INDIAN STATISTICAL INSTITUTE

Post Graduate Diploma in Business Analytics
End Semester Examination, 2025

Computing for Data Sciences - Part II
Maximum Marks: 49

Date of Examination: 19 December 2025, 4:15pm Time Allowed: 1.5 Hours

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Roll No:

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1. **Answer any SEVEN questions.** [7 marks each $\times$ 7 = 49 marks]

   a) What is the directional derivative of $\phi = x^2y + y^2z + z^2x$ at the point $(1, -1, 2)$ in the direction of vector $v = i + 2j - 3k$? Give one example of transformation that takes a set of non-linearly separable data points to corresponding linearly separable points for binary classification? [5+2=7]

b) (i) A perceptron receives two inputs, $x_1 = 2$, $x_2 = -1$, with weights. $w_1 = 0.6$, $w_2 = -0.8$, respectively and bias $b = 0.2$. Compute the perceptron output using the following activation function:

$$f(u) = \begin{cases} 1, & u \geq 0, \\ 0, & u < 0. \end{cases}$$

(ii) Provide steps to train a multilayer perceptron (MLP). [4+3=7]

Id	Refund status	Marital status	Taxable income	Class
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Table 1: Training data

c) Consider the data in Table 1 for modeling a binary classifier. Design the decision tree for the above dataset. Explain why the designed decision tree is better compared to other possible decision trees for the above dataset. What is a dendrogram? [4 + 2 + 1=7]

d) Derive the primal and dual formulations of hard margin SVM loss function. What are the advantages of dual formulation? [3+3+1=7]

e) Given a set of n-D data points, in $\mathcal{R}^D$, derive the 2-D vector p that explains the maximum variance when the n-D data points are projected along p. How are the singular values of a matrix A calculated? [5+2+7]

f) Following points are to be clustered using the k-means algorithm into three clusters:

$$A1(2,10),\ A2(2,5),\ A3(8,4),\ B1(5,8),\ B2(7,5).\ B3(6,4),\ C1(1,2),\ C2(4,9).$$

Assume Euclidean distance as the distance function. Initially $A1$, $B1$, and $C1$ are assigned as the centers of each cluster, respectively. Find the final three clusters. [7]

g) Use the iterative perceptron learning rule to correctly classify patterns p_1, p_2, p_3, p_4 with their target classes given by t_1, t_2, t_3, t_4, respectively.

$$\{\, p_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix},\ t_1 = 0 \,\}, \quad \{\, p_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix},\ t_2 = 1 \,\}, \quad \{\, p_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix},\ t_3 = 0 \,\}, \quad \{\, p_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},\ t_4 = 1 \,\}.$$

Assume initial weights and bias are zero. The perceptron uses the update rule as:

$$w^{new} = w^{old} + (t - y)p, \qquad b^{new} = b^{old} + (t - y), \quad \text{and} \quad y = \begin{cases} 1, & w^T p + b > 0, \\ 0, & w^T p + b \leq 0. \end{cases}$$

[7]

h) Write algorithmic steps to find the second maximum in a series of positive integers using a single pass. Show steps to swap two integer variables without using any temporary variable. [5+2=7]

END OF PAPER - Part II