Indian Statistical Institute

Probability Theory II

B.Stat (hons) 1st year,
Midsemestral examination

Date: Feb 24, 2025 Duration: 2 hrs 30 min.

Attempt all questions. The maximum you can score is 40. Justify all your steps. This is a closed book, closed notes examination. It is extremely important that you clearly write the results you are using. Vague scribbles, contradictory arguments will not be given any credit.

1. Let $X \sim Unif(0,1)$ and $Y|X = x \sim Unif(0,x)$. Find joint density of (X,Y). Hence or otherwise find $E(Y)$.

[5+5]

2. Prove or disprove, as appropriate, each of the following statements.

   (a) If X_1, X_2 are IID from $Expo(1)$ distribution, then $X_{(1)}, X_{(2)}$ are independent.

   [5]

   (b) If $F(x)$ is a CDF that is differentiable everywhere except at finitely many points, then the following function must be a density corresponding to it:

$$f(x) = \begin{cases} F'(x) & \text{if it exists} \\ 0 & \text{else} \end{cases}$$

[5]

3. If $X_1, ..., X_n$ are IID from $Unif(0,\theta)$ distribution for some $\theta > 0$, then find $E(X_{(n)})$.

[10]

4. If (X,Y) is uniform over the set $[1,2] \times [1,3]$, then find joint density of XY.

[10]

5. If (X,Y) has uniform distribution over the set $\{(x,y) : x,y \geq 0, \quad x+y \leq 1\}$, then find $cov(X,Y)$.

[5]