge and Romig's expression (2.1) and in view of complicated nature of $\bar{I}(c_1)$ in (2.3.1) as compared to Dodge and Romig's $\bar{I}(c)$ function in (2.2), a modified iterative procedure corresponding to the iterative procedure by Dodge and Romig (1959, pages 19) may be developed and used to determine optimal ASR plans. But we prefer to use a direct method taking into account the specification (1.3.3) and (1.3.4) and first generating the set of plans S and then taking arbitrary lot size (N) and computing the value of $\bar{I}$ at a fixed value of $\bar{p}$, for each of the plan in S. The plan having minimum value of AOI is taken as the optimal plan. In all the cases we have found that our method converges in finite steps in obtaining the optimal plan. It may be noted that the iterative method given by Dodge and Romig does converge but they also have not given any theoretical proof for convergence.

The systematic steps for the computational procedure are given below:

Step 1. Choose some c_1 and, for fixed β_1, obtain n from (1.3.3).

Step 2. Substitute the value of n so obtained in (1.3.4) and obtain c_2.

Step 3. Choose a sufficient number of values of c_1 with an interval of 1 and obtain the plans (n, c_1, c_2) corresponding to each value of c_1 as above.

Step 4. Choose an arbitrary lot size N and compute the value of AOI at a fixed process average $\bar{p}$, for each of the plans obtained in the step 3. Choose the plan having minimum value of AOI as the optimal plan as soon as its AOI value is exceeded by the sample size of the plan corresponding to the next higher value of c_1. In other cases, continue the computations till two local minima for AOI are covered and then choose from these two minima the plan with lower AOI as the optimal plan.

As it can be seen from the Figures 1 and 2, in some cases there exist two locally minimum AOI. The computations have been carried out till both the plans corresponding to the two locally minimum values of AOI are covered. Then the plan with the smaller value of AOI has been selected.

In cases where the sample size of the succeeding plans, during the computations, become larger than the AOI of the first locally optimal plan, the further computations were abandoned selecting the first locally optimal plan as the absolutely (gobal) optimal plan. This procedure is justified by the following:

Let n_{c_1} denotes the sample size corresponding to the plan $S(c_1) \varepsilon S$. It has been observed during the computation for the plans in S that $n_{c_1} < n_{c_1+1}$ for $c_1 \geq 0$. Hence, if $S(c_1^o)$ denote a plan such that $\Delta \bar{I}\ (c_1^o - 1) \leq 0 < \Delta \bar{I}(c_1^o)$ for the smallest value of $c_1 = c_1^o$ when the computations are carried out in increasing order of c_1 and if $\bar{I}(c_1^o) <$ $n_{c_1^o+1}$ then clearly $\bar{I}(c_1^o) < n_{c_1^o+1} < \bar{I}(c)$ for all $c \geq c_1^o + 1$ implying that $S(c_1^o)$ is the absolutely optimal plan.

As an illustration, table 1 in the end provides a set of 72 plans which have been obtained for the following specifications:

$$\text{Lot size (N)} = 100\text{–}10{,}000;\ p_1 = 0.05;\ \ p_2 = 0.10;$$

$$\text{Process average } (\bar{p}) = 0.01,\ 0.02 \text{ and } 0.03;$$

$$\beta_1 = 0.07;\ \ \beta_2 = 0.10.$$

2.3.2 A comparison with Dodge and Romig's LTPD plans

A comparison of Dodge and Romig's (DR) single sampling LTPD plan with the ''corresponding'' three-decision

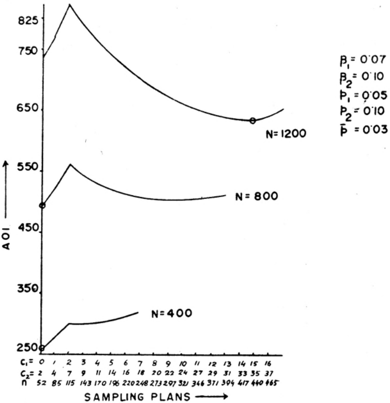

Fig. 2 THE CURVE OF AOI AGAINST INCREASING C_1 FOR VARYING LOT SIZE (N)

single sampling plan by attributes is carried out in table 2 in the end.

The values of parameters in the table 2, are chosen as follows:

For the three-decision plans, $p_1 = 0.10$; $p_2 = 0.15$; $\beta_1 = 0.10$; $\beta_2 = 0.10$; and for Dodge and Romig's (DR) plans, consumer's risk (β_1) is taken at 0.10 and the LTPD value at $p_1 = 0.10$. In both the cases lot sizes, same as in table 1, are chosen. Average amount of inspection (AOI) are computed at process averages 0.01, 0.02 and 0.03.

The concept of ''corresponding plan'' for the purpose of comparison in table 2 and also in further discussion, is as follows:

Given an optimal Dodge and Romig's single sampling LTPD plan (n, c) for specified values of $\bar{p}$, LTPD = p_1, consumer's risk $\beta_1 = 0.10$ and lot size N, then the ''corresponding'' three-decision single sampling plan by attributes is taken as (n, c_1, c_2) where $c_1 = c$ and c_2 is the solution of (1.3.4) for specified value of β_2 and values of n as in the above DR plan.

Similarly, given an optimal three-decision single sampling plan by attributes for given values of $\bar{p}$, N, p_1, p_2,

$\beta_1 = 0.10$ and β_2, then the ''corresponding'' Dodge and Romig's single sampling LTPD plan is taken as (n, c) where $c = c_1$.

The comparison carried out in table 2 shows that the Dodge and Romig's optimal single sampling LTPD plans involve higher values of AOI at specified values of $\bar{p}$ than the ''corresponding'' single sampling three-decision plan. The tabulation shows that an optimal three-decision plan involves a smaller value of AOI than the ''corresponding'' optimal DR plan.

We shall prove the following results:

Lemma 1. Let I_1^o be the value of AOI of a given optimal DR plan (n, c) for specified value of lot size (N), process average ($\bar{p}$), lot tolerance percent defective (LTPD) as p_1 and consumer's risk $\beta_1 = 0.10$ and in addition to the above let p_2, $p_1 < p_2$ and β_2 be specified and I_2 be the value of AOI of the corresponding three-decision ASR plan (n, c_1, c_2) where $c_1 = c$ then $I_2 \leq I_1^o$

Proof: Since $I_1^o = n + (N-n) \text{ Prob } (X > c_1 \mid N, \bar{p}, n, c_1)$

$$= n+(N-n) \text{ Prob}(c_1 < X \leq c_2 \mid N, \bar{p}, n, c_1, c_2) + (N-n) \text{ Prob}(X > c_2 \mid N, \bar{p}, n, c_2)$$

$$= I_2 + (N-n) \text{ Prob } (X > c_2 \mid N, \bar{p}, n, c_2)$$

$\therefore \; I_2 \leq I_1^o$ for $(N-n) \text{ Prob } (X > c_2 \mid N, \bar{p}, n, c_2) \geq 0$
where X is a random variable denoting number of defectives in the sample of n items.

□

Theorem 1. Let I_1^o and I_2^o be the values of AOI of a given optimal DR plan and an optimal three-decision ASR plan respectively for specified lot size (N), process average ($\bar{p}$), LTPD = p_1, consumer's risk $\beta_1 = 0.10$, p_2, $p_1 < p_2$, and β_2 then $I_2^o \leq I_1^o$.

Proof: Since, from definition of optimal three-decision ASR plan $I_2^o \leq I_2$, from lemma 1 the result $I_2^o \leq I_2 \leq I_1^o$ follows.

□

2.4 Case(b) : AMS plans

The following problem is considered : To determine n, c_1 and c_2 which satisfy the equations (1.3.3) and (1.3.4) and minimise average amount of inspection given by

$$\bar{I} = n + (N-n) \sum_{x=c_2+1}^{n} e^{-n\bar{p}} (n\bar{p})^x / x! \quad \ldots (2.4.1)$$

for specified values of N, $\bar{p}$, p_1, p_2, β_1 and β_2. An AMS sampling plan is to be operated as described in the section 2.2 and the three decisions are to be taken as given in the column three of the table there.

The AOI function (2.4.1) is similar to the AOI function of Dodge and Romig as given by (2.2) in the sense that both involve only one acceptance number, in the former case this being c_2. Therefore, the minimum value of AOI in (2.4.1) can be uniquely determined using only n and c_2 which satisfy (1.3.4 a). Subsequently, to obtain the value of c_1 corresponding to the above values of n and c_2 and satisfying (1.3.3), we have to solve (1.3.3) for specified value of β_1 using n determined earlier.

Thus, to determine n and c_2 for specified N, $\bar{p}$, p_2 and $\beta_2 = 0.10$ we can use the arguments of Dodge and Romig (1959) pages 19-20. In the present case, taking $k = \bar{p}/p_2$, $a = np_2$, $M = Np_2$, $P_c = \beta_2 = 0.10$, $c = c_2$ we can consider (M,k)-plane on the same line as Dodge and Romig to obtain c minimising $z = \bar{I}p_2$ and next, the corresponding value of 'a' from (1.3.4a) giving the value of the sample size $n = a/p_2$. The theory of (M,k)-plane having zones with identical acceptance numbers minimising z and the resulting boundary equation and iterative procedure will be exactly identical as given in the pages 19 and 20 in the above reference.

In view of the above, to determine n and c_2 for given N, $\bar{p}$, p_2 and $\beta_2 = 0.10$ we can directly use Dodge and Romig's

(1959) eight sampling inspection tables given in their appendix 4 pages 182-185 in the above reference for the values of p_2 = 0.5%, 1.0%, 2.0%, 3.0%, 4.0%, 5.0%, 7.0% and 10%.

Thus, given lot size N, process average $\bar{p}$, p_1, p_2, β_1 and β_2 (0.10) use the following steps to obtain an optimal three-decision AMS plan.

Step 1. Choose an arbitrary value of c_2.

Step 2. Obtain the value of n from DR table for LTPD = p_2, process average $\bar{p}$ and lot size N, consumer's risk = 0.10.

Step 3. Using the value of n as in step 2 and specified value of β_1 in (1.3.3) obtain the value of c_1. Thus, through steps 1-3 one gets the required plan (n, c_1, c_2).

Step 4. Keeping the value of n and c_2 fixed, as obtained above, i.e., without changing the AOI value, change c_1 by 1 and obtain the value of risk β_1 from (1.3.3). This will give a new plan (n, c_1+1, c_2) for the same value of N, $\bar{p}$, p_1, p_2 and β_2 but different value of β_1.

Using the above steps with different lot ranges and different value of c_2, a set of about 450 plans were obtained for lot range : 1-10,000; $\bar{p}$: 0-5%; p_1 = 0.05; p_2 = 0.10; p_2 = 0.10 and p_1 : 0.01% to 50% in Pandey (1972a). Since it is easy to obtain such plans using directly DR tables and the steps given above the above set of plans are not reproduced in the thesis and readers are referred to Pandey (1972a).

Concluding remarks

From the above discussion it is felt that in situations where Dodge and Romig's LTPD plans are applicable, it may be found more economical to discriminate between bad and very bad lots and use the plans given in the section 2.3, having much smaller AOI. However, situations may arise where bad lot may not be considered bad enough to be subjected to 100% inspection and, instead, it may be economical to encash them by selling at a lower price, only very bad lots being given 100% inspection. In the latter situation the plans presented in section 2.4 are to be used. It should not create much difficulty to decide as to which between the two types of plans should be used in a particular situation. This problem can be easily resolved by the user on the basis of the relevant cost parameters.

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CHAPTER THREE

SINGLE SAMPLING THREE-DECISION PLAN BY ATTRIBUTES PROVIDING AVERAGE QUALITY PROTECTION

3.1 Introduction

In the previous Chapter three-decision ASR and AMS plans providing lot quality protection on the line of DR single sampling two-decision plan providing lot quality protection were considered. For given lot size (N), Average Outgoing Quality Limit (AOQL) = p_L , process average ($\bar{p}$) Dodge and Romig (1959) have also considered among other plans the problem of determination of the value of n and c which minimise the value of $\bar{I}$ given in (2.2). These AOQL plans of Dodge and Romig are known to provide average quality protection.

Motivated by the above we consider, in this Chapter, following two variety of three-decision single sampling plan providing average quality protection for specified lot size (N), process average ($\bar{p}$) and AOQL (p_L) :

(i) Plans with 'accept-screen-reject out right' type of decision (ASR plans) and

(ii) Plans with 'accept-screen-accept with penalty to supplier' type of decision (ASP plans).

It is shown that these three-decision plans involve smaller average amount of inspection (AOI) as compared with the 'corresponding'' Dodge and Romig's plans. Some illustrative numerical examples for the construction of such plans are also given.

3.2 Three-decision ASR plans and its applicability

The concept of three-decision ASR plans with lot quality protection was introduced in Chapter 2. A single sampling three-decision ASR plan by attributes with average quality protection is defined in the manner as the earlier ASR plans, by the parameters n, c_1 and c_2 but it is operated with a slightly modified procedure as follows:

Take a random sample of n from a lot of size N and lot x be the observed number of defectives in the sample; then

if $0 \leq x \leq c_1$, accept the lot after rectifying or replacing the defectives in the sample,

if $c_1 < x \leq c_2$, accept the lot after screening,

if $c_2 < x \leq n$, reject the lot.

The plans discussed here are applicable under the following conditions:

(1) Items are inspected on a lot-by-lot basis and are in ample supply.

(2) Inspection does not involve any destructive or very costly testing i.e., 100% inspection is permissible.

(3) Any defective item, if detected, can be either replaced with good items or rectified.

(4) Quality protection is expressed in terms of average outgoing quality limit (AOQL).

3.3 The operating characteristic curve

For incoming lot quality p the probability of acceptance $P_a(p)$, probability of screening $P_s(p)$ and probability of rejection $P_r(p)$ under an ASR plan are defined as follows:

$$P_a(p) = B(c_1; n, p) \quad \cdots \quad \cdots \quad (3.3.1)$$

$$P_s(p) = B(c_2; n, p) - B(c_1; n, p) \quad \cdots \quad (3.3.2)$$

$$P_r(p) = 1 - B(c_2; n, p) \quad \cdots \quad \cdots \quad (3.3.3)$$

where $B(c; n, p) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$. For p not exceeding 0.10, Poisson approximation G(c, np) to B(c; n, p) is used by Dodge and Romig (1959) and Hald (1981) where $G(c,np) = \sum_{x=0}^{c} g(x,np)$, $g(x,np) = e^{-np} (np)^x / x!$. Following Hald (1981) page 38, we shall use the phrase ''under Poisson conditions'' when this approximation is employed. The results in this Chapter are obtained under Poisson conditions.

3.4 Optimal ASR plan

Let p denote the incoming lot quality. Assuming $p \leq 0.10$, which is a reasonable assumption for most of the industrial situations, the average amount of inspection for an ASR plan can be written as

$$I = n + (N-n) \sum_{x=c_1+1}^{c_2} g(x,np) \qquad \ldots \quad (3.4.1)$$

Since under the ASR plan the rejected lots which are in proportion $P_r(p)$, are discarded and do not contribute to average outgoing quality we have $(N-n)p$ defectives on the average in the accepted lots, which are in proportion $P_a(p)$, and zero defective in the lots having been screened , which are in proportion $P_s(p)$. Therefore, denoting average outgoing quality by p_A we have

$$Np_A = [(N-n)p \, P_a(p) + o \, P_s(p)] / [P_a(p) + P_s(p)]$$

from which we obtain

$$p_A = [(N-n)/N]\, p\, P_a(p) / [P_a(p) + P_s(p)] \quad \ldots \quad (3.4.2)$$

Alternately, we can write

$$p_A = p \frac{N - I_1}{N} \quad \ldots \quad \ldots \quad (3.4.3)$$

where I_1 is the average amount of inspection conditional that we consider only those lots which are either accepted or screened, i.e.,

$$I_1 = [n\, P_a(p) + N\, P_s(p)] / [P_a(p) + P_s(p)] \ldots (3.4.4)$$

The average outgoing quality limit (p_L) is the maximum value of p_A that will result under a given sampling plan, considering all possible values of p in the submitted product. The value of p for which this maximum value of p_A occurs is designated as p_m.

<u>Lemma 2</u>. For given lot size (N) and any ASR plan (n, c_1, c_2) the value of average outgoing quality limit (p_L) is given by

$$p_L = \left(\frac{1}{n} - \frac{1}{N}\right) \times G(c_1, x) / G(c_2, x) \quad \ldots \quad (3.4.5)$$

where x is the solution of

$$G(c_1,x)\,G(c_2,x) = xg(c_1,x)\,G(c_2,x) - xg(c_2,x)\,G(c_1,x) \quad \ldots (3.4.6)$$

Proof: Differentiating (3.4.3) with respect p and equating to zero we get

$$\frac{dp_A}{dp} = \frac{N - I_1}{N} - \frac{p}{N}\frac{dI_1}{dp} = 0 \quad \ldots (3.4.7)$$

Substituting for I_1 in (3.4.7) and simplifying we get

$$P_a^2(p)+P_a(p)P_s(p) = npg(c_1,np)\,[\,P_a(p)+P_s(p)\,] - npg(c_2,np)P_a(p) \quad \ldots (3.4.8)$$

Let the solution of (3.4.8) be $p = p_m$ and then the value of p_A for $p = p_m$ is equal to p_L. Denoting $np_m = x$ and substituting it for np in (3.4.2) and (3.4.8), under Poisson conditions, we get required results (3.4.5) and (3.4.6) respectively. □

Solving (3.4.6) by Newton-Raphson method of approximation we obtain the solution x, the value of $y = xG(c_1, x)/G(c_2, x)$, for the values of $c_1 = 0(1)20$ as given in the table 3 in the end. The smallest feasible value of c_2 is taken corresponding to a given c_1 in (3.4.6).

The value of c_1 giving minimum value of I at a specified process average $\bar{p}$ is obtained from the inequalities

$$\Delta \bar{I}\ (c_1-1) \leq 0 < \Delta \bar{I}\ (c_1) \qquad \ldots \qquad (3.4.9)$$

where $\bar{I}$ denotes the value of I when $p = \bar{p}$.

To determine optimal plan (n, c_1, c_2) we should obtain c_1 minimising $\bar{I}$ in the sense of (3.4.9) for given lot size (N) and process average $(\bar{p})$, and then consider this value of c_1 and corresponding value of c_2 and obtain the value of x and hence y satisfying (3.4.6), i.e., from the table 3. Next, the value of n can be obtained for specified value of p_L from:

$$n = Ny / [Np_L + y] \qquad \ldots \qquad (3.4.10)$$

To obtain c_1 minimising $\bar{I}$ for specified lot size (N), process average $(\bar{p})$ and AOQL (p_L) we can proceed on the line of Dodge and Romig (1959) pages 19 and 39 to obtain boundary condition under (3.4.9) as follows:

Let $\bar{M} = N\bar{p}$, $a = n\bar{p}$ and, for a given c_1, let the corresponding values of c_2, a and y be denoted by $c_2^{(c_1)}$, a_{c_1} and y_{c_1} respectively. Further, let $\bar{p}/p_L$ be denoted as k. Consider the $(\bar{M},k)$-plane. For a given value of

AOQL $= p_L$, and for any pair $(\bar{M},k)$, a particular pair (c_1, a_{c_1}) can be computed which makes $\bar{I}$ a minimum. Since the acceptance number c_1 assumes only discrete values, the same minimum value of $\bar{I}$ will correspond to many pairs $(\bar{M},k)$ for the same value of c_1. From this it is evident that on an $(\bar{M},k)$ - plane there exist zones in which the acceptance numbers are identical. To find the boundary lines of these zones it is noted that for certain pairs $(\bar{M},k)$ two pairs of (c_1, a_{c_1}) exist, giving the same value of $\bar{I}$. These values of c_1 are found to differ by 1 in all such cases. The boundary point for any two adjacent zones would be given under the optimal condition (3.4.9) as

$$\bar{M} = \frac{(a_{c_1+1}-a_{c_1})+a_{c_1} \sum_{c_1+1}^{c_2^{(c_1)}} g(r,a_{c_1})-a_{c_1+1} \sum_{c_1+2}^{c_2^{(c_1+1)}} g(r,a_{c_1+1})}{\sum_{c_1+1}^{c_2^{(c_1)}} g(r,a_{c_1}) - \sum_{c_1+1}^{c_2^{(c_1+1)}} g(r,a_{c_1+1})} \qquad \ldots \quad (3.4.11)$$

Thus, to obtain an optimal plan (n,c_1,c_2) one may consider c_1, c_2 and y satisfying (3.4.6) i.e., from table 3 and then, by a process of iteration, boundary points for the adjacent zones on the $(\bar{M},k)$ -plane can be obtained, subsequently sketching all the zones each with an identical value of c_1. This chart on the $(\bar{M},k)$ -plane gives the value of c_1 minimising $\bar{I}$ for specified $\bar{p}/p_1$ and lot size (N). The corresponding value of n is obtained from (3.4.10) as mentioned earlier.

A systematic iterative procedure to obtain the boundary point on the $(\bar{M},k)$-plane is given below:

Step 1 : Choose some arbitrary value of c_1.

Step 2 : Assume N as infinite, obtain n_{c_1} and n_{c_1+1} and hence a_{c_1} and a_{c_1+1} from (3.4.5), using the values of y_{c_1} and y_{c_1+1} from table 3.

Step 3 : Obtain the value of $\bar{M}$ and hence N from (3.4.11) using chosen values of c_1 and c_1+1 and corresponding values of $c_2^{(c_1)}$ and $c_2^{(c_1+1)}$ and the values of a_{c_1} and a_{c_1+1} as obtained in step 2.

Step 4 : From (3.4.5), using N as obtained in step 3 and the values of y_{c_1} and y_{c_1+1}, obtain new values of a_{c_1} and a_{c_1+1} .

Step 5 : Again obtain $\bar{M}$ from (3.4.11) using the new values of a_{c_1} and a_{c_1+1} obtained in step 4.

Step 6 : Obtain more accurate values of a_{c_1} and a_{c_1+1} from (3.4.5) using N as obtained in step 5.

Step 7 : Terminate the iteration for a fixed $k = \bar{p}/p_L$ when at two successive stages of iteration the solutions are almost identical.

3.4.1 Numerical example

As an example, let $\bar{p} = 0.005$, $p_L = 0.050$. For $k = \bar{p}/p_L = 0.100$ and $c_1 = 0,1,2,3$ and 4 and using the corresponding values of y from the table of x and y (table 3), the boundary points $\bar{M}$ for the adjacent zones were obtained as given in table 4, under the procedure of section 3.4 given earlier.

For illustration the optimal plan for $c_1 = 0$; $\bar{p} = 0.005$; $p_L = 0.05$ and $y = 0.414214$ is obtained as follows:

The value of $\bar{M}$ corresponding to $k = 0.100$ from table 4 is 1.404712 for $c_1 = 0$ and $c_1+1 = 1$. The value of the lot size at the boundary of the zone for $c_1 = 0$ is obtained by dividing the value of $\bar{M}$ by the value of $\bar{p}$ i.e.,

$$N = \frac{1.404712}{0.005} = 280.94$$

Using (3.4.5) we get the value of sample size as 8.05. The average lot size for the $c_1 = 0$ zone is taken as 144 and the lot range as 8 - 280.

3.5 Special plans (ASP plans)

In the previous section we have considered ASR plans with average quality protection. Under an ASR plan items are assumed to be in ample supply (section 3.2) and hence it is possible to afford outright rejection of a lot. But in practice situations arise where either items are in short supply or demand increases suddenly and existing sources of supply may not be adequate to meet the increased demand. The sudden increase in the demand may occur due to emergency imposed on by a war, epidemic or famine. In such cases it may not be possible to afford an outright rejection of a lot which is supplied. However, it may be desirable to discourage the supply of poor quality material by imposing some penalty on the supplier while accepting the lots which would have been rejected outright by the ASR plan. Such lots, as explained later, may be either used directly in restricted situations where defective items do not hamper much the end use or as substandard product or after screening at a convenient time. It may not be possible to screen such lots at the time of acceptance due to exigency. For this purpose the procedure of three-decision ASR plan may be modified as follows:

Take a random sample of size n from N and, if x denotes the number of defectives in n, then

if $0 \leq x \leq c_1$	accept after rectifying or replacing observed defectives in the sample;	
if $c_1 < x \leq c_2$	screen and accept;	... (3.5.1)
if $c_2 < x \leq n$	accept with penalty to supplier (for substandard or restricted use)	

We shall refer to such three-decision plan as ASP plan. By penalty we mean a monetary punishment to the supplier by a fixed amount. This amount may be determined by means of practical consideration of the seriousness of the bad lot. However, in some cases such a punishment may be modified so that at the first instance such a lot may be accepted with a warning to the supplier that he must supply next k successive lots good ones otherwise he may be subjected to suitable monetary punishment, where k is a fixed constant to be chosen suitably. Some aspect of utilisation of the lots accepted with penalty to the supplier under an ASP plan need clarification at this stage. For such lots following course of actions may be some of the possibilities:

(1) accept the lot as such for the purpose of use if defective items, though undesirable, do not pose a great handicap to functional requirements of the item.

(2) screen the lot before use if defective items are not usable, at a convenient time.

Sometimes such lots may be kept aside and used only when actual non-availability occurs due to short supply in view of high demand. If supply is of stochastic nature and the distribution is known reasonably well, it may be worth investigating an optimal decision procedure minimising regret or loss in respect of instantaneous or deferred use of such lots.

Assuming that the lots accepted with penalty under ASP plans are used as such, without being screened, the average amount of inspection is given by (3.4.1) and average outgoing quality $p_A(p)$ is given by

$$p_A(p) = p \frac{N-n}{N} (1 - \sum_{r=c_1+1}^{c_2} g(r,np)) \quad \ldots \quad (3.5.1a)$$

In this case, we have the following result, analogous to lemma 2.

Lemma 3. For given lot size (N) and any ASP plan (n, c_1, c_2) the value of average outgoing quality limit (p_L) is given by

$$p_L = \left(\frac{1}{n} - \frac{1}{N} \right) y \quad \ldots \quad (3.5.2)$$

where $y = x \left[1 - \sum_{r=c_1+1}^{c_2} g(r,x) \right]$ and x is the solution of

$$1 - \sum_{r=c_1+1}^{c_2} g(r,x) = x \left[g(c_1,x) - g(c_2,x) \right] \quad \ldots \quad (3.5.3)$$

Proof: Proceeding on the line of (3.4.7) - (3.4.8) we get

$$\frac{dp_A}{dp} = \frac{N-n}{N} \left[1 - \sum_{r=c_1+1}^{c_2} g(r,np) + np \, (g(c_2,np) - g(c_1,np)) \right] .. (3.5.4)$$

Solving (3.5.4) for p, suppose we get the solution $p = p'$. Let $p'n = x$; then we get (3.5.2) and (3.5.3) on the line of lemma 2.

□

Solving (3.5.3) by Newton-Raphson method for the value of $c_1 = 0$ to 21, we obtain the solution x and the value of y as given in the table 6.

Argueing as in the section 3.4 and proceeding along identical lines we get the boundary points $\bar{M}$ for the adjacent zones on the $(\bar{M},k)$-plane under the ASP plan for $c_1 = 0,1,2,3,4,5$ as given in table 7 when same values $\bar{p}$ and p_L as earlier are used.

Numerical example

Proceeding on the lines as in the previous section (3.4) some illustrative optimal ASP plans are obtained and are given in the table 8. As mentioned, these plans are only illustrative. An exhaustive table for such AOQL three-decision plans can be worked out following the steps given here.

