

Indian Statistical Institute
Probability Theory II
B.Stat (hons) 1st year,
Supplementary midsemestral examination

Date: Mar 28, 2025

Duration: 2 hrs 30 min.

Attempt all questions. The maximum you can score is 40. Justify all your steps. This is a closed book, closed notes examination. It is extremely important that you clearly write the results you are using. Vague scribbles, contradictory arguments will not be given any credit.

1. Let $X_1, \dots, X_n$ be IID $Expo(\lambda)$ for some $\lambda > 0$. Find $E(X_{(1)})$.[5]
2. Prove or disprove, as appropriate, each of the following statements.
 - (a) If $F(x, y)$ is a bivariate CDF discontinuous at some $(a, b) \in \mathbb{R}^2$, then it must be discontinuous at (x, b) for all $x > a$.[5]
 - (b) If X is a random variable with a density, then $X_+ := \max\{X, 0\}$ also has a density.[5]
3. Let $X_1, \dots, X_n$ be a random sample from $Unif(-\theta, \theta)$ distribution for some $\theta > 0$. Let R be the sample range (i.e., $R = X_{(n)} - X_{(1)}$). Show that for all $\epsilon > 0$ we have $P(R > 2\theta - \epsilon) > 0$.[10]
4. If (X, Y) is uniform over the unit disc in $\mathbb{R}^2$, then find density of $\sqrt{X^2 + Y^2}$.[10]
5. If X has uniform distribution over $(1, 2)$, and given X the random variable Y is conditionally distributed as $Unif(0, X^2)$, then find $corr(X, Y)$.[10]