Indian Statistical Institute
M. Tech (QR & OR) II Year - 2025-2026
Semester-I
End-semester examination
Subject: Business Analytics

Examination Date: 26th November 2025 Duration: 3 hours

Full Marks: 70
Note: Answer all questions. Scientific calculators are allowed.

1. You are given the following small dataset:

ID	Weather	Temperature	Play
1	Sunny	Hot	No
2	Sunny	Cool	Yes
3	Rainy	Hot	No
4	Rainy	Cool	No
5	Sunny	Pleasant	Yes

   (a) Construct a Decision Tree (DT) to predict Play (Yes/No) using the features Weather and Temperature. Show all intermediate calculations, including the computation of Gini impurity, and draw the final decision tree. Split the tree only if the node contains more than two examples.

   (b) Using your final decision tree, predict the class label for the instance where Weather = Rainy and Temperature = Pleasant.

   (c) Explain the overfitting problem in decision trees. Describe both pre-pruning and post-pruning techniques used to address it.

   (d) Since algorithms like Decision Trees and Support Vector Machines (SVM) are inherently designed for binary classification, explain how they can be extended to handle multi-class classification problems.

[8+2+6+4=20]

2. Consider a linearly separable support vector machine (SVM). Answer the following questions:

   (a) Why is the SVM called a maximum margin hyperplane classifier? Explain the concept in detail with appropriate mathematical formulation and a visual illustration.

   (b) Derive the dual optimization problem of the Support Vector Machine from its primal formulation. Clearly explain it in detail.

   (c) Explain the difference between a hard-margin and a soft-margin SVM.

   (d) What is the kernel trick in SVM? Specify the situations in which it is applied.

[4+8+4+4=20]

3. Consider a logistic regression model *without an intercept term*. You are given the following dataset:

$$x = [2,\ 1,\ 3]^\top, \qquad y = [1,\ 0,\ 1]^\top.$$

Assume the initial parameter value $\beta^{(0)} = 0$ and the learning rate $\alpha = 0.1$.

   (a) Derive the logistic loss function (negative log-likelihood) for this model. Explain why logistic regression requires an iterative optimization rather than least square estimates.

   (b) Derive the gradient of the loss with respect to β and compute the value of $\beta^{(1)}$ after one iteration of gradient descent. Show all intermediate calculations clearly.

   (c) Consider a disease classification problem in which only 5% of the patients in the dataset have the disease (positive class).

i. Discuss how you would address this class imbalance problem during model training.

ii. Which evaluation metrics should be used to properly assess model performance in this scenario, and why?

[3+7+(3+2)=15]

4. (a) Draw the profile of membership function for a fuzzy set called "Short men". Take your own values for different heights. What is the difference between Crisp Set & Fuzzy Set?

(b) Construct a business problem to use desirability optimization methodology for decision making.

(c) Formulate a goal programming problem with goal, aspiration levels and deviation variables.

[(4+2)+5+4=15]