Comparison with Dodge and Romig's plan

Given a Dodge and Romig's single sampling plan for a fixed value of the lot size (N), process average $(\bar{p})$ and

AOQL (p_L) with sample size n and acceptance number c, we shall define a ''corresponding'' three-decision plan (n, c_1, c_2) where $c_1 = c$ and $c_2 > c_1$, as earlier in Chapter 2. Such ''corresponding'' plans may be neither unique nor optimal in the sense of Dodge and Romig.

We shall state the following:

Lemma 4. For given lot size (N), process average ($\bar{p}$) and AOQL (p_L) let (n, c_1) be the optimal Dodge and Romig's plan with AOI as I_1^o and (n, c_1, c_2), $c_1 < c_2$ be the ''corresponding'' three-decision ASR plan with AOI as I_2, then $I_2 \leq I_1^o$.

Proof: As in case of lemma 1, we can write

$$I_1^o = I_2 + (N-n) \text{ Prob } (X > c_2 \mid N, \bar{p}, n, c_2)$$

which implies that $I_2 \leq I_1^o$ for $(N-n) \text{ Prob } (X > c_2 \mid N, \bar{p}, n, c_2) \geq 0$ where X is a random variable denoting number of defectives in the sample.

□

Theorem 2. For a given lot size (N), process average ($\bar{p}$) and average outgoing quality limit (p_L), let I_1^o and I_2^o be the value of AOI of the optimal DR plan and optimal three-decision ASR plan respectively then $I_2^o \leq I_1^o$.

Proof: Since $I_2^o \leq I_2$ the result follows immediately from lemma 4.

□

Corollary: For a given lot size (N), process average ($\bar{p}$) and average outgoing quality limit (p_L) if I_1^o and I_2^o be the value of AOI of an optimal DR plan and optimal three-decision ASP plan respectively, then $I_2^o \leq I_1^o$.

3.7 General remarks

The use of plans providing average quality protection in terms of AOQL are proposed by several authors such as Dodge (1943), Dodge and Romig (1959), Soundararajan (1972), Pandey (1977). However, AOQL has been criticised as an inadequate measure as it does not provide a sharp upper bound for average outgoing quality. Anscombe (1958) characterised AOQL as being only the statistician's guarantee and remarked that in practice it is not the consumer's requirement. He emphasised the need for an alternative measure. So far no adequate alternative measure has been found. In case of continuous sampling plan (CSP-1) of Dodge (1943), Hillier (1964) proposed average extra defective limit (AEDL) as an alternative measure to obtain a unique plan. But AEDL is based on AOQL concept and is mainly a way out to obtain a unique CSP-1 plan i.e., value of parameters i and f for given values of AOQL and incoming lot quality (p) rather than an alternative measure. It would be of interest to examine inflexion average quality i.e., value of $p(N-n)\ \mathrm{Prob}(0 \leq X \leq c \mid N, n, c, p)/N$ for $p = p_A^0$ as a sharper bound alternative to AOQL for a given three-decision plan where p_A^0 is one of the values of p-coordinate of the point of inflexion of $p\ p_a(p)$.

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CHAPTER FOUR

A CHARACTERISATION OF SINGLE SAMPLING THREE-DECISION PLANS BY ATTRIBUTES

4.1 Introduction

In earlier chapters (chapter 2 and 3) we discussed the problem of determination of unique three-decision plans which :

(1) satisfy certain desirable specifications in terms of operating characteristic function, and

(2) meet the desirable practical requirements of achieving minimum average amount of inspection at process average quality.

Although, the above requirements are quite vital in view of practical needs, an acceptance sampling plan can be uniquely determined or completely characterised also in terms of operating characteristic function alone. For example, a single sampling two-decision plan by attributes is completely characterised if one specifies two points on its O.C. Curve which may either be producer's and consumer's risks [Cameron (1952)] at given quality levels, or indifference quality level (IQL, i.e., quality level with probability of acceptance 0.50) and the slope of the O.C. curve at IQL

[Hamaker (1950)] . In the present Chapter (section 4.2) we have discussed a characterisation of single sampling three-decision plans in terms of operating characteristic function based on the point of inflexion [Pandey (1977)].

-.2 A characterisation of three-decision (ASR) plan

For incoming lot quality p the probability of acceptance $P_a(p)$, probability of screening $P_s(p)$ and probability of rejection $P_r(p)$ under an ASR plan are defined by (3.3.1), (3.3.2) and (3.3.3) respectively. As before (Chapter 2 section 2.3) we shall use Poisson approximation of the Binomial distribution in this section.

Further, we shall use the following notations:

p_s^* : p - coordinate of the maximum ordinate of the $P_s(p)$ curve against p.

p_a^o : p - coordinate of the point of inflexion of $P_a(p)$.

The values corresponding to p_a^o on $P_s(p)$ and $P_r(p)$ can be similarly introduced.

A suitable modification of these notations may be made in cases where more than one points of inflexion exist.

However, such modifications are not relevant to p_a^o since the point of inflexion of $P_a(p)$ for any positive integer c_1 is unique and is given by c_1/n. For $c_1 = 0$ the point of inflexion of $P_a(p)$ is not defined.

p_a^t : The point where the tangent at the point of inflexion of $P_a(p)$ meets the p-axis.

s_a^o : Slope of $P_a(p)$ at its point of inflexion.

p_A^o : p - coordinate of one of the points of inflexion of $p\,P_a(p)$.

s_A^o : Slope of the curve of $p\,P_a(p)$ at any one of its points of inflexion.

First, we shall prove the following theorem :

<u>Theorem 3</u>. For c_1 any positive integer in R^1 the function $f(c_1)$ given by

$$f(c_1) = (c_1-1)!\; c_1^{-c_1} \sum_{r=0}^{c_1} (c_1)^r/r! \quad \ldots (4.2.1)$$

is one-to-one strictly decreasing function of c_1.

Proof: Write

$$f(c_1) = \frac{c_1!}{c_1^{c_1+1}} \sum_{r=o}^{c_1} (c_1)^r/r!$$

where c_1 is any positive integer.

It is enough to prove that

$$f(c_1) > f(c_1+1) > 0 \qquad \ldots \qquad \ldots (4.2.2)$$

clearly, $f(c_1) > 0$ for all $c_1 \geq 1$.

$$f(c_1) = \frac{c_1!\, e^{c_1}}{c_1^{c_1+1}} \sum_{r=o}^{c_1} e^{-c_1} \frac{c_1^r}{r!}$$

$$= \frac{e^{c_1}\Gamma(c_1+1)}{c_1^{c_1+1}} \frac{\int_{c_1}^{\infty} e^{-x} x^{c_1} dx}{\Gamma(c_1+1)}$$

$$= \frac{e^{c_1}}{c_1^{c_1+1}} \int_{c_1}^{\infty} e^{-x} x^{c_1} dx$$

$$= e^{c_1} \int_1^{\infty} e^{-c_1 x} x^{c_1} dx \qquad \ldots \quad (4.2.3)$$

by making the substitution $x = c_1 x$.

Now, consider $f(c_1)$ to be defined by (4.2.3) for all real $c_1 > 0$. If we can prove that $f'(c_1) < 0$ for all c_1, it will imply, in particular, that $f(c_1) > f(c_1+1)$ for integral values of c_1. The function $f(c_1)$ is differentiable every where except when c_1 is an integer and the conditions for differentiation under integral sign are satisfied.

Now,

$$f'(c_1) = e^{c_1} \int_1^{\infty} e^{-c_1 x} x^{c_1} dx + e^{c_1} \left[\int_1^{\infty} \left(e^{-c_1 x} x^{c_1} \log x - e^{-c_1 x} x^{c_1+1} \right) dx \right]$$

$$= e^{c_1} \left[\int_1^{\infty} e^{-c_1 x} x^{c_1} \left[\log x - (x-1) \right] dx \right]$$

Denote by $\Phi(x)$ the function $[\log x - (x-1)]$

$$\frac{d\Phi(x)}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x} < 0 \quad \text{for } x > 1 \quad \text{i.e., } \Phi(x) \text{ is}$$

a decreasing function of x for $x > 1$.

But $\Phi(1) = 0$. Therefore $\Phi(x) < 0$ for $1 < x \leq \infty$ i.e., $f'(c_1) < 0$ which implies (4.2.2) for positive integral value of c_1 showing that $f(c_1)$ is strictly decreasing one-to-one function of c_1 (Fig.3).

We shall now prove the following characterisation theorem.

Theorem 4. The acceptance sampling three-decision ASR plan is completely characterised by p_a^o, p_a^t and p_s^* points on its O.C. Curve.

Proof: Let (n, c_1, c_2) and (n', c_1', c_2') be any two plans with values of p_a^o, p_a^t, p_s^*, and $p_a^{o'}$, $p_a^{t'}$, $p_s^{*'}$ respectively.

It is enough to show that $p_a^o = p_a^{o'}$, $p_a^t = p_a^{t'}$ and $p_s^* = p_s^{*'}$ imply $n = n'$, $c_1 = c_1'$ and $c_2 = c_2'$

From $p_a^o = p_a^{o'}$ we get

$$\frac{c_1}{n} = \frac{c_1'}{n'} \qquad \dots \qquad \dots \qquad (4.2.4)$$

and $p_a^t = p_a^{t'}$ gives

$$\frac{c_1}{n}\left[1+\frac{(c_1-1)!\,e^{c_1}}{c_1^{c_1}}\,G(c_1,c_1)\right] = \frac{c_1'}{n'}\left[1+\frac{(c_1'-1)!\,e^{c_1'}}{c_1'^{c_1'}}\,G(c_1',c_1')\right] \qquad \dots \qquad (4.2.5)$$

and from definition of p_s^* and $p_s^{*'}$ we get

$$(n\,p_s^*)^{c_2-c_1} = \frac{c_2!}{c_1!} \qquad \dots \qquad (4.2.6)$$

and

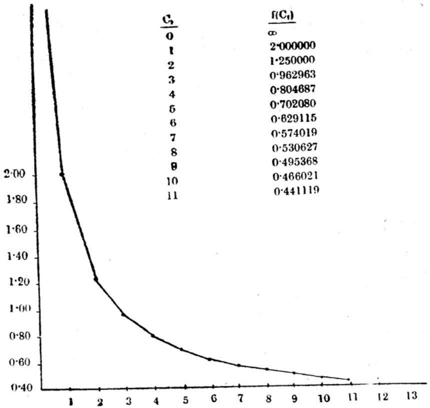

FIG.3 CURVE FOR $f(c_1)$ AGAINST c_1.

$$(n' \, p_s^{*'})^{c_2'-c_1'} = \frac{c_2'!}{c_1'!} \qquad \ldots \qquad (4.2.7)$$

From (4.2.4) and (4.2.5) we get

$$\frac{(c_1-1)!}{c_1^{c_1}} \sum_{r=o}^{c_1} \frac{(c_1)^r}{r!} = \frac{(c_1'-1)!}{c_1'^{c_1'}} \sum_{r=o}^{c_1'} \frac{(c_1')^r}{r!} \quad \ldots \quad (4.2.8)$$

$$\text{i.e.,} \qquad f(c_1) = f(c_1') \qquad \ldots \qquad \ldots \qquad (4.2.8a)$$

Since $f(c_1)$ is strictly decreasing one-to-one function of c_1 from theorem 3 there can not exist two integral values of c_1 for which (4.2.8a) holds and hence $c_1 = c_1'$ which in turn from (4.2.4) implies $n = n'$.

Further from (4.2.4), (4.2.6) and (4.2.7) we get

$$\ln k = \frac{1}{c_2'-c_1'} \ln \frac{c_2'!}{c_1'!} - \frac{1}{c_2-c_1} \ln \frac{c_2!}{c_1!} \qquad \ldots \quad (4.2.9)$$

where $k = c_1'/c_1$. Since $c_1' = c_1$, (4.2.8) implies

$$\frac{1}{c_2'-c_1'} \ln \frac{c_2'!}{c_1'!} = \frac{1}{c_2-c_1} \ln \frac{c_2!}{c_1!} \qquad \ldots \quad (4.2.10)$$

which means $c_2' = c_2$ proving the required result.

Corollary: The acceptance sampling three-decision plan by attributes is completely characterised by any one of the following combinations of parameters

(a) p_a^o, s_a^o and p_s^*

(b) p_A^o, s_A^o and p_s^*

The proof of this corollary is similar to theorem 4 and is omitted.

Concluding remarks

The characterisation results established in this Chapter may be used to obtain the three-decision plans which may be unique in the sense of two-decision plans by Cameron (1952). Such a determination is based on specified values of the characterising triplet of theorem 4. It does not require the economic criterion of either minimum average amount of inspection or minimum average total cost of inspection and decision discussed through Chapters 2, 3 and 5. Although, the three-decision plan obtained in this way, will not fulfil the practical requirements of Chapters 2, 3 and 5, it will be a reasonably good sampling procedure in the event of lack of information regarding costs and will gain currency of use provided one gets conversant with the physical concept of point of inflexion and related values, in practice.

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CHAPTER FIVE

BAYESIAN SINGLE SAMPLING THREE-DECISION PLANS FOR DISCRETE AND CONTINUOUS PRIOR DISTRIBUTIONS

5.1 Introduction

The purpose of this Chapter is to develop Bayesian single sampling three-decision plans by attributes under a linear cost model. A glance through the literature on sampling inspection shows that the main concepts controlling the choice of a sampling system can be best described in the words of Sitting (1951) as follows:

''The choice of a sampling system for Acceptance Sampling has been controlled by two concepts ever since statisticians have been engaged in this field - risk and cost. First, a given 'amount of protection' is laid down and then a sampling system is chosen from those available on the basis of achieving minimum 'amount of inspection' for the required amount of protection''.

As mentioned earlier, the most important system of plans providing lot quality and average quality protection and achieving minimum amount of inspection at process average quality is Dodge and Romig's (1929) system of plans.

In practice the costs are more tangible to the industrialist and the corresponding choice is easier to make than the choice of risk points and risks. This is mainly because the various decision costs, difficult though some of them may be to determine, are closer to the kind of data that an industrialist can supply on a rational basis than are the various risks and the risk points. In practice the plans based on even rough estimates of costs may be found quite satisfactory as Tippett (1958, pp.148) has pointed out in the following words:

''It would seem that schemes based on rough estimates of costs, or even informed guesses, are likely to be more satisfactory than those chosen in other ways''. These considerations have provided the main motivation for the work of the present Chapter.

During the fifties a good deal of valuable work was done on economic plans based on various decision costs. Some of them are by Anscombe (1950), Hamaker (1951), Sitting (1951), Weibull (1951), Champernowne (1953) and Horsnell (1957). In this Chapter in section 5.4 an attempt has been made to develop economic three-decision plans based on

(i) risks (restriction on the consumer's risks) and as well as

(ii) sampling and decision costs

Further, in section 5.5, economic three-decision plans using total sampling inspection costs and decision costs per lot, and under less restrictive conditions than in section 5.4, are developed.

For developing economic plans, it is appropriate to take some prior distribution for the item characteristic or the lot quality to evaluate the minimum expected value of the loss or cost function. As early as 1946 E.C. Molina and G.A. Barnard recommended that Bayes' theorem, which involves prior distributions, should occupy a prominent place in the theory of sampling inspection. Soon, for the Bayesian approach a more precise theoretical formulation and general mathematical expressions were provided by Lehman and Wald in 1947.

Since then several papers on Bayesian sampling inspection have appeared. Some of them are Barnard (1954), Guthrie and Johns (1959), Wetherill (1960), Pfanzagl (1963), Bernard (1963), Weiler (1965), Schafer (1966), Stange (1966), Soland (1968), Hald (1960b, 1965, 1967a, 1967b, 1968a, 1968b, 1973), Hald and Thyregod (1971), Thyregod (1974, 1975), Pandey (1974a).

In this Chapter a double binomial with parameters p' and p'' ($p' < p''$) with weights w_1 and w_2 respectively ($w_1+w_2=1$), a three point binomial with parameters p', p'' and p''' ($p' < p'' < p'''$) and weights w_1, w_2 and w_3 ($w_1+w_2+w_3 = 1$) respectively and, in the limiting case, a point binomial with parameter $\bar{p}$ have been used as prior distribution for the lot quality. In section 5.8 we also assume beta prior distribution with parameters π and t for the lot quality.

A linear cost model under the set up of three-decision plan is considered. Two types of Bayesian three-decision plans are developed, designated as (i) restricted (satisfying some restrictions on the consumer's risk) Bayesian three-decision plan and (ii) unrestricted Bayesian three-decision plan. In both the cases optimum plans developed here minimise expected total inspection and decision cost per lot. In the case of the restricted plans, the restriction has been taken in the form of specification on consumer's risk (β_1 and β_2) as in Chapter 2.

The Bayesian single sampling ASR plans developed in this Chapter, is defined by the parameters n, c_1 and c_2 and is to be operated as follows:

Take a random sample of n items from a lot size N and let x be the number of defectives in the sample then accept if $0 \leq x \leq c_1$;

screen if $c_1 < x \leq c_2$;

reject if $c_2 < x \leq n$.

These plans are applicable under situations as stated in the following section.

5.2 The conditions for applicability

Though inspection procedures with three-decision criteria are not yet in current use, it is not difficult to find practical situations where they can be usefully employed. For applicability of the three-decision ASR plans the following five conditions should be satisfied.

(1) Items are inspected on lot-by-lot basis,

(2) Defective items in accepted lots cause some damage which is measurable in economic terms,

(3) Inspection does not involve any destructive or very costly testing i.e., 100% inspection is permissible,

(4) Any defective item if detected can be either replaced with good item or rectified,

(5) The condition ($p_u < p_v$) for preferring three-decision plans, in terms of break-even qualities as defined in section 5.3 by (5.3.10) and (5.3.11) holds.

It is felt that the conditions (1)-(4) are generally satisfied in those industrial situations where two-decision i.e., acceptance-rectification or acceptance-rejection plans are in use. Thus, it is in fact condition (5) which we assume in addition for treatment of three-decision ASR plan in the present Chapter.

The economic advantages of the three-decision plans greatly depend on the weights i.e., w_1 and w_2. For example, in case of the double binomial prior distribution mentioned earlier, for higher values of w_2 three-decision plans will be more and more advantageous as compared with two-decision plans as illustrated in the figures 10 and 11.

The conditions for the applicability of three-decision plans of other types, e.g., AMS and ASP plans, can also be listed down as in case of ASR plans. For example, the conditions for the applicability of three-decision plans with ''accept - accept as moderately good - out right reject'' type of decision, i.e., AMR plans, are given below:

(a) Items are inspected on lot-by-lot basis.

(b) Acceptance of any moderately defective item is undesirable but, if accepted, it does not substantially hamper the functional requirements.

(c) Inspection involves either a destructive or costly testing i.e., 100% inspection is not permissible.

(d) The condition ($p_u < p_v$) for three-decision plan holds where p_u and p_v are defined by (5.3.10) and (5.3.11) respectively in the section 5.3.

To point out a practical situation where AMR type plans can be applied, we may mention the following situation actually experienced by the author during his work:

An organisation (around Calcutta) purchases various store items from different local suppliers. One of the store items is a sort of bandages used for medical dressing.

The organisation uses sampling inspection for the purpose of deciding the acceptance of the lot of bandages. The sample is subjected to the following two types of tests:

(i) scouring test (which is destructive test) and

(ii) test for determining weight per metre.

Any failure on the part of a bandage to meet the specifications in respect of the two characteristics is undesirable but it does not affect much the end use of the bandages. Depending on the test results a lot is either

(a) accepted as satisfactory or (b) accepted with some penalty to the supplier or (c) declared unsatisfactory and rejected outright.

The organisation has some amount of arbitrariness while deciding the percentage of penalty to the supplier. This is mainly because the inspection is actually done on the basis of a two-decision plan and a good deal of arbitrariness arises while classifying the unsatisfactory lots into acceptable lots with penalty and the lots which are outright rejectable ones. This was a situation where it was felt that a more objective three-decision acceptance sampling plan by variables, with ''accept-accept with penalty - reject outright'' type of decisions, i.e., an APR plan, would be relevant, provided that a condition similar to the condition (d) above was met. A rough computation of the various costs did, in fact, lead to this conclusion.

5.3 The cost-model

Let N and n denote the lot size and sample size and let X and x denote the number of defectives in the lot and the sample respectively. The acceptance number are denoted by c_1 and c_2 ($c_1 < c_2$).

The following linear cost function will be considered in the Chapter

$$h(X,x,N,n,c_1,c_2) = \begin{cases} nS_1+xS_2+(N-n)A_1+(X-x)A_2 & \text{for } x \leq c_1 \\ nS_1+xS_2+(N-n)T_1+(X-x)T_2 & \text{for } c_1 < x \leq c_2 \\ nS_1+xS_2+(N-n)R_1+(X-x)R_2 & \text{for } c_2 < x \leq n \end{cases}$$

... (5.3.1)

where nS_1 denotes the cost of inspection and xS_2 denotes the cost proportional to the number of defective items in the sample. In fact S_1 includes sampling and testing costs per item and S_2 denotes additional costs for an inspected defective item including the repair costs per item in case the defective items found in the sample are repaired. Thus the costs $nS_1 + xS_2$ associated with the sample give the costs of sampling inspection. For convenience in the subsequent mathematical treatment, we have assumed the same per defective unit cost S_2 for sampled items in each of these situations, although this cost may actually be different in the case of outright rejection.

Cost of acceptance is given by $(N-n)A_1 + (X-x)A_2$. The part $(N-n)A_1$, proportional to the number of items in the remainder of the lot, represent handling and similar costs, and A_1 will usually be very small. The part $(X-x)A_2$ is proportional to the number of defective items accepted and hence A_2 will often be considerable. If the accepted items are used as a basic raw material or components to manufacture some product, A_2 may include the manufacturing cost or the price of an item, the costs of handling a defective item in assembling and disassembling and damage done to other parts used in the assembly. In the case of inspection of finished goods A_2 may include the cost of repair, service and guarantees plus loss of good will.

Cost of screening is given by $(N-n)T_1 + (X-x)T_2$, and is composed of a part $(N-n)T_1$, proportional to the number of items in the remainder of the lot, and another part, $(X-x)T_2$, proportional to the number of defective items in the remainder of the lot. Here T_1 may include the costs of handling and the costs of inspection per item whereas T_2 may include the costs of rework or replacement and the costs of delay per defective items caused due to screening.

Costs of out right rejection are $(N-n)R_1 + (X-x)R_2$, and is composed of $(N-n)R_1$, proportional to the number of items in the remainder of the lot, and another part $(X-x)R_2$

which is proportional to the number of defective items in the remainder of the lot. R_1 may include the costs of storage per rejected item before disposal, the costs of handling per rejected item during disposal and the costs of delay in availability of raw material or the components for manufacturing or assembling per rejected item. R_2 is generally zero or negligible.

From Hald (1967a), for three different kind of lots, the cost function (5.3.1) becomes, by using a Taylor expansion around np + O ($\sqrt{n}$) for variable x and taking x = Np,

$$h \sim \begin{cases} n(S_1+S_2p) + (N-n)(A_1+A_2p) & \text{for } x \leq c_1 \\ n(S_1+S_2p) + (N-n)(T_1+T_2p) & \text{for } c_1 < x \leq c_2 \quad \ldots \ (5.3.2) \\ n(S_1+S_2p) + (N-n)(R_1+R_2p) & \text{for } c_2 < x \leq n \end{cases}$$

disregarding the terms of order $\sqrt{n}$. The average cost can be written as

$$K(N,n,c_1,c_2) = \int K(N,n,c_1,c_2,p)dW(p) \quad \ldots \ (5.3.3)$$

where W(p) denotes the cumulative prior distribution of p and

$$K(N,n,c_1,c_2,p) = n(S_1+S_2p) + (N-n) \left[(A_1+A_2p)\, P_a(p) + (T_1+T_2p)P_s(p) + (R_1+R_2p)P_r(p)\right] \quad \ldots \ (5.3.4)$$

The probabilities $P_a(p)$, $P_s(p)$ and $P_r(p)$ are given by (3.3.1), (3.3.2) and (3.3.3) respectively in Chapter 3. The expression (5.3.4) represents the average cost over all the lots for a given process average p and, since it is assumed that the process average is a random variable which follows the cumulative distribution W(p) the expression (5.3.3) gives the overall average cost for all values of p.

To simplify the notations let us introduce the following four cost functions:

$$k_s(p) = S_1 + S_2 p \quad \dots \quad \dots \quad (5.3.5)$$

$$k_a(p) = A_1 + A_2 p \quad \dots \quad \dots \quad (5.3.6)$$

$$k_t(p) = T_1 + T_2 p \quad \dots \quad \dots \quad (5.3.7)$$

$$k_r(p) = R_1 + R_2 p \quad \dots \quad \dots \quad (5.3.8)$$

defined for $0 \leq p \leq 1$. The corresponding averages k_s, k_a, k_t and k_r are defined by

$$k = \int_0^1 k(p)\,dW(p) \quad \dots \quad \dots \quad (5.3.9)$$

We shall make the following assumptions regarding the functions (5.3.5) - (5.3.8).

(1) All the functions are non-negative and none is identical to zero.

(2) $k_a(0) < k_t(0) < k_r(0)$ i.e., for a 100% good lot cost of acceptance per item is the least and the cost of screening per item will be quite low whereas the cost of rejection per item would be considerable.

(3) $k_a(1) > k_t(1) > k_r(1)$ i.e., for a 100% defective lot the cost of acceptance per item is the highest and the cost of screening would be considerable, whereas the cost of rejection per item is the least.

From the above assumptions it follows that $A_1 < T_1 < R_1$ and $A_1 + A_2 > T_1 + T_2 > R_1 + R_2$, and hence that (i) $A_2 > T_2 > R_2$, (ii) $A_2 - T_2 > T_1 - A_1 > 0$ and (iii) $T_2 - R_2 > R_1 - T_1 > 0$. Also, the equations $k_a(p) = k_t(p)$ and $k_t(p) = k_r(p)$ have the solution

$$p_u = (T_1-A_1)/(A_2-T_2), \quad 0 \leq p_u \leq 1 \qquad \ldots (5.3.10)$$

and

$$p_v = (R_1-T_1)/(T_2-R_2), \quad 0 \leq p_v \leq 1 \qquad \ldots (5.3.11)$$

respectively.

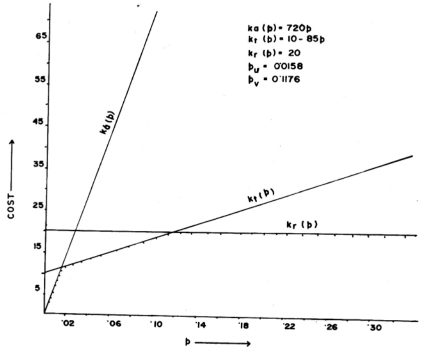


FIG. 4 COST FUNCTIONS WHEN $p_u < p_v$, $k_a(p) = 720p$, $k_t(p) = 10+85p$, $k_r(p) = 20$, $p_u = 0.0158$, $p_v = 0.1176$.

Since the intercepts A_1, T_1 and R_1 are in the order $0 \leq A_1 < T_1 < R_1$ and the slopes in the order $A_2 > T_2 > R_2 \geq 0$ the relative positions of the graphs of $k_a(p)$, $k_t(p)$ and $k_r(p)$ can either be as in Fig.4 or as in Fig.5.

