INDIAN STATISTICAL INSTITUTE

Final Examination:(2025-2026)

MStat. 2nd Year

Signal & Image Processing

Date: 22.11.2025 Maximum Marks: 100 Duration: 3 hours

Note: The examination is closed book. The marks for the Signal Processing and Image Processing Sections add up to 65, the maximum you can score in each Section is 50. Use of calculators is permitted.

Signal Processing

1. Consider the sequences

$$x_1[n] = \begin{cases} (n+1)(0.4)^n, & n = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2[n] = u[n] - u[n-2]$$

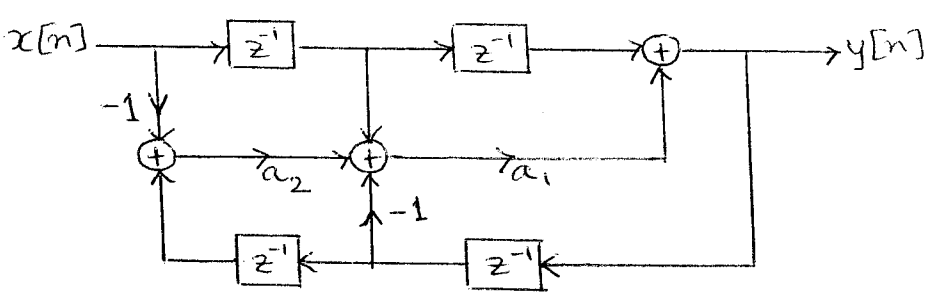
where $u[n]$ stands for the unit step sequence. Determine

(a) The result of the linear convolution of $x_1[n]$ and $x_2[n]$.

(b) The result of a 3-point circular convolution of $x_1[n]$ and $x_2[n]$.

[6+4=10]

2. Determine the system function for the block diagram shown below.



Can this system function act as an All-Pass System? Justify your answer.(7+2+4 =13)

3. An LTI system with impulse response $h[n]$ has poles at $0.7e^{\pm j\pi/4}$. Determine the pole locations of a system with impulse response $h[-n+1]$. State with justification if the new system is stable.[6+2+2=10]

4. A causal LTI system produces the output

$$y[n] = (0.4)^n u[n] - (0.3)(0.4)^{n-1} u[n-1]$$

for the input

$$x[n] = (0.2)^n u[n]$$

where $u[n]$ is the unit step sequence.

(a) Determine the system function.

(b) Determine the difference equation characterizing the system.

(c) Give a Direct Form I realization of this system.

[5+5+4=14]

5. Consider the following sequence

$$x[n] = \begin{cases} n, & 0 \le n \le 4 \\ 4, & 5 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

Let $X(e^{j\omega})$ be the Discrete Time Fourier Transform (DTFT) of $x[n]$. Consider the sequence

$$Y[k] = X(e^{j\omega})|_{\omega=\frac{2\pi k}{6}, k=0,\cdots,5}$$

Determine the inverse Discrete Fourier Transform (DFT) of $Y[k]$. You will get full credit if you do not explicitly compute the DFT. [7]

6. Give the flow diagram of a radix-2 decimation-in-time algorithm to compute the 4-point inverse DFT. [6]

7. Consider a causal IIR filter designed using the Impulse Invariance Method with T=0.4 seconds and system function:

$$H(z) = \frac{2}{1 - e^{-1.3} z^{-1}} + \frac{5}{1 - e^{-2.0} z^{-1}}$$

Determine the parent causal analog transfer function. [5]

Image Processing

1. The figure below shows a binary image with '1' representing the foreground:

0	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	1	0	0	0

   (a) Using a 3×3 structural element with the origin at its centre, determine the result of

      i. Opening this image (erosion followed by dilation).

      ii. Closing this image (dilation followed by erosion).

   (b) State with justification whether you would prefer to use a morphological operation over a smoothing kernel for noise removal in images.

[6+6+4=16]

2. Consider a color image of a region with vegetation and other geographical features. Only the vegetation is in different shades of green. You have to perform segmentation in order to detect the regions with vegetation and mark the rest of the image with gray colour. Give the steps of the approach that you would follow and your preferred image format for this purpose. [4+2=6]

3. For the 4×8 grayscale image shown below,

110	140	140	180	180	180	220	220
110	140	140	180	180	180	250	250
110	140	140	220	220	220	250	250
110	140	140	220	220	220	250	250

   (a) Compute the histogram and outline how you would obtain a threshold for binarizing the image.

   (b) Determine the entropy of this image.

   (c) Obtain a code for transmitting the grayscale image using Huffman coding and give the average code length.

[(6+6)+5+(7+3)=27]

4. State

(a) How a cyan coloured image would look in red light.

(b) Principal Component Analysis (PCA) is performed on a multi-spectral image X of 8 bands. Of the transformed bands, only 3 are used to reconstruct an approximation of the image, X_{new}. Give an expression for the resulting mean squared error (MSE) with reference to X.

(c) The criteria to be considered when selecting the block sizes in case of JPEG compression of an image.

(d) The steps of the approach for determining the edges (horizontal and vertical) of a noisy image providing justification for each step.

[2+6+4+4=16]