INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2025-26

Course Name : B.Stat. 3rd Year
Subject Name : Sample Surveys
Date : Sep 10, 2025 **Total Marks :** 30 **Duration :** 2 hrs 30 mins.

Answer Q.4 and any 2 from Q.1 to Q.3.

Symbols and notations are as usual. Only hand calculator is allowed.

1. (a) For an SRSWOR of size n drawn from a population with N units, show that the sample mean $\bar{y}$ is an unbiased estimator of population mean $\bar{Y}$ of a variable of interest y. Also with proof derive the $Var(\bar{y})$ and obtain an unbiased estimator of that.

   (b) A survey is to be conducted to estimate the population proportion of households having at least one student using smart phones and internet for online education in a small city, using SRSWOR. It is given that the total number of households in this city is 52943. Determine the minimum sample size required for this survey, if the population proportion is to be estimated with a maximum discrepancy of d taken as $d = e \times p = 10\%$ of p, where e is the relative permissible margin in error, at the 95% probability level.

   Modify your answer if you have information obtained from some recent previous result that there was **40%** households having such students using smart phones in the entire city and you believe this as a reasonable guess for determining the sample size for the survey in the current situation.

   [7+3=10]

2. (a) Show that under linear systematic sampling the sample mean $\bar{y}$ is an unbiased estimator for $\bar{Y}$ of a varible of interest y. Also prove that $V(\bar{y}) = \dfrac{S_y^2(N-1)}{nN}[1+(n-1)\rho_c]$, where S_y^2 is the population variance and ρ_c is the intraclass correlation coefficient. Assume that the sample size is a factor of the population size.

   (b) Derive the condition when systematic sampling performs better than the SRSWOR sampling. State with justification how the derived condition can be achieved in order to get the systematic sampling much more efficient than the simple random sampling. [7+3=10]

3. (a) Obtain with proof an unbiased estimator for the population mean based on a stratified random sample with SRSWOR for within stratum sampling. Also obtain the variance and variance estimator of this estimator.

   (b) Obtain the allocation of sample sizes by optimum allocation methods (with cost consideration). State the interpretation of this type of sample size allocation. [5+5=10]

4. In a sample survey for estimating the total number of standards of pepper (i.e. pepper production as measured in some measuring system) in a tehsil, a two-stage cluster sampling scheme was adopted. The tehsil had the total **56** villages, a sample of **8** villages was selected with **SRSWOR** at the first stage and from each sample village **5** clusters of a fixed number of fields producing pepper each were drawn with **SRSWOR** at the second stage. Data on the number of clusters in the sample villages and on the number of pepper standards in the sample clusters are given in the following table.

Based on this sample survey data, estimate unbiasedly the total number of pepper standards in the tehsil and obtain its relative standard error (rse) in percentage.

[10]

No proof is needed. Only write the formulae and computation results.

Table:– Pepper data obtained from 5 sample clusters selected in sample villages

Sample Vill.	#Clusters	Pepper(clust1)	Pepper(clust2)	Pepper(clust3)	Pepper(clust4)	Pepper(clust5
1	27	430	402	363	975	389
2	24	586	1234	100	368	344
3	14	1164	546	3060	1724	1274
4	116	693	218	836	1218	575
5	25	191	270	4502	4184	243
6	118	1036	1333	1179	728	1957
7	147	1555	254	950	382	355
8	26	910	452	129	122	243