(4) $k_s(p) \geq k_m(p)$ for $0 \leq p \leq 1$ where $k_m(p)$ is defined as follows:

If $p_u < p_v$, $k_m(p)$ is given by (5.3.12)

$$k_m(p) = \begin{cases} k_a(p) & \text{for } p < p_u \\ k_t(p) & \text{for } p_u \leq p < p_v \\ k_r(p) & \text{for } p \geq p_v \end{cases} \quad \ldots \quad (5.3.12)$$

If $p_u \geq p_v$, $k_m(p)$ is given by (5.3.13)

$$k_m(p) = \begin{cases} k_a(p) & \text{for } p < p_w \\ k_r(p) & \text{for } p \geq p_w \end{cases} \quad \ldots \quad (5.3.13)$$

where $p_w = (R_1 - A_1)/(A_2 - R_2)$, $0 \leq p_w \leq 1$ .. (5.3.14)

From these it is obvious that if $p_u < p_v$ and the assumptions made in (1) - (4) are true, the minimum unavoidable cost will be given by (5.3.12) as shown in Fig.4 and, if $p_u \geq p_v$, it would be given by (5.3.13) as shown in Fig.5.

In this Chapter we shall assume that $p_u < p_v$. For $p_u < p_v$ the average minimum cost k_m is defined as

$$k_m = \int_0^{p_u} k_a(p)\,dW(p) + \int_{p_u}^{p_v} k_t(p)\,dW(p) + \int_{p_v}^{1} k_r(p)\,dW(p) \quad \ldots(5.3.15)$$

k_m given by (5.3.15) represents the average cost per item when the following decision rule is used:

accept all lots from processes with $p < p_u$;

screen all lots from processes with $p_u \leq p < p_v$;

reject all lots from processes with $p \geq p_v$.

The proportions p_u and p_v are called break-even qualities and these values can be used to classify the lots in three quality grades as follows:

Grade A : lot of quality $p < p_u$

Grade B : lot of quality p, $p_u \leq p < p_v$ ...(5.3.16)

Grade C : lot of quality $p \geq p_v$

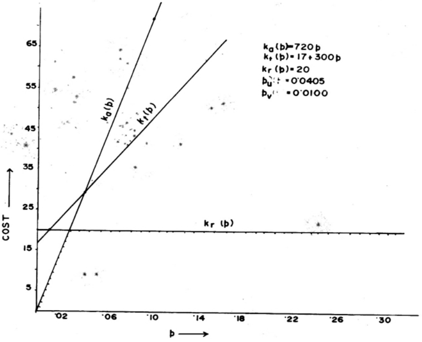


FIG. 5 COST FUNCTIONS WHEN $p_u \geq p_v$, $k_a(p) = 720p$, $k_t(p) = 17+300p$, $k_r(p) = 20$, $p_u = 0.0405$, $p_v = 0.0100$.

Using Hald's (1967a) technique we define the standardised form of (5.3.3) as

$$R(N,n,c_1,c_2) = [K(N,n,c_1,c_2) - N k_m] / (k_s - k_m) \quad \ldots (5.3.17)$$

which gives

$$R = n + \frac{(N-n)}{(k_s-k_m)} [(A_2-T_2)[\int_0^{p_u} (p_u-p)(1-P_a(p))\, dW(p) + \int_{p_u}^{1} (p-p_u)\, P_a(p) dW(p)] + (T_2-R_2)[\int_0^{p_v} (p_v-p) P_r(p) dW(p) + \int_{p_v}^{1} (p-p_v)(1-P_r(p))\, dW(p)]] \quad \ldots \quad \ldots \quad (5.3.18)$$

As mentioned earlier, in this thesis we shall confine our attention to some discrete prior distributions of the process average - double and point binomial [Pandey (1974a)], triple binomial and, continuous prior distribution of the process average - Beta distribution [Pandey (1973)], to develop three-decision sampling plan.

The terminologies of ''double binomial'' and ''point binomial'' used here are due to Hald (1960). However, according to Hald (1981) we may use the phrase one point prior, two point prior and three point prior as the case may be.

The double binomial distribution is a weighted average of two binomials with parameters p' and p'', p' < p'' and weights w_1 and w_2, $w_1 + w_2 = 1$ i.e., the process average has a two point distribution. A triple binomial is similarly defined.

To justify the assumption regarding the prior distributions of the lot quality p, at least from the practical point of view, the following may be added on the basis of Hald (1981) pages 16-19.

Regarding one point prior, in the first place, it should be noted that it is impossible to keep a process in control with a fixed process average, $\bar{p}$, say, for a long time. The process goes out of control and produces outliers i.e., lots with poorer quality. The sampling plan is, then, to be designed to detect the outliers and keep the probability of acceptance of an outlier low. When lot size (N) becomes large the lot quality distribution tends to one-point distribution of the process average because the standard deviation $\overline{pq}/N$ becomes negligible or zero.

The two-point prior may be considered as a modification of one-point prior in the sense that the first component represents product of normal quality (p') and the second represents outliers of poorer quality (p''). We assume, in this case, that the outliers occur at random and have a binomial distribution corresponding to the second component of the prior.

Three-point prior may be, similarly, considered as a modification of two point prior in the sense that the first component represents product of normal quality (p'), the second and third components represent outliers of quality levels p'' and p''' respectively. Here, outliers are assumed to occur at random and have binomial distribution corresponding to the second and third components. If there are three different sources of supply with different levels of quality one may use sampling plan to distinguish between the products of three levels.

For beta prior distribution, a process in control with respect to beta prior will occasionally go out of control and some lots of poorer quality will be produced before the process is adjusted. This gives rise to a situation which can be described as beta prior with outliers. The model of beta prior with outliers is usually a more realistic model than one point prior because it is extremely difficult to keep a process in control with a fixed process average.

One-point prior may be considered also as a limiting form of beta prior with mean $\bar{p} = \pi/(\pi+t)$ and $\sigma_p^2 = \bar{p}\bar{q}/(\pi+t+1)$ when $\bar{p}$ is fixed and $\pi+t \longrightarrow \infty$ where π and t denote the parameters of the beta prior.

We shall assume that $p' < p_u < p_v < p''$. The other cases such as $p' < p_u < p'' < p_v$; $p_u < p' < p'' < p_v$; $p_u < p' < p_v < p''$ etc., can be dealt on a similar line.

The standardised cost function (5.3.18) can be written as

$$R(N,n,c_1,c_2) = n + (N-n)\,[\lambda_{11}(1-P_a(p')) + \lambda_{12}\,P_a(p'') + \lambda_{21}\,P_r(p') + \lambda_{22}(1-P_r(p''))] \quad \ldots \ (5.3.19)$$

where

$$\begin{aligned} \lambda_{11} &= w_1\,[k_t(p') - k_a(p')]\,/\,(k_s-k_m) \\ \lambda_{12} &= w_2\,[k_a(p'') - k_t(p'')]\,/\,(k_s-k_m) \quad \ldots \ (5.3.20) \\ \lambda_{21} &= w_1\,[k_r(p') - k_t(p')]\,/\,(k_s-k_m) \\ \lambda_{22} &= w_2\,[k_t(p'') - k_r(p'')]\,/\,(k_s-k_m) \end{aligned}$$

under the inequalities assumed above, all the $\lambda_{ij} > 0$.

Thus the average decision loss per item is a linear combination of the probabilities $1-P_a(p')$, $P_a(p'')$, $P_r(p')$ and $1-P_r(p'')$.

For $\lambda_{ij} > 0$ we should have $p_u < p_v$ which we have assumed.

Under the three-point binomial prior distribution we shall assume that $p' < p_u < p'' < p_v < p'''$. The standardised cost function under a three-point binomial distribution can be written as

$$R(N,n,c_1,c_2) = n + (N-n)\,[\mu_{11}(1-P_a(p')) + \mu_{12}P_a(p'') + \mu_{13}P_a(p''') + \mu_{21}P_r(p') + \mu_{22}P_r(p'') + \mu_{23}(1-P_r(p'''))] \quad \ldots(5.3.21)$$

where

$$\begin{aligned}
\mu_{11} &= w_1\,[k_t(p') - k_a(p')]\,/\,(k_s-k_m) \\
\mu_{12} &= w_2\,[k_a(p'') - k_t(p'')]\,/\,(k_s-k_m) \\
\mu_{13} &= w_3\,[k_a(p''') - k_t(p''')]\,/\,(k_s-k_m) \\
\mu_{21} &= w_1\,[k_r(p') - k_t(p')]\,/\,(k_s-k_m) \\
\mu_{22} &= w_2\,[k_r(p'') - k_t(p'')]\,/\,(k_s-k_m) \qquad \ldots(5.3.22) \\
\mu_{23} &= w_3\,[k_t(p''') - k_r(p''')]\,/\,(k_s-k_m)
\end{aligned}$$

Under beta prior distribution with parameters π and t for the lot quality p, the standardised cost function is given by

$R(N,n,c_1,c_2)$

$$= n+\frac{(N-n)}{(k_s-k_m)}\Big[(A_2-T_2)\Big[\frac{p_u}{B(\pi,t)}\int_0^1 p^{\pi-1}q^{t-1}I_p(c_1+1,n-c_1)dp$$

$$-\frac{\pi}{(\pi+t)}\int_0^1 \frac{p^{\pi}q^{t}}{B(\pi+1,t)} I_p(c_1+1,n-c_1)dp+\frac{\pi}{(\pi+t)}-\frac{\pi}{(\pi+t)}I_{p_u}(\pi+1,t)$$

$$-p_u+p_u I_{p_u}(\pi,t)\Big] +(T_2-R_2)\Big[\frac{p_v}{B(\pi,t)}\int_0^1 p^{\pi-1}q^{t-1} I_p(c_2+1,n-c_2)dp$$

$$-\frac{\pi}{(\pi+t)}\int_0^1 \frac{p^{\pi}q^{t}}{B(\pi+1,t)} I_p(c_2+1,n-c_2)dp+\frac{\pi}{(\pi+t)}$$

$$-\frac{\pi}{(\pi+t)} I_{p_v}(\pi+1,t) - p_v + p_v I_{p_v}(\pi,t)\Big]\Big] \quad \ldots \quad (5.3.23)$$

$$\text{where } I_p(\pi,t) = [B(\pi,t)]^{-1}\int_0^p t^{\pi-1}(1-t)^{t-1}dt,\ 0 \le p \le 1 \quad \ldots \quad (5.3.24)$$

From amongst the large number of plans satisfying specified values of the lot percent defective (LTPD), consumer's risk (β = 0.10), lot size (N) and process average ($\bar{p}$), Dodge and Romig's LTPD systems of plans select the unique plan which minimises average amount of inspection at process average quality $\bar{p}$. Similar to the above plan we developed three-decision plan by attributes [Pandey(1972a)] in Chapter 2. In this Chapter the criterion of minimum total average cost per lot has been chosen to select the optimum plan.

As explained earlier, in most practical situations the quality of items being produced by a process may be assumed either to be fluctuating on two or three levels in the manner as described above or to be stable around an average level. Further, in some cases it may be desired to have sampling inspection plans which either have some specified probability of misclassification (consumer's risk) at some specified levels of quality or are free from any such restriction. As a continuous case mostly beta distribution has been proposed as prior in the literature. Motivated by the above considerations, we shall devote the subsequent sections to the following five areas of the single sampling three-decision plan:

(1) Double binomial restricted Bayes solution - Bayesian single sampling plan with double binomial as a prior

distribution and with specified values of the consumer's risk (β_1 and β_2). The risk R given by (5.3.19) is minimised under these conditions. These plans will be called restricted Bayesian three-decision single sampling plans by attributes with double binomial as the prior distribution and are discussed in section 5.4.

(2) Double binomial unrestricted Bayes solution - Bayesian single sampling plan with double binomial as prior distribution. Next, in section 5.5, plans (n,c_1,c_2) minimising R given by (5.3.19) and with no restriction in terms of consumer's risks, are developed. These plans will be referred to as unrestricted Bayesian single sampling three-decision plans by attributes.

(3) Three-point binomial unrestricted Bayes solution - Bayesian single sampling plan with three-point binomial as prior distribution. In section 5.6, plans (n,c_1,c_2) minimising R given by (5.3.21) and with no restriction in terms of consumer's risks, are developed.

(4) Point binomial Bayes solution - The extreme case where the double binomial reduces to a point binomial as a prior distribution (either $w_1 = 0$ or $w_2 = 0$) is discussed in section 5.7.

(5) Beta prior unrestricted Bayes solution - Bayesian single sampling with beta as a prior distribution. In section 5.8, plans (n, c_1, c_2) minimising R given by (5.3.23) are developed.

5.- Double binomial restricted Bayes solution

In this section we shall assume that the prior distribution is a double binomial with parameters p' and p'', $p' < p''$ and weights w_1 and w_2, $w_1 + w_2 = 1$ i.e., the process average has a two point distribution. The standardised average cost is given by (5.3.19). The problem is to find out (n, c_1, c_2) minimising (5.3.19) and satisfying (1.3.3) and (1.3.4), where β_1 and β_2 have specified values (Fig.6).

Let S be the set of plans (n, c_1, c_2) satisfying (1.3.3) and (1.3.4). For any plan (n, c_1, c_2) in S, if any one of n, c_1 and c_2 is fixed the other two are uniquely determined (Chapter 2 section 2.3). For example, if $p_1 = 0.05$; $p_2 = 0.10$; $\beta_1 = 0.07$ and $\beta_2 = 0.10$, then for $c_1=0$, say, $B(c_1; n, p_1) = \beta_1$ gives a unique $n = 52$ and $B(c_2; n, p_2) = \beta_2$ gives $c_2 = 2$ with $n = 52$ where the two conditions are satisfied as closely as possible. Thus a plan in S can be uniquely defined according to any one of n, c_1 and c_2. As before, we have chosen c_1 for this purpose.

Let $S(c_1)$ denote a plan (n, c_1, c_2). Since the choice of any one of the variables n, c_1, c_2 fixes the other two in the present case, we have decided to take c_1 as the independent variable of the triplet, the regret function $R(N, n, c_1, c_2)$ can be written for this case as $R_1(N, c_1)$. Using the notations of (3.3.1) - (3.3.3) we write $R_1(N, c_1)$ from (5.5.19) as

$$R_1(N,c_1) = n(c_1) + (N-n(c_1)) [\lambda_{11}(1 - B(c_1;\, n(c_1),\, p')) + \lambda_{12} B(c_1;\, n(c_1), p'') + \lambda_{21}(1-B(c_2(c_1);\, n(c_1), p')) + \lambda_{22} B(c_2(c_1);\, n(c_1),\, p'')] \quad \ldots \quad \ldots (5.4.1)$$

where $n(c_1)$ denotes the sample size in a plan (n, c_1, c_2) and the sample size $n(c_1)$ and second acceptance number c_2 are determined by the choice of first acceptance number c_1, as indicated earlier.

For a fixed N, the value of c_1 minimising $R_1(N, c_1)$ is determined from the inequality

$$\Delta R_1 (N,\, c_1-1) \leq 0 < \Delta R_1 (N,\, c_1) \quad \ldots (5.4.2)$$

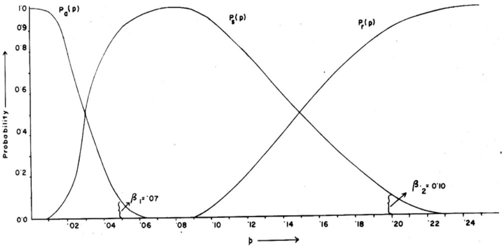


FIG.6 OPERATING CHARACTERISTIC CURVE FOR THE PLAN (n, c_1, c_2) WITH $n = 220$, $c_1 = 6$, $c_2 = 36$.

We have

$$\Delta R_1(N,c_1) = [1-\lambda_{11}-\lambda_{21}+\lambda_{11}\ B(c_1+1;\ n(c_1),\ p') -$$
$$\lambda_{12}\ B(c_1+1;\ n(c_1),\ p'') +$$
$$\lambda_{21}\ B(c_2(c_1+1);\ n(c_1+1),\ p') -$$
$$\lambda_{22}\ B(c_2(c_1+1);\ n(c_1+1),\ p'')]\ \Delta n(c_1) -$$
$$(N-n(c_1))\ [\lambda_{11}\ \Delta B(c_1;\ n(c_1),\ p') -$$
$$\lambda_{12}\ \Delta B(c_1;\ n(c_1),\ p'')+\lambda_{21}\ \Delta B(c_2(c_1);n(c_1),p') -$$
$$\lambda_{22}\ \Delta B(c_2(c_1);\ n(c_1),\ p'')] \qquad \ldots \quad (5.4.3)$$

To obtain the bounds for the lot size for which the plan (n,c_1,c_2) satisfying (5.4.2) is the optimal plan we shall define the following auxiliary function from (5.4.3):

$$N_{c_1} = n(c_1)+ [\frac{1}{\lambda_{22}} - \frac{\lambda_{11}}{\lambda_{22}} - \frac{\lambda_{21}}{\lambda_{22}} + \frac{\lambda_{11}}{\lambda_{22}}\ B(c_1+1;\ n(c_1+1),\ p') -$$
$$\frac{\lambda_{12}}{\lambda_{22}}\ B(c_1+1;\ n(c_1+1),p'')+\frac{\lambda_{21}}{\lambda_{22}}\ B(c_2(c_1+1);\ n(c_1+1),p') -$$
$$B\ (c_2(c_1+1);\ n(c_1+1),\ p'')]\ \frac{\Delta n(c_1)}{U(c_1)} \qquad \ldots \quad (5.4.4)$$

where

$$U(c_1) = [\frac{\lambda_{11}}{\lambda_{22}} \Delta B(c_1; n(c_1), p') - \frac{\lambda_{12}}{\lambda_{22}} \Delta B(c_1; n(c_1), p'')$$

$$+ \frac{\lambda_{21}}{\lambda_{22}} \Delta B(c_2(c_1); n(c_1), p') - \Delta B(c_2(c_1); n(c_1), p'')]$$

Clearly $\Delta R_1(N, c_1) = \lambda_{22} U(c_1) (N_{c_1} - N)$ and

$$\Delta R_1(N, c_1-1) = \lambda_{22} U(c_1-1)(N_{c_1-1} - N) \text{ where } \lambda_{22} \geq 0.$$

The function $U(c_1)$ is related to the function $G(c_1)$, which will be defined in (5.4.11) and used subsequently, as follows:

$$\lambda_{22} U(c_1) = - \Delta G(c_1)$$

Although it has not been possible to study the monocity of $G(c_1)$ analytically, extensive computations show that it is a monotonically decreasing function of c_1 (Fig.7) and, hence, that $U(c_1) > 0$ for all values of c_1.

Therefore, it follows from (5.4.2) that the plan (n, c_1, c_2) is optimal for the lot size N if

$$N_{c_1-1} \leq N < N_{c_1} \qquad \ldots \quad (5.4.5)$$

It is clear from (5.3.19) that for fixed c_1 and the λ_{ij}'s $R_1(N, c_1)$ is always an increasing linear function of N.

Consider two plans - plan 1: (n_1, c_1', c_2') and plan 2: (n_2, c_1'', c_2'') and let (N_1, N_1') be the range of values of N where plan 1 is optimal and (N_2, N_2') be the range of values where plan 2 is optimal, according to (5.4.5). For plan 1, $R_1(N)$ increases when N rises from N_1 to N_1' and for plan 2 it increases when N increases from N_2 to N_2'. Let $N_1 < N_2 < N_1' < N_2'$; then (N_2, N_1') is the range of overlap in N. We shall now examine the question as to which of the two plans - plan 1 or plan 2 - should be preferred in (N_2, N_1').

Since $N_1 < N_2 < N_1' < N_2'$ and it is given that

$$\min\left\{R_1(N,n_1,c_1',c_2'), R(N,n_2,c_1'',c_2'')\right\} = \begin{cases} R_1(N,n_1,c_1',c_2') & \text{for } N \in M_1 \\ R_1(N,n_2,c_1'',c_2'') & \text{for } N \in M_2 \end{cases}$$

where $M_1 = \left\{N;\ N_1 \leq N \leq N_1'\right\}$ and $M_2 = \left\{N;\ N_2 \leq N \leq N_2'\right\}$, and further $R_1(N)$ is increasing linear function of N, the $R_1(N)$ functions for plan 1 and plan 2 must intersect, as explained later in section 5.9. , at some point in (N_2, N_1'), the range of overlap (see Fig.8). At the point of intersection in (N_2, N_1')

the values of $R_1(N)$ for the two plans must be equal i.e.,

$$R_1(N,n_1,c_1',c_2') = R_1(N,n_2,c_1'',c_2'') \qquad \dots \quad (5.4.6)$$

which gives

$$N_{12} = \frac{(n_2-n_1)(1-\lambda_{11})+n_2\left[\delta_1(n_2,c_1'')-\lambda_{21}+\delta_2(n_2,c_2'')\right]-n_1\left[\delta_1(n_1,c_1')-\lambda_{21}+\delta_2(n_1,c_2')\right]}{\left[\delta_1(n_2,c_1'') + \delta_2(n_2,c_2'') - \delta_1(n_1,c_1') - \delta_2(n_1,c_2')\right]}$$

$$\dots \quad (5.4.7)$$

where $\delta_i(n,c_j) = \lambda_{i1}B(c_j;n,p') - \lambda_{i2}B(c_j;n,p'')$, $i = 1,2$ ... (5.4.8)

Thus

$$R_1(N,n_1,c_1',c_2') \lesseqgtr R_1(N,n_2,c_1'',c_2'') \text{ according as } N \lesseqgtr N_{12} \quad \dots \quad (5.4.9)$$

and hence in (N_2,N_{12}) we should prefer plan 1 to plan 2 and in (N_{12},N_1') we should prefer plan 2 to plan 1.

Let the regret function $R_1(N,c_1)$ be written as follows:

$$R_1(N,c_1) = \begin{cases} N; & 0 \leq N \leq n(c_1) \\ n(c_1) + (N-n(c_1))\,G(c_1); & n(c_1) < N \end{cases} \qquad \dots \quad (5.4.10)$$

where $G(c_1)$ denotes the expected decision loss (standardised) and is defined as follows:

$$G(c_1) = \lambda_{11}(1-B(c_1;\, n(c_1),\, p'))+ \lambda_{12}B(c_1; n(c_1),\, p'') +$$

$$\lambda_{21}(1-B(c_2(c_1);n(c_1),p'))+ \lambda_{22}B(c_2(c_1);n(c_1),p'') \quad \ldots \quad (5.4.11)$$

As stated earlier, we have not been able to study analytically the nature of $G(c_1)$ as function of c_1. The results of extensive numerical computations, as illustrated by Fig.7, show that $G(c_1)$ is a non-increasing function of c_1 with falling rate of decrease i.e., $\Delta G(c_1) < 0$ and $\Delta^2 G(c_1) > 0$, and further $G(c_1) < 1$ for all the values of c_1.

Consider the function

$$R_1^o(N,c_1) = \underset{S(c_1)\,\varepsilon\, S}{\text{Inf.}}\ R_1(N,c_1)$$

For any c_1 the plan $S(c_1)\ \varepsilon\ S$ is optimal for lot range $N_{c_1-1} \leq N < N_{c_1}$. The function $R_1^o(N,c_1)$ is a concave function of N according to (5.4.10).

Writing $N_{c_1} = n(c_1) + \dfrac{1-G(c_1+1)}{-\ \Delta G(c_1)}\ \Delta n(c_1)$ we note that

$N_{c_1} > n(c_1)$. N_{c_1} is an increasing function of $n(c_1)$. Hence, as mentioned earlier, for increasing values of c_1 of various optimal plans in S, the corresponding lot size ranges would be moving to the right, possibly overlapping according to (5.4.9). For discussion on uniqueness of the solution of this section later, we shall simplify the notation by denoting c_1 as c and redefine the point of intersection N(c, c+1) of $R_1(N,c)$ and $R_1(N,c+1)$ as

$$N(c,c+1) = \frac{n(c+1)(1-G(c+1)) - n(c)(1-G(c))}{-\Delta G(c)} \quad .. (5.4.12)$$

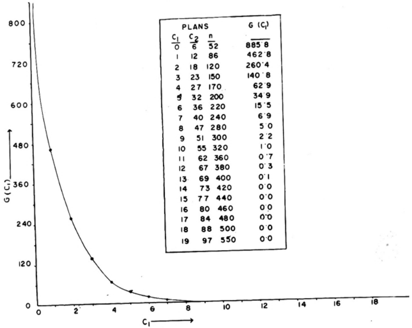
..1 Numerical computation of optimal plans

The optimal plans can be systematically tabulated as follows:

Step 1 : Take some arbitrary value of c_1 and obtain a plan, say $S(c_1) \epsilon S$, by using (1.3.3) and (1.3.4).

Step 2 : For the plan $S(c_1)$ so obtained, compute the value of N_{c_1} and N_{c_1-1} using (5.4.4).

Step 3 : Choose $c_1 = 0,1,2,3,\ldots$ systematically and proceed as in steps 1-2 and tabulate the sampling plans and the corresponding bounds for the lot sizes.



PLANS c_1	c_2	n	$G(c_1)$
0	6	52	885·8
1	12	86	462·8
2	18	120	260·4
3	23	150	140·8
4	27	170	62·9
5	32	200	34·9
6	36	220	15·5
7	40	240	6·9
8	47	280	5·0
9	51	300	2·2
10	55	320	1·0
11	62	360	0·7
12	67	380	0·3
13	69	400	0·1
14	73	420	0·0
15	77	440	0·0
16	80	460	0·0
17	84	480	0·0
18	88	500	0·0
19	97	550	0·0

FIG.7 EXPECTED DECISION LOSS (STANDARDISED) AS A FUNCTION OF c_1 FOR BAYESIAN PLAN WITH DOUBLE BINOMIAL AS A PRIOR DISTRIBUTION $p' = 0.01$, $p'' = 0.15$ AND $w_1 = 0.93$, $w_2 = 1-w_2$ ($G(c_1)$ IN THE UNITS OF 10^{-3}).

Step 4 : For two plans with overlapping N-intervals use (5.4.7) and (5.4.9) to select the optimal plans.

Thus from the tabulated plans an optimal plan for a given lot size can be obtained against the lot range in which the given lot size falls. For illustration let us consider the following example:

Example 1 : To obtain a single sampling three-decision plan by attributes minimising the total average cost per lot when the following information is given :

The four cost functions are $k_s(p) = 23 + 35\,p$; $k_a(p) = 720\,p$; $k_t(p) = 10 + 85\,p$; $k_r(p) = 20$; the coefficients denote costs per item in money units i.e., the cost of sampling and testing is 23 money units per item in the sample, cost of accepting a defective item is 720 money units and the cost involved in rejecting an item outright is 20 money units etc. (see section 5.3).

Let us further assume that lots are generated with probability $w_1 = 0.93$ from a binomially controlled process with $p' = 0.01$ and with probability $w_2 = 0.07$ from the process with $p'' = 0.15$. Also, suppose it is given that the probability of misclassification as a superior grade lot is 0.07 for a lot containing 5% defective items and it is 0.10 for a lot containing 20% defective items.

The break even qualities work out as $p_u = 0.0158$ and $p_v = 0.1176$. From (5.3.9) and (5.3.12) $k_s = 23.693$; $k_m = 8.096$; $k_s - k_m = 15.597$. Substituting the relevant values in the expression of λ_{ij} as defined in (5.3.20) we obtain $\lambda_{11} = 0.217638$; $\lambda_{12} = 0.382606$; $\lambda_{21} = 0.545586$ and $\lambda_{22} = 0.012342$. To indicate the computational procedure let us choose $c_1 = 1$ in the step 1. From (1.3.3) and (1.3.4) we get $c_2 = 12$ and $n = 86$. To compute the bounds for the lot sizes which may be specified and for which (86,1,12) is the optimal three-decision plan, we shall use (5.4.4).

The necessary quantities to compute N_{c_1} for $c_1 = 1$ are: $\Delta B(c_1;\, n(c_1),\, p') = 0.093020$; $\Delta B(c_1; n(c_1), p'') = -0.000010$; $\Delta B(c_2(c_1);\, n(c_1), p') = 0$; $\Delta B(c_2(c_1); n(c_1), p'') = 0.096490$; $U = 1.544128$; $B(c_1+1;\, n(c_1+1),\, p') = 0.880360$; $B(c_2(c_1+1); n(c_1+1), p') = 1$; $B(c_1+1; n(c_1+1), p'') = 0$; $B(c_2(c_1+1); n(c_1+1), p'') = 0.562450$.

