Indian Statistical Institute

Mid-Semestral Examination, M.Tech. (CS), 2024-25

Distributed Computing

Date: 27.02.2025 **Duration: 2 hours** **Maximum marks: 30**

The total question marks is 40. Answer as many as you can, but the maximum score achievable is 30. Only precise and to-the-point answers will get full marks.

1. In distributed computing, the $KT0$ model refers the initial (K)nowledge of all nodes is restricted (T)ill radius 0 (i.e., just the local knowledge). The $KT1$ model refers the initial (K)nowledge of all nodes is restricted (T)ill radius 1 (i.e., just the knowledge of their neighbors).

   (a) Suppose we have an algorithm that works in the $KT0$ model. Will it work in the $KT1$ model without any changes? [2]

   (b) Suppose the time complexity of solving a problem in the $KT1$ model is T. Then what does it imply regarding the time complexity of the problem in the $KT0$ model? [4]

   (c) Suppose the message complexity of solving a problem in the $KT1$ model is M. Then what can be the message complexity of the problem in $KT0$ model? [4]

2. You are given an 8-node cycle with nodes $v_1, v_2, \ldots, v_8$. For $1 < i < 8$, node v_i is connected to nodes v_{i-1} and v_{i+1}. Node v_1 is connected to nodes v_8 and v_2 and node v_8 is connected to nodes v_7 and v_1. The IDs of these nodes are as follows: $\texttt{ID}_{v_1} = 1$, $\texttt{ID}_{v_2} = 6$, $\texttt{ID}_{v_3} = 7$, $\texttt{ID}_{v_4} = 2$, $\texttt{ID}_{v_5} = 3$, $\texttt{ID}_{v_6} = 8$, $\texttt{ID}_{v_7} = 4$, and $\texttt{ID}_{v_8} = 5$. Consider the execution of the deterministic Leader Election algorithm that runs in $O(n)$ rounds and uses $O(n \log n)$ messages, on this input cycle. Recall that this algorithm runs in phases and in Phase i, every active node sends two messages, one in the clockwise direction and the other in the counter-clockwise direction, and these messages are expected to travel for 2^i hops and return. The goal of this algorithm is to elect the node with lowest ID as the leader.

   Show which nodes remain *active* after each of Phases 0, 1, and 2 of this algorithm. Your answer should consist of 3 sets of nodes: those which are active after Phase 0, those that are active after Phase 1, and those that are active after Phase 2. [10]

3. Given a n-node complete network G with distinct edge weights, give a distributed algorithm to compute the MST of G in $O(\log n)$ rounds. Analyze the message complexity of your algorithm. [10]

4. You are given an n-node cycle. Fix an arbitrary node v. Calculate the probability that v becomes permanently colored in the first round of execution of Luby's $(\Delta+1)$-coloring algorithm. What is the expected number of nodes that will participate in the second round of the algorithm? [10]

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