M. Stat. I Year

Sample Survey and Design of Experiments – Final Exam – 2024-25

Total Score: 150 **Date: 07/05/2025** **Allotted time: 150 min**

- This examination has 5 questions. Time allotted is 2 hours and 30 minutes.
- Total marks on the paper are 150. This will be scaled down to 50.
- The answer to the first question must be submitted electronically via email to the instructor (smajumd2@gmail.com) by 11:59 PM today (07/05/2025).
- Discussing about the first question is allowed (except in the examination hall). However, everyone must write their own answer to the question, and must submit their own answer scripts.
- No discussion is allowed in the examination hall during the examination period.
- You are allowed one handwritten formula sheet (both sides).
- Calculator, phone, tablet, or any other electronic accessory is not allowed.
- Clearly write your roll number, subject, and year on the first page of your answer sheet, as well as any extra pages you use.
- Show all your work in the answer sheet. Do not submit the formula sheet with your answer sheet.

---------- Best of Luck ----------

1. Assignment given in class, to be submitted separately via email to the instructor. [30]

2. Consider a symmetric balanced incomplete block design (BIBD) with v treatments and r replicates. [15+5+10]

   a. Show that its C-matrix is $C = \frac{v(r-1)}{v-1}(I_v - v^{-1}J_{vv})$.
   b. Find all the eigenvalues of C and their multiplicities.
   c. For any vector $x \in \mathbb{R}^v$, show that $\overline{\tau x} - \bar{\tau}\bar{x}$ is estimable for this design, where τ is the vector of treatment effects, $\overline{\tau x} = \frac{1}{v}\sum \tau_i x_i, \bar{\tau} = \frac{1}{v}\sum \tau_i, \bar{x} = \frac{1}{v}\sum x_i$.

3. Answer the following questions: [8+12+10]

   a. Show that $x^2 + x + 2$ is irreducible in $GF(3)$.
   b. Using a, or otherwise, construct $GF(3^2)$.
   c. Consider a Latin Square design of order 9 constructed using $GF(3^2)$. Find its incidence matrix N, and using that, concur whether a Latin Square Design can be considered a BIBD or not. (Hint: You don't actually have to write down the Latin Square matrix of order 9. Think about the properties of a Latin Square Design instead).

4. A cotton candy factory owner has 7 machines producing cotton candy in his factory. However, due to power consumption regulations, he can only use 3 machines a day. The workers operating the machines also vary day to day in a week. Lately, he suspects that machine #5 is not producing as well as the other machines on average. He approaches you to design an experiment for him, and also to devise statistical tests that he can use to infer about his suspicions. [8+14+8]

   a. What would be a suitable design here? Justify every choice you make here.
   b. Write down the design of your choice and how you constructed it.
   c. Write down the suitable hypothesis (or hypotheses) to test here. Write down the corresponding contrast(s). Address any estimability concern.

5. Consider the following design:

$$
\begin{array}{ccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0
\end{array}
$$

[5+10+15]

   a. Identify this design as a fractional factorial design, i.e., if this is a $\frac{1}{s^k}(s^m)$ design, then identify s, m, k.
   b. What is the cardinality of the set of defining contrasts here? Identify at least one treatment combination that is confounded here.
   c. Given that it is an Orthogonal Array, what is the resolution of this design? Construct a fractional factorial design of order $\frac{1}{s^k}(s^{m+1})$ from this. What is the resolution of this new design?