Substituting these values in (5.4.7) we get

$$N_{c_1} = 86 + (78.351966) \times 34 / 1.544128 = 1811.224103$$

Similarly $N_{c_1-1} = 847$.

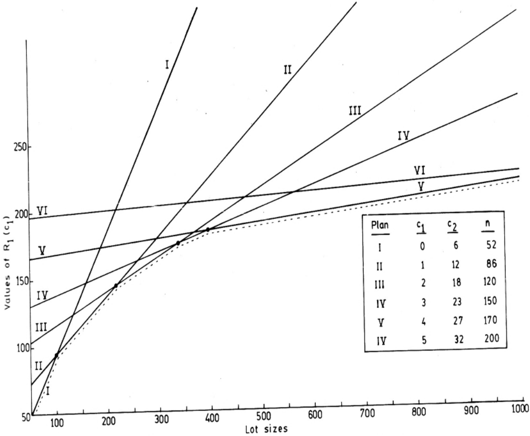


Fig.8: THE VALUES OF R_1 (c_1) FOR VARYING LOT SIZES

Thus for any given lot size N in the range $847 \leq N \leq 1811$ the optimal plan is given by (86, 1, 12). Proceeding further we obtain a set of three-decision plans for $c_1=0,1,2,3$ etc. with their corresponding lot ranges.

Next, consider the plans (52,0,6) and (86,1,12) under the step 4 for their possible overlap. From (5.4.10) we get for the overlap

$$n(0)(1-G(0)) + NG(0) = n(1)(1-G(1)) + NG(1)$$

where $n(0) = 52$; $G(0) = 0.8858$; $n(1) = 86$; $G(1) = 0.4628$

and we obtain

$N = N_{12} = 95.1791$ which can be also obtained from (5.4.7). Thus, according to (5.4.9) the plan (52, 0, 6) is optimal for $52 \leq N \leq 95$. Similarly, we find that the plan (86, 1, 12) is optimal for $96 \leq N \leq 210$ and so on (Fig.8)

The optimal single sampling restricted ASR Bayesian plans for double binomial prior with $p' = 0.01$; $p'' = 0.15$; $w_1 = 0.93$; $w_2 = 0.07$ and $p_1 = 0.05$; $p_2 = 0.20$; $\beta_1 = 0.07$; $\beta_2 = 0.10$ and the cost parameters same as in the example are obtained for $c_1 = 0,1,2,3,4$ and 5 and are given in the table 9. (Fig.8)

To compute the average total cost per item we have taken $\bar{N}$ as average lot size where $\bar{N}$ is the geometric mean of the two limits of lot size range corresponding to a sampling plan in the table 9. For the subsequent discussions also in the Chapter we shall use the geometric mean $\bar{N}$ in this manner. Average cost of acceptance without inspection per item is 14.26 money units. The percentage saving due to acceptance sampling plan increases with the lot size.

5.5 Double binomial unrestricted Bayes solution

To determine (n, c_1, c_2) minimising $R(N, n, c_1, c_2)$ given by (5.3.19) is the problem of unrestricted Bayesian solution. The plans are unrestricted in the sense that they are not required to satisfy any restriction on their operating characteristic function i.e., there is no limitation in terms of risks. The plans providing minimum R are to be used if the minimum R is less than the costs of accepting or rejecting all lots without inspection. As mentioned earlier we assume $\lambda_{11} > 0$, $\lambda_{12} > 0$, $\lambda_{21} > 0$ and $\lambda_{22} > 0$ i.e., $p' < p_u < p_v < p''$.

The value of (n, c_1, c_2) minimising R must satisfy the following three conditions:

$$\Delta_{c_1} R(N,n,c_1-1,c_2) \leq 0 < \Delta_{c_1} R(N,n,c_1,c_2), \; 0 \leq c_1, \; c_2 \leq n, \; c_1 \leq c_2 \qquad \ldots\ldots (5.5.1)$$

$$\Delta_{c_2} R(N,n,c_1,c_2-1) \leq 0 < \Delta_{c_2} R(N,n,c_1,c_2), \; 0 \leq c_1, \; c_2 \leq n, \; c_1 \leq c_2 \qquad \ldots\ldots (5.5.2)$$

$$\Delta_n R(N,n-1,c_1,c_2) \leq 0 < \Delta_n R(N,n,c_1,c_2), \; c_1 \leq c_2 \leq n \leq N \qquad \ldots\ldots (5.5.3)$$

Since $\Delta_{c_1} B(c_1;n,p') = -p'b(c_1;n,p')$ we get from (5.3.19)

$$\Delta_{c_1} R(N,n,c_1,c_2) = (N-n)\,[-\lambda_{11}b(c_1+1;n,p') + \lambda_{12}b(c_1+1;n,p'')] \qquad \ldots\ldots (5.5.4)$$

$$\Delta_{c_2} R(N,n,c_1,c_2) = (N-n)\,[-\lambda_{21}b(c_2+1;n,p') + \lambda_{22}b(c_2+1;n,p'')] \qquad \ldots\ldots (5.5.5)$$

$$\begin{aligned}\Delta_n R(N,n,c_1,c_2) = 1 - [&\lambda_{11}(1-B(c_1;n,p')) + \lambda_{12}B(c_1;n,p'') + \\ &\lambda_{21}(1-B(c_2;n,p')) + \lambda_{22}B(c_2;n,p'')] + \\ &(N-n-1)\,[\lambda_{11}p'b(c_1;n,p') - \lambda_{12}p''b(c_1;n,p'') + \\ &\lambda_{21}p'b(c_2;n,p') - \lambda_{22}p''b(c_2;n,p'')] \quad \ldots (5.5.6)\end{aligned}$$

Solving the inequalities with respect to n and N we find that a Bayesian three-decision plan must satisfy the following three inequalities given by (5.5.7), (5.5.8) and (5.5.11)

$$\alpha_1 + \beta c_1 \leq n < \alpha_1 + \beta\,(c_1+1) \quad \ldots \quad (5.5.7)$$

$$\alpha_2 + \beta c_2 \leq n < \alpha_2 + \beta\,(c_2+1) \quad \ldots \quad (5.5.8)$$

where $c_1 \leq c_2$

$$\alpha_i = \log \frac{\lambda_{i2}}{\lambda_{i1}} \Big/ \log \frac{q'}{q''}, \quad i = 1, 2 \quad \ldots \quad (5.5.9)$$

$$\beta = \log \frac{p'' q'}{p' q''} \Big/ \log \frac{q'}{q''} \quad \ldots \quad \ldots \quad (5.5.10)$$

and $F(n-1,c_1,c_2) \leq N < F(n,c_1,c_2)$ ... ... (5.5.11)

where $F(n,c_1,c_2)$ is defined by

$$F(n,c_1,c_2) = (n+1) + \frac{-\lambda_{11}+\lambda_{11}B(c_1;n,p')-\lambda_{12}B(c_1;n,p'')-\lambda_{21}+\lambda_{21}B(c_2;n,p')-\lambda_{22}B(c_2;n,p'')}{-\lambda_{11}p'b(c_1;n,p')+\lambda_{12}p''b(c_1;n,p'')-\lambda_{21}p'b(c_2;n,p')+\lambda_{22}p''b(c_2;n,p'')}$$

... (5.5.12)

For two plans (n_1,c_1',c_2') and (n_2,c_1'',c_2'') satisfying (5.5.7) and (5.5.8) and having overlapping N-intervals according to (5.5.12),

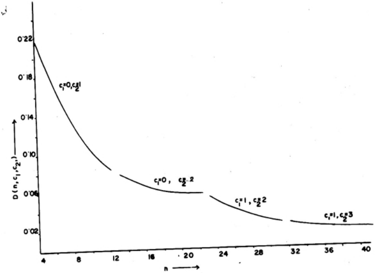


FIG. 9 THE EXPECTED DECISION LOSS (STANDARDISED) AS FUNCTION OF THE SAMPLE SIZE FOR "DOUBLE BINOMIAL" PRIOR DISTRIBUTION WITH $p' = 0.01$, $p'' = 0.15$ AND $w_1 = 0.93$, $w_2 = 1-w_1$.

we solve the equation $R(N,n_1,c_1',c_2') = R(N,n_2,c_1'',c_2'')$ for N to get the expression for N_{12} as given in (5.4.7). Then using (5.4.9) we can select one of the two plans (n_1,c_1',c_2') and (n_2,c_1'',c_2'') as an optimal plan

In this case the three-decision plan (n,c_1,c_2) is not required to satisfy the conditions (1.3.3) and (1.3.4).

Write $R(N,n,c_1,c_2)$ as

$$R(N,n,c_1,c_2) = \begin{cases} N, & 0 \leq N \leq n \\ n(1-D(n,c_1,c_2))+D(n,c_1,c_2)N, & n < N \end{cases} \quad ..(5.5.13)$$

where $D(n,c_1,c_2)$ denotes the expected standardised decision loss and is given by

$$D(n,c_1,c_2) = \lambda_{11}(1-B(c_1;n,p'))+\lambda_{12}B(c_1,n,p'') + \lambda_{21}(1-B(c_2;n,p'))+\lambda_{22}B(c_2;n,p'') \quad ... (5.5.14)$$

The graph of $R(N,n,c_1,c_2)$ as given by (5.5.13) is, for a fixed (n,c_1,c_2), a polygonal line consisting of two segments N and $n + (N-n)\, D(n,c_1,c_2)$.

Consider

$$R^{o}(N) = \inf_{\Omega = [n,c_1,c_2]} R(N,n,c_1,c_2) \quad ... (5.5.15)$$

where Ω denotes the set of plans $\{(n,c_1,c_2);\ 0 \leq c_1 \leq c_2 \leq n.\}$ For $0 \leq N \leq n$, the minimum in (5.5.13) is N and the optimal procedure, then, is as good as inspecting all the items i.e., $n = N$. We shall assume that $n < N$ and then, $R(N,n,c_1,c_2)$ is a concave piecewise linear function of the lot size N.

From (5.5.7) and (5.5.8) we note that for a given n we have a pair of (c_1,c_2) i.e. c_1 and c_2 depend on n. But the pair (c_1,c_2) remains same for a set of values of n satisfying (5.5.7) and (5.5.8). In fact the value of $D(n,c_1,c_2)$ changes for changing triplet (n,c_1,c_2) and hence it is difficult to study the nature of $D(n,c_1,c_2)$ analytically and also empirically for all the range of values of the triplet. Figure 9 indicates the nature of $D(n,c_1,c_2)$ for a few fixed pairs (c_1,c_2) and corresponding changing values of n.

5.5.1 Numerical computation of optimal three-decision and comparison with two decision plans

The plans discussed in 5.5 above can be tabulated as follows:

Step 1 : Compute $\lambda_{ij}, i,j = 1,2$ from the expressions given in (5.3.20) and hence compute α_1, α_2 from (5.5.9) and β from (5.5.10).

Step 2 : Choose some arbitrary c_1 (say $c_1 = 0$) and, using the value of α_1, α_2 and β as obtained in step 1, compute the lower limit $l(c_1)$ and the upper limit $u(c_1)$ for n from (5.5.7) such that $l(c_1) \leq n \leq u(c_1)$.

Step 3 : Take c_2 as c_1+1 and compute the lower limit $l(c_2)$ and the upper limit $u(c_2)$ for n from (5.5.8) such that $l(c_2) \leq n \leq u(c_2)$.

Step 4 : In case $l(c_2) \leq u(c_1)$ choose c_2 as c_1+1 and the values of n satisfying both (5.5.7) and (5.5.8) would be in the closed interval $l(c_1,c_2) \leq n \leq u(c_1,c_2)$ where

$$l(c_1,c_2) = \max\,[\,l(c_1),\ l(c_2)]$$

and $$u(c_1,c_2) = \min\,[\,u(c_1),\ u(c_2)]$$

Increase c_2 systematically till we have $l(c_2) \leq u(c_1)$. For $l(c_2) > u(c_1)$, increase c_1 systematically till we reach $l(c_2) \leq u(c_1)$.

Step 5 : Choose $c_1 = 0,1,2, \ldots\ldots$ systematically and proceed as in the steps 2-4 and list the value of n satisfying (5.5.7) and (5.5.8) corresponding to each pair of c_1 and c_2.

Step 6 : For the plans listed in the step 5, using (5.5.11) compute the bounds (N-intervals) of the lot sizes for which these plans satisfy the local optimality conditions (5.5.7),(5.5.8) and (5.5.11).

Step 7 : For two plans (n_1,c_1',c_2') and (n_2,c_1'',c_2'') having overlapping N-intervals select one of these plans as an optimal plan by using (5.4.7) and (5.4.9).

Example 2

Using the costs as given in the example 1 of section (5.4.2) and assuming that the lots are generated with probability $w_1 = 0.93$ from a binomially controlled process with $p' = 0.01$ and with probability $w_2 = 0.07$ from the process with $p'' = 0.15$, the steps 1-7 gave the values of c_1,c_2,n and N as in the table 10 where $\alpha_1 = +3.696568$, $\alpha_2 = -24.850612$ and $\beta = 18.761415$. The values of δ_1 and δ_2 as defined in (5.4.8) are also provided in the table 10 corresponding to each combination of c_1,c_2,n and the N-interval.

Using the values of δ_1 and δ_2 as given in the table 10 the points of intersection according to (5.4.7) are computed and are given in the table 11.

Combining the results of the tables 10 and 11 and using the step 7, we obtain the system of optimum plans given in the table 12.

Let $\bar{N}$ denote the geometric mean of the two limits of a lot size range in the table 12. The values of the cost per item using lot size as $\bar{N}$ have been computed and are given in the table 12.

The percentage saving effected by the use of the three-decision plan as compared with the average cost involved in accepting an item without inspection is found to be increasing for the increasing lot sizes. It can be seen from the table 12 that the above percentage saving in cost is at least 40 percent i.e., for the lot size of 2115. The computational results given in the table 12 show that the optimum sample size increases with the lot size and the optimal decision loss per non-sampled item decreases with the sample size (Fig.9).

The probability of acceptance and probability of screening are computed for the incoming lot quality $p' = 0.01$ and the probability of rejection and probability of screening are computed for the lot quality $p'' = 0.15$ for each of the optimal plans in the table 12. The probability of acceptance at p' increases with the optimal sample size whereas the probability of screening decreases with the increasing lot sizes which may turn out to be a desirable economic feature.

The probability of rejecting a lot of quality $p'' = 0.15$ out right, increases with the increasing optimal sample size whereas the probability of screening a lot of this quality increases in the beginning (upto $\bar{N} = 141$) and then decreases for higher lot sizes.

The close values of $K/\bar{N}$ in the table 12 tempt one to work out a system of nearly optimum plans containing only one plan for each pair of decision numbers c_1 and c_2. Such plans are worked out and are provided in the table 13.

The procedure of simplification used to obtain the nearly optimal plans in the table 13 is to compute n as $[l(c_1,c_2) + u(c_1,c_2)] / 2$ where $l(c_1,c_2)$ and $u(c_1,c_2)$ are the values as obtained in the step 4 such that the closed interval $l(c_1,c_2) \leq n \leq u(c_1,c_2)$ gives the possible optimal values of sample sizes corresponding to a particular pair c_1 and c_2. The N-intervals are computed by using (5.4.7).

A comparison between the table 12 and table 13 shows that (i) the system in table 13 provides considerable simplification of the optimal system and, (ii) nearly optimum plans are just as satisfactory as the optimum plans for the most practical purposes.

It may be of some interest to examine economic advantages of a three-decision Bayesian plan over two-decision Bayesian plan empirically. For this purpose, in the example 3 we shall consider the situation similar to one in example 1 but with a two-decision, say, acceptance - screening, plan.

We shall work out the average total cost of inspection and decision per item for such a plan and will attempt a comparison between these values and the corresponding values for a three-decision plan as obtained in the table 12.

Example 3

Let us assume that the prior distribution of the lot quality and the three cost functions viz., cost of inspection, cost of acceptance and the cost of screening, be the same as in example 1 and the following sampling procedure be used:

Take a sample of size n from a lot of N items and let x denote the number of defectives observed in n then:

$$\begin{aligned} &\text{accept the lot if } x \leq c \\ &\text{screen the lot if } x > c \end{aligned} \qquad \ldots (5.5.23)$$

The cost function K(N,n,c,p) can be written as

$$K(N,n,c,p) = n(S_1+S_2p) + (N-n)\ [(A_1+A_2p)P_a(p)+(T_1+T_2p)P_s(p)] \qquad \ldots \quad (5.5.24)$$

where the symbols have the usual meaning and $P_a(p) = B(c;n,p)$ and $P_s(p) = 1-P_a(p)$ denote the probability of acceptance and the probability of screening respectively.

The unavoidable minimum cost $k_m(p)$ would be defined as

$$k_m(p) = k_a(p')w_1 + k_t(p'')w_2 \qquad \ldots \quad (5.5.25)$$

and the standardised cost function as

$$R(N,n,c) = n + (N-n)\ [\lambda_1(1-P_a(p')) + \lambda_2\ P_a(p'')] \quad \ldots (5.5.26)$$

where

$$\lambda_1 = w_1\ [k_t(p') - k_a(p')]\ /(k_s-k_m)$$

$$\lambda_2 = w_2\ [k_a(p'') - k_t(p'')]\ /(k_s-k_m)$$

It is required to obtain (n,c) which minimises (5.5.26). The necessary expressions to determine such two-decision optimal plans are :

$$\alpha + \beta c \leq n < \alpha + \beta(c+1) \qquad \ldots \quad (5.5.27)$$

where

$$\alpha = \log \frac{\lambda_2}{\lambda_1} \Big/ \log \frac{q'}{q''}$$

$$\beta = \log \frac{p'' q'}{p' q''} \Big/ \log \frac{q'}{q''}$$

and $$F(n-1,c) \leq N < F(n,c) \qquad \ldots \quad \ldots (5.5.28)$$

where $F(n,c)$ is defined by

$$F(n,c) = n + 1 + \frac{1-\lambda_1+\lambda_1 P_a(p')-\lambda_2 P_a(p'')}{-\lambda_1 p' b(c;n,p') + \lambda_2 p'' b(c;n,p'')}$$

The points of intersection for the two plans (n',c') and (n'',c'') having overlapping N-intervals is given by

$$N_{12} = \frac{(n''-n')(1-\lambda_1) + n''\delta(n'',c'') - n'\delta(n',c')}{\delta(n'',c'') - \delta(n',c')} \quad \ldots (5.5.29)$$

where $\delta(n,c) = \lambda_1 B(c;n,p') - \lambda_2 B(c;n,p'')$.

For this example, $\lambda_1 = 0.217638$, $\lambda_2 = 0.382606$, $\alpha = 3.696567$ and $\beta = 18.761415$. Using the expressions (5.5.27)-(5.5.29) and the values in the tables 14 and 15 and proceeding on the lines similar to the steps 1-7 of this section (i.e., after suitable modification of the steps for two-decision plan), we obtain the optimal plans as given in the table 16.

The Figure 10 shows the average total cost of inspection plus the decision cost per item using the corresponding values of $\bar{N}$ from tables, 12 (three-decision plans) and 16 (two-decision plans) and taking $p' = 0.01$; $p'' = 0.15$; $w_1 = 0.93$ and $w_2 = 0.07$ as the double binomial prior distribution. The Figure 10 clearly shows that the three-decision plan is more economical than the two-decision plan.

The advantages of the three-decision plan as compared with the two-decision plan are more pronounced for higher values of w_2 i.e., when considerable number of lots with deteriorated or very bad quality levels are submitted for inspection, the three-decision plan will have definitely substantial reduction in the values of $K/\bar{N}$ as compared with two-decision plan. To illustrate it numerically in example 4, we shall consider the three-decision plans and two-decision plans for the same set of cost function as in the example 2 and 3 but with higher value of w_2. Let us take $p' = 0.01$; $p'' = 0.15$; $w_1 = 0.10$ and $w_2 = 0.90$ as a double binomial prior distribution.

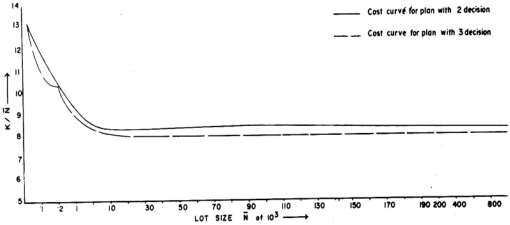


FIG. 10 COST CURVES FOR PLANS WITH TWO AND THREE DECISION CRITERIA $w_1 = 0.93$, $w_2 = 0.07$.

Example 4

The necessary quantities to obtain the three-decision and two-decision (accept-screen type) plans for the double binomial prior distribution mentioned above and for the cost functions $k_s(p)$, $k_a(p)$, $k_t(p)$ and $k_r(p)$ same as in the example 2 and 3 are :

$$k_s = 27.76; \quad k_m = 18.72$$

$\lambda_{11} = 0.403762$; $\lambda_{12} = 8.487278$; $\lambda_{21} = 0.101217$;

$\lambda_{22} = 0.273783$; $\alpha_1 = 19.974527$; $\alpha_2 = 6.526389$;

$\beta = 18.761415$ and $\alpha = \alpha_1$; $\lambda_1 = \lambda_{11}$; $\lambda_2 = \lambda_{12}$

Using the above values of λ_{ij}'s, α_1, α_2 and β in (5.5.7), (5.5.8) and (5.4.7) we obtain optimum three-decision plans through the tables 17,18 and 19. Similarly, using the above values of α, β, λ_1 and λ_2 in (5.5.27), (5.5.28) and (5.5.29) we obtain the optimum two-decision acceptance - screening plan through the tables 20,21 and 22. The Figure 11 shows the average total cost of inspection plus the decision cost per item against the average lot sizes ($\bar{N}$) for the both three-decision and two-decision optimal plans (tables 19 and 22).

It is clear from the Figure 11 that the three-decision plan is now, much economical than the two-decision plan.

5.6 Three-point binomial unrestricted Bayes solution

Determination of (n,c_1,c_2) minimising $R(N,n,c_1,c_2)$ given by (5.3.21) is the problem considered in this section. We shall assume that $\mu_{ij} > 0$, $i,j = 1,2,3$ i.e., $p' < p_u < p'' < p_v < p'''$

The value of (n,c_1,c_2) minimising R given by (5.3.21) must statisfy the conditions (5.5.1), (5.5.2) and (5.5.3) where

$$\Delta_{c_1}R(N,n,c_1,c_2) = (N-n)\,[-\mu_{11}b(c_1+1;n,p') + \mu_{12}b(c_1+1;n,p'')+ \mu_{13}b(c_1+1;\,n,\,p''')] \quad \ldots (5.6.1)$$

$$\Delta_{c_2}R(N,n,c_1,c_2) = (N-n)\,[-\mu_{21}b(c_2+1;n,p') - \mu_{22}b(c_2+1;n,p'') + \mu_{23}b(c_2+1;\,n,\,p''')] \quad \ldots (5.6.2)$$

$$\begin{aligned}\Delta_n R(N,n,c_1,c_2) = 1- &[\mu_{11}(1-B(c_1;n,p'))+ \mu_{12}B(c_1;n,p'') + \\ &\mu_{13}B(c_1;n,p'')+ \mu_{21}(1-B(c_2;n,p')) + \\ &\mu_{22}(1-B(c_2;n,p'')) + \mu_{23}B(c_2;n,p''')] + \\ &(N-n-1)\,[\mu_{11}p'b(c_1;n,p')-\mu_{12}p''b(c_1;n,p'')- \\ &\mu_{13}p'''b(c_1;n,p''') + \mu_{21}p'b(c_2;n,p') + \\ &\mu_{22}p''b(c_2;n,p'')-\mu_{23}p'''b(c_2;n,p''')] \quad \ldots (5.6.3)\end{aligned}$$

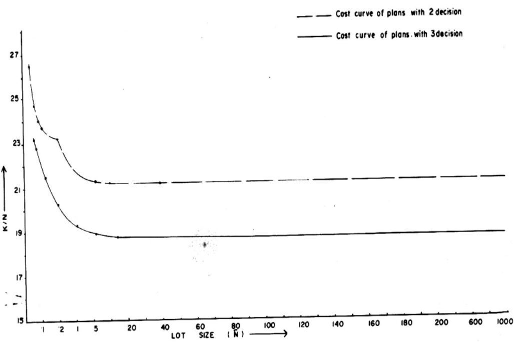


FIG. 11 COST CURVES FOR PLANS WITH TWO AND THREE DECISION CRITERIA FOR $w_1 = 0.10$, $w_2 = 0.90$.

Solving the inequalities (5.5.1), (5.5.2) and (5.5.3) in the present context, with respect to n and N by using (5.6.1)-(5.6.3) we find that a Bayesian three-decision plan (n, c_1, c_2) must satisfy the following inequalities given by (5.6.4), (5.6.5) and (5.6.6)

$$f_1\,(c_1, n) \leq 0 < f_1\,(c_1+1, n) \quad \ldots \quad (5.6.4)$$

$$f_2\,(c_2, n) \leq 0 < f_2\,(c_2+1, n) \quad \ldots \quad (5.6.5)$$

where

$$f_1(c_1, n) = -\mu_{11} b(c_1; n, p') + \mu_{12} b(c_1; n, p'') + \mu_{13} b(c_1; n, p''') \text{ and}$$

$$f_2(c_2, n) = -\mu_{21} b(c_2; n, p') - \mu_{22} b(c_2; n, p'') + \mu_{23} b(c_2; n, p''')$$

$$F(n-1), c_1, c_2) \leq N < F(n, c_1, c_2) \quad \ldots \quad (5.6.6)$$

where

$$\begin{aligned} F(n, c_1, c_2) = n+1+ \; & [1-\mu_{11}+\mu_{11}B(c_1; n, p')-\mu_{12}B(c_1; n, p'') \\ & - \mu_{13}B(c_1; n, p''')-\mu_{21}+\mu_{21}B(c_2; n, p') - \mu_{22} \\ & + \mu_{22}B(c_2; n, p'')-\mu_{23}B(c_2, n, p''')] / [-\mu_{11}p' b(c_1; n, p') \\ & + \mu_{12}p'' b(c_1; n, p'')+\mu_{13}p''' b(c_1; n, p''')-\mu_{21}p' b(c_2; n, p') \\ & - \mu_{22}p'' b(c_2; n, p'')+ \mu_{23}p''' b(c_2; n, p''')] \quad \ldots \quad (5.6.7) \end{aligned}$$

For two plans(n_1, c_1', c_2') and (n_2, c_1'', c_2'') satisfying (5.6.4)-(5.6.6) and having overlapping N-intervals we solve $R(N, n_1, c_1', c_2') = R(N, n_2, c_1'', c_2'')$ giving

$$N_{12} = [(n_2-n_1)(1-\mu_{11}-\mu_{21}-\mu_{22})+n_2\left\{\delta_1(n_2,c_1'') + \delta_2(b_2,c_2'')\right\}- n_1\left\{\delta_1(n_1,c_1') + \delta_2(n_1,c_2')\right\}] / [\delta_1(n_2,c_1'') + \delta_2(n_2,c_2'') - \delta_1(n_1,c_1') - \delta_2(n_1,c_2')] \quad \ldots \quad (5.6.8)$$

where

$$\delta_1(n,c_j) = \mu_{11}B(c_j;n,p') - \mu_{12}B(c_j;n,p'') - \mu_{13}B(c_j;n,p''') \quad \ldots \quad (5.6.9)$$

and

$$\delta_2(n,c_j) = \mu_{21}B(c_j;n,p') + \mu_{22}B(c_j;n,p'') - \mu_{23}B(c_j;n,p''') \quad \ldots \quad (5.6.10)$$

To select one of the two plans (n_1, c_1', c_2') and (n_2, c_1'', c_2'') as optimal plan in appropriate range of the lot sizes we shall use (5.4.9).

