INDIAN STATISTICAL INSTITUTE

Analysis I : B. Stat. 1st year
Semester Examination: 2025-26
17.11.2025

Maximum Marks: 50 Maximum Time: 3 hrs.

Answer all the questions but maximum you can score is 50.

(1) (a) If $a > 0$, then prove that $\lim_{n \to \infty} a^{1/n} = 1$. [5]

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. If $f(x) = f(x^2)$, for all $x \in \mathbb{R}$, then prove that f is a constant function. [5]

(2) (a) Suppose $\{a_n\}$ is a sequence of real numbers. If $\sum_{n=1}^{\infty} na_n$ converges then prove that the series $\sum_{n=1}^{\infty} na_{n+1}$ also converges. [6]

(b) Prove that for any $\alpha \in (1, 2)$, the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{(n+1)^{\alpha}},$$

converges. [6]

(3) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a bounded, continuous function. Prove that there exists $x_0 \in \mathbb{R}$, such that $f(x_0) = x_0$. [6]

(4) Let B be a bounded subset of $\mathbb{R}$ and $f : B \to \mathbb{R}$ be uniformly continuous. Prove that $f(B)$ is a bounded subset of $\mathbb{R}$. [6]

(5) Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function. If $\lim_{x \to \infty} f'(x) = l \in \mathbb{R}$, then prove that $\lim_{x \to \infty} f(x)/x = l$. [6]

(6) Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function and $f^{(2)}(x) \geq 0$, for all $x \in \mathbb{R}$. If f is a bounded function then prove that it is a constant function. [6]

(7) Let $f : [0, 1] \to \mathbb{R}$ be such that the third derivative $f^{(3)}$ is continuous on $[0, 1]$. Suppose that $f(0) = f'(0) = f^{(2)}(0) = f'(1) = f^{(2)}(1) = 0$ and $f(1) = 1$. Prove that there exists $c \in (0, 1)$ such that $f^{(3)}(c) \geq 24$. [6]