INDIAN STATISTICAL INSTITUTE

Semester Examination : 2024-2025

B. Stat. (Hons.) III Year

Subject : SQC & OR

Full Marks: 100 **Time: 3 hours** **Date of Examination: 5th May 2025**

INSTRUCTIONS

A) This is a closed-book closed-notes examination.

B) This paper carries 108 marks. You may answer as much as you like; but the maximum you can score is 100.

C) Relevant tables are given with the question paper.

1. Consider the queuing model $M/M/c$. Define $W_q(0)$ as the probability that a customer has zero delay (i.e. no delay) in the queue.
   - **a.** Find the expression for $W_q(0)$.
   - **b.** Calls to a technical support center arrive according to a Poisson process with rate 30 per hour. The time for a support person to serve one customer is exponentially distributed with a mean of 5 minutes. The support center has three technical staff to assist callers. What is the probability that a customer is able to immediately access a support staff without being delayed on hold? Assume that customers do not abandon their calls.
   - **c.** Find the length of the queue for the model described in (**b**) above.
   - **d.** This call center has been taken over by an MNC and the three technical staff are transferred. A very experienced engineer has been hired whose service time is normally distributed with a mean of 1.5 minutes and a standard deviation of 6 minutes. Find the average number of customers waiting in the line in this case.

   [Hint: Recall that $\frac{1}{p_0} = \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!\left(1-\frac{\rho}{c}\right)}$, where the symbols have their usual meaning.**]**

[5+5+5+5=20]

2. A company has two grades of inspectors, **I** and **II**, who are to be assigned for a quality control inspection. It is required that at least 1800 pieces be inspected per 8-hour day.
   Grade I inspectors can check pieces at the rate of 25 per hour, with an accuracy of 98% at a wage rate of Rs 4/- per hour.
   Grade II inspectors can check pieces at the rate of 15 per hour, with an accuracy of 95% at a wage rate of Rs 3/- per hour.
   Each time an error is made by an inspector, the cost to the company is Rs 2/-.

The company has available for inspection job eight Grade I inspectors, and ten Grade II inspectors.
The company wants to determine the optimal assignment of inspectors which will minimize the total cost of the inspection.
Formulate this as an LP problem.

[15]

3. Obtain the strategies for both the players A and B and the value of the game if the pay-off matrix of A is given by the following:-

$$\begin{pmatrix} 6 & 1 & 3 & 2 & 3 \\ 5 & 7 & -5 & 1 & 4 \\ 8 & 4 & -1 & 2 & 2 \\ 3 & 3 & -2 & 2 & 5 \end{pmatrix}$$

[15]

4. Consider the following double sampling plan, where the symbols have their usual meaning:

$$N = 500;\ n_1 = 50;\ c_1 = 1;\ r_1 = 4;\ n_2 = 100;\ c_2 = 3;\ r_2 = 5.$$

   a. What type of inspection is being advocated by this plan?
   b. Describe its operation.
   c. Find probability of acceptance of a lot that has fraction defective $p = 0.05$.
   d. Find the average sample number for this plan.

[2+5+10+3=20]

5. Measurements of the thickness of support beams are taken by a single operator. The same beam is measured twice by the operator. Twenty such beams are measured. He finds the standard deviation of these measurement values to be 3.743 mm; while the average range of these beams is 0.2800 mm. From engineering considerations, the specification on the thickness are 50 ± 8 mm. However, in order to capture the market, the management has set the thickness target to be 51 mm.

   a. Assuming the process to be in control, estimate the precision –to-tolerance ratio for the measuring instrument.
   b. Is the gage capable?
   c. Give an estimate of the product variability.
   d. Mention the aspect(s) of variability which can be captured by this study?
   e. Mention the aspect(s) of variability – if any - which cannot be captured by this study and how they can be overcome?
   f. What is the observed process potential?
   g. Assuming the thickness to be normally distributed, what is the process fall-out?
   h. Calculate the capability index in which the management would be interested.

[4+1+2+2+2+2+4+3=20]

6. Answer the following questions as best as you can:-

i). The j^{th} constraint in the dual of an LPP is satisfied as strict inequality by the optimal solution. The j^{th} variable of the primal will assume a value _____.

ii). If the j^{th} constraint in the primal is an equality, then the corresponding dual variable is _____ .

iii). If, the optimum of an LPP occurs at $X = (1,\ 0,\ 0, 2)$ and $Y = (0,\ 1,\ 0,\ 3)$, then the optimum also occurs at _____ .

iv). If in any simplex iteration the minimum ratio rule fails, then the LPP has _____ .

v). AOQL means _____ .

vi). Consumer's risk of 10% means that _____ .

vii). If ***nothing*** is known concerning the pattern of variation of a set of numbers, we can calculate the standard deviation of this set of numbers and state that the sample mean$\pm$3 the calculated standard deviation will include ***at least*** ____% of all the numbers.

viii). Who is reputed to have said: *All models are wrong, but some are useful.*

ix). Two person zero-sum game means that the _____ .

x). Who is considered as the father of game theory?

xi). Who is considered as the father of SQC?

xii). Who introduced the notion of loss while discussing concepts of quality?

xiii). Customer behavior in which she moves from one queue to another (in the hope of reducing her waiting time) in a multiple channel situation is _____ .

xiv). When is the World Quality Day celebrated?

xv). When is the World Consumer Rights Day celebrated?

xvi). When is the World Standards Day celebrated?

xvii). When measurements show a lack of statistical control, the standard error of the average _____.

xviii). Who is credited with having developed the PDCA wheel?

[1 × 18 = 18]

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Appendix VI Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits			Chart for Standard Deviations: Factors for Center Line		Chart for Standard Deviations: Factors for Control Limits				Chart for Ranges: Factors for Center Line		Chart for Ranges: Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_1	B_2	B_3	B_4	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$

$$A = \frac{3}{\sqrt{n}}, \qquad A_3 = \frac{3}{c_4\sqrt{n}}, \qquad c_4 = \frac{4n-4}{4n-3}.$$

$$B_1 = 1 - \frac{3}{c_4\sqrt{2(n-1)}}, \qquad B_2 = 1 + \frac{3}{c_4\sqrt{2(n-1)}},$$

$$B_3 = c_4 - \frac{3}{\sqrt{2(n-1)}}, \qquad B_4 = c_4 + \frac{3}{\sqrt{2(n-1)}}.$$