Numerical computation of optimal plans

Clearly, for the three-point binomial prior it is not possible to determine the bounds of the optimal sample size n, explicitly in terms of the decision numbers c_1 and c_2, see (5.6.4) and (5.6.5). Therefore, to determine n satisfying (5.6.4) and (5.6.5) the method of systematic search for chosen values of c_1 and c_2 is to be used.

The optimal three-decision Bayesian plans, in this case, can be tabulated as follows:

Step 1 : Compute μ_{ij}, $i = 1,2$; $j = 1,2,3$.

Step 2 : Choose $c_1=0$ and $c_2=c_1+1$ and obtain the smallest n satisfying (5.6.4) and (5.6.5).

Step 3 : Compute the bounds (N-intervals) for the lot sizes for the plans obtained in the step 2 using (5.6.7).

Step 4 : Using the chosen values of c_1 and c_2 increase n systematically and if it satisfies (5.6.4) and (5.6.5) obtain the N-interval as in the step 3 and proceed until a value of n which does not satisfy (5.6.4) and (5.6.5) is encountered.

Step 5 : Increase c_1 and c_2 systematically and obtain the corresponding values of n and proceed as before.

Step 6 : For two plans having overlapping N-intervals select the optimal plan using (5.6.8) and (5.4.9).

Example 5 illustrates the computations.

Example 5

Assume the various costs are same as in example 1. Let the lots be generated under a three-point binomial with probabilities $w_1 = 0.90$, $w_2 = 0.08$ and $w_3 = 0.02$ and the corresponding lot qualities as $p' = 0.01$, $p'' = 0.05$ and $p''' = 0.15$ respectively.

The necessary quantities to obtain the plans are :

$\mu_{11} = 0.210617$; $\mu_{12} = 0.111560$; $\mu_{13} = 0.109316$

$\mu_{21} = 0.527986$, $\mu_{22} = 0.029493$; $\mu_{23} = 0.003526$

Using these values and following the steps 1-6 we obtain a set of illustrative optimal three-decision plans provided in the table 23.

5.7 Point binomial Bayes solution

Assume that the prior distribution of the lot quality is point binomial with the parameter $\bar{p}$. The cost function, under the assumption 4 (section 5.3), has its minimum value when the following decision rule is used :

accept all lots from the processes with $\bar{p} < p_u$
screen all lots from the processes with $\bar{p}$, $p_u \leq \bar{p} < p_v$
reject all lots from the processes with $\bar{p} \geq p_u$

... (5.7.1)

where p_u and p_v are the break-even qualities as defined in section 5.3.

The above prior distribution amounts to having a full information of the lot quality.

In such a case the optimal procedure is given by (5.7.1) resulting in the following decision costs :

$$k_m(\bar{p}) = \begin{cases} k_a(\bar{p}) & \text{if } \bar{p} < p_u \\ k_t(\bar{p}) & \text{if } p_u \leq \bar{p} < p_v \\ k_r(\bar{p}) & \text{if } \bar{p} \geq p_v \end{cases} \quad \ldots (5.7.2)$$

where we assume, as mentioned earlier, $p_u < p_v$.

.8 Beta prior unrestricted Bayes solution

Several continuous prior distributions may be used for incoming lot quality p. Some of them are Beta, mixed-Beta, Polya and mixed-Polya. We assume, here, Beta prior for illustration. The treatments in the other cases, will proceed on similar lines.

.8.1 Determination of optimal plans

The problem is to determine the values of n, c_1 and c_2 minimising the value of R given by (5.3.23). It can be shown, on the lines similar to section (5. 9), the Bayesian solution in this case also is unique. To avoid repetitions of the steps of section (5. 9) and in particular, involving more complicated mathematical expressions in the present case, the proof for the uniqueness of the solution is omitted.

Since the optimal plan (n, c_1, c_2) is unique the solution of local optimum conditions will provide global opimum solution in respect of n, c_1 and c_2.

The value of c_1 minimising R given by (5.3.23) must satisfy

$$\Delta_{c_1} R(N, n, c_1 - 1, c_2) \leq 0 < \Delta_{c_1} R(N, n, c_1, c_2) \quad \ldots (5.8.1)$$

On simplification, we obtain from (5.8.1) the following bound on the sample size n

$$(\pi + c_1)\, p_u^{-1} - d \leq n < (\pi + c_1 + 1)\, p_u^{-1} - d \quad \ldots (5.8.2)$$

where $d = \pi + t$. Simplifying a similar condition with respect to c_2 we get another bound for n

$$(\pi + c_2)\, p_v^{-1} - d \leq n < (\pi + c_2 + 1)\, p_v^{-1} - d \quad \ldots (5.8.3)$$

Similarly, simplifying optimum condition with respect to n we obtain a bound for the lot size N

$$F(n-1,c_1,c_2) \leq N < F(n,c_1,c_2) \quad \ldots \quad (5.8.4)$$

where $F(n,c_1,c_2)$ is given by

$$F(n,c_1,c_2) = n+1+[D_1/D_2] \quad \ldots \quad (5.8.5)$$

The expressions for D_1 and D_2 are complicated functions of n, c_1, c_2, p_u and p_v and are given in the Appendix to this Chapter.

The opimal Bayesian ASR plans, in this case, can be obtained in the following steps:

Step 1 : Compute p_u and p_v.

Step 2 : Choose c_1 systematically as 0,1,2,.....etc., and the corresponding value of c_2 $(c_2 \geq c_1)$ and obtain upper and lower limits for the sample size n.

Step 3 : For a chosen pair of c_1 and c_2 list down the values of the sample sizes which are common to the intervals obtained from (5.8.2) and (5.8.3).

Step 4 : Compute the lot size range using (5.8.4), corresponding to each value of sample size listed under the step 3.

5.8.2 Numerical computation of optimal ASR plans

Assume that $w(p)$ is the Beta density with parameters $\pi = 1.5$ and $t = 0.5$ with expected value $\bar{p} = 0.75$. The cost functions are $k_s(p) = 45 + 65\,p$; $k_a(p) = 150\,p$; $k_t(p) = 20 + 50\,p$ and $k_r(p) = 30$.

Here $k_s = 45 + 65\pi/d = 93.75$, $k_m = 27.2858$, $p_u = 0.20$, $p_v = 0.40$.

The value of k_m is obtained by numerical integration using curvature formula (see Rao et.al.(1975)). Using steps 1-4 we get a set of illustrative optimal plans given in the table 24. In the above illustrative example, it may be noted that all the conditions (1-4) of section 5.3 are satisfied. Even though $E(p) = \bar{p} = \pi/d = 0.75$ which does not lie between p_u and p_v the problem admits optimal plans. Some of these plans are given in the table 24. Thus, if the conditions of the section 5.3 are met one can obtain optimal plans by taking any suitable values of π and t and not only those values for which $p_u \leq \bar{p} \leq p_v$.

5.8.3 General remark

As mentioned earlier optimal Bayesian ASR plans can be worked out for other continuous distributions on the similar lines. The bounding approaches used to obtain optimal plans in this section are the common feature of the entire Chapter 5. However, it is also possible to obtain c_1 and c_2 analytically as a function of n under certain conditions of regularity on the posterior risk or loss and then a unidimensional search can be used over n to obtain optimal plans against various lot sizes systematically as in Guthrie and Johns (1959) and Thyregod (1974). Since the optimal plans obtained through the two approaches are not found significantly different we have confined us to bounding approaches which are direct ones.

5.9 An open problem

The numerical computation of the optimal plans in the examples considered for the following cases yielded unique optimal solutions which are listed in the tabular form in the end of the thesis:

(i) Two-point prior restricted Bayes solution (sec.5.4).

(ii) Two-point prior unrestricted Bayes solution (sec. 5.5).

(iii) Three-point prior unrestricted Bayes solution (sec. 5.6)

(iv) Beta prior unrestricted Bayes solution (Sec.5.8)

What about showing analytically existence of unique optimal solution in these cases? Let us consider, for example, the case of two point prior restricted Bayes solution. The uniqueness of optimal Bayes solution, in this case, can be proved analytically provided

(a) the function G(c) as defined in (5.4.11) is analytically shown as a decreasing function of c with falling rate of decrease i.e., $\Delta_c G(c) < 0$ and $\Delta_c^2 G(c) > 0$. and

(b) the point of intersection N(c,c+1) of $R_1(N,c)$ and $R_1(N,c+1)$ as defined in (5.4.12) is analytically shown as an increasing function of c i.e., $\Delta_c N(c,c+1) > 0$.

We have not been able to prove (a) and (b) analytically and it is noted from Hald (1960) pages 312 that it has not been possible to prove (b) analytically even in the case of two-decision plans.

We pose (a) and (b) as "open conjectures". However, we have carried out extensive numerical computations and found that both the conjectures (a) and (b) are true for the range of values of c taken. Our numerical results in respect of (a) and (b) are illustrated in the Fig.7 and Fig.8 respectively.

In the light of the above numerical investigations if we accept (a) and (b) as true, then, the proof for uniqueness proceeds rigorously as follows:

Lemma 5 : Let $f_1(x) = a_1 + b_1 x$ and $f_2(x) = a_2 + b_2 x$, where $0 < a_1 < a_2$ and $b_1 > b_2 > 0$, be defined in R^2. Then the two functions intersect at $x_0 > 0$ such that $f_1(x) < f_2(x)$ for $0 \leq x < x_0$ and $f_1(x) \geq f_2(x)$ for $x \geq x_0$.

Proof: Since $b_1 > b_2 > 0$ the two functions $f_1(x)$ and $f_2(x)$ must intersect at some point (x_0, y_0) in R^2. Suppose $x_0 \leq 0$. Then, at x_0 we have $a_2 - a_1 = (b_1 - b_2)x_0 \leq 0$ contradicting $0 < a_1 < a_2$. Hence $x_0 > 0$.

Now, $a_1 < a_2$, the function $f_1(x)$ will meet $f_2(x)$ from below implying the results of the lemma. □

Lemma 6 : Let $R_1(N,c) = n(c)(1-G(c)) + G(c)N$. For any $0 \leq c' < c'' \leq n$ there exists an unique $N_o > 0$ such that $R_1(N_o,c') = R_1(N_o,c'')$. Further, we have $R_1(N,c'') > R_1(N,c')$ for all $0 \leq N < N_o$ and $R_1(N,c'') < R_1(N,c')$ for $N > N_o$.

Proof : As mentioned earlier in the Chapter 2 for the plans in S, $n(c+k) > n(c)$ for any $k = 1,2,\ldots$. We take $n(c'') > n(c')$ and note that $n(c'')(1-G(c'')) > n(c')(1-G(c'))$ and $G(c'') < G(c')$. The required results follow from lemma 5 by putting $a_1 = n(c')(1-G(c'))$, $a_2 = n(c'')(1-G(c''))$, $b_1 = G(c')$ and $b_2 = G(c'')$.

□

Theorem 5 : Let $c_o > 1$ and let N_o be such that $R_1(N_o,c_o) = R_1(N_o, c_o+1)$. Then $c = c_o+1$ is the unique value which satisfies the condition $\Delta R_1(N_o,c_o) \leq 0 < \Delta R_1(N_o,c_o+1)$.

Proof : By hypothesis we have $\Delta R_1(N_o,c_o) = 0$. We have by lemma 6, $\Delta R_1(N,c_o+1) > 0$ for all $0 \leq N < N(c_o+1, c_o+2)$. Since $N_o = N(c_o,c_o+1) < N(c_o+1, c_o+2)$, we have $\Delta R_1(N_o,c_o+1)>0$. Consider any $c > c_o+1$. For all $N < N(c,c+1)$ we have $\Delta R_1(N,c) > 0$. Since $N_o < N(c,c+1)$ we have $\Delta R_1(N_o,c) > 0$ for all $c > c_o+1$. Now, consider any $0 \leq c < c_o$. By lemma 6, $\Delta R_1(N,c) < 0$ for all $N > N(c,c+1)$.

Since $N_o = N(c_o,c_o+1) > N(c,c+1)$ we have $\Delta R_1(N_o,c) < 0$

□

<u>Theorem 6</u> : For any $\bar{N} > 0$ there exists an unique c_o such that $\Delta R_1(\bar{N}, c_o) \leq 0 < \Delta R_1(\bar{N}, c_o+1)$.

<u>Proof</u> : For simplicity of notation let $N_k = N(k, k+1)$. If $N_k \leq \bar{N} < N_{k+1}$, let $c_o = k$. We have from the proof of the theorem 5 for $N_k = M$, $\Delta R_1(M,k) = 0$ and $\Delta R_1(N,c) < 0$ for all $c < k$ and $N > N_c$. But $\bar{N} \geq N_k > N_c$ and hence $\Delta R_1(\bar{N},c) < 0$. Similarly, we can show that $\Delta R_1(\bar{N},c) > 0$ for all $c \geq k$. □

This completes the proof that the solution is unique.

5.10 Concluding remarks

Bayesian three decision plans discussed in this Chapter have a definite economic advantages over the two-decision plans under certain situations as indicated in sec. 5.5. If the optimal three-decision plans, for most practical situations, are made available in the tabular form, the users may find it quite convenient.

Reasonable estimates of the parameters λ_{ij} and μ_{ij} can be easily obtained by having a good feed-back information system for the process. To facilitate the use, exhaustive tables are being compiled separately. However, in view of the economic advantages of these plans over two-decision plans, even in the absence of tables, the plans can be easily worked out with the help of an electronic computer from the systematic steps given in the various sections of the present Chapter.

We have assumed a very simple form of prior distribution for 'p', purely based on practical considerations as discussed by Hald (1981). We considered a process producing either a single value, $100\bar{\bar{p}}$, of percent of defectives (section 5.7) or two values, $100p'$ or $100p''$, of percent of defectives (section 5.4 and 5.5) or three values, $100p'$, $100p''$ and $100p'''$, of percent of defectives (section 5.6). In the section 5.8, as an extreme generalisation of the above, we have considered a continuous prior viz., beta.

APPENDIX TO CHAPTER FIVE

Expression for D_1 and D_2 (Section 5.8.1)

$$D_1 = 1 - [(A_2-T_2)\{p_u G(\pi,t,n,c_1) - G(\pi+1,t,n,c_1) + \pi(1-I_{p_u}(\pi+1,t))/(\pi+t) - p_u(1-I_{p_u}(\pi,t))\} + (T_2-R_2)\{p_v G(\pi,t,n,c_2) - G(\pi+1,t,n,c_2) + \pi(1-I_{p_v}(\pi+1,t))/(\pi+t) - p_v(1-I_{p_v}(\pi,t))\}] / (k_s-k_m)$$

where $I_p(\pi,t) = \int_0^p z^{\pi-1}(1-z)^{t-1}dz/B(\pi,t), \quad 0 \leq p \leq 1$

$$G(\pi,t,n,c) = \int_0^1 p^{\pi-1} q^{t-1} I_p(c+1,n-c)dp/B(\pi,t)$$

$$B(\pi,t) = \int_0^1 p^{\pi-1}(1-p)^{t-1}dp, \quad \pi > 0, \quad t > 0$$

$$D_2 = [(A_2-T_2)\{B(\pi+c_1+2,n+t-c_1)/(n-c_1)B(\pi,t)B(c_1+1,n-c_1) - B(\pi+c_1+1,n+t-c_1)p/(n-c_1)B(\pi,t)B(c_1+1,n-c_1)\} - (T_2-R_2)\{B(\pi+c_2+2,n+t-c_2)/(n-c_2)B(\pi,t)B(c_2+1,n-c_2) - B(\pi+c_2+1,n+t-c_2)p/(n-c_2)B(\pi,t)B(c_2+1,n-c_2)\}] / (k_s-k_m)$$

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Sampling Inspection Tables

TABLE 1. THE SINGLE SAMPLING THREE-DECISION ASR PLAN BY ATTRIBUTES [Case(a)]

$p_1 = 0.05$; $p_2 = 0.10$; $\beta_1 = 0.07$; $\beta_2 = 0.10$

Lot size	process average = 0.01				process average = 0.02				process average = 0.03			
	n	c_1	c_2	AOI	n	c_1	c_2	AOI	n	c_1	c_2	AOI
100	52	0	2	70.8	52	0	2	79.1	52	0	2	80.3
150	52	0	2	90.4	52	0	2	107.3	52	0	2	109.8
200	85	1	4	108.8	52	0	2	135.5	52	0	2	139.4
250	85	1	4	119.2	52	0	2	163.7	52	0	2	168.9
300	85	1	4	129.6	85	1	4	188.4	52	0	2	198.4
350	85	1	4	139.9	143	3	9	209.3	52	0	2	227.9
400	115	2	7	146.0	143	3	9	225.4	52	0	2	257.4
450	115	2	7	151.5	170	4	11	241.3	52	0	2	286.9
500	115	2	7	156.9	170	4	11	254.0	52	0	2	316.4
600	115	2	7	167.8	196	5	14	277.1	52	0	2	375.4
700	143	3	9	174.3	220	6	16	294.1	52	0	2	434.5
800	143	3	9	180.0	220	6	16	309.5	52	0	2	493.5
900	143	3	9	185.6	220	6	16	325.0	346	11	27	538.4
1000	143	3	9	191.2	220	6	16	340.4	346	11	27	573.1
1200	170	4	11	199.8	248	7	18	369.6	440	15	35	631.4
1400	170	4	11	205.6	297	9	22	389.6	440	15	35	681.7

Contd.....

(Table 1 - Contd.....)

Lot size	process average = 0.01				Process average = 0.02				process average = 0.03			
	n	c_1	c_2	AOI	n	c_1	c_2	AOI	n	c_1	c_2	AOI
1600	170	4	11	211.3	297	9	22	406.3	440	15	35	732.1
1800	170	4	11	217.1	321	10	24	419.9	440	15	35	782.5
2000	196	5	14	222.4	321	10	24	433.2	465	16	37	832.1
3000	196	5	14	237.1	417	14	33	473.6	650	24	55	948.2
4000	220	6	16	247.0	417	14	33	495.6	789	30	72	1063.1
5000	220	6	16	254.1	417	14	33	517.4	900	35	78	1117.9
7000	220	6	16	268.3	440	15	35	555.3	1000	40	79	1262.4
10000	248	7	18	285.4	490	17	41	598.0	1000	40	79	1393.6

TABLE 2. AOI VALUES FOR THREE-DECISION SINGLE SAMPLING ASR PLANS BY ATTRIBUTES [Case(a)] AND THE CORRESPONDING DODGE AND ROMIG'S PLANS

Lot Size	Process average = 0.01						
	Three decision plans				DR plans		
	n	c_1	c_2	AOI	n	c	AOI
100	20	0	1	33.2	20	0	34.6
150	22	0	1	44.8	22	0	47.4
200	38	1	2	45.9	38	1	46.9
250	38	1	2	48.4	38	1	49.8
300	38	1	2	50.8	38	1	52.5
350	38	1	2	53.2	38	1	55.3
400	38	1	2	55.7	38	1	58.0
450	38	1	2	58.1	38	1	60.8
500	38	1	2	60.6	38	1	63.6
600	38	1	2	65.5	38	1	69.1
700	38	1	2	70.4	38	1	74.7
800	38	1	2	75.3	38	1	80.3
900	50	2	4	61.6	50	2	61.7
1000	50	2	4	63.0	50	2	63.1
1200	50	2	4	65.7	50	2	65.9
1400	50	2	4	68.4	50	2	68.6
1600	50	2	4	71.2	50	2	71.4
1800	50	2	4	73.9	50	2	74.2
2000	50	2	4	76.6	50	2	76.9
3000	50	2	4	90.3	50	2	90.8
4000	50	2	4	104.0	50	2	104.6
5000	50	2	4	117.7	50	2	118.4
7000	65	3	5	93.5	65	3	93.9
10000	65	3	5	105.9	65	3	106.4

Contd....

(Table 2 - Contd...)

Lot Size	Process average = 0.02						
	Three decision plans				DR plans		
	n	c_1	c_2	AOI	n	c	AOI
100	20	0	1	43.9	20	0	46.6
150	35	1	2	49.0	35	1	52.8
200	35	1	2	55.1	35	1	60.5
250	50	2	4	65.0	50	2	65.7
300	50	2	4	68.7	50	2	69.6
350	50	2	4	72.5	50	2	73.5
400	50	2	4	76.2	50	2	77.4
450	50	2	4	80.0	50	2	81.4
500	50	2	4	83.7	50	2	85.3
600	65	3	5	86.1	65	3	87.1
700	65	3	5	90.0	65	3	91.3
800	65	3	5	93.9	65	3	95.4
900	65	3	5	97.9	65	3	99.5
1000	65	3	5	101.8	65	3	103.7
1200	80	4	7	104.8	80	4	105.0
1400	80	4	7	109.2	80	4	109.5
1600	80	4	7	113.6	80	4	114.0
1800	80	4	7	118.0	80	4	118.4
2000	80	4	7	122.5	80	4	122.9
3000	80	4	7	144.6	80	4	145.3
4000	90	5	9	127.0	90	5	127.3
5000	90	5	9	136.4	90	5	136.9
7000	105	6	10	141.4	105	6	141.5
10000	105	6	10	157.4	105	6	157.4

Contd.....

(Table 2 - Contd...)

Lot Size	Process average = 0.03						
	Three decision plans				DR plans		
	n	c_1	c_2	AOI	n	c	AOI
100	33	1	2	47.2	33	1	50.4
150	48	2	3	60.0	48	2	65.7
200	65	3	5	66.0	65	3	74.5
250	65	3	5	86.7	65	3	89.3
300	65	3	5	92.6	65	3	95.8
350	65	3	5	98.5	65	3	102.3
400	65	3	5	104.4	65	3	108.9
450	75	4	7	102.4	74	4	103.1
500	75	4	7	106.0	74	4	106.8
600	80	4	7	126.8	80	4	128.2
700	90	5	9	121.9	90	5	122.9
800	90	5	9	127.1	90	5	128.3
900	90	5	9	132.4	90	5	133.7
1000	90	5	9	137.6	90	5	139.0
1200	105	6	10	147.4	105	6	147.9
1400	105	6	10	155.2	105	6	155.7
1600	105	6	10	163.0	105	6	163.5
1800	105	6	10	171.7	105	6	171.3
2000	105	6	10	178.5	105	6	179.1
3000	115	7	12	180.9	115	7	181.1
4000	130	8	14	195.3	130	8	195.4
5000	130	8	14	212.2	130	8	212.2
7000	140	9	15	208.1	140	9	208.1
10000	150	10	16	207.3	150	10	207.4

TABLE 3. THE VALUES OF x SATISFYING (3.4.7) AND CORRESPONDING VALUES OF y.

c_1	c_2	x	y	c_1	c_2	x	y
0	2	1.414213	0.414214	11	13	11.215362	8.158714
1	3	2.236375	0.952182	12	14	12.140244	8.953887
2	4	3.092727	1.558027	13	15	13.068499	9.756142
3	5	3.965123	2.208438	14	16	14.000797	10.564794
4	6	4.849036	2.891019	15	17	14.930630	11.379249
5	7	5.742012	3.598241	16	18	15.864714	12.199019
6	8	6.641530	4.325121	17	19	16.800401	13.023669
7	9	7.547888	5.068162	18	20	17.737896	13.852822
8	10	8.459299	5.824794	19	21	18.677091	14.686149
9	11	9.374142	6.593061	20	22	19.617890	15.523357
10	12	10.292835	7.371440				

TABLE 4. THE VALUES OF $\bar{M}$ ON THE BOUNDARIES FOR ASR PLANS

c_1	c_1+1	$\bar{M}$	y
0	1	1.404712	0.414214
1	2	16.389329	0.952182
2	3	136.381060	1.558027
3	4	978.750790	2.208438
4	5	6430.144500	2.891019

TABLE 5. SOME ILLUSTRATIVE OPTIMAL ASR SINGLE SAMPLING PLANS FOR $\bar{p} = 0.005$ AND $p_L = 0.05$

lot size	$\bar{N}$	c_1	c_2	n
8 - 280	144	0	2	8
281 - 3277	1779	1	3	18
3278 - 27276	15277	2	4	31
27277 - 195750	111514	3	5	44
195751 - 1286028	740890	4	6	57

TABLE 6. THE VALUES OF x SATISFYING (3.5.4) AND THE CORRESPONDING VALUES OF y.

c_1	c_2	x	y	c_1	c_2	x	y
0	3	1.500000	0.433158	11	15	9.646886	7.463453
1	4	1.900000	0.907877	12	16	10.494637	8.206485
2	6	2.444306	1.395127	13	17	11.348001	8.957475
3	7	3.183143	1.981920	14	18	12.206446	9.716324
4	8	3.944987	2.600902	15	19	13.069517	10.481834
5	9	4.724959	3.152097	16	20	13.936821	11.634342
6	10	5.519693	3.912079	17	21	14.808015	12.031571
7	11	6.326748	4.595737	18	22	15.682820	12.569263
8	12	7.144284	5.294427	19	23	16.534527	13.602765
9	13	7.970872	6.006078	20	24	17.442188	14.313414
10	14	8.805380	6.729958	21	25	18.326320	15.027411

TABLE 7. THE VALUES OF $\bar{M}$ ON THE BOUNDARIES FOR ASP PLANS

c_1	c_1+1	$\bar{M}$	y
0	1	1.1779	0.4332
1	2	14.0262	0.9079
2	3	166.5458	1.3951
3	4	1320.0289	1.9819
4	5	7926.4795	2.6009
5	6	90196.6000	3.1521

TABLE 8. SOME ILLUSTRATIVE OPTIMAL ASP SINGLE SAMPLING PLANS

lot size	$\bar{N}$	c_1	c_2	n
8 - 236	144	0	3	8
237 - 2805	1521	1	4	18
2806 - 33310	18058	2	6	28
33311 - 264007	148659	3	7	40
264008 - 1585296	924652	4	8	52
1585297 - 18039320	9812308	5	9	63

TABLE 9. THE OPTIMAL SINGLE SAMPLING RESTRICTED ASR BAYESIAN PLANS FOR DOUBLE BINOMIAL PRIOR DISTRIBUTION WITH $p'=0.01$; $p''=0.15$; $w_1=0.93$; $w_2=1-w_1$; $p_1=0.05$; $p_2=0.20$; $\beta_1=0.07$ AND $\beta_2=0.10$.

lot size	Sample size n	Decision number c_1	Decision number c_2	$\bar{N}$	$K/\bar{N}$	(%) saving
52 - 95	52	0	6	70	23.23	*
96 - 210	86	1	12	142	20.38	*
211 - 335	120	2	18	266	17.36	*
336 - 390	150	3	23	362	15.84	*
391 - 1204	170	4	27	686	12.70	10.94
1205 - 1214	200	5	32	1204	10.73	24.75

* Acceptance without inspection is more economical.

TABLE 10. THE VALUES OF c_1, c_2 AND n OBTAINED FROM (5.5.7), (5.5.8) AND (5.5.12) FOR $w_1 = 0.93$ AND $w_2 = 0.07$ USING (5.4.7).

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
0	1	4	27 - 32	0.009339	0.534273
		5	33 - 39	0.037207	0.534743
		6	40 - 48	0.060602	0.535206
		7	49 - 58	0.080198	0.535634
		8	59 - 71	0.096568	0.536007
		9	72 - 86	0.110198	0.536313
		10	87 - 105	0.121503	0.536541
		11	106 - 129	0.130833	0.536686
		12	130 - 159	0.138488	0.536744
0	2	13	149 - 179	0.144723	0.536901
		14	180 - 220	0.149752	0.537407
		15	221 - 272	0.153759	0.537902
		16	273 - 342	0.156901	0.538380
		17	343 - 439	0.159309	0.538837
		18	440 - 581	0.161096	0.539268
		19	582 - 809	0.162359	0.539670
		20	810 - 1222	0.163178	0.540041
		21	1223 - 2196	0.163622	0.540379
		22	2197 - 7133	0.163751	0.540683

Contd....

(Table 10 - Contd....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
1	2	23	177 - 199	0.166776	0.540953
		24	200 - 226	0.171919	0.541189
		25	227 - 259	0.176423	0.541390
		26	260 - 297	0.180349	0.541558
		27	298 - 343	0.183759	0.541692
		28	344 - 399	0.186704	0.541795
		29	400 - 469	0.189234	0.541866
		30	470 - 557	0.191395	0.541907
		31	558 - 670	0.193225	0.541920
1	3	32	568 - 667	0.194763	0.542071
		33	668 - 793	0.196039	0.542329
		34	794 - 956	0.197085	0.542567
		35	957 - 1174	0.197926	0.542786
		36	1175 - 1477	0.198584	0.542985
		37	1478 - 1923	0.199083	0.543166
		38	1924 - 2643	0.199438	0.543330
		39	2644 - 3985	0.199669	0.543476
		40	3986 - 7345	0.199787	0.543605
		41	7346 - 30576	0.199808	0.543719
2	3	42	572 - 645	0.201141	0.543818
		43	646 - 731	0.202686	0.543902
		44	732 - 833	0.204045	0.543973
		45	834 - 954	0.205236	0.544031
		46	955 - 1100	0.206276	0.544077
		47	1101 - 1277	0.207179	0.544110
		48	1278 - 1497	0.207960	0.544133
		49	1498 - 1774	0.208630	0.544145
		50	1775 - 2134	0.209202	0.544147

(Contd...)

(Table 10 - Contd...)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
2	4	51	1709 - 2003	0.209685	0.544237
		52	2004 - 2376	0.210088	0.544342
		53	2377 - 2859	0.210420	0.544436
		54	2860 - 3506	0.210688	0.544522
		55	3507 - 4412	0.210898	0.544599
		56	4413 - 5763	0.211057	0.544669
		57	5764 - 7986	0.211169	0.544730
		58	7987 - 12297	0.211240	0.544785
		59	12298 - 24164	0.211273	0.544833
3	4	60	1470 - 1656	0.211282	0.544874
		61	1657 - 1873	0.211869	0.544910
		62	1874 - 2127	0.212387	0.544941
		63	2128 - 2427	0.212843	0.544966
		64	2428 - 2783	0.213244	0.544986
		65	2784 - 3212	0.213594	0.545002
		66	3213 - 3735	0.213899	0.545013
		67	3736 - 4384	0.214163	0.545020
		68	4385 - 5206	0.214390	0.545024
3	5	69	4162 - 4839	0.214584	0.545026
		70	4840 - 5679	0.214749	0.545071
		71	5680 - 6744	0.214886	0.545111
		72	6745 - 8129	0.214999	0.545148
		73	8130 - 9996	0.215090	0.545181
		74	9997 - 12636	0.215161	0.545210
		75	12637 - 16638	0.215214	0.545236
		76	16639 - 23393	0.215251	0.545259
		77	23394 - 37167	0.215273	0.545280
		78	37168 - 80505	0.215282	0.545297

(Contd....)

(Table 10 - Contd...)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
4	5	79	4091 - 4618	0.215338	0.545313
		80	4619 - 5232	0.215545	0.545326
		81	5233 - 5951	0.215727	0.545337
		82	5952 - 6799	0.215888	0.545346
		83	6800 - 7809	0.216030	0.545353
		84	7810 - 9027	0.216153	0.545359
		85	9028 - 10515	0.216261	0.545363
		86	10516 - 12367	0.216354	0.545365
		87	12368 - 14723	0.216435	0.545366
4	6	88	11461 - 13344	0.216503	0.545371
		89	13345 - 15687	0.216561	0.545388
		90	15688 - 18666	0.216610	0.545404
		91	18667 - 22564	0.216649	0.545418
		92	22565 - 27860	0.216681	0.545430
		93	27861 - 35440	0.216706	0.545442
		94	35441 - 47146	0.216725	0.545452
		95	47147 - 67532	0.216737	0.545460
		96	67533 - 111786	0.216744	0.545468
		97	111787 - 280257	0.216747	0.545475
5	6	98	11160 - 12617	0.216786	0.545480
		99	12618 - 14313	0.216861	0.545485
		100	14314 - 16299	0.216928	0.545489
		101	16300 - 18645	0.216986	0.545492
		102	18646 - 21443	0.217037	0.545495
		103	21444 - 24822	0.217082	0.545497
		104	24823 - 28965	0.217121	0.545498
		105	28966 - 34139	0.217155	0.545499
		106	34140 - 40756	0.217184	0.545499

(Contd...)

(Table 10 - Contd...)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
5	7	107	31061 - 36221	0.217209	0.545503
		108	36222 - 42662	0.217230	0.545510
		109	42663 - 50891	0.217247	0.545516
		110	50892 - 61729	0.217262	0.545521
		111	61730 - 76593	0.217273	0.545526
		112	76594 - 98165	0.217282	0.545530
		113	98166 - 132201	0.217288	0.545534
		114	132202 - 193708	0.217293	0.545537
		115	193709 - 338046	0.217295	0.545540
		116	338047 - 1074693	0.217295	0.545543
6	7	117	30041 - 33998	0.217318	0.545545
		118	33999 - 38604	0.217346	0.545547
		119	38605 - 44005	0.217370	0.545548
		120	44006 - 50392	0.217392	0.545549
		121	50393 - 58025	0.217411	0.545550
		122	58026 - 67266	0.217427	0.545551
		123	67267 - 78631	0.217441	0.545552
		124	78632 - 92887	0.217454	0.545552
		125	92888 - 111226	0.217465	0.545552
6	8	126	83254 - 97247	0.217474	0.545554
		127	97248 - 114782	0.217482	0.545557
		128	114783 - 137310	0.217488	0.545559
		129	137311 - 167205	0.217493	0.545561
		130	167206 - 208645	0.217497	0.545563
		131	208646 - 269731	0.217500	0.545564
		132	269732 - 368506	0.217503	0.545566
		133	368507 - 554957	0.217504	0.545567
		134	554958 - 1037924	0.217505	0.545568
		135	1037925 - 5255216	0.217505	0.545569

(Contd....)

(Table 10 - Contd...)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
7	8	136	80035 - 90648	0.217516	0.545570
		137	90649 - 103014	0.217527	0.545571
		138	103015 - 117530	0.217544	0.545572
		139	117531 - 134727	0.217551	0.545572
		140	134728 - 155325	0.217557	0.545572
		141	155326 - 180332	0.217557	0.545572
		142	180333 - 211201	0.217563	0.545572
		143	211202 - 250110	0.217567	0.545572
		144	250111 - 300485	0.217571	0.545572
7	9	145	221289 - 258953	0.217575	0.545574
		146	258954 - 306371	0.217577	0.545575
		147	306372 - 367664	0.217580	0.545575
		148	367665 - 449686	0.217582	0.545576
		149	449687 - 564736	0.217583	0.545577
		150	564737 - 737324	0.217584	0.545578
		151	737325 - 1024329	0.217585	0.545578
		152	1024330 - 1594833	0.217586	0.545579
		153	1594834 - 3276047	0.217586	0.545579
8	9	154	187124 - 211473	0.217587	0.545579
		155	211474 - 239686	0.217591	0.545580
		156	239687 - 272594	0.217595	0.545580
		157	272595 - 311284	0.217599	0.545580
		158	311285 - 357209	0.217602	0.545580
		159	357210 - 412353	0.217604	0.545580
		160	412354 - 479511	0.217607	0.545581
		161	479512 - 562747	0.217609	0.545581
		162	562748 - 668222	0.217611	0.545581

(Contd...)

(Table 10 - Contd...)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
8	10	163	502932 - 584373	0.217612	0.545581
		164	584374 - 685201	0.217613	0.545581
		165	685202 - 812787	0.217614	0.545582
		166	812788 - 978829	0.217615	0.545582
		167	978830 - 1203085	0.217616	0.545582
		168	1203086 - 1521781	0.217617	0.545583
		169	1521782 - 2009309	0.217617	0.545583
		170	2009310 - 2846502	0.217617	0.545583
		171	2846503 - 4617132	0.217617	0.545583
		172	4617133 - 10842024	0.217617	0.545583
9	10	173	490845 - 555053	0.217618	0.545583
		174	555054 - 629534	0.217620	0.545584
		175	629535 - 716530	0.217621	0.545584
		176	716531 - 818989	0.217623	0.545584
		177	818990 - 940867	0.217624	0.545584
		178	940868 - 1087611	0.217625	0.545584
		179	1087612 - 1266941	0.217626	0.545584
		180	1266942 - 1490189	0.217627	0.545584
		181	1490190 - 1774713	0.217627	0.545584
9	11	182	1319079 - 1535285	0.217628	0.545584
		183	1535286 - 1804106	0.217628	0.545584
		184	1804107 - 2146163	0.217629	0.545584
		185	2146164 - 2594620	0.217629	0.545585
		186	2594621 - 3206457	0.217629	0.545585
		187	3206458 - 4088583	0.217630	0.545585
		188	4088584 - 5467939	0.217630	0.545585
		189	5467940 - 7925832	0.217630	0.545585
		190	7925833 - 13532746	0.217630	0.545585
		191	13532747 - 39113025	0.217630	0.545585

(Contd...)

(Table 10 - Contd....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
10	11	192	1280539 - 1448935	0.217630	0.545585
		193	1448936 - 1644513	0.217631	0.545585
		194	1644514 - 1873304	0.217632	0.545585
		195	1873305 - 2143270	0.217632	0.545585
		196	2143271 - 2465159	0.217633	0.545585
		197	2465160 - 2853863	0.217633	0.545585
		198	2853864 - 3330653	0.217633	0.545585
		199	3330654 - 3927047	0.217634	0.545585
		200	3927048 - 4691870	0.217634	0.545585
10	12	201	3444970 - 4017000	0.217634	0.545585
		202	4017001 - 4731533	0.217634	0.545585
		203	4731534 - 5646220	0.217634	0.545585
		204	5646221 - 6855081	0.217634	0.545585
		205	6855082 - 8522645	0.217635	0.545585
		206	8522646 - 10965447	0.217635	0.545585
		207	10965448 - 14880491	0.217635	0.545586
		208	14880492 - 22162869	0.217635	0.545586
		209	22162870 - 40420712	0.217635	0.545586
		210	40420713 - 170380640	0.217635	0.545586

TABLE 11. THE POINTS OF INTERSECTION FROM (5.5.7), (5.5.8) AND (5.8.12) FOR $w_1 = 0.93$ AND $w_2 = 0.07$ USING (5.4.7).

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
0	1	12	0	2	13	155.7
0	2	13	1	2	23	374.8
0	2	14	1	2	24	380.0
0	2	15	1	2	25	380.0
0	2	15	1	2	26	363.7
0	2	16	1	2	27	366.8
0	2	17	1	2	28	366.8
0	2	17	1	2	29	369.4
0	2	18	1	2	30	371.4
0	2	18	1	2	31	381.2
1	2	31	1	3	32	607.4
1	2	31	1	3	33	363.1
1	3	33	2	3	43	1229.3
1	3	33	2	3	44	1155.6
1	3	34	2	3	45	1162.1
1	3	34	2	3	46	1140.9
1	3	35	2	3	47	1156.0
1	3	36	2	3	48	1163.6
1	3	37	2	3	49	1165.1
1	3	37	2	3	50	1196.6
2	3	50	2	4	51	1179.0
2	3	50	2	4	52	1883.9
2	4	52	3	4	62	3482.3
2	4	52	3	4	63	3289.8
2	4	53	3	4	64	3297.0
2	4	53	3	4	65	3246.7

(Contd...)

(Table 11 - Contd...)

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
2	4	54	3	4	66	3281.5
2	4	55	3	4	67	3297.4
2	4	55	3	4	68	3361.2
3	4	68	3	5	69	5151.6
3	4	68	3	5	70	4977.3
2	5	70	4	5	80	9562.4
3	5	70	4	5	81	8893.3
3	5	71	4	5	82	8945.8
3	5	72	4	5	83	8955.2
3	5	72	4	5	84	8848.2
3	5	73	4	5	85	8928.0
3	5	74	4	5	86	8962.7
3	5	74	4	5	87	9152.0
4	5	87	4	6	88	13767.1
4	5	87	4	6	89	13583.3
4	6	89	5	6	99	25255.8
4	6	89	5	6	100	23574.3
4	6	90	5	6	101	23779.2
4	6	91	5	6	102	23730.5
4	6	91	5	6	103	23513.4
4	6	92	5	6	104	23699.7
4	6	93	5	6	105	23794.9
4	6	93	5	6	106	24378.9
5	6	106	5	7	107	34571.1
5	6	106	5	7	108	35176.7
5	7	108	6	7	117	73252.3
5	7	108	6	7	118	65445.8

(Contd...)

(Table 11 - Contd...)

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
5	7	108	6	7	119	61886.8
5	7	109	6	7	120	61889.2
5	7	109	6	7	121	60699.1
5	7	110	6	7	122	61633.3
5	7	111	6	7	123	61952.3
5	7	112	6	7	124	61954.2
5	7	113	6	7	125	61638.7
6	7	125	6	8	126	61016.3
6	7	125	6	8	127	91017.3
6	8	127	7	8	137	169597.2
6	8	127	7	8	138	142968.7
6	8	128	7	8	139	144850.2
6	8	128	7	8	140	146455.6
6	8	129	7	8	141	160113.8
6	8	130	7	8	142	160115.8
6	8	131	7	8	143	160117.4
6	8	131	7	8	144	164674.6
7	8	144	7	9	145	166798.2
7	8	144	7	9	146	222350.2
7	9	146	8	9	156	434907.3
7	9	146	8	9	157	407535.1
7	9	147	8	9	158	407537.3
7	9	147	8	9	159	413923.5
7	9	148	8	9	160	400133.6
7	9	149	8	9	161	400135.4
7	9	149	8	9	162	406386.0
7	9	162	8	10	163	1000131.0

(Contd....)

(Table 11 - Contd....)

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
8	9	162	8	10	164	1000132.0
8	10	164	9	10	174	1000144.0
8	10	164	9	10	175	1000145.0
8	10	165	9	10	176	1000148.0
8	10	166	9	10	177	1000150.0
8	10	166	9	10	178	1000151.0
8	10	167	9	10	179	1000153.0
8	10	168	9	10	180	1091062.9
8	10	168	9	10	181	1181970.8
9	10	181	9	11	182	1000169.0
9	10	181	9	11	183	2000157.0
9	11	183	10	11	193	3500163.0
9	11	183	10	11	194	2200167.6
9	11	184	10	11	195	2750164.8
9	11	184	10	11	196	2400169.6
9	11	185	10	11	197	3000167.0
9	11	186	10	11	198	3000168.0
9	11	187	10	11	199	3000172.0
9	11	187	10	11	200	3250170.8
10	11	200	10	12	201	3250170.8
10	11	200	10	12	202	3250170.8

TABLE 12. THE OPTIMAL SINGLE SAMPLING UNRESTRICTED ASR BAYESIAN PLANS FOR w_1=0.93 AND w_2=0.07.

N	n	c_1	c_2	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$	$P_s(p')$	$P_r(p'')$	$P_s(p'')$
							(in percentage)			
27-32	4	0	1	29	13.20	7.43	96.06	3.00	10.95	36.85
33-39	5	0	1	35	12.88	9.68	95.10	4.80	16.48	39.15
40-48	6	0	1	43	12.52	12.20	94.15	5.71	22.35	39.93
49-58	7	0	1	53	12.15	14.80	93.21	6.59	28.34	39.60
59-71	8	0	1	64	11.83	17.04	92.27	7.46	34.28	38.47
72-86	9	0	1	78	11.50	19.35	91.35	8.30	40.05	36.79
87-105	10	0	1	95	11.20	21.46	90.44	9.14	45.57	34.74
106-129	11	0	1	116	10.92	23.42	89.53	9.95	50.78	32.48
130-155	12	0	1	141	10.68	25.10	88.64	10.74	55.65	30.12
156-179	13	0	2	167	10.48	26.50	87.75	12.22	30.80	57.11
180-220	14	0	2	198	10.30	27.77	86.87	13.09	35.21	54.51
221-272	15	0	2	245	10.10	29.17	86.01	13.95	39.58	51.69
273-342	16	0	2	305	9.92	30.43	85.15	14.80	43.86	48.71
343-366	17	0	2	354	9.81	31.20	84.29	15.64	48.02	45.66
367-369	28	1	2	367	9.79	31.35	96.82	2.91	81.29	12.43
370-371	29	1	2	370	9.78	31.42	96.60	3.10	83.16	11.35
372-381	30	1	2	376	9.77	31.49	96.39	3.28	84.86	10.34
382-607	31	1	2	481	9.51	33.31	96.16	3.47	86.41	9.39
608-636	32	1	3	621	9.29	34.85	95.93	4.04	72.79	23.54
637-793	33	1	3	710	9.19	35.55	95.70	4.27	75.05	21.76
794-956	34	1	3	871	9.06	36.46	95.46	4.51	77.15	20.06
957-1155	35	1	3	1051	8.95	37.23	95.21	4.74	79.12	18.45
1156-1163	47	2	3	1159	8.91	37.51	98.83	1.04	93.64	4.29
1164-1165	48	2	3	1164	8.90	37.59	98.76	1.10	94.28	3.89
1166-1196	49	2	3	1180	8.90	37.59	98.69	1.16	94.87	3.52

Contd....

([illegible] 12 - Contd....)

N	n	c_1	c_2	$\bar{N}$	K/$\bar{N}$	(%) saving	$P_a(p')$	$P_s(p')$	$P_r(p'')$	$P_s(p'')$
							(in percentage)			
[illegible]7-1778	50	2	3	1458	8.78	38.43	98.62	1.22	95.40	3.19
[illegible]9-1883	51	2	4	1830	8.67	39.20	98.54	1.44	89.78	8.97
[illegible]84-2376	52	2	4	2115	8.61	39.62	98.46	1.52	90.69	8.21
[illegible]7-2859	53	2	4	2606	8.54	40.11	98.38	1.60	91.54	7.50
[illegible]60-3281	54	2	4	3063	8.49	40.48	98.30	1.68	92.31	6.84
[illegible]82-3297	66	3	4	3289	8.47	40.60	99.56	0.39	97.74	1.53
[illegible]98-3361	67	3	4	3329	8.47	40.60	99.54	0.41	97.97	1.39
[illegible]62-4977	68	3	4	4090	8.41	41.02	99.51	0.43	98.18	1.25
[illegible]78-5679	70	3	5	5316	8.35	41.44	99.46	0.54	96.16	3.39
[illegible]80-6744	71	3	5	6189	8.32	41.65	99.43	0.56	96.52	3.09
[illegible]5-8129	72	3	5	7404	8.29	41.86	99.40	0.59	96.84	2.81
[illegible]30-8927	73	3	5	8519	8.27	42.00	99.37	0.62	97.14	2.55
[illegible]28-8962	85	4	5	8944	8.27	42.00	99.83	0.15	99.17	0.56
[illegible]63-9151	86	4	5	9056	8.26	42.07	99.82	0.15	99.26	0.51
[illegible]52-13583	87	4	5	11149	8.24	42.21	99.81	0.16	99.33	0.46
[illegible]84-15687	89	4	6	14597	8.21	42.43	99.79	0.20	98.55	1.29
[illegible]88-18666	90	4	6	17112	8.20	42.50	99.78	0.21	98.69	1.17
[illegible]67-22564	91	4	6	20523	8.18	42.64	99.77	0.22	98.81	1.06
[illegible]65-23699	92	4	6	23125	8.18	42.64	99.76	0.24	98.92	0.96
[illegible]00-23794	104	5	6	23746	8.17	42.70	99.93	0.06	99.69	0.21
[illegible]95-24378	105	5	6	24084	8.17	42.70	99.93	0.06	99.72	0.19
[illegible]9-34571	106	5	6	29031	8.17	42.70	99.93	0.06	99.75	0.17
[illegible]2-35176	107	5	7	34872	8.16	42.78	99.92	0.08	99.39	0.54
[illegible]7-42662	108	5	7	38739	8.15	42.85	99.92	0.08	99.45	0.49
[illegible]3-50891	109	5	7	46595	8.14	42.92	99.91	0.08	99.50	0.45
[illegible]2-61633	110	5	7	56005	8.14	42.92	99.98	0.09	99.55	0.40
[illegible]4-61638	122	6	7	61635	8.13	42.99	99.97	0.02	99.87	0.09

Contd.....

(Table 12 - Contd....)

N	n	c_1	c_2	$\bar{\bar{N}}$	K/$\bar{N}$	(%) saving	$P_a(p')$	$P_s(p')$	$P_r(p'')$	$P_s(p'')$
							(in percentage)			
61639-91016	125	6	7	74900	8.13	42.99	99.97	0.02	99.91	0.06
91017-97247	126	6	8	91629	8.12	43.06	99.97	0.03	99.77	0.21
97248-114782	127	6	8	102899	8.12	43.06	99.97	0.03	99.79	0.19
114783-137310	128	6	8	125542	8.11	43.13	99.97	0.03	99.81	0.17
137311-160113	129	6	8	148274	8.11	43.13	99.99	0.01	99.83	0.15
160114-160115	141	7	8	160114	8.11	43.13	99.99	0.01	99.95	0.03
160116-160117	142	7	8	160116	8.11	43.13	99.99	0.01	99.96	0.03
160118-164674	143	7	8	162380	8.11	43.13	99.99	0.01	99.96	0.03
164675-166798	144	7	8	165733	8.11	43.13	99.99	0.01	99.96	0.02
166799-222350	145	7	9	192581	8.11	43.13	99.99	0.01	99.91	0.08
222351-306371	146	7	9	261001	8.10	43.20	99.99	0.01	99.92	0.07
306372-367664	147	7	9	335621	8.10	43.20	99.99	0.01	99.93	0.07
367665-400133	148	7	9	383555	8.10	43.20	99.99	0.01	99.93	0.06
400134-400135	160	8	9	400134	8.10	43.20	100.00	0.00	99.98	0.01
400136-406386	161	8	9	403248	8.10	43.20	100.00	0.00	99.98	0.01
406387-668222	162	8	9	521111	8.10	43.20	100.00	0.00	99.98	0.01
668223-685201	164	8	10	676658	8.10	43.20	100.00	0.00	99.97	0.03
685202-812787	165	8	10	746272	8.10	43.20	100.00	0.00	99.97	0.03
812788-978829	166	8	10	891953	8.10	43.20	99.99	0.01	99.97	0.03
978830-1000153	167	8	10	989434	8.10	43.20	99.99	0.01	99.97	0.02
1000154-1000169	179	9	10	1000161	8.10	43.20	100.00	0.00	99.99	0.00
1000170-1535285	182	9	11	1239171	8.10	43.20	100.00	0.00	99.98	0.01
1535286-1804106	183	9	11	1664277	8.10	43.20	100.00	0.00	99.99	0.01
1804107-2146163	184	9	11	1967716	8.10	43.20	100.00	0.00	99.99	0.01
2146164-2594620	185	9	11	2359762	8.10	43.20	100.00	0.00	99.99	0.01
2594621-3000168	186	9	11	2790035	8.10	43.20	100.00	0.00	99.99	0.01
3000169-3000172	198	10	11	3000170	8.10	43.20	100.00	0.00	100.00	0.00
3000173-3250170	199	10	11	3122670	8.09	43.27	100.00	0.00	100.00	0.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

TABLE 13. THE NEARLY OPTIMAL SINGLE SAMPLING UNRESTRICTED ASR BAYESIAN PLANS FOR $w_1 = 0.93$ AND $w_2 = 0.07$.

N	n	c_1	c_2	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$ (in percentage)	$P_s(p')$ (in percentage)	$P_r(p'')$ (in percentage)	$P_s(p'')$ (in percentage)
1-146	8	0	1	12	19.17	*	92.27	7.46	34.28	38.47
147-363	18	0	2	230	10.22	28.33	83.45	16.48	52.03	42.60
364-609	27	1	2	470	9.54	33.10	97.03	2.73	79.26	13.58
610-1134	37	1	3	831	9.10	36.18	94.71	5.24	82.64	15.52
1135-1782	46	2	3	1422	8.79	38.25	98.90	0.99	92.93	4.73
1783-3331	55	2	4	2437	8.56	39.97	98.22	1.76	93.02	6.23
3332-4721	64	3	4	3966	8.42	40.95	99.61	0.35	97.20	1.87
4722-8950	74	3	5	6500	8.32	41.65	99.34	0.65	97.41	2.32
8951-13140	83	4	5	10845	8.24	42.21	99.85	0.13	98.98	0.69
3141-23514	93	4	6	17578	8.19	42.56	99.75	0.25	99.03	0.87
3515-35801	102	5	6	29014	8.16	42.78	99.94	0.05	99.62	0.26
5802-60498	112	5	7	46539	8.14	42.92	99.90	0.10	99.63	0.33
0499-97192	121	6	7	76681	8.12	43.06	99.98	0.02	99.86	0.10
7193-117769	131	6	8	106987	8.12	43.06	99.96	0.04	99.86	0.13
7770-409222	141	7	8	219531	8.11	43.13	99.99	0.01	99.95	0.03
9223-555699	158	8	9	476869	8.10	43.20	100.00	0.00	99.98	0.02
5700-1125149	168	8	10	790724	8.10	43.20	99.99	0.01	99.98	0.02
5150-1428735	177	9	10	1267888	8.10	43.10	100.00	0.00	99.99	0.01
8736-3000168	187	9	11	2070373	8.10	43.20	100.00	0.00	99.99	0.01
0169-5000175	196	10	11	3873160	8.10	43.20	100.00	0.00	100.00	0.00

* Acceptance without inspection is more economic.

TABLE 14. THE VALUES OF c, n AND N OBTAINED FROM (5.5.27) AND (5.5.28) FOR $w_1 = 0.93$ AND $w_2 = 0.07$.

c	n	N	σ (n, c)
0	4	27 - 33	0.009339
	5	34 - 41	0.037207
	6	42 - 50	0.060602
	7	51 - 60	0.080198
	8	61 - 73	0.096567
	9	74 - 88	0.110198
	10	89 - 107	0.121502
	11	108 - 131	0.130833
	12	132 - 160	0.138488
	13	161 - 198	0.144723
	14	199 - 247	0.149751
	15	248 - 313	0.153759
	16	314 - 407	0.156900
	17	408 - 544	0.159309
	18	545 - 766	0.161096
	19	767 - 1173	0.162359
	20	1174 - 2148	0.163178
	21	2149 - 7383	0.163622
1	23	185 - 208	0.166775
	24	209 - 236	0.171919
	25	237 - 270	0.176422
	26	271 - 309	0.180349
	27	310 - 356	0.183758
	28	357 - 411	0.186703
	29	412 - 479	0.189234
	30	480 - 562	0.191394

(Contd....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
1	31	563 - 666	0.193225
	32	667 - 798	0.194762
	33	799 - 969	0.196039
	34	970 - 1200	0.197085
	35	1201 - 1523	0.197925
	36	1524 - 2006	0.198584
	37	2007 - 2795	0.199082
	38	2796 - 4306	0.199438
	39	4307 - 8309	0.199668
	40	8310 - 48319	0.199787
2	42	603 - 679	0.201141
	43	680 - 768	0.202686
	44	769 - 873	0.204045
	45	874 - 995	0.205236
	46	996 - 1141	0.206275
	47	1142 - 1315	0.207179
	48	1316 - 1525	0.207959
	49	1526 - 1783	0.208630
	50	1784 - 2104	0.209202
	51	2105 - 2511	0.209685
	52	2512 - 3043	0.210420
	53	3044 - 3762	0.210420
	54	3763 - 4779	0.210688
	55	4780 - 6319	0.210898
	56	6320 - 8908	0.211056
	57	8909 - 14129	0.211169
	58	14130 - 30002	0.211239

(Contd.....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
3	60	1560 - 1755	0.211282
	61	1756 - 1981	0.211869
	62	1982 - 2242	0.212387
	63	2243 - 2548	0.212843
	64	2549 - 2907	0.213244
	65	2908 - 3331	0.213594
	66	3332 - 3839	0.213899
	67	3840 - 4453	0.214163
	68	4454 - 5205	0.214390
	69	5206 - 6144	0.214584
	70	6145 - 7340	0.214748
	71	7341 - 8908	0.214885
	72	8909 - 11042	0.214998
	73	11043 - 14098	0.215089
	74	14099 - 18813	0.215160
	75	18814 - 26997	0.215214
	76	26998 - 44608	0.215251
	77	44609 - 109560	0.215273
4	79	4356 - 4907	0.215338
	80	4908 - 5544	0.215544
	81	5545 - 6283	0.215727
	82	6284 - 7144	0.215888
	83	7145 - 8156	0.216029
	84	8157 - 9353	0.216153
	85	9354 - 10785	0.216261
	86	10786 - 12518	0.216354
	87	12519 - 14647	0.216434
	88	14648 - 17310	0.216503

(Contd.....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
4	89	17311 - 20718	0.216561
	90	20719 - 25215	0.216609
	91	25216 - 31389	0.216649
	92	31390 - 40354	0.216681
	93	40355 - 54492	0.216706
	94	54493 - 79982	0.216724
	95	79983 - 139409	0.216737
	96	139410 - 433503	0.216744
5	98	11908 - 13428	0.216786
	99	13429 - 15183	0.216861
	100	15184 - 17217	0.216927
	101	17218 - 19590	0.216986
	102	19591 - 22377	0.217037
	103	22378 - 25680	0.217082
	104	25681 - 29635	0.217121
	105	29636 - 34430	0.217155
	106	34431 - 40334	0.217184
	107	40335 - 47748	0.217209
	108	47749 - 57288	0.217230
	109	57289 - 69966	0.217247
	110	69967 - 87559	0.217262
	111	87560 - 113508	0.217273
	112	113509 - 155467	0.217282
	113	155468 - 234580	0.217288
	114	234581 - 438667	0.217292
	115	438668 - 2164130	0.217295

(Contd....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
6	117	32088 - 36208	0.217317
	118	36209 - 40962	0.217345
	119	40963 - 46475	0.217370
	120	46476 - 52909	0.217391
	121	52910 - 60474	0.217410
	122	60475 - 69450	0.217427
	123	69451 - 80215	0.217441
	124	80216 - 93300	0.217454
	125	93301 - 109465	0.217464
	126	109466 - 129851	0.217473
	127	129852 - 156241	0.217482
	128	156242 - 191603	0.217488
	129	191604 - 241259	0.217493
	130	241260 - 315814	0.217497
	131	315815 - 439865	0.217500
	132	439866 - 686479	0.217502
	133	686480 - 1412523	0.217504
	134	1412524 - 50871382	0.217505
7	136	85530 - 96558	0.217516
	137	96559 - 109286	0.217526
	138	109287 - 124055	0.217536
	139	124056 - 141306	0.217544
	140	141307 - 161616	0.217551
	141	161617 - 185752	0.217557
	142	185753 - 214764	0.217562
	143	214765 - 250124	0.217567
	144	250125 - 293973	0.217571
	145	293974 - 349543	0.217575

(Contd....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
7	146	349544 - 421964	0.217577
	147	421965 - 519905	0.217580
	148	519906 - 659262	0.217582
	149	659263 - 872696	0.217583
	150	872697 - 1239663	0.217584
	151	1239664 - 2017045	0.217585
	152	2017046 - 4760449	0.217585
8	154	200560 - 226015	0.217587
	155	226016 - 255266	0.217591
	156	255267 - 289043	0.217595
	157	289044 - 328272	0.217598
	158	328273 - 374147	0.217602
	159	374148 - 428234	0.217604
	160	428235 - 492633	0.217607
	161	492634 - 570231	0.217609
	162	570232 - 665111	0.217610
	163	665112 - 783258	0.217612
	164	783259 - 933806	0.217613
	165	933807 - 1131457	0.217614
	166	1131458 - 1401481	0.217615
	167	1401482 - 1791337	0.217616
	168	1791338 - 2401893	0.217617
	169	2401894 - 3492064	0.217617
	170	3492065 - 5987798	0.217617
	171	5987799 - 17518308	0.217617

(Contd.....)

(Table 14 - Contd.....)

c	n	N	δ (n, c)
9	173	526126 - 593128	0.217618
	174	593129 - 670168	0.217620
	175	670169 - 759202	0.217621
	176	759203 - 862716	0.217622
	177	862717 - 983927	0.217624
	178	983928 - 1127079	0.217625
	179	1127080 - 1297889	0.217626
	180	1297890 - 1504270	0.217627
	181	1504271 - 1757505	0.217627
	182	1757506 - 2074270	0.217628
	183	2074271 - 2480330	0.217628
	184	2480331 - 3017729	0.217629
	185	3017730 - 3760118	0.217629
	186	3760119 - 4849393	0.217629
	187	4849394 - 6598659	0.217630
	188	6598660 - 9862224	0.217630
	189	9862225 - 18095132	0.217630
	190	18095133 - 78780136	0.217630
10	192	1372404 - 1547777	0.217630
	193	1547778 - 1749578	0.217631
	194	1749579 - 1983018	0.217632
	195	1983019 - 2254750	0.217632
	196	2254751 - 2573416	0.217632
	197	2573417 - 2950470	0.217633
	198	2950471 - 3401440	0.217633
	199	3401441 - 3947956	0.217634
	200	3947957 - 4621105	0.217634
	201	4621106 - 5467293	0.217634
	202	5467294 - 6559068	0.217634

(Contd.....)

(Table 14 - Contd.....)

c	n	N	δ (n, c)
10	203	.6559069 - 8016642	0.217634
	204	8016643 - 10054861	0.217634
	205	10054862 - 13099323	0.217635
	206	13099324 - 18129056	0.217635
	207	18129057 - 28009965	0.217635
	208	28009966 - 56268951	0.217635
	209	56268952 - 835379968	0.217635
11	211	3563301 - 4020266	0.217635
	212	4020267 - 4546550	0.217635
	213	4546551 - 5156002	0.217635
	214	5156003 - 5866367	0.217636
	215	5866368 - 6700802	0.217636
	216	6700803 - 7690150	0.217636
	217	7690151 - 8876487	0.217636
	218	8876488 - 10318826	0.217636
	219	10318827 - 12102699	0.217636
	220	12102700 - 14357086	0.217636
	221	14357087 - 17286196	0.217636
	222	17286197 - 21233931	0.217637
	223	21233932 - 26828320	0.217637
	224	26828321 - 35351848	0.217637
	225	35351849 - 49895101	0.217637
	226	49895102 - 80279236	0.217637
	227	80279237 - 183280877	0.217637
12	229	8181890 - 9215948	0.217637
	230	9215949 - 10402273	0.217637
	231	10402274 - 11769869	0.217637
	232	11769870 - 13355451	0.217637

(Contd....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
12	233	13355452 - 15206254	0.217637
	234	15206255 - 17384197	0.217637
	235	17384198 - 19972200	0.217637
	236	19972201 - 23084077	0.217637
	237	23084078 - 26880634	0.217637
	238	26880635 - 31596992	0.217637
	239	31596993 - 37591510	0.217637
	240	37591511 - 45439205	0.217637
	241	45439206 - 56125310	0.217637
	242	56125311 - 71491433	0.217637
	243	71491434 - 95425758	0.217637
	244	95425759 - 137801995	0.217637
	245	137801996 - 233208071	0.217637
	246	233208072 - 647592070	0.217637
13	248	21084078 - 23758087	0.217637
	249	23758088 - 26828491	0.217637
	250	26828492 - 30371765	0.217638
	251	30371766 - 34485057	0.217638
	252	34485058 - 39293886	0.217638
	253	39293887 - 44963576	0.217638
	254	44963577 - 51716748	0.217638
	255	51716749 - 59860952	0.217638
	256	59860953 - 69834042	0.217638
	257	69834043 - 82282123	0.217638
	258	82282124 - 98201090	0.217638
	259	98201091 - 119211735	0.217638
	260	119211736 - 148142881	0.217638
	261	148142882 - 190417159	0.217638

(Contd....)

(Table 14 - Contd.....)

c	n	N	δ (n, c)
13	262	190417160 - 257911939	0.217638
	263	257911940 - 382638913	0.217638
	264	382638914 - 690803749	0.217638
	265	690803750 - 713421106	0.217638
14	267	54180575 - 61077215	0.217638
	268	61077216 - 69003581	0.217638
	269	69003582 - 78160992	0.217638
	270	78160993 - 88806158	0.217638
	271	88806159 - 101272186	0.217638
	272	101272187 - 116000002	0.217638
	273	116000003 - 133586764	0.217638
	274	133586765 - 154863008	0.217638
	275	154863009 - 181020590	0.217638
	276	181020591 - 213835226	0.217638
	277	213835227 - 256076605	0.217638
	278	256076606 - 312320612	0.217638
	279	312320613 - 390713355	0.217638
	280	390713356 - 507303810	0.217638
	281	507303811 - 698706754	0.217638
	282	698706755 - 1070592237	0.217638
	283	1070592238 - 2105774513	0.217638
	284	2105774514 - 20285434939	0.217638
15	286	138896541 - 156644237	0.217638
	287	156644238 - 177062157	0.217638
	288	177062158 - 200679613	0.217638
	289	200679614 - 228174201	0.217638
	290	228174202 - 260429101	0.217638

(Contd.....)

(Table 14 - Contd....)

c	n	N	δ (n, c)
15	291	260429102 - 298619391	0.217638
	292	298619392 - 344346011	0.217638
	293	344346012 - 399851182	0.217638
	294	399851183 - 468379464	0.217638
	295	468379465 - 554813808	0.217638
	296	554813809 - 666866319	0.217638
	297	666866320 - 817483874	0.217638
	298	817483875 - 1030210469	0.217638
	299	1030210470 - 1352837733	0.217638
	300	1352837734 - 1899473839	0.217638
	301	1899473840 - 3026727137	0.217638
	302	3026727138 - 6708310314	0.217638
	⋮	⋮ ⋮	⋮

TABLE 15. THE POINTS OF INTERSECTION FROM (5.5.27) AND (5.5.28) FOR $w_1 = 0.93$ AND $w_2 = 0.07$ USING (5.5.29).

c'	n'	c"	n"	N_{12}
0	13	1	23	443.4
0	14	1	24	444.5
0	14	1	25	409.4
0	15	1	26	412.9
0	15	1	27	401.2
0	16	1	28	406.2
0	17	1	29	406.6
0	17	1	30	411.5
0	18	1	31	412.7
0	18	1	32	424.3
0	19	1	33	425.7
0	19	1	34	442.1
0	20	1	35	443.2
0	20	1	36	463.3
0	20	1	37	484.7
0	21	1	38	487.0
0	21	1	39	511.4
1	39	2	54	1390.7
1	39	2	55	1454.1
1	39	2	56	1522.0
1	40	2	57	1523.9
1	40	2	58	1601.7
2	58	3	74	4128.5
2	58	3	75	4324.4
2	58	3	76	4533.8
3	76	4	91	10795.0
3	76	4	92	11254.1

(Contd....)

(Table 15 - Contd....)

c'	n'	c"	n"	N_{12}
3	76	4	93	11749.0
3	77	4	94	11782.3
3	77	4	95	12361.0
4	95	5	110	28655.7
4	95	5	111	30501.8
4	95	5	112	31276.6
4	96	5	113	30938.2
4	96	5	114	32517.2
5	114	6	129	74730.0
5	114	6	130	78151.8
5	114	6	131	81833.5
5	115	6	132	82229.4
5	115	6	133	86227.9
5	115	6	134	90579.2
6	134	7	151	212623.0
6	134	7	152	225122.0
7	152	8	168	500142.0
8	168	9	181	1300150.0
8	168	9	182	1272880.0
8	168	9	183	1363790.0
8	169	9	184	1250160.0
8	169	9	185	1333490.0
8	170	9	186	1333490.0
8	170	9	187	1307850.0
8	171	9	188	1307850.0
8	171	9	189	1384780.0
9	189	10	204	3750170.0
9	189	10	205	3200180.0

(Contd....)

(Table 15 - Contd....)

c'	n'	c"	n"	N_{12}
9	189	10	206	3400180.0
9	190	10	207	3400180.0
9	190	10	208	3600180.0
9	190	10	209	3800180.0
10	209	11	226	8500200.0
10	209	11	227	9000200.0
11	227	12	243	∞
11	227	12	244	∞
11	227	12	245	∞
12	245	13	260	15000200.0
12	245	13	261	16000200.0
12	245	13	262	17000200.0
12	246	13	263	17000200.0
12	246	13	264	18000200.0
13	264	14	279	∞
13	264	14	280	∞
13	264	14	281	∞
13	265	14	282	∞
14	282	15	297	∞
14	282	15	298	∞
14	282	15	299	∞
14	283	15	300	∞
14	284	15	302	∞

TABLE 16. THE OPTIMAL SINGLE SAMPLING TWO-DECISION BAYESIAN PLAN FOR $w_1 = 0.93$ AND $w_2 = 0.07$.

N	n	c	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$ (in percentage)	$P_s(p")$ (in percentage)
27-33	4	0	29	13.21	7.36	96.06	47.80
34-41	5	0	37	12.80	10.24	95.10	55.63
42-50	6	0	45	12.46	12.62	94.15	62.29
51-60	7	0	55	12.11	15.08	93.21	67.94
61-73	8	0	66	11.81	17.18	92.27	72.75
74-88	9	0	80	11.51	19.28	91.35	76.84
89-107	10	0	97	11.22	21.32	90.44	80.31
108-131	11	0	118	10.95	23.21	89.53	83.27
132-160	12	0	145	10.69	25.03	88.64	85.78
161-198	13	0	178	10.47	26.58	87.75	87.91
199-247	14	0	221	10.26	28.05	86.87	89.72
248-313	15	0	278	10.06	29.45	86.01	91.26
314-406	16	0	357	9.88	30.71	85.15	92.57
407-	28	1	407	9.80	31.28	96.82	93.73
408-411	29	1	409	9.79	31.35	96.60	94.51
412-413	30	1	412	9.79	31.35	96.39	95.20
414-424	31	1	418	9.78	31.42	96.16	95.80
425-426	32	1	425	9.78	31.42	95.93	96.34
427-442	33	1	434	9.77	31.49	95.70	96.80
443-	34	1	443	9.77	31.49	95.46	97.21
444-463	35	1	453	9.76	31.56	95.21	97.57
464-485	36	1	474	9.73	31.77	94.97	97.88
486-487	37	1	486	9.73	31.77	94.71	98.16
488-511	38	1	499	9.72	31.84	94.45	98.40

Contd......

(Table 16 - Contd.....)

N	n	c	$\bar{N}$	K/$\bar{N}$	(%) saving	$P_a(p')$	$P_s(p'')$
						(in percentage)	
512-1391	39	1	843	9.27	34.99	94.19	98.61
1392-1454	54	2	1422	8.98	37.03	98.30	99.15
1455-1522	55	2	1488	8.96	37.17	98.22	99.25
1523-1524	56	2	1523	8.95	37.24	98.13	99.34
1525-1602	57	2	1563	8.95	37.24	98.04	99.42
1603-4128	58	2	2572	8.73	37.38	97.95	99.49
4129-4324	74	3	4225	8.60	39.69	99.34	99.73
4325-4534	75	3	4428	8.59	39.76	99.31	99.76
4535-10795	76	3	6996	8.49	40.46	99.28	99.79
10796-11254	91	4	11022	8.43	40.88	99.77	99.87
11255-11749	92	4	11499	8.43	40.88	99.76	99.87
11750-11782	93	4	11765	8.42	40.95	99.75	99.90
11783-12361	94	4	12068	8.42	40.95	99.74	99.91
12362-28655	95	4	18821	8.38	41.23	99.72	99.92
28656-30502	110	5	29564	8.35	41.44	99.91	99.95
30503-30938	111	5	30719	8.35	41.44	99.91	99.96
30939-32517	113	5	31718	8.35	41.44	99.90	99.97
32518-74730	114	5	49295	8.33	41.58	99.89	99.97
74731-78152	129	6	76422	8.32	41.65	99.97	99.98
78153-81833	130	6	79971	8.32	41.65	99.96	99.98
81834-82229	131	6	82031	8.31	41.73	99.96	99.99
82230-86228	132	6	84205	8.31	41.73	99.96	99.99
86229-90579	133	6	88377	8.31	41.73	99.96	99.99
90580-212623	134	6	138778	8.30	41.80	99.96	99.99
212624-225122	151	7	218783	8.30	41.80	99.98	100.00
225123-500142	152	7	335549	8.30	41.80	99.98	100.00

Contd......

(Table 16 - Contd.....)

N	n	c	$\bar{\bar{N}}$	K/$\bar{\bar{N}}$	(%) saving	$P_a(p')$	$P_s(p'')$
						(in percentage)	
500143-1250160	168	8	790733	8.29	41.86	99.99	100.00
1250161-1307850	184	9	1278680	8.29	41.86	100.00	100.00
1307851-3200180	187	9	2045814	8.29	41.86	100.00	100.00
3200181-3400180	205	10	3298665	8.29	41.86	100.00	100.00
3400181-3600180	206	10	3498751	8.29	41.86	100.00	100.00
3600181-3800180	208	10	3698828	8.29	41.86	100.00	100.00
3800181-8500200	209	10	5683511	8.29	41.86	100.00	100.00
8500201-9000200	226	11	8746628	8.29	41.86	100.00	100.00
9000201-15000200	227	11	11619157	8.29	41.86	100.00	100.00
15000201-16000200	260	13	15492134	8.29	41.86	100.00	100.00
16000201-17000200	261	13	16492623	8.29	41.86	100.00	100.00

TABLE 17. THE VALUES OF c_1, c_2, n AND N OBTAINED FROM (5.5.7), (5.5.8) AND (5.5.12) FOR $w_1 = 0.10$ AND $w_2 = 0.90$.

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
0	1	26	60 - 68	0.186846	0.076045
		27	69 - 79	0.202347	0.078595
		28	80 - 93	0.215088	0.080816
		29	94 - 109	0.225486	0.082743
		30	110 - 130	0.233899	0.084408
		31	131 - 157	0.240627	0.085841
		32	158 - 194	0.245927	0.087066
		33	195 - 245	0.250019	0.088107
		34	246 - 320	0.253067	0.088986
		35	321 - 440	0.255289	0.089720
		36	441 - 662	0.256759	0.090327
		37	663 - 1200	0.257611	0.090822
		38	1201 - 4287	0.257942	0.091217
1	2	45	166 - 189	0.323041	0.092908
		46	190 - 218	0.328613	0.093687
		47	219 - 253	0.333287	0.094370
		48	254 - 297	0.337177	0.094966
		49	298 - 354	0.340383	0.095484
		50	355 - 428	0.342992	0.095934
		51	429 - 528	0.345081	0.096320
		52	529 - 670	0.346715	0.096651
		53	671 - 885	0.347954	0.096933
		54	886 - 1246	0.348848	0.097169
		55	1247 - 1970	0.349441	0.097366

Contd.....

(Table 17 - Contd.....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
1	2	56	1971 - 4125	0.349772	0.097527
		57	4126 - 243569	0.349875	0.097656
2	3	63	376 - 429	0.371348	0.097989
		64	430 - 493	0.373689	0.098287
		65	494 - 571	0.375682	0.098550
		66	572 - 668	0.377367	0.098781
		67	669 - 789	0.378780	0.098984
		68	790 - 944	0.379951	0.099160
		69	945 - 1149	0.380910	0.099313
		70	1150 - 1429	0.381682	0.099445
		71	1430 - 1832	0.382287	0.099559
		72	1833 - 2457	0.382746	0.099655
		73	2458 - 3549	0.383074	0.099736
		74	3550 - 5921	0.383288	0.099804
		75	5922 - 14895	0.383400	0.099860
3	4	82	973 - 1116	0.391574	0.100043
		83	1117 - 1291	0.392423	0.100149
		84	1292 - 1503	0.393148	0.100242
		85	1504 - 1765	0.393763	0.100324
		86	1766 - 2096	0.394280	0.100395
		87	2097 - 2521	0.394710	0.100458
		88	2522 - 3085	0.395063	0.100512
		89	3086 - 3865	0.395346	0.100559
		90	3866 - 5005	0.395568	0.100599
		91	5006 - 6819	0.395736	0.100634
		92	6820 - 10127	0.395854	0.100663
		93	10128 - 18015	0.395929	0.100687
		94	18016 - 60778	0.395965	0.100706

Contd.....

(Table 17 - Contd.....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
4	5	101	2548 - 2936	0.399128	0.100781
		102	2937 - 3405	0.399441	0.100819
		103	3406 - 3977	0.399708	0.100853
		104	3978 - 4686	0.399935	0.100883
		105	4687 - 5581	0.400127	0.100909
		106	5582 - 6740	0.400286	0.100932
		107	6741 - 8290	0.400416	0.100951
		108	8291 - 10456	0.400521	0.100969
		109	10457 - 13677	0.400602	0.100983
		110	13678 - 18940	0.400663	0.100996
		111	18941 - 29034	0.400706	0.101006
		112	29035 - 56059	0.400732	0.101015
		113	56060 - 357877	0.400742	0.101022
5	6	120	6715 - 7754	0.401984	0.101053
		121	7755 - 9008	0.402101	0.101067
		122	9009 - 10541	0.402200	0.101079
		123	10542 - 12445	0.402285	0.101090
		124	12446 - 14861	0.402357	0.101100
		125	14862 - 18007	0.402416	0.101108
		126	18008 - 22250	0.402464	0.101116
		127	22251 - 28253	0.402503	0.101122
		128	28254 - 37350	0.402533	0.101127
		129	37351 - 52685	0.402556	0.101132
		130	52686 - 83863	0.402571	0.101136
		131	83864 - 181088	0.402579	0.101139
6	7	138	15390 - 17701	0.403024	0.101148
		139	17702 - 20461	0.403075	0.101154

Contd....

(Table 17 - Contd....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
6	7	140	20462 - 23797	0.403119	0.101159
		141	23798 - 27885	0.403156	0.101164
		142	27886 - 32981	0.403188	0.101168
		143	32982 - 39475	0.403215	0.101172
		144	39476 - 47989	0.403237	0.101175
		145	47990 - 59582	0.403255	0.101178
		146	59583 - 76209	0.403270	0.101180
		147	76210 - 101943	0.403281	0.101182
		148	101944 - 146885	0.403289	0.101184
		149	146886 - 244921	0.403294	0.101185
		150	244922 - 623784	0.403297	0.101187
7	8	157	40436 - 46534	0.403476	0.101191
		158	46535 - 53830	0.403495	0.101193
		159	53831 - 62665	0.403511	0.101195
		160	62666 - 73521	0.403526	0.101197
		161	73522 - 87110	0.403538	0.101198
		162	87111 - 104521	0.403548	0.101199
		163	104522 - 127518	0.403556	0.101201
		164	127519 - 159156	0.403563	0.101202
		165	159157 - 205230	0.403569	0.101203
		166	205231 - 278245	0.403573	0.101204
		167	278246 - 411080	0.403576	0.101204
		168	411081 - 727056	0.403577	0.101205
		169	727057 - 2435759	0.403578	0.101205
8	9	176	105850 - 121874	0.403650	0.101207
		177	121875 - 141083	0.403658	0.101208

Contd.....

(Table 17 - Contd.....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
8	9	178	141084 - 164399	0.403664	0.101208
		179	164400 - 193147	0.403669	0.101209
		180	193148 - 229288	0.403674	0.101210
		181	229289 - 275871	0.403678	0.101210
		182	275872 - 337903	0.403681	0.101211
		183	337904 - 424227	0.403684	0.101211
		184	424228 - 552080	0.403686	0.101211
		185	552081 - 760171	0.403687	0.101212
		186	760172 - 1157240	0.403688	0.101212
		187	1157241 - 2211353	0.403689	0.101212
		188	2211354 - 13286787	0.403689	0.101212
9	10	195	276013 - 317982	0.403718	0.101213
		196	317983 - 368396	0.403721	0.101213
		197	368397 - 429763	0.403723	0.101214
		198	429764 - 505702	0.403726	0.101214
		199	505703 - 601637	0.403727	0.101214
		200	601638 - 726098	0.403729	0.101214
		201	726099 - 893316	0.403730	0.101214
		202	893317 - 1128969	0.403731	0.101215
		203	1128970 - 1484600	0.403732	0.101215
		204	1484601 - 2081205	0.403732	0.101215
		205	2081206 - 3285702	0.403733	0.101215
		206	3285703 - 6981463	0.403733	0.101215
10	11	213	625088 - 717169	0.403744	0.101215
		214	717170 - 826785	0.403745	0.101215
		215	826786 - 958769	0.403746	0.101216

Contd.....

(Table 17 - Contd.....)

c_1	c_2	n	N	$\delta_1(n,c_1)$	$\delta_2(n,c_2)$
10	11	216	958770 - 1119925	0.403746	0.101216
		217	1119926 - 1320152	0.403748	0.101216
		218	1320153 - 1574448	0.403748	0.101216
		219	1574449 - 1906704	0.403749	0.101216
		220	1906705 - 2357466	0.403749	0.101216
		221	2357467 - 3001560	0.403750	0.101216
		222	3001561 - 3994102	0.403750	0.101216
		223	3994103 - 5717775	0.403750	0.101216
		224	5717776 - 9442121	0.403750	0.101216
		225	9442122 - 23432468	0.403750	0.101216

TABLE 18. THE POINTS OF INTERSECTION FROM (5.5.7), (5.5.8) AND (5.5.12) FOR $w_1 = 0.10$ AND $w_2 = 0.90$ USING (5.4.7).

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
0	1	32	1	2	45	174.7
0	1	32	1	2	46	135.6
0	1	33	1	2	47	177.3
0	1	34	1	2	48	178.1
0	1	34	1	2	49	182.8
0	1	35	1	2	50	184.2
0	1	35	1	2	51	190.4
0	1	36	1	2	52	191.9
0	1	37	1	2	53	192.9
0	1	37	1	2	54	200.9
0	1	38	1	2	55	202.0
0	1	38	1	2	56	210.8
0	1	38	1	2	57	220.0
1	2	57	2	3	74	524.6
1	2	57	2	3	75	550.0
2	3	75	3	4	91	1284.9
2	3	75	3	4	92	1246.4
2	3	75	3	4	93	1411.4
3	4	93	4	5	108	3159.8
3	4	93	4	5	109	3302.0
3	4	93	4	5	110	3452.8
3	4	94	4	5	111	3455.3
3	4	94	4	5	112	3628.6
3	4	94	4	5	113	3812.6
4	5	113	5	6	130	8851.2
4	5	113	5	6	131	9313.2

(Contd....)

(Table 18 - Contd....)

c_1'	c_2'	n_1	c_1''	c_2''	n_2	N_{12}
5	6	131	6	7	147	21596.4
5	6	131	6	7	148	22636.2
5	6	131	6	7	149	23772.3
6	7	149	7	8	164	52585.3
6	7	149	7	8	165	54745.2
6	7	149	7	8	166	57184.4
6	7	150	7	8	167	57571.0
6	7	150	7	8	168	60540.8
7	8	168	8	9	183	132900.2
7	8	168	8	9	184	139287.0
7	8	168	8	9	185	145455.5
7	8	169	8	9	186	145456.7
7	8	169	8	9	187	152699.5
7	8	169	8	9	188	161173.4
8	9	188	9	10	205	361878.9
8	9	188	9	10	206	383154.8
9	10	206	10	11	222	889083.3
9	10	206	10	11	223	944638.2
9	10	206	10	11	224	1000193.0

TABLE 19. THE OPTIMAL SINGLE SAMPLING UNRESTRICTED ASR BAYESIAN PLANS FOR $w_1 = 0.10$ AND $w_2 = 0.90$.

N	n	c_1	c_2	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$	$P_s(p')$	$P_r(p'')$	$P_s(p'')$
							(in percentage)			
60-68	26	0	1	63	23.29	76.21	77.00	20.22	91.83	6.71
69-79	27	0	1	73	22.85	76.66	76.23	20.79	92.84	5.92
80-93	28	0	1	86	22.39	77.13	75.47	21.35	93.73	5.22
94-109	29	0	1	101	21.99	77.54	74.72	21.89	94.51	4.59
110-130	30	0	1	119	21.62	77.92	73.97	22.42	95.20	4.04
131-157	31	0	1	143	21.25	78.30	73.23	22.93	95.80	3.55
158-174	32	0	1	165	21.00	78.55	72.50	23.43	96.34	3.11
175-177	45	1	2	175	21.46	78.08	92.54	6.43	97.35	2.06
178-	47	1	2	177	21.44	78.10	91.95	6.88	97.93	1.62
179-182	48	1	2	180	21.41	78.13	91.66	7.10	98.17	1.44
183-184	49	1	2	183	21.39	78.16	91.36	7.33	98.39	1.27
185-190	50	1	2	187	21.36	78.19	91.06	7.56	98.58	1.13
191-	51	1	2	190	21.34	78.21	90.75	7.79	98.75	1.00
192-	52	1	2	191	21.36	78.19	90.44	8.02	98.90	0.88
193-200	53	1	2	196	21.32	78.22	90.13	8.25	99.03	0.78
201 -	54	1	2	200	21.30	78.25	89.82	8.49	99.15	0.69
202-210	55	1	2	205	21.28	78.27	89.50	8.72	99.25	0.61
211-220	56	1	2	215	21.20	78.35	89.18	8.95	99.34	0.53
221-524	57	1	2	340	20.36	79.21	88.86	9.18	99.42	0.47
525-549	74	2	3	536	20.03	79.54	96.16	3.18	99.73	0.21
550-1284	75	2	3	840	19.59	79.99	96.03	3.27	99.76	0.19
1285-1346	91	3	4	1315	19.38	80.21	98.66	1.11	99.87	0.10
1347-1411	92	3	4	1378	19.35	80.24	98.61	1.15	99.89	0.09
1412-3159	93	3	4	2111	19.14	80.45	98.56	1.19	99.90	0.08
3160-3302	108	4	5	3230	19.04	80.56	99.52	0.40	99.94	0.05
3303-3452	109	4	5	3376	19.03	80.57	99.51	0.41	99.95	0.04

Contd......

(Table 19 - Contd.....)

N	n	c_1	c_2	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$	$P_s(p')$	$P_r(p'')$	$P_s(p'')$
							(in percentage)			
3453-3455	111	4	5	3453	19.02	80.58	99.47	0.44	99.96	0.03
3456-3628	112	4	5	3540	19.02	80.58	99.45	0.46	99.96	0.03
3629-3812	113	4	5	3719	19.00	80.60	99.42	0.47	99.97	0.03
3813-8851	130	5	6	5809	18.93	80.67	99.79	0.17	99.98	0.01
8852-9313	131	5	6	9079	18.85	80.75	99.78	0.18	99.99	0.01
9314-21596	147	6	7	14182	18.81	80.79	99.92	0.06	99.99	0.01
21597-22636	148	6	7	22110	18.78	80.82	99.92	0.06	99.99	0.01
22637-23772	149	6	7	23197	18.78	80.82	99.92	0.07	99.99	0.00
23773-52585	164	7	8	33356	18.76	80.84	99.97	0.02	100.00	0.00
52586-54745	165	7	8	53654	18.75	80.85	99.97	0.02	100.00	0.00
54746-57184	166	7	8	55951	18.75	80.85	99.97	0.02	100.00	0.00
57185-57571	167	7	8	57377	18.75	80.85	99.97	0.03	100.00	0.00
57572-60541	168	7	8	59037	18.75	80.85	99.97	0.03	100.00	0.00
60542-132900	183	8	9	89699	18.74	80.86	99.99	0.01	100.00	0.00
132901-139287	184	8	9	136056	18.73	80.87	99.99	0.01	100.00	0.00
139288-145455	185	8	9	142338	18.73	80.87	99.99	0.01	100.00	0.00
145456-	186	8	9	145455	18.73	80.87	99.99	0.01	100.00	0.00
145457-152699	187	8	9	149034	18.73	80.87	99.99	0.01	100.00	0.00
152700-161173	188	8	9	156879	18.73	80.87	99.99	0.01	100.00	0.00
161174-361878	205	9	10	241506	18.73	80.87	100.00	0.00	100.00	0.00
361879-383154	206	9	10	372364	18.72	80.88	99.99	0.00	100.00	0.00
383155-889083	222	10	11	583657	18.72	80.88	100.00	0.00	100.00	0.00
889084-944638	223	10	11	916440	18.72	80.88	100.00	0.00	100.00	0.00
944639-1000193	224	10	11	972019	18.72	80.88	100.00	0.00	100.00	0.00

Note: Cost of acceptance without inspection = 97.92 money units.

TABLE 20. THE VALUES OF c, n AND N OBTAINED FROM (5.5.27) AND (5.5.28) FOR $w_1 = 0.10$ AND $w_2 = 0.90$.

c	n	N	δ (n, c)
0	20	30 - 33	0.001277
	21	34 - 38	0.047319
	22	39 - 44	0.085992
	23	45 - 50	0.118407
	24	51 - 57	0.145506
	25	58 - 66	0.168092
	26	67 - 77	0.186846
	27	78 - 90	0.202347
	28	91 - 107	0.215087
	29	108 - 127	0.225486
	30	128 - 154	0.233898
	31	155 - 189	0.240627
	32	190 - 238	0.245927
	33	239 - 309	0.250019
	34	310 - 420	0.253087
	35	421 - 615	0.255289
	36	616 - 1038	0.256759
	37	1039 - 2621	0.257611
1	39	90 - 99	0.262074
	40	100 - 111	0.276481
	41	112 - 125	0.288894
	42	126 - 141	0.299549
	43	142 - 160	0.308668
	44	161 - 183	0.316442
	45	184 - 210	0.323041
	46	211 - 244	0.328613

Contd....

(Table 20 - Contd....)

c	n	N	σ (n, c)
1	47	245 - 286	0.333287
	48	287 - 340	0.337177
	49	341 - 408	0.340383
	50	409 - 500	0.342992
	51	501 - 627	0.345081
	52	628 - 814	0.346715
	53	815 - 1110	0.347954
	54	1111 - 1648	0.348848
	55	1649 - 2913	0.349441
	56	2914 - 9288	0.349772
2	58	226 - 252	0.352583
	59	253 - 284	0.357484
	60	285 - 320	0.361742
	61	321 - 364	0.365429
	62	365 - 415	0.368611
	63	416 - 477	0.371548
	64	478 - 551	0.373681
	65	552 - 642	0.375682
	66	643 - 756	0.377367
	67	757 - 900	0.373779
	68	901 - 1086	0.379951
	69	1087 - 1336	0.380910
	70	1337 - 1686	0.381682
	71	1687 - 2206	0.382287
	72	2207 - 3051	0.382746
	73	3052 - 4656	0.383074
	74	4657 - 8828	0.383288

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
2	75	8829 - 45341	0.383400
3	77	569 - 641	0.384863
	78	642 - 726	0.386604
	79	727 - 825	0.388122
	80	826 - 942	0.389442
	81	943 - 1080	0.390587
	82	1081 - 1246	0.391574
	83	1247 - 1447	0.392423
	84	1448 - 1693	0.393148
	85	1694 - 2000	0.393763
	86	2001 - 2390	0.394280
	87	2391 - 2898	0.394710
	88	2899 - 3584	0.395062
	89	3585 - 4553	0.395346
	90	4554 - 6017	0.395568
	91	6018 - 8466	0.395736
	92	8467 - 13361	0.395854
	93	13362 - 27898	0.395929
	94	27899 - 1494586	0.395965
4	96	1468 - 1662	0.396673
	97	1663 - 1890	0.397307
	98	1891 - 2155	0.397862
	99	2156 - 2469	0.398345
	100	2470 - 2840	0.398765
	101	2841 - 3285	0.399128
	102	3286 - 3824	0.399440
	103	3825 - 4486	0.399708

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
4	104	4487 - 5312	0.399935
	105	5313 - 6366	0.400127
	106	6367 - 7749	0.400286
	107	7750 - 9629	0.400416
	108	9630 - 12318	0.400521
	109	12319 - 16457	0.400602
	110	16458 - 23607	0.400663
	111	23608 - 38834	0.400706
	112	38835 - 93141	0.400732
5	114	3390 - 3839	0.400804
	115	3840 - 4359	0.401072
	116	4360 - 4965	0.401307
	117	4966 - 5673	0.401513
	118	5674 - 6507	0.401692
	119	6508 - 7497	0.401849
	120	7498 - 8684	0.401984
	121	8685 - 10124	0.402100
	122	10125 - 11895	0.402200
	123	11896 - 14113	0.402285
	124	14114 - 16955	0.402357
	125	16956 - 20705	0.402416
	126	20706 - 25854	0.402464
	127	25855 - 33321	0.402503
	128	33322 - 45064	0.402533
	129	45065 - 66122	0.402556
	130	66123 - 114592	0.402571
	131	114593 - 341948	0.402579

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
6	133	8898 - 10088	0.402632
	134	10089 - 11466	0.402732
	135	11467 - 13068	0.402820
	136	13069 - 14943	0.402898
	137	14944 - 17152	0.402965
	138	17153 - 19777	0.403024
	139	19778 - 22926	0.403075
	140	22927 - 26752	0.403119
	141	26753 - 31472	0.403156
	142	31473 - 37406	0.403188
	143	37407 - 45048	0.403215
	144	45049 - 55207	0.403237
	145	55208 - 69299	0.403255
	146	69300 - 90056	0.403270
	147	90057 - 123513	0.403281
	148	123514 - 186185	0.403289
	149	186186 - 345233	0.403294
	150	345234 - 1551094	0.403297
7	152	23348 - 26481	0.403327
	153	26482 - 30107	0.403365
	154	30108 - 34325	0.403398
	155	34326 - 39261	0.403428
	156	39262 - 45082	0.403453
	157	45083 - 52004	0.403476
	158	52005 - 60325	0.403495
	159	60326 - 70458	0.403511
	160	70459 - 82996	0.403526
	161	82997 - 98825	0.403538

Contd.....

(Table 20 - Contd....)

c	n	N	δ (n, c)
7	162	98826 - 119333	0.403548
	163	119334 - 146816	0.403556
	164	146817 - 185387	0.403563
	165	165388 - 243196	0.403568
	166	243197 - 339027	0.403573
	167	339028 - 528038	0.403576
	168	528039 - 1072438	0.403577
	169	1072439 - 18245953	0.403578
8	171	61077 - 69678	0.403594
	172	69679 - 78772	0.403608
	173	78773 - 89821	0.403621
	174	89822 - 102761	0.403632
	175	102762 - 118033	0.403642
	176	118034 - 136223	0.403650
	177	136224 - 158132	0.403658
	178	158133 - 184881	0.403664
	179	184882 - 218098	0.403669
	180	218099 - 260236	0.403674
	181	260237 - 315184	0.403678
	182	315185 - 389491	0.403681
	183	389492 - 495130	0.403684
	184	495131 - 656575	0.403686
	185	656576 - 932929	0.403687
	186	932930 - 1512682	0.403688
	187	1512683 - 3496136	0.403689
9	189	140597 - 159144	0.403690
	190	159145 - 180521	0.403697

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
9	191	180522 - 205276	0.403702
	192	205277 - 234103	0.403707
	193	234104 - 267898	0.403711
	194	267899 - 307833	0.403715
	195	307834 - 355481	0.403718
	196	355482 - 413000	0.403721
	197	413001 - 483439	0.403723
	198	483440 - 571257	0.403726
	199	571258 - 683255	0.403727
	200	683256 - 830346	0.403729
	201	830347 - 1031245	0.403730
	202	1031246 - 1320962	0.403731
	203	1320963 - 1773513	0.403732
	204	1773514 - 2577284	0.403732
	205	2577285 - 4395076	0.403733
	206	4395077 - 12396673	0.403733
10	208	364917 - 413064	0.403734
	209	413065 - 468577	0.403736
	210	468578 - 532901	0.403739
	211	532902 - 607871	0.403740
	212	607872 - 695860	0.403742
	213	695861 - 799993	0.403744
	214	799994 - 924482	0.403745
	215	924483 - 1075145	0.403746
	216	1075146 - 1260267	0.403747
	217	1260268 - 1492075	0.403748
	218	1492076 - 1789425	0.403748

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
10	219	1789426 - 2183016	0.403749
	220	2183017 - 2726467	0.403749
	221	2726468 - 3522660	0.403750
	222	3522661 - 4797290	0.403750
	223	4797291 - 7160214	0.403750
	224	7160215 - 13028723	0.403750
	225	13028724 - 52477831	0.403750
11	227	943741 - 1068325	0.403751
	228	1068326 - 1212051	0.403752
	229	1212052 - 1378715	0.403753
	230	1378716 - 1573159	0.403754
	231	1573160 - 1801671	0.403754
	232	1801672 - 2072573	0.403755
	233	2072574 - 2397141	0.403755
	234	2397142 - 2791058	0.403756
	235	2791059 - 3276839	0.403756
	236	3276840 - 3888032	0.403756
	237	3888033 - 4676986	0.403757
	238	4676987 - 5730234	0.403757
	239	5730235 - 7201935	0.403757
	240	7201936 - 9396145	0.403757
	241	9396146 - 13007735	0.403757
	242	13007736 - 20048111	0.403757
	243	20048112 - 39770472	0.403757
	244	39770473 - 414991121	0.403757
12	246	2432948 - 2754460	0.403758
	247	2754461 - 3125627	0.403758

Contd.....

(Table 20 - Contd.....)

c	n	N	σ (n, c)
12	248	3125628 - 3556416	0.403758
	249	3556417 - 4059594	0.403759
	250	4059595 - 4651807	0.403759
	251	4651808 - 5355206	0.403759
	252	5355207 - 6199978	0.403759
	253	6199979 - 7228408	0.403760
	254	7228409 - 8501697	0.403760
	255	8501698 - 10111980	0.403760
	256	10111981 - 12204829	0.403760
	257	12204830 - 15024767	0.403760
	258	15024768 - 19016830	0.403760
	259	19016831 - 25085488	0.403760
	260	25085489 - 35394459	0.403760
	261	35394460 - 56729579	0.403760
	262	56729580 - 126992061	0.403760
13	264	5534872 - 6254788	0.403760
	265	6254789 - 7082637	0.403760
	266	7082638 - 8039099	0.403760
	267	8039100 - 9150333	0.403761
	268	9150334 - 10449975	0.403761
	269	10449976 - 11982081	0.403761
	270	11982082 - 13805593	0.403761
	271	13805594 - 16001329	0.403761
	272	16001330 - 18683326	0.403761
	273	18683327 - 22018081	0.403761
	274	22018082 - 26258912	0.403761
	275	26258913 - 31811414	0.403761

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
13	276	31811415 - 39368638	0.403761
	277	39368639 - 50221313	0.403761
	278	50221314 - 67080339	0.403761
	279	67080340 - 96773077	0.403761
	280	96773078 - 162842243	0.403761
	281	162842244 - 437435780	0.403761
14	283	14192910 - 16041705	0.403761
	284	16041706 - 18169184	0.403761
	285	18169185 - 20629387	0.403761
	286	20629388 - 23490924	0.403761
	287	23490925 - 26842378	0.403762
	288	26842379 - 30800328	0.403762
	289	30800329 - 35521606	0.403762
	290	32521607 - 41222631	0.403762
	291	41222632 - 48211110	0.403762
	292	48211111 - 56940414	0.403762
	293	56940415 - 68108074	0.403762
	294	68108075 - 82846736	0.403762
	295	82846737 - 103127494	0.403762
	296	103127495 - 132713878	0.403762
	297	132713879 - 179801851	0.403762
	298	179801852 - 266273485	0.403762
	299	266273486 - 476749965	0.403762
	300	476749966 - 1752652728	0.403762
15	302	36316922 - 41056503	0.403762
	303	41056504 - 46514833	0.403762

Contd.....

(Table 20 - Contd.....)

c	n	N	δ (n, c)
15	304	46514834 - 52833059	0.403762
	305	52833060 - 60191059	0.403762
	306	60191060 - 68822073	0.403762
	307	68822074 - 79034571	0.403762
	308	79034572 - 91245907	0.403762
	309	91245908 - 106035869	0.403762
	310	106035870 - 124235360	0.403762
	311	124235361 - 147080355	0.403762
	312	147080356 - 176494981	0.403762
	313	176494982 - 215650549	0.403762
	314	215650550 - 270175147	0.403762
	315	270175148 - 351109443	0.403762
	316	351109444 - 483481217	0.403762
	317	483481218 - 738737818	0.403762
	318	738737819 - 1435452562	0.403762
	319	1435452563 - 11038902489	0.403762

TABLE 21. THE POINTS OF INTERSECTION FROM (5.5.27) AND (5.5.28) FOR $w_1 = 0.10$ AND $w_2 = 0.90$ USING (5.5.29).

c'	n'	c''	n''	N_{12}
0	27	1	39	199.4
0	28	1	40	198.6
0	29	1	41	196.5
0	29	1	42	186.2
0	30	1	43	187.3
0	31	1	44	187.5
0	31	1	45	187.1
0	32	1	46	188.6
0	33	1	47	189.2
0	33	1	48	193.6
0	34	1	49	194.9
0	34	1	50	201.1
0	35	1	51	202.7
0	36	1	52	203.7
0	36	1	53	212.0
0	37	1	54	213.0
0	37	1	55	222.4
1	55	2	70	510.0
1	55	2	71	531.7
1	55	2	72	554.7
1	56	2	73	555.9
1	56	2	74	582.1
1	56	2	75	609.5
2	75	3	92	1429.2
2	75	3	93	1500.4
2	75	3	94	1575.3

(Contd....)

(Table 21 - Contd....)

c'	n'	c"	n"	N_{12}
3	94	4	111	3668.8
3	94	4	112	3858.5
4	112	5	128	8985.5
4	112	5	129	9420.9
4	112	5	130	9888.3
5	130	6	145	22048.7
5	130	6	146	23008.6
5	130	6	147	24062.1
5	131	6	148	24063.3
5	131	6	149	25294.0
6	149	7	165	58531.8
6	149	7	166	61069.4
6	149	7	167	63966.9
6	150	7	168	64423.8
6	150	7	169	67753.2
7	169	8	186	154703.0
7	169	8	187	162319.0
8	187	9	203	372269.0
8	187	9	204	395524.0
8	187	9	205	409266.0
9	205	10	220	937693.0
9	205	10	221	941370.0
9	205	10	222	1000190.0
9	206	10	223	1000190.0
9	206	10	224	1059020.0
10	224	11	239	2143070.0
10	224	11	240	2285930.0

(Contd....)

(Table 21 - Contd....)

c'	n'	c"	n"	N_{12}
10	224	11	241	2428780.0
10	224	11	242	2571640.0
10	225	11	243	2571640.0
10	225	11	244	2714500.0
11	244	12	261	5666900.0
11	244	12	262	6000230.0
12	262	13	279	17000200.0
12	262	13	280	18000200.0
13	280	14	295	7500280.0
13	280	14	296	8000280.0
13	280	14	297	8520280.0
13	281	14	298	17000300.0
13	281	14	299	18000300.0
14	299	15	314	∞
14	299	15	315	∞
14	299	15	316	∞
14	300	15	317	∞
14	300	15	318	∞
14	300	15	319	∞

TABLE 22. THE OPTIMAL SINGLE SAMPLING TWO-DECISION BAYESIAN PLANS FOR w_1=0.10 AND w_2 = 0.90

N	n	c	N	K/$\bar{N}$	(%) saving	$P_a(p')$	$P_s(p'')$
						(in percentage)	
30 - 33	20	0	31	26.51	72.92	81.79	96.12
34 - 38	21	0	35	26.17	73.27	80.97	96.71
39 - 44	22	0	41	25.75	73.70	80.16	97.20
45 - 50	23	0	47	25.38	74.08	79.36	97.62
51 - 57	24	0	53	25.06	74.41	78.57	97.98
58 - 66	25	0	61	24.71	74.76	77.78	98.28
67 - 77	26	0	71	24.36	75.12	77.00	98.54
78 - 90	27	0	83	24.03	75.46	76.23	98.76
91 - 107	28	0	98	23.71	75.78	75.47	98.94
108 - 127	29	0	117	23.41	76.09	74.72	99.10
128 - 154	30	0	140	23.14	76.37	73.97	99.24
155 - 187	31	0	170	22.88	76.63	73.23	99.35
188 - 189	46	1	188	23.12	76.39	92.25	99.48
190 - 193	47	1	191	23.09	76.42	91.95	99.55
194 -	48	1	193	23.07	76.44	91.66	99.61
195 - 201	49	1	197	23.04	76.47	91.36	99.66
202 -	50	1	201	23.02	76.49	91.06	99.71
203 -	51	1	202	23.02	76.49	90.75	99.75
204 - 212	52	1	207	22.99	76.52	90.44	99.78
213 -	53	1	212	22.97	76.54	90.13	99.81
214 - 222	54	1	217	22.95	76.56	89.82	99.84
223 - 510	55	1	337	22.39	77.13	89.50	99.86
511 - 531	70	2	520	22.16	77.36	96.67	99.90
532 - 554	71	2	542	22.12	77.41	96.41	99.91
555 -	72	2	555	22.11	77.42	96.42	99.92

Contd.....

(Table 22 - Contd.....)

N	n	c	$\bar{N}$	K/$\bar{N}$	(%) saving	$P_a(p')$ (in percentage)	$P_s(p'')$
556 - 582	73	2	568	22.09	77.44	96.29	99.93
583 - 609	74	2	595	22.06	77.47	96.16	99.94
610 - 1429	75	2	933	21.77	77.77	96.03	99.95
1430 - 1500	92	3	1464	21.63	77.91	98.61	99.97
1501 - 1575	93	3	1537	21.61	77.93	98.56	99.98
1576 - 3668	94	3	2404	21.47	78.07	98.50	99.98
3669 - 3858	111	4	3762	21.40	78.14	99.47	99.99
3859 - 8985	112	4	5888	21.33	78.22	99.45	99.99
8986 - 9420	128	5	9200	21.29	78.26	99.81	99.99
9421 - 9888	129	5	9651	21.29	78.26	99.80	100.00
9889 - 22048	130	5	14765	21.26	78.29	99.79	100.00
22049 - 23008	145	6	22523	21.24	78.31	99.93	100.00
23009 - 24062	146	6	23529	21.24	78.31	99.93	100.00
24063 -	147	6	24062	21.24	78.31	99.92	100.00
24064 - 25294	148	6	24671	21.24	78.31	99.92	100.00
25295 - 58531	149	6	38477	21.22	78.33	99.93	100.00
58532 - 61069	165	7	59787	21.21	78.34	99.97	100.00
61070 - 63966	166	7	62501	21.21	78.34	99.97	100.00
63967 - 64423	167	7	64194	21.21	78.34	99.97	100.00
64424 - 67753	168	7	66067	21.21	78.34	99.97	100.00
67754 - 154703	169	7	102380	21.20	78.35	99.97	100.00
154704 - 162319	186	8	158465	21.20	78.35	99.99	100.00
162320 - 372269	187	8	245818	21.20	78.35	99.99	100.00
372270 - 395524	203	9	383720	21.20	78.35	100.00	100.00
395525 - 409266	204	9	402336	21.20	78.35	100.00	100.00
409267 - 937693	205	9	619489	21.20	78.35	100.00	100.00

Contd.....

(Table 22 - Contd.....)

N	n	c	$\bar{N}$	$K/\bar{N}$	(%) saving	$P_a(p')$	$P_s(p'')$
						(in percentage)	
937694 - 941370	220	10	939530	21.20	78.35	100.00	100.00
941371 - 1000190	221	10	970334	21.20	78.35	100.00	100.00
1000191 -	222	10	1000190	21.20	78.35	100.00	100.00
1000192 - 1059020	223	10	1029185	21.20	78.35	100.00	100.00
1059021 - 2143070	224	10	1506504	21.20	78.35	100.00	100.00
2143071 - 2285930	239	11	2213348	21.20	78.35	100.00	100.00
2285931 - 2428780	240	11	2356273	21.19	78.36	100.00	100.00
2428781 - 2571640	241	11	2499189	21.19	78.36	100.00	100.00
2571641 - 2714500	242	11	2642105	21.19	78.36	100.00	100.00
2714501 - 5666900	244	11	3922092	21.19	78.36	100.00	100.00
5666901 - 6000230	261	12	5831184	21.19	78.36	100.00	100.00
6000231 - 7500280	262	12	6708458	21.19	78.36	100.00	100.00
7500281 - 8000280	295	14	7746247	21.19	78.36	100.00	100.00
8000281 - 8520280	296	14	8256187	21.19	78.36	100.00	100.00
8520281 - 17000300	297	14	12035253	21.19	78.36	100.00	100.00
17000301 - 18000300	298	14	17493156	21.19	78.36	100.00	100.00

TABLE 23. THE OPTIMAL SINGLE SAMPLING UNRESTRICTED ASR BAYESIAN PLANS FOR THREE-POINT BINOMIAL PRIOR WITH $p' = 0.01$; $p'' = 0.05$; $p''' = 0.15$; $w_1 = 0.90$; $w_2 = 0.08$ AND $w_3 = 0.02$.

Lot size	n	c_1	c_2	δ_1	δ_2
59 - 69	2	0	1	0.041573	0.553796
70 - 82	3	0	1	0.054382	0.553613
83 - 98	4	0	1	0.065461	0.553350
99 - 116	5	0	1	0.075051	0.553003
117 - 138	6	0	1	0.038352	0.552571
139 - 164	7	0	1	0.090541	0.552053
165 - 197	8	0	1	0.096766	0.551451
198 - 237	9	0	1	0.102156	0.550767
238 - 290	10	0	1	0.106819	0.550003
291 - 360	11	0	1	0.110850	0.549161
361 - 457	12	0	1	0.114330	0.548253
458 - 600	13	0	1	0.117328	0.547273
601 - 832	14	0	1	0.119904	0.546229
833 - 1270	15	0	1	0.122109	0.545124
1271 - 2410	16	0	1	0.123988	0.543962
2411 - 12550	17	0	1	0.125580	0.542746
12551 and above	18	0	1	0.127573	0.541532

TABLE 24. THE OPTIMAL SINGLE SAMPLING UNRESTRICTED ASR BAYESIAN PLANS FOR BETA PRIOR WITH $\pi = 1.5$ AND $t = 0.5$

Lot size	n	c_1	c_2
8 - 694	8	0	2
695 - 2841	9	0	3
2842 - 4552	10	1	3
4553 - 6605	15	2	5
⋮	⋮	⋮	⋮

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